ferent test excitations provided that the n voltage vectors are independent, i.e., the matrix in (3) is nonsingular.

The proof of the above theorem is obvious because if the matrix in (3) is nonsingular, then it can be inverted to produce $Y_1, \cdots, Y_n$. However, several interesting observations can be made. *First, the branches in the cutset must be branches of a tree or else the columns of the matrix in* (3) *will not be independent.* This restriction immediately rules out parallel elements. *Secondly, at least one of the test sources must be contained in the cutset.* If this is not true, then the right-hand side of (3) is zero. Since the Y_i's are not zero, this means that the test voltage vectors are dependent. For example, if all of the admittances are connected to a common node, then at least one of the test current sources must be connected to this node, as illustrated in fig. 10 of [1], or else an independent set of voltages vectors will not be obtained.

III. Controlled Sources

Controlled sources can easily be handled in the method. Voltage-controlled current sources are treated the same as admittance branches. However, the matrix in (3) will be singular if the controlling voltage is linearly dependent on one or more of the other voltages in this matrix. For example, the controlling voltage must not be the voltage across one of the admittances in the cutset, otherwise (3) will contain two identical columns. However, this dependency can be removed, if this admittance can be computed from a previous cutset calculation so that by substitution its voltage can be eliminated from present cutset equations. A similar result can be obtained using the differential form of Tellegen's theorem [4]. Given the branch constraint $I_\beta = g_m V_\alpha$, a term $(V_\alpha + \Delta V_\alpha)\hat{V}_\beta \nabla g_m$ appears on the left side of (11) in [1]. Since all branches are shorted in the adjoint circuit, except for the branches in the cutset, then either $\hat{V}_\beta$ is zero if the controlled source is not in the cutset, or $\hat{V}_\beta = \hat{V}_{p1}$ in (12) of [1].

Current-controlled current sources ($I_\beta = \beta I_\alpha$) should not be included in the cutset unless the controlling current I_α can be measured or computed. For example, if I_α is the current through an admittance which has been previously calculated, then $I_\alpha^{(j)}$ can be determined for each test condition from the node voltages (i.e., $I_\alpha^{(j)} = Y_a V_\alpha^{(j)}$) and β can be computed. In the adjoint circuit approach the term $(I_\alpha + \Delta I_\alpha)\hat{V}_\beta \Delta\beta$ appears [4]. If this source is not contained in the cutset, then $\hat{V}_\beta = 0$. If it is contained in the cutset, then again $I_\alpha + \Delta I_\alpha$ must be measured or computed in order to determine $\Delta\beta$.

Finally, controlled voltage sources must *not* be contained in any of the cutsets, unless their currents can be measured. However, in the case of voltage-controlled voltage sources ($V_\beta = \mu V_\alpha$), the parameter μ can be computed if all node voltages are accessible. If such a source has a series impedance and the internal node is not accessible, then the Norton transformation can be used and the source becomes a voltage-controlled current source in parallel with an impedance. In the adjoint circuit approach controlled voltage source terms in (11), [1], are eliminated by simply replacing the controlled voltage source with an open-circuit in the adjoint circuit.

II. Summary

The above derivation of the algorithm is simpler than that used in [1]. However, when all nodes are accessible, the operational amplifier scheme of component isolation described in [1] is more practical because it is easier to implement in the automatic test equipment and handles both linear and nonlinear components with ease. Finally, the equations derived in [1] on the basis of the differential form of Tellegen's theorem have been used to generate a new algorithm for the location of single and multiple faults when not all of the nodes are accessible [4]–[6].

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Least-Squares Low-Pass Filters with Nonmonotonic Response

HENRIQUE S. MALVAR AND LUIZ P. CALÔBA

***Abstract*—A new class of all-pole low-pass filter functions, derived from the least-squares approximation technique, is introduced. The magnitude response is compared with those of some other all-pole filters, like the Butterworth, LSM, and Chebyshev ones. The three subclasses of the new class seem to be a suitable choice for the approximation function in filter synthesis.**

I. Introduction

In spite of its mathematical simplicity, the Butterworth low-pass filter is not widely used in practical filter synthesis, since it does not provide a high selectivity. Other monotonic filters, such as the L [1], [2], H [2], and LSM [3] ones, have been developed, with better stopband performance. However, in these filters, while the stopband attenuation is improved, the passband performance becomes worse. It is natural, therefore, to search for a filter that has the desirable features of the Butterworth filter, but with increased stopband attenuation, even though a nonmonotonic one. This was done recently by Jovanović and Rabrenović [4], using the minimisation of the ratio of the reflected power to the transmitted power in the passband, and a particular form for their polynomials.

The main objective of this work is to use the least-squares approximation technique to find solutions to the approximation problem, without the restriction of monotonic response.

II. The Q_n Polynomials

The magnitude squared function of the Q filter has the form:

$$|H(j\omega)|^2 \equiv \frac{1}{1+\epsilon^2 Q_n^2(\omega)}. \tag{1}$$

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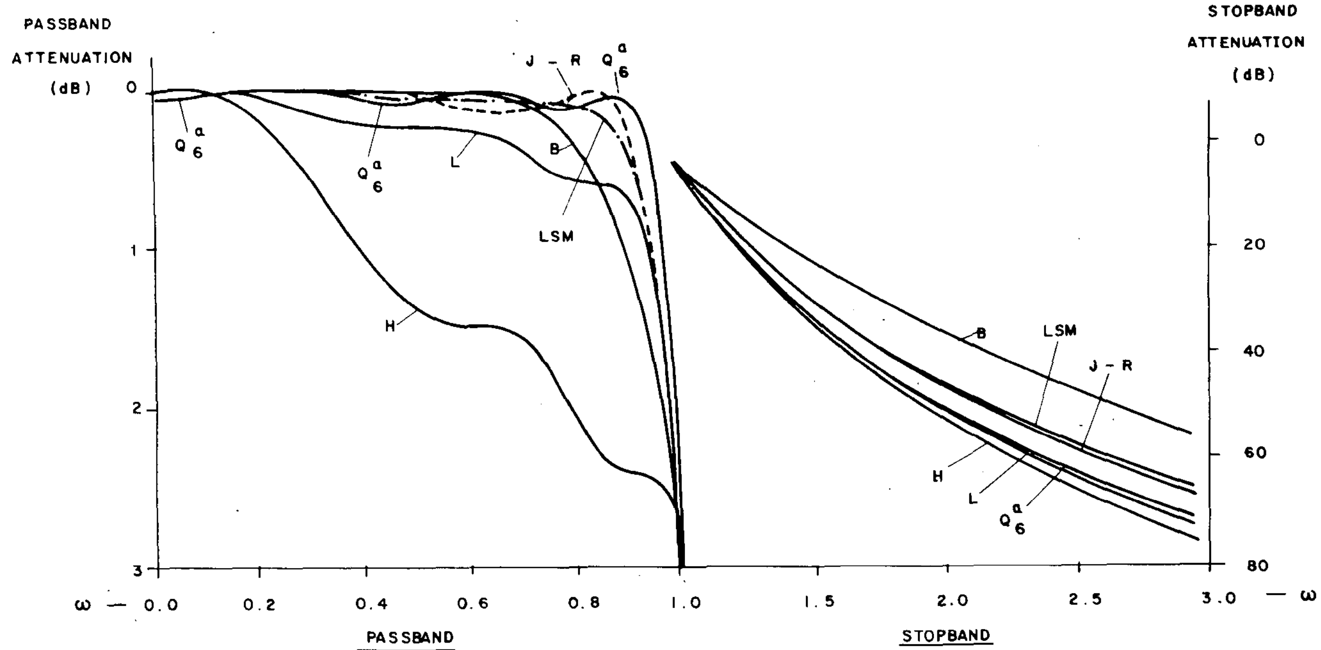

Fig. 1. Attenuation curves (decibels × frequency) for the filters of the first group (low attenuation for $\omega \to 0$). The filters, all of sixth order, are: B-Butterworth, L-Papoulis, H-Halpern, LSM-Least-squares monotonic, J–R-Jovanović and Rabrenović, and Q_6^a ($\in = 1$).

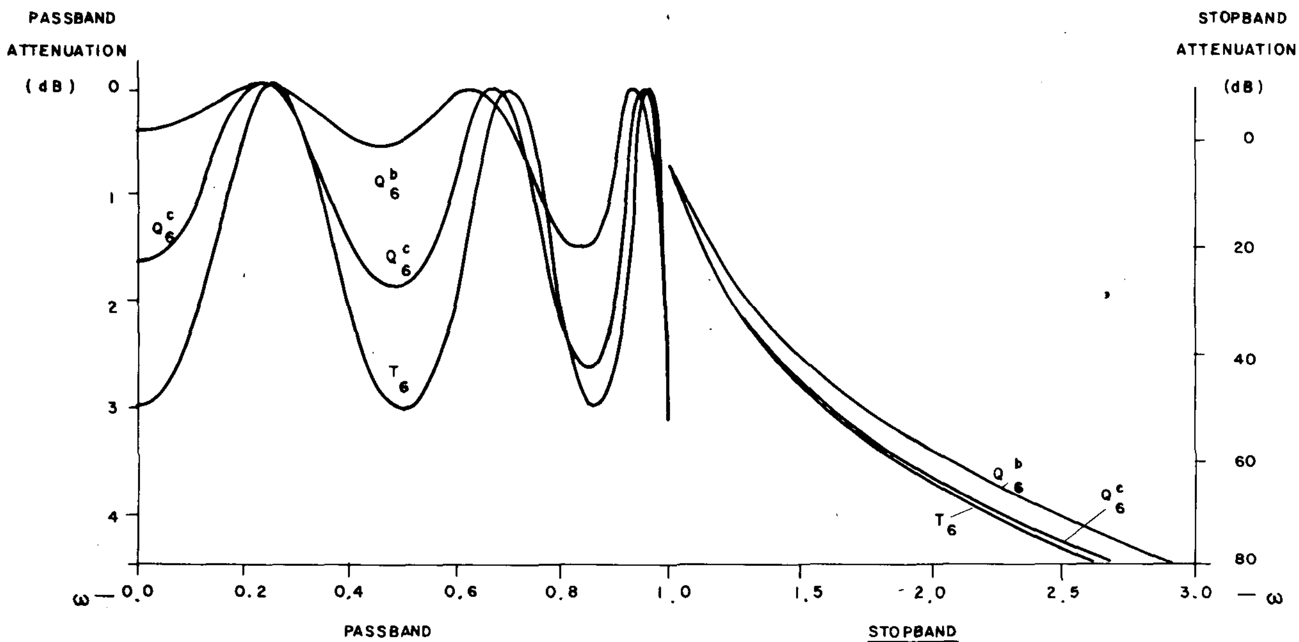

Fig. 2. Attenuation curves (decibels × frequency) for the filters of the second group (high attenuation for $\omega \to 0$). The filters, all of sixth order, are: Q_6^b, Q_6^c, and T_6 (Chebyshev); for all filters, $\in = 1$.

where $|H(j\omega)|$ is the magnitude of the filter transfer function, $Q_n(\omega)$ is the Q polynomial of nth degree, and $\in^2$ is a parameter that controls the loss.

As is well known, $Q_n(\omega)$ must be an even polynomial [5]. Therefore, we can write

$$Q_n(\omega) = \sum a_i \omega^i, \qquad i = n, n-2, \cdots, n \bmod 2 \tag{2}$$

where n mod 2 denotes the remainder of $n \div 2$.

The minimum ratio of the reflected power to the transmitted power in the passband is obtained by minimising the integral [6]:

$$E = \int_0^1 Q_n(\omega)\, d\omega. \tag{3}$$

Clearly, some constraint must be imposed to the coefficients of the Q_n polynomial, otherwise we would find the trivial solution $a_i = 0$, $\forall i$. In this work, we consider the following constraints:

1) $Q_n(1) = 1$ (4)

2) $\partial Q_n(\omega)/\partial\omega = n^2$, for $\omega = 1$ (5)

and

3) $a_n = 2^{n-1}$. (6)

The first restriction defines the passband as $\omega \leqslant 1$. The second and third constraints are characteristics of the Chebyshev polynomials [7].

If we solve the minimization problem using only one of the restrictions, we can define three subclasses of the Q_n polynomials, Q_n^a, Q_n^b, and Q_n^c, corresponding to the constraints 1, 2, and 3, respectively.

The coefficients a_i can be found by using the Lagrange's multiplier [8], that leads to:

$$\frac{\partial}{\partial a_j}[E+\lambda F_k(A)]=0, \qquad j=n, n-2, \cdots, n \bmod 2$$

$$F_k(A)=0 \tag{7}$$

where k denotes the kth restriction, A is the set of the coefficients a_i, λ is the Lagrange's multiplier, and

$$F_1(A)=1-\sum a_i \tag{8}$$

$$F_2(A)=n^2-\sum i a_i \tag{9}$$

$$F_3(A)=a_n-2^{n-1} \tag{10}$$

and

$$\frac{\partial E}{\partial a_j}=2\cdot\sum\frac{a_i}{(i+j)+1}. \tag{11}$$

It is important to note that Q_n^b and Q_n^c do not satisfy (4), that is, their passbands are not $\omega \leqslant T$. Therefore, we must normalize them (by making a frequency scaling), after their calculation, in order to have all the Q_n filters with a passband $\omega \leqslant 1$.

The magnitude responses of these filters are shown in Figs. 1 and 2, for $n=6$ and $\in = 1$. The filters considered for the comparison were:

first group (Fig. 1)—B (Butterworth), L (Papoulis [1]), H (Halpern [2]), LSM (Rakovich & Litovsky [3]), J & R (Jovanović & Rabrenović [4]) and Q_6^a.

second group (Fig. 2)—Q_6^b, Q_6^c, and T_6 (Chebyshev).

III. Conclusions

As was shown in Figs. 1 and 2, the Q_n polynomials have good magnitude characteristics, mainly the subclass Q_n^a, that seems to be one of the best choices for the approximation function when the attenuation in the passband is the most important factor. The other subclasses (Q_n^b and Q_n^c) seem to be more suitable for applications that originally required the Chebyshev filter, but there's a designer's wish for an approximation with a more smooth characteristic in the passband, but with almost the same performance in the stopband, for a given $\in$.

It is also expected good sensitivity characteristics for filters designed with the Q_n polynomials, because of the Orchard's argument [9].

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Q-Enhancement and Extension of the Stablity Range of Generalized Immitance Converters

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***Abstract*—A compensation technique is proposed which will extend the useful bandwidth of Antoniou-type immittance converters for the realization of *RC*-active filters with high pole-*Q* towards higher frequencies. Using operational amplifiers with very high unity-gain-frequency, stability problems are likely to occur. A method for an essential improvement of the stability margin is presented.**

I. Introduction

Because most LC ladder networks have very low sensitivities in pass- and stopband, it is convenient to derive active RC-filters from these doubly terminated reactance filters. The required network-elements—simulated inductors or supercapacitors—can be realized very effectively by means of generalized immittance converters. A converter type described by Antoniou [1] has proved to be very useful.

II. Quality Factor

Fig. 1 shows the Antoniou-type converter.

As it has been shown elsewhere [2]–[4], it is advantageous to choose $G_3 = G_4$, in order to eliminate most of the influence of the nonideal operational amplifiers.

From port 1 we obtain an input admittance

$$Y_{E1}=\frac{1}{s}\cdot\frac{G_2G_4G_6}{G_3C_5}=\frac{1}{sL} \tag{1}$$

i.e., a simulated inductance, if the op amps are ideal. At port 2 a supercapacitive input admittance

$$Y_{E2}=s^2\cdot\frac{C_1C_5G_3}{G_2G_4}=s^2D \tag{2}$$

is seen if the op amps are ideal.

For both one-port admittances, Y_{E1} and Y_{E2}, the maximum Q-factor can be derived.

Assuming $A(s)=1/(1/A_0+s/GB)$, where A_0 is the dc gain and GB the gain-bandwidth product, this leads to

$$[Q(\omega)]_{\max}=Q(\omega_0)=-\frac{\operatorname{Im}\{Y_{E1}\}}{\operatorname{Re}\{Y_{E1}\}}=\frac{A_0}{4}\cdot\frac{1}{1+A_0\left(\frac{\omega_0}{GB}\right)^2} \tag{3}$$

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