AMS/IP

Studies in Advanced Mathematics

Volume 32

Lectures on Mean Curvature Flows

Xi-Ping Zhu

American Mathematical Society • International Press

Contents

1 The curve shortening flow for convex curves **1**
1.1 Shrinking to a Point 2
1.2 Asymptotic Behavior 6

2 The Short Time Existence and The Evolution Equation of Curvatures **15**
2.1 Local Existence 17
2.2 Evolution of Metric and Curvature 18
2.3 Pinching Estimate 20

3 Contraction of Convex Hypersurfaces **25**
3.1 Some Facts on Convex Hypersurface 25
3.2 MCF FOR CONVEX HYPERSURFACES 29

4 Monotonicity and Self-Similar Solutions **35**
4.1 Type I Limits 35
4.2 The Classification of Self-similar Solutions 39

5 Evolution of Embedded Curves or Surfaces (I) **47**
5.1 Isoperimetric Estimates 48
5.2 Blow-up Argument 50
5.3 Convexity Theorem 52

6 Evolution of Embedded Curves and Surfaces (II) **55**
6.1 Curves with Finite Total Absolute Curvature 55
6.2 Long Time Existence for Complete Curves 60

7 Evolution of Embedded Curves and Surfaces (III) **67**
7.1 The Evolution Equation of Gradient Function 68
7.2 Gradient Estimates 69
7.3 Curvature Estimates 71
7.4 Long Time Existence for Entire Graphs 75

8 Convexity Estimates for Mean Convex Surfaces **77**
8.1 Evolution Equations 78
8.2 L^p Estimates 80
8.3 De Giorgi Iteration Argument 84

9 Li-Yau Estimates and Type II Singularities **89**
9.1 Translating Soliton 90
9.2 Li-Yau Type Inequality 91
9.3 Type II Limits 97

10 The Mean Curvature Flow in Riemannian Manifolds **101**
10.1 Hypersurfaces in Riemannian Manifolds 101
10.2 Evolution Equations 104

11 Contracting Convex Hypersurfaces in Riemannian Manifolds **109**
11.1 The Pinching Estimates 109
11.2 A Geometric Lemma 112
11.3 Huisken Theorem 115

12 Definition of Center of Mass for Isolated Gravitating Systems **123**
12.1 Approximately Round Surfaces 124
12.2 Existence of Constant Mean Curvature Surfaces 128
12.3 Center of Gravity 137

References **145**

Index **149**