# Lepton flavor non-universality in $B$-meson decays from a U(2) flavor model

**Adam Falkowski,**[a] **Marco Nardecchia**[b] **and Robert Ziegler**[c,d]

[a]*Laboratoire de Physique Théorique, Bat. 210, Université Paris-Sud,*
*91405 Orsay, France*
[b]*DAMTP, University of Cambridge,*
*Wilberforce Road, Cambridge, CB3 0WA, U.K.*
[c]*Sorbonne Universités, UPMC Univ. Paris 06, UMR 7589, LPTHE,*
*F-75005, Paris, France*
[d]*CNRS, UMR 7589, LPTHE,*
*F-75005, Paris, France*

*E-mail:* adam.falkowski@th.u-psud.fr, M.nardecchia@damtp.cam.ac.uk, robert.ziegler@lpth.jussieu.fr

ABSTRACT: We address the recent anomalies in semi-leptonic $B$-meson decays using a model of fermion masses based on the U(2) flavor symmetry. The new contributions to $b \to s\ell\ell$ transitions arise due to a tree-level exchange of a $Z'$ vector boson gauging a U(1) subgroup of the flavor symmetry. They are controlled by a single parameter and are approximately aligned to the Standard Model prediction, with constructive interference in the $e$-channel and destructive interference in the $\mu$-channel. The current experimental data on semi-leptonic $B$-meson decays can be very well reproduced without violating existing constraints from flavor violation in the quark and lepton sectors. Our model will be tested by new measurements of $b \to s\ell\ell$ transitions and also by future electroweak precision tests, direct $Z'$ searches, and $\mu$-$e$ conversion in nuclei.

# Contents

## 1 Introduction

While direct searches for new particles at the LHC have so far been inconclusive, recent results from the LHCb collaboration on semi-leptonic $B$-meson decays [1–3] might provide the first indirect hint of new physics beyond the Standard Model (SM). Starting with the $3\sigma$ anomaly in $B \to K^*\mu^+\mu^-$ angular observables [1], several other observables involving $b \to s\ell\ell$ transitions have produced significant deviations from the SM predictions. The most notable is the ratio of $B^\pm \to K^\pm\mu^+\mu^-$ to $B^\pm \to K^\pm e^+e^-$ branching ratios measured as $R_K = 0.745^{+0.097}_{-0.082}$ [2] (deviating from the SM prediction by $2.6\sigma$), as in this case the SM prediction $R_K \approx 1$ can be calculated with a very good accuracy. The measured branching fraction of $B_s \to \phi\mu^+\mu^-$ [3] is also low compared to the SM prediction.

These anomalies could well be the result of statistical fluctuations, experimental problems, underestimated hadronic uncertainties, or a combination of all three. Nevertheless, it is intriguing that many of the discrepancies can be simultaneously explained by assuming new physics contributions to the Wilson coefficients of 4-fermion operators with a b- and s-quark and 2 leptons [4–15]. Such contributions can be easily generated in explicit models through the exchange of a new $Z'$ gauge boson [16–26] or leptoquarks [11, 27–34].

In order to address the $R_K$ anomaly, lepton flavor universality has to be broken, which *generically* also implies lepton flavor violation (LFV) [30, 35–38]. It is tempting to connect these patterns of flavor violation to the SM flavor puzzle, i.e. the experimentally observed hierarchical structure of Yukawa couplings. Some papers in the literature have attempted to obtain this connection in the context of partial compositeness [19, 25, 28] or leptoquark models [30]. Other works [18, 22] considered gauged abelian symmetries that are able to reproduce some features of the CKM matrix (but not the SM quark mass hierarchies). However, up to now, no model has been proposed that directly connects the anomalies to the generation of fermion masses and mixings.

The purpose of this work is to provide a predictive model of this kind. We address the anomalies in $b \to s\ell\ell$ transitions in the context of a light $Z'$ vector boson, whose couplings to fermions are governed by an underlying $U(2)_F$ symmetry that explains fermion masses and mixings. The original $U(2)$ models proposed in the context of supersymmetry [39, 40] have been disfavored by precision measurements in the $B$-factories [41], as they predicted the relation $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ which was not borne out experimentally. However, it is not difficult to modify this prediction with a more general $U(1)_F$ charge assignment, as demonstrated in ref. [42].

The model that we are presenting here is essentially a non-supersymmetric version of the one in ref. [42], in which the dominant source of deviations from the SM is due to the tree-level exchange of the $Z'$ gauge boson associated to the $U(1)_F$ flavor group. Similarly to the supersymmetric model, the couplings of the $Z'$ are approximately $U(2)_F$ symmetric, and flavor violating effects in the quark sector are suppressed by the small CKM mixing angles involving the 3rd generation. In contrast to ref. [42], we do not demand that the $U(1)_F$ charges are compatible with $SU(5)$ grand unification. This generalization gives us more freedom in the charged lepton sector to address the observed anomalies in $b \to s\ell\ell$ transitions. Once this is achieved, the parametric freedom in the model is to a large extent fixed by matching to the observed quark and charged lepton masses and quark mixing angles.

In our scenario, the deviations in $b \to s\ell\ell$ observables arise from a simple pattern of $Z'$ contributions to the 4 relevant Wilson coefficients $C_{9,10}^{ee,\mu\mu}$. Namely, the new contributions are aligned with the SM one (i.e. approximately left-handed) and controlled by a *single* parameter (the ratio of the $U(1)_F$ gauge coupling and the $Z'$ mass multiplied by the $b_L$-$d_L$ mixing angle) that sets its overall magnitude. Moreover, they interfere constructively in the electron channel and destructively in the muon channel. As a consequence, we predict a simple pattern for the relevant $b \to s\ell\ell$ amplitudes: in the electron channels the SM predictions are rescaled by a factor $r_e > 1$, whereas in the muon channels they are rescaled by a correlated factor $r_\mu < 1$. The current experimental data on $b \to s\ell\ell$ transitions determine the overall normalization of the $Z'$ contribution. This in turn fixes the predictions for other flavor-violating observables up to $\mathcal{O}(1)$ coefficients that span the parameter space of our model. Comparing that with existing constraints from $\Delta F = 2$ and LFV observables, we obtain bounds on these $\mathcal{O}(1)$ coefficients. The strongest ones come from $B_s$ and kaon mixing, electroweak precision measurement in LEP-2, and, especially, from $\mu$-$e$ conversion in nuclei. These bounds disfavor large regions of the parameter space,

but they nevertheless leave enough room to address the $B$-meson anomalies. The corollary is that our scenario will be decisively tested not only by upcoming new data from LHCb, but also from near future tests of LFV in $\mu \to 3e$ decays and $\mu$-$e$ conversion in nuclei. Last but not least, if the $Z'$ boson couples to fermions with electroweak strength, it is within the kinematical reach of LHC.

This paper is organized as follows. In section 2 we define the setup of the U(2)$_F$ flavor model and use its predictions for fermion masses and mixings to determine the couplings of the $Z'$ gauge boson to fermions. In section 3 we demonstrate that the resulting contributions from tree-level $Z'$ exchange to Wilson coefficients controlling $b \to s\ell\ell$ transitions allow one to address the $B$-meson anomalies. In section 4 we study other constraints on the parameter space, and show that electroweak precision tests in LEP-2 and $\mu$-$e$ conversion in nuclei provide important constraints. We conclude in section 5. In appendix A we provide analytical results for the eigenvalues and mixing angles of the quark Yukawa matrices.

## 2    The model

In this section we define our model with a U(2)$_F$ flavor symmetry. We first study its predictions concerning the fermion masses and mixings and demonstrate that the observed patterns in the quark and lepton sector can be reproduced. Then we discuss the physics of the $Z'$ boson associated to the U(1)$_F$ factor of the flavor group. This degree of freedom will be the origin of lepton flavor violation in the $B$-meson sector that we discuss in the next section.

### 2.1    Flavor symmetries

We first consider an extension of the SM with the global symmetry U(2)$_F \equiv$ SU(2)$_F \times$ U(1)$_F$ acting in the fermion's generation space. Here we restrict to the effective description involving only SM fields and spurions parametrizing the breaking of SU(2)$_F \times$ U(1)$_F$. We assume that the additional degrees of freedom needed to UV-complete this theory are heavy enough not to play role in the low-energy dynamics, i.e. the cutoff-scale $\Lambda$ of the effective theory is in the multi-TeV range. The first two generations transform as a doublet under SU(2)$_F$, and the third generation is an SU(2)$_F$ singlet. The U(1)$_F$ charges of all fermions are treated as free parameters for a while; they will be fixed later to reproduce the observed mass and mixing hierarchies. The Higgs field is a total flavor singlet. The breaking of the flavor symmetry is described by two scalar spurions: $\phi$ transforming as $\mathbf{2}_{X_\phi}$, and $\chi$ transforming as $\mathbf{1}_{-1}$. These fields acquire the following vacuum expectation values (VEVs):

$$\langle\phi\rangle = \begin{pmatrix} \epsilon_\phi\Lambda \\ 0 \end{pmatrix}, \qquad \langle\chi\rangle = \epsilon_\chi\Lambda, \tag{2.1}$$

where we assume $\epsilon_{\phi,\chi} \ll 1$. We also define $\tilde{\phi} \equiv i\sigma^2\phi^*$ which transforms as $\mathbf{2}_{-X_\phi}$. In table 1 we list the field content and their general transformation properties under the flavor group. In the next sections we will specify the U(1)$_F$ charges $X_i^F$ needed to reproduce fermion masses and mixings.

| | $Q_a$ | $U_a$ | $D_a$ | $L_a$ | $E_a$ | $Q_3$ | $U_3$ | $D_3$ | $L_3$ | $E_3$ | $H$ | $\phi_a$ | $\chi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SU(2)$_F$ | **2** | **2** | **2** | **2** | **2** | **1** | **1** | **1** | **1** | **1** | **1** | **2** | **1** |
| U(1)$_F$ | $X_1^Q$ | $X_1^U$ | $X_1^D$ | $X_1^L$ | $X_1^E$ | $X_3^Q$ | $X_3^U$ | $X_3^D$ | $X_3^L$ | $X_3^E$ | 0 | $X_\phi$ | $-1$ |

**Table 1**. The field content and U(2)$_F$ quantum numbers.

As the fermions are in general charged under U(2)$_F$, Yukawa couplings require additional spurion insertions in order to be U(2)$_F$-invariant. This leads to non-renormalizable interaction suppressed by appropriate powers of $\Lambda$. After inserting the spurion VEVs the cutoff dependence drops out, and Yukawa hierarchies arise from powers of the small parameters $\epsilon_{\phi,\chi}$. The resulting Yukawa matrices are of the form

$$
y_f \approx \begin{pmatrix}
h_{11}^f \epsilon_\phi^2 \epsilon_\chi^{|X_1^\mathcal{D}+X_1^\mathcal{S}-2X_\phi|} & h_{12}^f \epsilon_\chi^{|X_1^\mathcal{D}+X_1^\mathcal{S}|} & h_{13}^f \epsilon_\phi \epsilon_\chi^{|X_1^\mathcal{D}+X_3^\mathcal{S}-X_\phi|} \\
-h_{12}^f \epsilon_\chi^{|X_1^\mathcal{D}+X_1^\mathcal{S}|} & h_{22}^f \epsilon_\phi^2 \epsilon_\chi^{|X_1^\mathcal{D}+X_1^\mathcal{S}+2X_\phi|} & h_{23}^f \epsilon_\phi \epsilon_\chi^{|X_1^\mathcal{D}+X_3^\mathcal{S}+X_\phi|} \\
h_{31}^f \epsilon_\phi \epsilon_\chi^{|X_3^\mathcal{D}+X_1^\mathcal{S}-X_\phi|} & h_{32}^f \epsilon_\phi \epsilon_\chi^{|X_3^\mathcal{D}+X_1^\mathcal{S}+X_\phi|} & h_{33}^f \epsilon_\chi^{|X_3^\mathcal{D}+X_3^\mathcal{S}|}
\end{pmatrix} , \tag{2.2}
$$

where $\mathcal{D} = Q, L$, and $\mathcal{S} = U, D, E$. In each entry we omitted terms suppressed by more powers of $\epsilon_{\phi,\chi}$ coming from higher-dimensional terms in the effective theory. The absolute value appears because only positive powers of $\chi$ or $\chi^*$ are allowed in the effective Lagrangian. Note that, in contrast to the supersymmetric U(2) model in ref. [42], there are no holomorphy constraints, which leads to a more general Yukawa pattern.

We move to discussing the consequences of the Yukawa pattern in eq. (2.2) for the fermion masses and mixing.

## 2.2 Quark masses and mixings

In the quark sector, we fix $X_3^Q = X_3^U = 0$, so that the top Yukawa coupling is not suppressed. Furthermore, we impose the following constraints on the charges:

$$
X_\phi < 0, \quad X_1^Q + X_\phi \geq 0, \quad X_1^U + X_\phi \geq 0, \quad X_3^D \geq 0, \quad X_1^D + X_\phi \geq 0. \tag{2.3}
$$

With these constraints, we find the following up- and down-quark Yukawa matrices:

$$
y_u \approx \begin{pmatrix}
0 & h_{12}^u \epsilon_{12}^u & 0 \\
-h_{12}^u \epsilon_{12}^u & h_{22}^u \epsilon_{23}^u \epsilon_{32}^u & h_{23}^u \epsilon_{23}^u \\
0 & h_{32}^u \epsilon_{32}^u & h_{33}^u
\end{pmatrix} , \quad
y_d \approx \begin{pmatrix}
0 & h_{12}^d \epsilon_{12}^u \frac{\epsilon_{32}^d}{\epsilon_{32}^u} & 0 \\
-h_{12}^d \epsilon_{12}^u \frac{\epsilon_{32}^d}{\epsilon_{32}^u} & h_{22}^d \epsilon_{23}^u \epsilon_{32}^d & h_{23}^d \epsilon_{23}^u \epsilon_{33}^d \\
0 & h_{32}^d \epsilon_{32}^d & h_{33}^d \epsilon_{33}^d
\end{pmatrix} , \tag{2.4}
$$

where we have defined

$$
\epsilon_{12}^u \equiv \epsilon_\chi^{X_1^Q+X_1^U}, \quad \epsilon_{23}^u \equiv \epsilon_\phi \epsilon_\chi^{X_1^Q+X_\phi}, \quad \epsilon_{32}^u \equiv \epsilon_\phi \epsilon_\chi^{X_1^U+X_\phi}, \quad \epsilon_{32}^d \equiv \epsilon_\phi \epsilon_\chi^{X_1^D+X_\phi}, \quad \epsilon_{33}^d \equiv \epsilon_\chi^{X_3^D}. \tag{2.5}
$$

The $y_{11}$, $y_{13}$, and $y_{31}$ entries are not exactly zero but one can show they yield subleading corrections to quark masses and mixings relatively suppressed at least by $\epsilon_\phi^2$. Thus, effectively, three texture zeros appear in the Yukawa matrix, much as in the supersymmetric

models [42]. It was pointed out long ago [39] that the presence of these three texture zeros leads to relations among quark masses and mixings that work remarkably well from the phenomenological point of view.

Yukawa matrices of the form in eq. (2.4) can be diagonalized in a fully analytic way. However, it is more convenient to first illustrate the most important points in perturbative analysis. Indeed, since mixing angles in the left-handed (LH) quark sector are known to be small, one can use them as the small parameter in which the eigenvalues and the remaining mixing angles are expanded. Ignoring $\mathcal{O}(1)$ coefficients, this gives the following rough estimates for the eigenvalues and the CKM matrix:

$$y_t \sim 1\,, \qquad y_c \sim \epsilon_{23}^u \epsilon_{32}^u\,, \qquad y_u \sim \frac{\epsilon_{12}^u \epsilon_{12}^u}{\epsilon_{23}^u \epsilon_{32}^u}\,,$$

$$y_b \sim \epsilon_{33}^d\,, \qquad y_s \sim \epsilon_{23}^u \epsilon_{32}^d\,, \qquad y_d \sim y_u \frac{\epsilon_{32}^d}{\epsilon_{32}^u}\,,$$

$$V_{cb} \sim \epsilon_{23}^u\,, \qquad V_{us} \sim \frac{\epsilon_{12}^u}{\epsilon_{23}^u \epsilon_{32}^u}\,, \qquad V_{ub} \sim \frac{\epsilon_{12}^u}{\epsilon_{32}^u}\,. \qquad (2.6)$$

We thus have 5 small parameters that set the order of magnitude of 8 observable quantities. Expressed in powers of the Cabibbo angle $\lambda \approx 0.2$, the magnitudes of the parameters consistent with experiment is

$$\epsilon_{23}^u \sim \epsilon_{32}^u \sim \epsilon_{32}^d \sim \epsilon_{33}^d \sim \lambda^2, \quad \epsilon_{12}^u \sim \lambda^5, \qquad (2.7)$$

where the $y_s/y_c$ hierarchy must be explained by order 1 factors. The parametric size of rotation angles and matrices (see appendix for our conventions) is then given by

$$s_{12}^{Lu} \approx s_{12}^{Ru} \sim \lambda^2\,, \qquad s_{13}^{Lu} \sim \lambda^4\,, \qquad s_{13}^{Ru} \sim \lambda^4\,, \qquad s_{23}^{Lu} \sim \lambda^2\,, \qquad s_{23}^{Ru} \sim \lambda^2\,,$$
$$s_{12}^{Ld} \approx s_{12}^{Rd} \sim \lambda\,, \qquad s_{13}^{Ld} \sim \lambda^3\,, \qquad s_{13}^{Rd} \sim \lambda\,, \qquad s_{23}^{Ld} \sim \lambda^2\,, \qquad s_{23}^{Rd} \sim 1 \qquad (2.8)$$

and

$$V_L^u \sim V_R^u \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}\,, \qquad V_L^d \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\,, \qquad V_R^d \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}\,. \qquad (2.9)$$

The constraints in eq. (2.7) are recovered (up to a mismatch in $\epsilon_{12}^u$ that again is ascribed to order one factors) when the $U(1)_F$ charges are fixed as

$$X_3^Q = X_3^U = 0, \qquad X_1^Q = X_1^U = X_1^D = X_3^D = -X_\phi = 1\,, \qquad (2.10)$$

and the spurion VEVs are of the order $\epsilon_\chi \lesssim \epsilon_\phi \sim \lambda^2$. One robust conclusion is that, given the observed masses and mixings, the $U(1)_F$ charges in the RH down sector should be universal. This has important consequences for phenomenology, as we will discuss later on.

We now improve on the above rough estimates, taking into account the $\mathcal{O}(1)$ coefficients. For our purpose, it is convenient to express the observables in terms of physical quark masses and the unitary rotations that connect the flavor and mass basis. To this

end we pick 4 rotation angles: $s_{23}^{Lu}$, $s_{23}^{Ru}$, $s_{23}^{Ld}$, $s_{23}^{Rd}$. As we show in appendix, the remaining rotation angles, up to percent corrections, can be expressed in terms of these 4 angles and the quark mass ratios:

$$s_{12}^{Lu} \approx -s_{12}^{Ru} \approx \sqrt{\frac{m_u}{m_c}}, \qquad s_{13}^{Lu} \approx -s_{23}^{Lu} s_{12}^{Lu}, \qquad\qquad s_{13}^{Ru} \approx s_{23}^{Ru} s_{12}^{Lu},$$

$$s_{12}^{Ld} \approx -s_{12}^{Rd} \approx \sqrt{\frac{m_d}{m_s}} \sqrt{c_{23}^{Rd}}, \quad s_{13}^{Ld} \approx -s_{23}^{Ld} s_{12}^{Ld} \left( 1 - \frac{s_{23}^{Rd}}{c_{23}^{Rd} s_{23}^{Ld}} \frac{m_s}{m_b} \right), \quad s_{13}^{Rd} \approx \frac{s_{23}^{Rd}}{c_{23}^{Rd}} s_{12}^{Ld}. \quad (2.11)$$

Using eq. (2.11), the CKM elements up to phase factors can be expressed as:

$$V_{us} \approx \sqrt{\frac{m_d}{m_s}} \sqrt{c_{23}^{Rd}}, \quad V_{cb} \approx s_{23}^{Ld} - s_{23}^{Lu}, \quad V_{ub} \approx \sqrt{\frac{m_u}{m_c}} \left( s_{23}^{Ld} - s_{23}^{Lu} \right) - \sqrt{\frac{m_d m_s}{m_b^2}} \frac{s_{23}^{Rd}}{c_{23}^{Rd}}. \quad (2.12)$$

In the original U(2) models [39, 40] $s_{23}^{Rd}$ was taken to be small, $s_{23}^{Rd} \sim V_{cb}$. From eq. (2.12), this leads to the prediction $|V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}$ which deviates from experimental data by more than $3\sigma$. However, with a large RH 2-3 rotation angle, $s_{23}^{Rd} \sim c_{23}^{Rd} \sim 1/\sqrt{2}$, the CKM angles can be well fit (see also refs. [41, 42]). The other 3 rotation angles parametrizing the model can be small, $s_{23}^{Lu} \sim s_{23}^{Ru} \sim s_{23}^{Ld} \sim |V_{cb}|$.

One can explicitly verify that the charge assignment in eq. (2.10) allows one to fit the masses and mixings in quark sector with coefficients $h_{ij}^q$ that are indeed $\mathcal{O}(1)$, and in turn check the validity of the above parametrization. For our fit, we take the masses and mixings calculated in the SM at the scale 10 TeV [43]. With the Yukawa matrices

$$y_u \approx \begin{pmatrix} 0 & 4.9 \cdot \epsilon_\chi^2 & 0 \\ -4.9 \cdot \epsilon_\chi^2 & 3.7 \cdot \epsilon_\phi^2 & 0.89 \cdot \epsilon_\phi \\ 0 & 1.3 \cdot \epsilon_\phi & 0.79 \end{pmatrix}, \quad y_d \approx \begin{pmatrix} 0 & 3.6 \cdot \epsilon_\chi^2 & 0 \\ -3.6 \cdot \epsilon_\chi^2 & -0.62 \cdot \epsilon_\phi^2 & 4.9 \cdot \epsilon_\phi \epsilon_\chi \\ 0 & -0.20 \cdot \epsilon_\phi & 2.5 \cdot \epsilon_\chi \end{pmatrix}, \quad (2.13)$$

and the spurions VEVs

$$\epsilon_\chi \approx 0.0040, \qquad \epsilon_\phi \approx 0.035. \quad (2.14)$$

One can reproduce the observed masses and mixings within the experimental errors. One can also check that the above Yukawa matrices give values for the 2-3 mixing angles

$$|s_{23}^{Lu}| \approx 1.0 \cdot |V_{cb}|, \quad |s_{23}^{Ru}| \approx 1.5 \cdot |V_{cb}|, \quad |s_{23}^{Ld}| \approx 2.1 \cdot |V_{cb}|, \quad |s_{23}^{Rd}| \approx 0.60. \quad (2.15)$$

Moreover the other rotation angles are in very good agreement with the approximate expression in eq. (2.11). We will use eq. (2.15) and eq. (2.14) as a reference point for the natural values of the mixing angles in phenomenological analyses below.

In summary, the quark masses and mixing angles can be successfully fit in our $U(2)_F$ flavor model with $\mathcal{O}(1)$ Yukawa coefficients in the Lagrangian. The remaining freedom can be parametrized by four rotation angles. Their precise values depend on the $\mathcal{O}(1)$ coefficients, however, barring large cancellations, their order of magnitude is fixed: $s_{23}^{Lu} \sim s_{23}^{Ru} \sim s_{23}^{Ld} \sim |V_{cb}|$, and $s_{23}^{Rd} \sim 1$.

## 2.3 Charged lepton masses

In the lepton sector we focus on the charged lepton masses, and we ignore here the neutrino masses.[1] Therefore the rotation angles in the charged lepton sector are not constrained by phenomenology, which leaves more freedom in the choice of the model parameters. The simplest possibility is to take the $U(1)_F$ lepton charges to be compatible with $SU(5)$ grand unification, that is to say, the same as for the down-type quarks [42]. However, one can show that such a choice does not allow to address the lepton non-universality in $b \to s\ell\ell$ transitions, which is the primary goal in this paper. Therefore we make a different choice of the $U(1)_F$ charges:

$$X_1^L = -3 - X_1^E, \qquad X_3^L = 2 - X_1^E, \qquad X_3^E = 4 + X_1^E, \qquad (2.16)$$

where $X_1^E$ does not enter into the lepton mass matrix and is left unspecified for the moment. This choice leads to the following lepton Yukawa matrix:

$$y_e \approx \begin{pmatrix} h_{11}^e \epsilon_\phi^2 \epsilon_\chi & h_{12}^e \epsilon_\chi^3 & h_{13}^e \epsilon_\phi \epsilon_\chi^2 \\ -h_{12}^e \epsilon_\chi^3 & 0 & h_{23}^e \epsilon_\phi \\ h_{31}^e \epsilon_\phi \epsilon_\chi^3 & h_{32}^e \epsilon_\phi \epsilon_\chi & 0 \end{pmatrix}, \qquad (2.17)$$

where the 2-2 and 3-3 diagonal elements are suppressed by $\epsilon_\phi^2 \epsilon_\chi^5$ and $\epsilon_\chi^6$, and thus can be neglected. The consequence is that the muon and tau Yukawa couplings are set by the 23 and 32 off-diagonal elements. Indeed, diagonalizing eq. (2.17) yields the Yukawa couplings

$$y_e \approx h_{11}^e \epsilon_\phi^2 \epsilon_\chi, \quad y_\mu \approx \epsilon_\phi \epsilon_\chi h_{32}^e, \quad y_\tau \approx h_{23}^e \epsilon_\phi. \qquad (2.18)$$

Using $\epsilon_{\phi,\chi}$ in eq. (2.14), the correct lepton masses are recovered by fixing three $\mathcal{O}(1)$ coefficients as $h_{11}^e \approx 0.57, h_{23}^e \approx 0.29, h_{32}^e \approx 4.3$. The rotation angles are then determined by the remaining $\mathcal{O}(1)$ coefficients:

$$s_{12}^{Le} \approx \frac{h_{13}^e}{h_{23}^e} \epsilon_\chi^2 \approx 5.5 \times 10^{-5} h_{13}^e, \qquad s_{12}^{Re} \sim \frac{h_{31}^e}{h_{32}^e} \epsilon_\chi^2 \approx 4.2 \times 10^{-6} h_{31}^e,$$

$$s_{23}^{Le} \approx 1, \qquad\qquad\qquad s_{23}^{Re} \approx 0,$$

$$s_{13}^{Le} \approx \frac{h_{12}^e}{h_{32}^e} \frac{\epsilon_\chi^2}{\epsilon_\phi} \approx 1.1 \times 10^{-4} h_{12}^e, \qquad s_{13}^{Re} \approx -\frac{h_{12}^e}{h_{23}^e} \frac{\epsilon_\chi^3}{\epsilon_\phi} \approx -6.2 \times 10^{-6} h_{12}^e. \qquad (2.19)$$

In summary, the charged lepton masses can be well reproduced with the $U(1)_F$ charge assignment in eq. (2.16). The resulting structure of the Yukawa matrix in eq. (2.17) leads to a large LH mixing between the 2nd and 3rd generation, $s_{23}^{Le} \approx 1$, and a small RH 2-3 rotation, $s_{23}^{Re} \approx 0$. The remaining freedom is the charge $X_1^E$ and the three $\mathcal{O}(1)$ coefficients $h_{12}^e, h_{13}^e, h_{31}^e$ that set the magnitude of the 1-2 and 1-3 mixing angles. We will use this freedom later when addressing the $b \to s$ anomalies in a way that avoids phenomenological constraints.

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[1]This is mainly due to simplicity; there are no obvious obstacles to reproduce neutrino masses and mixings with Dirac neutrinos upon adding RH neutrinos.

## 2.4  Z'-boson

We now extend the model by promoting $U(1)_F$ to a local symmetry (as in ref. [42]). We assume that the associated gauge boson is relatively light, with a mass in the TeV range. Note that the $U(1)_F$ symmetry without additional fermions is necessarily anomalous if the model explains fermion mass hierarchies. This is due to the relation [44–46],

$$\det y_u y_d \sim \epsilon_\chi^{4X_1^Q + 2X_1^U + 2X_1^D + 2X_3^Q + X_3^U + X_3^D} \equiv \epsilon_\chi^{C_3}, \tag{2.20}$$

where $C_3$ is the anomaly coefficient of the mixed $SU(3)^2 U(1)_F$ anomaly. As $U(1)_F$ is spontaneously broken by the VEVs of $\phi$ and $\chi$ at a scale $v' \sim \epsilon \Lambda$, we assume that the anomaly is cancelled by unspecified dynamics (involving new chiral fermions) at the scale $\Lambda_{UV} \lesssim 4\pi v'$. Since the new gauge boson has a mass given by $M_{Z'} = g'v'$, it can easily be the lightest new degree of freedom when $g'$ is sufficiently small. We therefore ignore the additional heavy dynamics and concentrate on the effects of the $Z'$ gauge boson.

In the flavor basis, $Z'$ couples to each fermion proportionally to its $U(1)_F$ charge $X_i^a$,

$$\mathcal{L} \supset g' Z'_\mu \left[ X_i^Q Q_i^\dagger \overline{\sigma}^\mu Q_i + X_i^U U_i^\dagger \overline{\sigma}^\mu U_i + X_i^D D_i^\dagger \overline{\sigma}^\mu D_i + X_i^L L_i^\dagger \overline{\sigma}^\mu L_i + X_i^E E_i^\dagger \overline{\sigma}^\mu E_i \right]. \tag{2.21}$$

We have fixed these charges (except for $X_1^E$) to fit the observed fermion mass hierarchies. That fit also determines the unitary rotations that connect the flavor and the mass basis. Therefore, flavor non-universal effects mediated by $Z'$ are predicted in our model, up to an overall normalization determined by the $Z'$ mass and gauge coupling, and up to the freedom of choosing $X_1^E$ and order one Yukawa factors. In particular, the $SU(2)_F$ structure for the first two generations implies that flavor changing effects are entirely determined by the 3rd row of the rotation matrices. In the mass basis, the $Z'$ couplings take the form

$$\mathcal{L} \supset g' \Delta_{L,R}^{f_i f_j} f_i^\dagger \overline{\sigma}^\mu f_j Z'_\mu, \tag{2.22}$$

$$\Delta_L^{f_i f_j} = X_1^{\mathcal{D}} \left[ \delta_{ij} - \frac{X_1^{\mathcal{D}} - X_3^{\mathcal{D}}}{X_1^{\mathcal{D}}} (V_L^f)_{3i} (V_L^f)_{3j}^* \right], \qquad \mathcal{D} = Q, L, \tag{2.23}$$

$$\Delta_R^{f_i f_j} = X_1^{\mathcal{S}} \left[ \delta_{ij} - \frac{X_1^{\mathcal{S}} - X_3^{\mathcal{S}}}{X_1^{\mathcal{S}}} (V_R^f)_{3i}^* (V_R^f)_{3j} \right], \qquad \mathcal{S} = U, D, E. \tag{2.24}$$

Note that flavor-violating couplings are proportional to the charge difference $X_1 - X_3$. As a consequence, with the charge assignments in eq. (2.10) there is no flavor violation in the RH down sector: $\Delta_R^{d_i d_j} = \delta_{ij}$. For the LH down quarks we find

$$\Delta_L^{d_i d_j} \approx \begin{pmatrix} 1 & -s_{13}^{Ld} s_{23}^{Ld} & s_{13}^{Ld} \\ -s_{13}^{Ld} s_{23}^{Ld} & (c_{23}^{Ld})^2 & s_{23}^{Ld} \\ s_{13}^{Ld} & s_{23}^{Ld} & (s_{23}^{Ld})^2 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^4 \end{pmatrix}. \tag{2.25}$$

Since $s_{13}^{Ld} \ll s_{23}^{Ld}$ (see eq. (2.11)), the largest flavor violating effect of $Z'$ is in $b \to s$ quark transitions. This will be handy for addressing the recent $B$-meson anomalies, as we will discuss in the next section. For LH and RH up quarks we find

$$\Delta_X^{u_i u_j} \approx \begin{pmatrix} 1 & -s_{13}^{Xu} s_{23}^{Xu} & s_{13}^{Xu} \\ -s_{13}^{Xu} s_{23}^{Xu} & (c_{23}^{Xu})^2 & s_{23}^{Xu} \\ s_{13}^{Xu} & s_{23}^{Xu} & (s_{23}^{Xu})^2 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda^6 & \lambda^4 \\ \lambda^6 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^4 \end{pmatrix}, \tag{2.26}$$

where $X = \{L, R\}$.

In the charged lepton sector the $Z'$ couplings depend on the charge $X_1^E$ and to good approximation only on the 1-3 rotation angles.

$$\Delta_L^{e_i e_j} \approx \begin{pmatrix} -3 - X_1^E & 5s_{13}^{Le} & 0 \\ 5s_{13}^{Le} & 2 - X_1^E & 0 \\ 0 & 0 & -3 - X_1^E \end{pmatrix}, \quad \Delta_R^{e_i e_j} \approx \begin{pmatrix} X_1^E & 0 & 4s_{13}^{Re} \\ 0 & X_1^E & 0 \\ 4s_{13}^{Re} & 0 & 4 + X_1^E \end{pmatrix}. \quad (2.27)$$

The diagonal muon $Z'$ coupling is different from the electron and tau one at leading order, which is due to the 2-3 inversion in the LH sector. This feature of our model will allow us later to address the anomalies in $B$-meson decays involving muons and electrons. In the RH sector electrons and muons have approximately the same coupling to $Z'$ and the dominant flavor non-universal effects must involve the tau lepton. The largest flavor violating effects occur in the LH $\mu$-$e$ and RH $\tau$-$e$ transitions. Note that the rotation angles setting the magnitude of these lepton-flavor-violating effects are fixed up to $\mathcal{O}(1)$ factors and expected to be tiny, see eq. (2.19). As a result, lepton flavor violation in our model is suppressed at least by a factor of order $\epsilon_\chi^2/\epsilon_\phi \approx 5 \times 10^{-4}$ as compared to violation of lepton flavor universality.

## 3 Phenomenology of $b \to s\ell\ell$ transitions

We now turn to the predictions for $b \to s\ell\ell$ transitions. Our main goal is to address the recently observed violation of lepton flavor universality in $B$-meson decays [2]. In our model, this anomaly is due to the exchange of a $U(1)_F$ $Z'$ boson with mass in the multi-TeV range.

Low-energy observables are controlled by the 4-fermion effective operators that arise from integrating out the $Z'$ at tree level,

$$\mathcal{L}_{\text{eff}} \supset -\frac{g'^2}{2M_{Z'}^2} \left[ \Delta_L^{f_i f_j} \bar{f}_i \bar{\sigma}_\mu f_j + \Delta_R^{f_i f_j} f_i^c \sigma_\mu \bar{f}_j^c \right] \left[ \Delta_L^{f_k f_l} \bar{f}_k \bar{\sigma}_\mu f_l + \Delta_R^{f_k f_l} f_k^c \sigma_\mu \bar{f}_l^c \right]. \quad (3.1)$$

The 4-fermion operators in eq. (3.1) include the ones relevant for $B \to K\ell\ell$ decays which are customarily parametrized by the following effective Hamiltonian (see e.g. ref. [21]):

$$\mathcal{H}_{\text{eff}} \supset -\frac{\alpha}{\pi} \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \bar{s}\gamma_\mu P_L b \right] \left[ C_9^{\ell\ell'} \bar{\ell}\gamma^\mu \ell' + C_{10}^{\ell\ell'} \bar{\ell}\gamma^\mu \gamma^5 \ell' \right] + \text{h.c.} \quad (3.2)$$

Note that the analogous 4-fermion operators with RH quarks are not generated in our model ($\tilde{C}_9^{\ell\ell'} = \tilde{C}_{10}^{\ell\ell'} = 0$), as a consequence of the universal $U(1)_F$ charge assignment in the RH down sector. Matching eq. (3.1) and eq. (3.2), the Wilson coefficients are given by

$$C_9^{\ell\ell'} = -\frac{\pi g'^2}{\sqrt{2} M_{Z'}^2 G_F V_{tb} V_{ts}^* \alpha} \Delta_L^{sb} \left( \Delta_R^{\ell\ell'} + \Delta_L^{\ell\ell'} \right), \quad C_{10}^{\ell\ell'} = -\frac{\pi g'^2}{\sqrt{2} M_{Z'}^2 G_F V_{tb} V_{ts}^* \alpha} \Delta_L^{sb} \left( \Delta_R^{\ell\ell'} - \Delta_L^{\ell\ell'} \right). \quad (3.3)$$

Focusing for now on the lepton flavor conserving operators with electrons or muons, we have

$$\begin{aligned} C_9^{ee} &\approx 0.19k\,, & C_{10}^{ee} &\approx -0.19 \left( 1 + 2/3 X_1^E \right) k\,, \\ C_9^{\mu\mu} &\approx -0.13k\,, & C_{10}^{\mu\mu} &\approx 0.13 \left( 1 - X_1^E \right) k\,, \end{aligned} \quad (3.4)$$

where we defined the parameter $k$ as

$$k \equiv \left( \frac{20\,\text{TeV}}{M_{Z'}/g'} \right)^2 \left( \frac{s_{23}^{Ld}}{|V_{cb}|} \right) , \tag{3.5}$$

and we used the numerical values $|V_{ts}| \approx |V_{cb}| \approx 0.041$, $\alpha(m_b) \approx 1/133$.

The explicit expressions for the Wilson coefficients in eq. (3.4) imply that large corrections to $B$-meson decays involving muons are correlated with comparable corrections to the analogous observables involving electrons. While new physics in the muonic sector alone gives the most economical explanation of the LHCb anomalies (including $R_K$ [12, 14]), it has been emphasized that large corrections in electron channels are not only allowed but could also (slightly) improve the goodness of the global fit [13].

We can now identify the parameter space of our model where the measurements of semileptonic $b \to s\ell\ell$ transitions with $\ell = e, \mu$ are best reproduced. As can be seen from eq. (3.4), these observables depend on just 2 parameters: $k$ defined in eq. (3.5), and the U(1)$_F$ charge $X_1^E$. For the moment, we treat them as free parameters, various precision constraints will be discussed in the next section. In order to find the best fit region for $k$ and $X_1^E$, we use the result of ref. [13]. The authors provide the results of the fit in the 2D planes $(C_9^{ee}, C_9^{\mu\mu})$, $(C_{10}^{ee}, C_{10}^{\mu\mu})$, $(C_9^{\mu\mu}, C_{10}^{\mu\mu})$ and $(C_9^{ee}, C_{10}^{ee})$. Ignoring possible correlations in the full 4D likelihood, we identify the allowed range for $k$ for discrete values of $X_1^E$ by requiring to simultaneously remain inside the 68% or 95% confidence level regions in every 2D plane. We obtain:

| $X_1^E$ | $1\sigma$ Region | $2\sigma$ Region |
|---|---|---|
| $-3$ | – | $k \in [0.5, 2.1]$ |
| $-2$ | – | $k \in [0.5, 3.1]$ |
| $-1$ | – | $k \in [0.6, 5.1]$ |
| $0$ | $k \in [2.7, 4.2]$ | $k \in [0.8, 6.6]$ |
| $1$ | – | $k \in [1.2, 4.9]$ |
| $2$ | – | – |

This simplified analysis suggests that, while a reasonable fit to the $b \to s\ell\ell$ data is possible for a range of $X_1^E$, the best case scenario is $X_1^E = 0$. *In the rest of this paper we focus on that particular choice.* In this case, the new physics contributions mediated by $Z'$ are purely left-handed, $C_9^{ee} = -C_{10}^{ee}$, $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$, and we can derive constraints on $k$ using the 2-parameter fit of ref. [13] for precisely this case. This way, we find that the $1\sigma$ confidence interval is $k \in [1.9, 4.9]$ and the $2\sigma$ one is $k \in [0.2, 6.5]$. The allowed region of the parameters $C_9^{ee}$-$C_9^{\mu\mu}$ parameter space overlaid with the prediction of our model is displayed in figure 1.

We turn to discussing predictions of our model. The case with $X_1^E = 0$ is particularly simple because the SM contributions to the effective Hamiltonian in eq. (3.2) are also purely left-handed, $C_9^{SM} \approx -C_{10}^{SM}$, with $C_9^{SM}(m_b) \approx 4.2$. Therefore the new physics contributions interfere constructively with the SM in the $e$-channel and destructively in the $\mu$-channel, resulting in a simple rescaling of $B$-meson decay rates by the factors $r_e$ and $r_\mu$ that are the
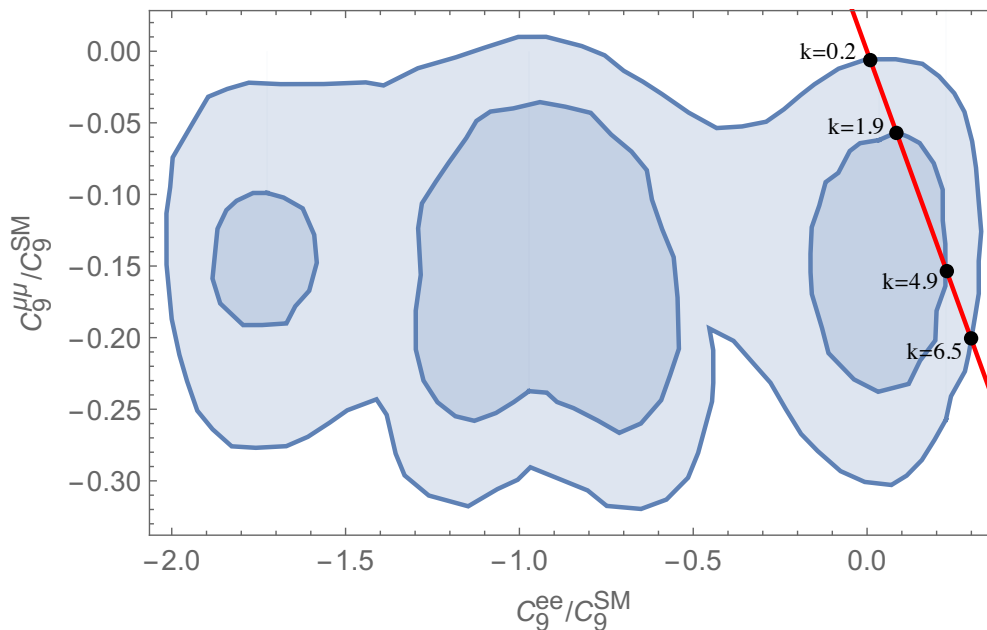
**Figure 1**. Global fit results from [13], the blue (light blue) domain corresponds to the $1\sigma$ ($2\sigma$) region. The red line is our model prediction for $X_1^E = 0$ and varying $k$.

same for all $b \to see$ and $b \to s\mu\mu$ processes,[2]

$$r_e \approx (1 + 0.044k)^2, \qquad r_\mu \approx (1 - 0.029k)^2 \,. \tag{3.6}$$

For $k \in [1.9, 4.9]$ we thus predict an enhancement in all the electron channels by $r_e \in [1.17, 1.48]$, and a suppression in all the muon channels by $r_\mu \in [0.73, 0.89]$. As a reference, in table 2 we show the measured values and the SM predictions for various $b \to s\ell\ell$ observables. It is remarkable that most observables in the muon channel shows a deficit compared to the SM predictions, while the ones in the electron channels show some (albeit not statistically significant) enhancement.

Lepton flavor universality is often tested by measuring ratios of branching fractions of semileptonic $B$-meson decays. In our model we have

$$R_X = \frac{\mathcal{B}(B \to X\mu^+\mu^-)}{\mathcal{B}(B \to Xe^+e^-)} \approx \frac{r_\mu}{r_e} = \left(\frac{1 - 0.029k}{1 + 0.044k}\right)^2 \,, \tag{3.7}$$

where $X = K, K^*, \phi, X_s$. The interval $k \in [1.9, 4.9]$ corresponds $R_X \in [0.50, 0.76]$, which should be compared to the LHCb measurement $R_K = 0.745^{+0.097}_{-0.082}$. Future improvements in the precision of $R_K$ and other measurements will be crucial for testing our model, since we predict a rather low value for these observable. This is actually supported by measurements of inclusive $B \to X_s\ell\ell$ decay ratios from BaBar ($R_{X_s} = 0.58 \pm 0.19$) [47]

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[2]This a good approximation in the limit where contributions from fully-hadronic operators can be neglected.

| Observable | $q^2$ [GeV$^2$] | SM prediction | Measurement | Ratio |
|---|---|---|---|---|
| $10^6 (\text{GeV})^2 \times \mathcal{B}(B^+ \to X_s e^+ e^-)$ | [1.0,6.0] | $1.73 \pm 0.12$ | $1.93 \pm 0.55$ [47] | $1.12 \pm 0.33$ |
| $10^6 (\text{GeV})^2 \times \mathcal{B}(B^+ \to X_s e^+ e^-)$ | [14.2,25.0] | $0.20 \pm 0.06$ | $0.56 \pm 0.19$ [47] | $2.80 \pm 1.27$ |
| $10^9 \times \mathcal{B}(B_s \to \mu^+ \mu^-)$ | – | $3.54 \pm 0.27$ | $2.9 \pm 0.7$ [48, 49] | $0.8 \pm 0.2$ |
| $10^9 (\text{GeV})^2 \langle \frac{d\mathcal{B}}{dq^2} \rangle (B^0 \to K^0 \mu^+ \mu^-)$ | [1.1,6.0] | $31.7 \pm 9.4$ | $18.7 \pm 3.6$ [50] | $0.59 \pm 0.21$ |
| $10^9 (\text{GeV})^2 \langle \frac{d\mathcal{B}}{dq^2} \rangle (B^0 \to K^0 \mu^+ \mu^-)$ | [15.0,22.0] | $13.6 \pm 2.0$ | $9.5 \pm 1.7$ [50] | $0.70 \pm 0.16$ |
| $10^9 (\text{GeV})^2 \langle \frac{d\mathcal{B}}{dq^2} \rangle (B^+ \to K^+ \mu^+ \mu^-)$ | [1.1,6.0] | $34.8 \pm 10.3$ | $24.2 \pm 1.4$ [50] | $0.70 \pm 0.21$ |
| $10^9 (\text{GeV})^2 \langle \frac{d\mathcal{B}}{dq^2} \rangle (B^+ \to K^+ \mu^+ \mu^-)$ | [15.0,22.0] | $14.8 \pm 2.0$ | $12.1 \pm 0.7$ [50] | $0.82 \pm 0.12$ |
| $10^9 (\text{GeV})^2 \langle \frac{d\mathcal{B}}{dq^2} \rangle (B^+ \to K^* \mu^+ \mu^-)$ | [1.1,6.0] | $50.5 \pm 28.6$ | $36.6 \pm 8.7$ [50] | $0.72 \pm 0.45$ |
| $10^9 (\text{GeV})^2 \langle \frac{d\mathcal{B}}{dq^2} \rangle (B^+ \to K^* \mu^+ \mu^-)$ | [15.0,19.0] | $61.5 \pm 34.8$ | $39.5 \pm 8.5$ [50] | $0.64 \pm 0.39$ |
| $10^6 (\text{GeV})^2 \times \mathcal{B}(B^+ \to X_s \mu^+ \mu^-)$ | [1.0,6.0] | $1.66 \pm 0.12$ | $0.66 \pm 0.88$ [47] | $0.40 \pm 0.53$ |
| $10^6 (\text{GeV})^2 \times \mathcal{B}(B^+ \to X_s \mu^+ \mu^-)$ | [14.2,25.0] | $0.24 \pm 0.07$ | $0.60 \pm 0.31$ [47] | $2.50 \pm 1.48$ |

**Table 2**. The measured values and the SM predictions for various $b \to s\ell\ell$ observables used in the fit of ref. [13].

| Decay | Branching ratio | Ref. | Type |
|---|---|---|---|
| $B^+ \to K^+ \mu^+ \mu^-$ | $(4.29 \pm 0.22) \times 10^{-9}$ | [2] | LFC |
| $B^+ \to K^+ \mu^\pm \tau^\mp$ | $< 4.8 \times 10^{-5}$ | [53] | LFV |
| $B^+ \to K^+ e^\pm \tau^\mp$ | $< 3.0 \times 10^{-5}$ | [53] | LFV |
| $B^+ \to K^+ e^\pm \mu^\mp$ | $< 9.1 \times 10^{-8}$ | [53] | LFV |
| $B_s \to \mu^+ \mu^-$ | $(2.9 \pm 0.7) \times 10^{-9}$ | [48, 49] | LFC |
| $B_s \to e^\pm \mu^\mp$ | $< 1.1 \times 10^{-8}$ | [54] | LFV |

**Table 3**. Experimental constraints on the branching fraction of several lepton flavor conserving (LFC) and lepton flavor violating (LFV) $B$-meson decays.

and Belle ($R_{X_s} = 0.42 \pm 0.25$) [51], although with large errors. Another test of lepton-non-universality is provided by double ratios [52] such as $R_{K^*}/R_K$. In our model, as in any scenario with $\tilde{C}_{9,10} = 0$, all these double ratios are predicted to be equal to one.

Finally, we comment on the predictions concerning LFV $B$-meson decays. The Wilson coefficients of 4-fermion operators mediating these decays are suppressed by additional powers of small parameters, e.g.

$$C_9^{\mu e} \approx -C_{10}^{\mu e} \approx -0.3 \, s_{13}^{Le} \, k \,. \tag{3.8}$$

From eq. (2.19), typical values of the lepton mixing angle are $s_{13}^{Le} \sim 10^{-3}$-$10^{-5}$. This implies the rate of lepton flavor violating decays is suppressed by at least 6 orders of magnitude compared to the lepton flavor conserving ones. Given the present sensitivity summarized in table 3, this will not be observable in the near future. LFV decays involving tau leptons are even more suppressed.

## 4 Constraints from flavor violation and direct searches

In this section we discuss constraints on the parameters of our model from $\Delta F = 2$ flavor transitions, LFV decays of leptons, LEP-1 and LEP-2 electroweak precision observables, and direct $Z'$ searches at the LHC. We will show that it is possible to address the observed violation of lepton-flavor universality in $B$-meson decays without violating these constraints.

### 4.1 Constraints from $\Delta F = 2$ observables

The $Z'$ boson exchange generates 4-quark operators mediating $\Delta F = 2$ transitions. In the notation of e.g. ref. [55] these are denoted $O_1$ (with 4 LH quarks), $\tilde{O}_1$ (with 4 RH quarks), and $O_5$ (with 2 LH and 2 RH quarks). Since $\Delta_R^{d_i d_j} = 0$, for the down-type quark only $O_1$ is generated. Their (in general complex) Wilson coefficients are given by

$$C_1^K = \frac{g'^2 \Delta_L^{sd} \Delta_L^{sd}}{2 M_{Z'}^2}, \qquad C_1^{B_d} = \frac{g'^2 \Delta_L^{bd} \Delta_L^{bd}}{2 M_{Z'}^2}, \qquad C_1^{B_s} = \frac{g'^2 \Delta_L^{bs} \Delta_L^{bs}}{2 M_{Z'}^2}, \qquad (4.1)$$

and numerically one has (using $m_d/m_s \approx 0.05$)

$$C_1^K = \frac{1.8 \times 10^{-10}}{\text{TeV}^2} \left( \frac{s_{23}^{Ld}}{|V_{cb}|} \right)^3 c_{23}^{Rd} k, \qquad (4.2)$$

$$C_1^{B_d} = \frac{1.1 \times 10^{-7}}{\text{TeV}^2} \left( \frac{s_{23}^{Ld}}{|V_{cb}|} \right) c_{23}^{Rd} k, \qquad C_1^{B_s} = \frac{2.1 \times 10^{-6}}{\text{TeV}^2} \left( \frac{s_{23}^{Ld}}{|V_{cb}|} \right) k, \qquad (4.3)$$

where $k$ is defined in eq. (3.5) and it needs to be $O(1)$ for the model to address the $B$-meson anomalies. We have also approximated $s_{13}^{Ld} \approx -s_{23}^{Ld} s_{12}^{Ld}$, which slightly overestimates the Wilson coefficients, see eq. (2.11). These expressions have to be compared to the bounds from $K$-mixing taken from ref. [55] (Im) and ref. [56] (Re), and the bounds from $B$-mixing taken from ref. [57]:

$$\text{Im} \, C_1^K < \frac{3.4 \times 10^{-9}}{\text{TeV}^2}, \qquad \text{Re} \, C_1^K < \frac{9.6 \times 10^{-7}}{\text{TeV}^2}, \qquad (4.4)$$

$$|C_1^{B_d}| < \frac{1.4 \times 10^{-6}}{\text{TeV}^2}, \qquad |C_1^{B_s}| < \frac{1.8 \times 10^{-5}}{\text{TeV}^2}. \qquad (4.5)$$

This shows that for $s_{23}^{Ld} \sim V_{cb}$ and $k$ in the experimentally preferred range $k \in [1.9, 4.9]$ the bounds from $K$, $B$ and $B_s$ mixing are satisfied, even for an $O(1)$ phase in $\text{Im} \, C_K^1$.

Turning to the up sector, 4-fermion operators with both LH and RH fermions are generated, as a result of a non-universal $\text{U}(1)_F$ charge assignment. In particular, for the $\Delta C = 2$ operators we have (with $m_u/m_c \approx 0.002$)

$$C_1^D = \frac{7.1 \times 10^{-12}}{\text{TeV}^2} \left( \frac{s_{23}^{Lu}}{|V_{cb}|} \right)^4 \left( \frac{|V_{cb}|}{s_{23}^{Ld}} \right) k,$$

$$\tilde{C}_1^D = \frac{7.1 \times 10^{-12}}{\text{TeV}^2} \left( \frac{s_{23}^{Ru}}{|V_{cb}|} \right)^4 \left( \frac{|V_{cb}|}{s_{23}^{Ld}} \right) k,$$

$$C_5^D = \frac{2.8 \times 10^{-11}}{\text{TeV}^2} \left( \frac{s_{23}^{Lu}}{|V_{cb}|} \right)^2 \left( \frac{s_{23}^{Ru}}{|V_{cb}|} \right)^2 \left( \frac{|V_{cb}|}{s_{23}^{Ld}} \right) k. \qquad (4.6)$$

Hence, D-meson mixing is further suppressed compared to K-meson mixing by the small mass ratio $m_u/m_c$. Given the bounds from ref. [58] (Im) and ref. [56] (Abs):

$$\operatorname{Im} C_1^D < \frac{0.9 \times 10^{-8}}{\mathrm{TeV}^2} \,, \qquad\qquad |C_1^D| < \frac{7.2 \times 10^{-7}}{\mathrm{TeV}^2} \,,$$

$$\operatorname{Im} \tilde{C}_1^D < \frac{0.9 \times 10^{-8}}{\mathrm{TeV}^2} \,, \qquad\qquad |\tilde{C}_1^D| < \frac{7.2 \times 10^{-7}}{\mathrm{TeV}^2} \,,$$

$$\operatorname{Im} C_5^D < \frac{0.4 \times 10^{-8}}{\mathrm{TeV}^2} \,, \qquad\qquad |C_5^D| < \frac{4.8 \times 10^{-7}}{\mathrm{TeV}^2} \,, \tag{4.7}$$

there are no further bounds on our model from $D-$mixing.

## 4.2 Semileptonic decays in $b \to d$ and $s \to d$ transitions

We now turn to semileptonic decays involving $b \to d$ and $s \to d$ transitions with electrons or muons in the final state. The relevant Wilson coefficients for the associated 4-fermion operators for $b \to d$ transitions are given by

$$C^{(bd)(ee)} = \frac{g'^2 \Delta_L^{bd} \Delta_L^{ee}}{2\, M_{Z'}^2} = \frac{3.4 \times 10^{-5}}{\mathrm{TeV}^2} \, k \,,$$

$$C^{(bd)(\mu\mu)} = \frac{g'^2 \Delta_L^{bd} \Delta_L^{\mu\mu}}{2\, M_{Z'}^2} = -\frac{2.3 \times 10^{-5}}{\mathrm{TeV}^2} \, k \,, \tag{4.8}$$

and for the $s \to d$ transitions by

$$C^{(sd)(ee)} = \frac{g'^2 \Delta_L^{sd} \Delta_L^{ee}}{2\, M_{Z'}^2} = -\frac{1.4 \times 10^{-6}}{\mathrm{TeV}^2} \left(\frac{s_{23}^{Ld}}{|V_{cb}|}\right) k \,,$$

$$C^{(sd)(\mu\mu)} = \frac{g'^2 \Delta_L^{sd} \Delta_L^{\mu\mu}}{2\, M_{Z'}^2} = \frac{9.4 \times 10^{-7}}{\mathrm{TeV}^2} \left(\frac{s_{23}^{Ld}}{|V_{cb}|}\right) k \,, \tag{4.9}$$

where we approximated $\Delta_L^{bd} \approx -s_{23}^{Ld} \sqrt{m_d/m_s}$ and $\Delta_L^{sd} \approx \left(s_{23}^{Ld}\right)^2 \sqrt{m_d/m_s}$. The upper bounds are summarized in the table 4, adapted from the case of composite leptoquarks [28]. From this table, it is easy to verify that for the experimentally preferred range $k \in [1.9, 4.9]$ all the bounds are satisfied.

## 4.3 Lepton flavor violation

From eq. (2.27), the largest LFV $Z'$ couplings are the ones to LH muons and electrons. These are constrained by several precise measurements of LFV $\mu \to e$ transitions. First, we have the $\mu \to 3e$ decay with the branching fraction

$$\mathrm{BR}(\mu \to 3e) = \frac{g'^4 v^4}{4 M_{Z'}^4} |\Delta_L^{e\mu}|^2 \left(2|\Delta_L^{ee}|^2 + |\Delta_R^{ee}|^2\right)$$

$$\approx 3.2 \times 10^{-14} \, k^2 \left(\frac{s_{13}^{Le}}{1.1 \times 10^{-4}}\right)^2 \left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)^2 \,, \tag{4.10}$$

where $v = 246\,\mathrm{GeV}$. This should be compared with the experimental limit from ref. [59]:

$$\mathrm{BR}(\mu \to 3e) < 1.0 \times 10^{-12} \,. \tag{4.11}$$

| Decay | $(ij)(kl)$ | $\|\Delta_L^{d_i d_j}\Delta_L^{e_k e_l}\|/(\sqrt{2}\,\frac{M_{Z'}}{1\text{TeV}})^2$ |
|---|---|---|
| $K_S \to e^+e^-$ | $(21)(11)$ | $< 1.0$ |
| $K_L \to e^+e^-$ | $(21)(11)$ | $< 2.7 \times 10^{-3}$ |
| $K_S \to \mu^+\mu^-$ | $(21)(22)$ | $< 5.1 \times 10^{-3}$ |
| $K_L \to \mu^+\mu^-$ | $(21)(22)$ | $< 3.6 \times 10^{-5}$ |
| $K^+ \to \pi^+e^+e^-$ | $(21)(11)$ | $< 6.7 \times 10^{-4}$ |
| $K_L \to \pi^0 e^+e^-$ | $(21)(11)$ | $< 1.6 \times 10^{-4}$ |
| $K^+ \to \pi^+\mu^+\mu^-$ | $(21)(22)$ | $< 5.3 \times 10^{-3}$ |
| $B_d \to \mu^+\mu^-$ | $(31)(22)$ | $< 3.9 \times 10^{-3}$ |
| $B^+ \to \pi^+e^+e^-$ | $(31)(11)$ | $< 2.8 \times 10^{-4}$ |
| $B^+ \to \pi^+\mu^+\mu^-$ | $(31)(22)$ | $< 2.3 \times 10^{-4}$ |

**Table 4**. Upper bounds on Wilson coefficients from leptonic and semi-leptonic $K$ and $B$ decays with $s \to d$ and $b \to d$ transitions.

This limit can be violated for larger values of the parameter $k$. In particular, for $X_1^E = 0$ and $k$ in the experimentally favored range $k \in [1.9, 4.9]$ we get the constraint on the mixing angles:

$$\left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)\left(\frac{s_{13}^{Le}}{1.1 \times 10^{-4}}\right) < [3.0, 1.2]\,. \tag{4.12}$$

This can be satisfied for $s_{23}^{Ld} \sim |V_{cb}|$ if the $\mathcal{O}(1)$ Yukawa coupling controlling $s_{13}^{Le}$ is $\lesssim 1$.

A stronger constraint on LFV comes from $\mu$-$e$ conversion in nuclei. Borrowing the formulas e.g from refs. [60–62], for $X_1^E = 0$ the conversion rate in gold, titanium and aluminium nuclei is given by:

$$\text{CR}(\mu \to e, \text{Au}) \approx 4.0 \times 10^{-12}\left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)^2\left(\frac{s_{13}^{Le}}{1.1 \times 10^{-4}}\right)^2 k^2\,,$$

$$\text{CR}(\mu \to e, \text{Ti}) = 3.8 \times 10^{-12}\left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)^2\left(\frac{s_{13}^{Le}}{1.1 \times 10^{-4}}\right)^2 k^2\,,$$

$$\text{CR}(\mu \to e, \text{Al}) = 1.9 \times 10^{-12}\left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)^2\left(\frac{s_{13}^{Le}}{1.1 \times 10^{-4}}\right)^2 k^2\,. \tag{4.13}$$

This should be compared with the bounds from refs. [63, 64]:

$$\text{CR}(\mu \to e, \text{Au}) < 7.0 \times 10^{-13}, \quad \text{CR}(\mu \to e, \text{Ti}) < 4.3 \times 10^{-12}\,. \tag{4.14}$$

For the parameter $k$ in the range favored by the $B$-meson anomalies, $k \in [1.9.4.9]$, this leads to the constraint on a combination of mixing angles in our model

$$\left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)\left(\frac{s_{13}^{Le}}{1.1 \times 10^{-4}}\right) < [0.22, 0.08]\,. \tag{4.15}$$

Formally, $s_{13}^{Le}$ is a free parameter, therefore eq. (4.15) can always be satisfied with an appropriate choice of the lepton Yukawa couplings. However, our philosophy is to explain the flavor hierarchies with all Yukawa couplings in eq. (2.2) being $\mathcal{O}(1)$, in which case the natural value is $s_{13}^{Le} \sim 10^{-4}$. In this respect, eq. (4.15) forces us into a less natural corner of the parameter space and suggest a value of $k$ close to the lower $1\sigma$ boundary. We note that the experimental sensitivity to the $\mu$-$e$ conversion rate is expected to improve by many orders of magnitude in the near future [65–67]. In case of a null result, our model will no longer be an attractive solution to the $B$-meson anomalies.

## 4.4 Electroweak precision tests

Integrating out $Z'$ induces lepton-number *conserving* 4-fermion operators which can be constrained by electroweak precision tests. Here, we focus on the 4-lepton operators which give the strongest bounds due to large U(1)$_F$ charges of leptons. At leading order, these do not affect Z-pole observables measured in LEP-1 and SLC, but they can be constrained by off-Z-pole fermion scattering in LEP-2. We parametrize these operators as

$$\mathcal{L}_{\text{eff}} \supset \sum_{\ell \in e,\mu,\tau} \left[ \frac{[c_{LL}]_{e\ell}}{v^2}(\bar{e}\bar{\sigma}^\mu e)(\bar{\ell}\bar{\sigma}_\mu \ell) + \frac{[c_{LR}]_{e\ell}}{v^2}(\bar{e}\bar{\sigma}^\mu e)(\ell^c \sigma_\mu \bar{\ell}^c) + \frac{[c_{RR}]_{ef}}{v^2}(e^c \sigma^\mu \bar{e}^c)(\ell^c \sigma_\mu \bar{\ell}^c) \right]. \tag{4.16}$$

For $X_1^E = 0$ the non-zero Wilson coefficients are

$$[c_{LL}]_{ee} = -\frac{g'^2 v^2}{2M_{Z'}^2}(\Delta_L^{ee})^2 = -6.8 \times 10^{-4} k \left( \frac{|V_{cb}|}{s_{23}^{Ld}} \right), \tag{4.17}$$

$$[c_{LL}]_{e\mu} = -\frac{4}{3}[c_{LL}]_{ee}, \quad [c_{LL}]_{e\tau} = 2[c_{LL}]_{ee}, \quad [c_{LR}]_{e\tau} = -\frac{8}{3}[c_{LL}]_{ee}. \tag{4.18}$$

Note that the sign of each contribution is fixed, in particular the contribution to $[c_{LL}]_{ee}$ is always negative in our model. We calculated the impact of these operators on the LEP-2 observables quoted in ref. [68]. We used the total cross section and asymmetries of $e^+ e^- \to \mu^+ \mu^-, \tau^+ \tau^-$ measured at the center-of-mass energies $\sqrt{s} \in [130, 207]$ GeV, as well as the differential cross-sections of $e^+ e^- \to e^+ e^-$ at $\sqrt{s} \in [189, 207]$ GeV. This way we obtain the 95% CL constraint:

$$k \left( \frac{|V_{cb}|}{s_{23}^{Ld}} \right) \leq 1.1. \tag{4.19}$$

For $k$ in the experimentally favored range $k \in [1.9, 4.9]$, eq. (4.19) requires somewhat larger values of $s_{23}^{Ld}$, of order 2-4 $|V_{cb}|$. This leads to some tension with the bound from CP violation in kaon mixing in eq. (4.4), assuming $\mathcal{O}(1)$ phases entering $C_1^K$. Much as the LFV bound, these constraints point to rather low $k \approx 2$.

We also comment on the corrections to lepton flavor conserving muon decays. Loops with a $Z'$ boson result in the following 1-loop correction to the $\mu \to e\nu_\mu \nu_e$ decay width [21]:

$$\frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu \to e\nu\nu)_{\text{SM}}} = 1 - \epsilon, \tag{4.20}$$

where

$$\epsilon \approx -\frac{3g'^2}{4\pi^2}\Delta_L^{ee}\Delta_L^{\mu\mu}\frac{m_W^2\log(M_{Z'}^2/m_W^2)}{M_{Z'}^2} = 1.5\times10^{-5}k\left(\frac{|V_{cb}|}{s_{23}^{Ld}}\right)\log\left(\frac{M_{Z'}}{m_W}\right). \qquad (4.21)$$

The muon lifetime measurement does not constrain new physics by itself, because it is used to extract the SM input parameter $G_F$ (equivalently, the Higgs VEV $v$). However, indirectly, new physics contributions to $G_F$ shift other observables (for example $m_W$, Z-pole asymmetries, etc.) away from the SM predictions. To estimate the resulting constraints, we note that the effect in eq. (4.20) is equivalent to introducing the 4-lepton operator $\frac{[c_{LL}]_{1221}}{v^2}\bar{\ell}_1\bar{\sigma}_\mu\ell_2\bar{\ell}_2\bar{\sigma}_\mu\ell_1$, with the Wilson coefficient $[c_{LL}]_{1221} = \epsilon$. The constraint on this Wilson coefficient from the Z-pole observables can be read off using the global likelihood function quoted in ref. [69]. If only this one operator affects the Z-pole observables, the constraint reads $-0.8\times10^{-3} < [c_{LL}]_{1221} < 2\times10^{-3}$ at 95% CL. The resulting constraints on the parameter of our model are weaker than the ones from off-Z-pole measurements in LEP-2.

## 4.5   $Z'$ searches in colliders

Finally, the parameter space of our model is constrained by direct searches for resonances in colliders. Since addressing the $B$-meson anomalies requires $M_{Z'}/g' \sim 20\,\text{TeV}$, the $Z'$ boson predicted by our model is within the kinematic reach of LHC for $g'$ of electroweak strength or smaller. Note that the direct searches probe separately the $Z'$ mass and coupling constant, unlike all previously discussed observables that depended on these parameters only via the combination $M_{Z'}/g'$. Given the charge assignments in eq. (2.10) and eq. (2.16), the branching fraction of $Z'$ into dilepton final states is significant. In particular, for $X_1^E = 0$, we have

$$\text{Br}(Z' \to ee) \approx 14\%, \quad \text{Br}(Z' \to \mu\mu) \approx 6\%, \qquad (4.22)$$

and the strongest constraints are expected from the di-electron channel. In figure 2 we plot the constraints in the $M_{Z'}$-$g'$ plane based on the CMS search for di-electron resonances in the LHC at $\sqrt{s} = 8\,\text{TeV}$ [70]. These constraints imply $M_{Z'} \gtrsim 3\,\text{TeV}$ and $g' \gtrsim 0.1$ in the region of the parameter space favored by the $B$-meson anomalies. Note that the direct limits are complementary to the indirect ones from LEP-2. The latter would allow us to address the $B$-meson anomalies with a light ($m_{Z'} \lesssim 2\,\text{TeV}$) and very weakly coupled $Z'$; such possibility is however excluded by the resonance searches.

## 5   Summary and conclusions

In this work we have addressed recent anomalies in $b \to s\ell\ell$ transitions. This was achieved by a tree-level exchange of a light $Z'$ vector boson, whose couplings to the SM fermions are governed by an underlying U(2)$_F$ flavor symmetry that explains fermion masses and mixings. While the U(2)$_F$ quantum numbers of quarks are determined by quark masses and CKM angles, there is more freedom in the charged lepton sector. However, requiring that the $b \to s\ell\ell$ anomalies are fit within $1\sigma$ essentially selects unique U(2)$_F$ quantum numbers for charged leptons. The only free parameters are then the $Z'$ mass, the associated gauge
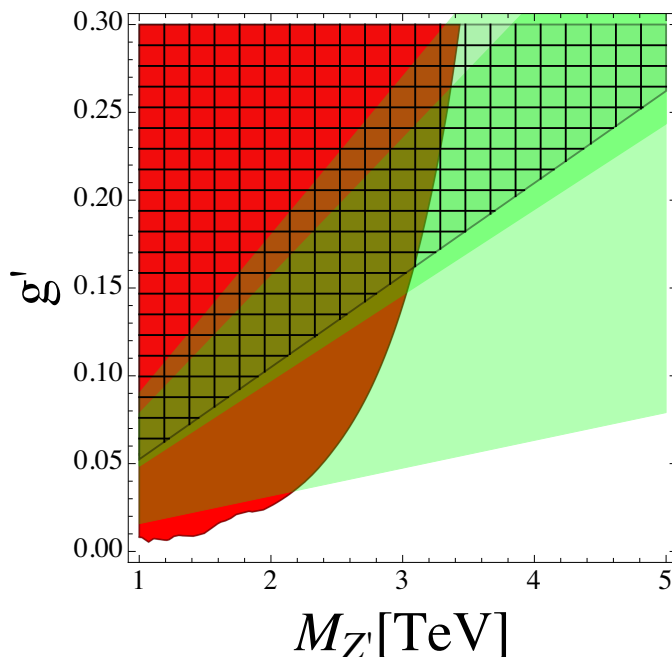
**Figure 2**. For $X_1^E = 0$, the region of the $M_{Z'}$-$g'$ plane of our model excluded by resonance searches at the LHC (red). We also show the indirect constraints from the 2-fermion production in LEP-2 (black mesh). The green regions correspond to $s_{23}^{Ld} = 2|V_{cb}|$ and the parameter $k$ in eq. (3.5) in the range favored by the $B$-meson anomalies at $1\,\sigma$ $k \in (1.9, 4.9)$ (darker) and at $2\,\sigma$ $k \in (0, 2, 6.5)$ (lighter).

coupling and a handful of $\mathcal{O}(1)$ parameters controlling flavor violation. As a result of the $U(2)_F$ symmetry structure, the magnitude of the latter is set by rotation angles involving the third generation, which implies that flavor violation in the quark sector is strongly suppressed by small CKM angles.

In our model, $Z'$ couples only to LH electrons and muons and has universal couplings to RH down quarks. Therefore, the Wilson coefficients relevant for the $b \to s\ell\ell$ transitions satisfy $C_9^{ee,\mu\mu} = -C_{10}^{ee,\mu\mu}$, and are therefore approximately aligned with the SM contribution. Furthermore, the ratio $C_9^{ee}/C_9^{\mu\mu}$ is fixed by the $U(2)_F$ quantum numbers. As a result, $b \to s\ell\ell$ transitions are governed by a single parameter $k$

$$ k \equiv \left( \frac{20\,\text{TeV}}{M_{Z'}/g'} \right)^2 \left( \frac{s_{23}^{Ld}}{|V_{cb}|} \right) , \tag{5.1} $$

controlling the overall magnitude of the $Z'$ contributions. Interference of the new contributions with the SM is constructive in the electron channel and destructive in the muon channel. This leads to the prediction that the semi-leptonic $B$-meson decay rates are rescaled by factors $r_e$ and $r_\mu$ that are common for all $b \to see$ and $b \to s\mu\mu$ processes, respectively. We find

$$ r_e \approx (1 + 0.044k)^2, \qquad r_\mu \approx (1 - 0.029k)^2, \qquad R_{X=K,K^*,\phi,X_s} = \left( \frac{1 - 0.029k}{1 + 0.044k} \right)^2 . \tag{5.2} $$

– 18 –

Using the fits to all available $B$-meson data from the previous literature, we determined the $1\sigma$ confidence interval for $k$:

$$k \in [1.9, 4.9] \,, \tag{5.3}$$

and thus we predict the rescaling factors in the electron and muon channels, along with their ratio $R_X$

$$r_e \in [1.17, 1.48] \,, \qquad r_\mu \in [0.73, 0.89] \,, \qquad R_X \in [0.50, 0.76] \,. \tag{5.4}$$

The parameter $k$ also controls flavor-violating processes, but only in conjunction with quark and lepton rotation angles which are fixed up to $\mathcal{O}(1)$ parameters. The strongest constraints on these parameters arise from $\mu$-$e$ conversion in nuclei. They can be satisfied if the mixing angle $s_{13}^{Le}$ is somewhat suppressed by these $O(1)$ parameters, and/or the mixing angle $s_{23}^{Ld}$ is somewhat enhanced. These considerations also favor smaller $k$, close to the lower limit of the $1\sigma$ confidence interval. Electroweak precision test from LEP-2 lead to similar conclusions. Explicitly, all bounds are satisfied for $k \approx 2$ and the reference values in eq. (2.15), for an $\mathcal{O}(1)$ coefficient $\lesssim 0.5$ in $s_{13}^{Le}$.

All in all, there exist regions of the parameter space of our model where the $B$-meson anomalies are explained and other experimental constraints are satisfied. The final verdict will be provided by further of lepton flavor universality in LHCb and B-factories, as well as by future experiments looking for $\mu - e$ conversion in nuclei. Apart from indirect searches, the $Z'$ boson is likely within the reach of the LHC run-2, and should first show up in the di-electron channel, as a result of its large coupling to electrons.

## Acknowledgments

## A    Exact quark rotation angles

In the flavor basis the Yukawa terms read

$$\mathcal{L} \subset +Q^T y_u U H + Q y_d D \tilde{H} + L^T y_e E \tilde{H} \,. \tag{A.1}$$

where each $y_f$ is a $3 \times 3$ matrix with in general complex elements. Here, for simplicity, we assume all entries are real, as this will allows us to obtain compact formulas for the

eigenvalues and the mixing angles. Going to the mass eigenstate basis involves unitary rotations defined by

$$y_u = V_L^u \, y_u^{\text{diag}} \, (V_R^u)^T \,, \qquad y_d = V_L^d \, y_d^{\text{diag}} \, (V_R^d)^T \,, \qquad y_e = V_L^e \, y_e^{\text{diag}} \, (V_R^e)^T \,. \qquad \text{(A.2)}$$

We parametrize the rotations as

$$V_L = V_{13}^L V_{12}^L V_{23}^L \,, \qquad V_R = V_{13}^R V_{12}^R V_{23}^R \,, \qquad \text{(A.3)}$$

where $(I = L, R)$ and

$$V_{12}^I = \begin{pmatrix} c_{12}^I & s_{12}^I & 0 \\ -s_{12}^I & c_{12}^I & 0 \\ 0 & 0 & 1 \end{pmatrix} \,, \qquad V_{13}^I = \begin{pmatrix} c_{13}^I & 0 & s_{13}^I \\ 0 & 1 & 0 \\ -s_{13}^I & 0 & c_{13}^I \end{pmatrix} \,, \qquad V_{23}^I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^I & s_{23}^I \\ 0 & -s_{23}^I & c_{23}^I \end{pmatrix} \,. \qquad \text{(A.4)}$$

We are interested in the Yukawa matrix of the form

$$y = \begin{pmatrix} 0 & y_{12} & 0 \\ -y_{12} & y_{22} & y_{23} \\ 0 & y_{32} & y_{33} \end{pmatrix} \,, \qquad y_{\text{diag}} = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \,, \qquad y_1 \ll y_2 \ll 1 \,. \qquad \text{(A.5)}$$

Plugging this form into the eigenvalue equations, from the 1-1, 1-3, 3-1, 1-2, and 2-1 entries we get 4 independent equations which are

$$c_{13}^L c_{13}^R s_{12}^L s_{12}^R \epsilon_{22} + c_{13}^R s_{12}^R s_{13}^L \epsilon_{32} + c_{13}^L s_{12}^L s_{13}^R \epsilon_{23} + s_{13}^L s_{13}^R \epsilon_{33} + c_{12}^L c_{12}^R c_{13}^L c_{13}^R y_1 = 0 \,, \qquad \text{(A.6)}$$

$$c_{13}^L c_{13}^R s_{12}^L \epsilon_{23} + c_{13}^R s_{13}^L \epsilon_{33} - c_{13}^L s_{12}^L s_{12}^R s_{13}^R \epsilon_{22} - s_{12}^R s_{13}^L s_{13}^R \epsilon_{32} - c_{12}^L c_{12}^R c_{13}^L s_{13}^R y_1 = 0 \,, \qquad \text{(A.7)}$$

$$c_{13}^L c_{13}^R s_{12}^R \epsilon_{32} - c_{13}^R s_{12}^L s_{12}^R s_{13}^L \epsilon_{22} + c_{13}^L s_{13}^R \epsilon_{33} - s_{12}^L s_{13}^L s_{13}^R \epsilon_{23} - c_{12}^L c_{12}^R c_{13}^R s_{13}^L y_1 = 0 \,, \qquad \text{(A.8)}$$

$$c_{12}^R c_{13}^L s_{12}^L \epsilon_{22} + c_{12}^L c_{13}^R s_{12}^R \epsilon_{22} + c_{12}^R s_{13}^L \epsilon_{32} + c_{12}^L s_{13}^R \epsilon_{23} - c_{12}^R c_{13}^L s_{12}^L y_1 - c_{12}^L c_{13}^R s_{12}^R y_1 = 0 \,, \qquad \text{(A.9)}$$

where we have defined

$$\epsilon_{22} = s_{23}^L s_{23}^R + c_{23}^L c_{23}^R y_2 \,, \qquad\qquad \epsilon_{23} = c_{23}^R s_{23}^L - c_{23}^L s_{23}^R y_2 \,, \qquad \text{(A.10)}$$

$$\epsilon_{32} = c_{23}^L s_{23}^R - c_{23}^R s_{23}^L y_2 \,, \qquad\qquad \epsilon_{33} = c_{23}^L c_{23}^R + s_{23}^L s_{23}^R y_2 \,, \qquad \text{(A.11)}$$

with

$$\epsilon_{22} \epsilon_{33} - \epsilon_{23} \epsilon_{32} = y_2 \,. \qquad \text{(A.12)}$$

From the first three equations one finds

$$s_{12}^R = -\frac{s_{13}^R \epsilon_{33}}{c_{13}^R \epsilon_{32}} \,, \qquad\qquad s_{12}^L = -\frac{s_{13}^L \epsilon_{33}}{c_{13}^L \epsilon_{23}} \,, \qquad \text{(A.13)}$$

and

$$c_{12}^L c_{12}^R y_1 + \frac{s_{12}^L s_{12}^R y_2}{\epsilon_{33}} = 0 \,. \qquad \text{(A.14)}$$

Using this last equation, one gets from the 4th equation the solution

$$c_{13}^R = \frac{\epsilon_{33} \left[ (c_{13}^L)^2 \epsilon_{23}^2 y_1^2 + (s_{13}^L)^2 \left( -\epsilon_{33}^2 y_1^2 + y_2^2 \right) \right]}{c_{13}^L \epsilon_{23}^2 y_1 y_2} \,. \qquad \text{(A.15)}$$

Using this solution and eqs. (A.13) in eq. (A.14), one finally can solve for $(c_{13}^L)^2$ in terms of $\epsilon_{ij}$ and $y_{1,2}$. Although the exact solutions are not very complicated, one can approximate these expressions using $y_1 \ll y_2 \ll 1$ to get the final solution (with a consistent choice of signs):

$$s_{13}^L \approx \sqrt{\frac{\epsilon_{23}^2 y_1}{\epsilon_{33} y_2}} \qquad c_{13}^L \approx \sqrt{1 - \frac{\epsilon_{23}^2 y_1}{\epsilon_{33} y_2}} \qquad (A.16)$$

$$s_{13}^R \approx \sqrt{\frac{\epsilon_{32}^2 y_1}{\epsilon_{33} y_2}} \qquad c_{13}^R \approx \sqrt{1 - \frac{\epsilon_{32}^2 y_1}{\epsilon_{33} y_2}} \qquad (A.17)$$

$$s_{12}^R = -\frac{s_{13}^R \epsilon_{33}}{c_{13}^R \epsilon_{32}} \qquad c_{12}^R = \text{sign}(-\epsilon_{23}\epsilon_{32})\sqrt{1 - (s_{12}^R)^2} \qquad (A.18)$$

$$s_{12}^L = -\frac{s_{13}^L \epsilon_{33}}{c_{13}^L \epsilon_{23}} \qquad c_{12}^L = \sqrt{1 - (s_{12}^L)^2}. \qquad (A.19)$$

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