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# Lepton-Flavour Nonconservation and the Type of Neutrino Masses

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The lepton-flavour nonconservation is studied in the left-right symmetric gauge theory with both Majorana and Dirac mass terms of neutrinos. The tree graphs with physical Higgs scalar exchange and the one-loop graphs with light (left-handed) and heavy (right-handed) gauge boson exchange contributing to $\mu\to e\gamma$ and $\mu\to ee\bar{e}$ are evaluated. The one-loop graphs with intermediate heavy Majorana neutrinos are shown to give more important effects than those with light Majorana neutrinos. The latter contributions to the rare decay rates are negligibly small, independently of the value of $\eta\equiv M^2_{W_L}/M^2_{W_R}$, because of the strong suppression by powers of light neutrino masses $m_{\nu L}$. The former contributions are not directly related to $m_{\nu L}$ but depend on $\eta$. They can lead to relatively large values of the rare decay rates, unless $\eta$ is inhibitingly small. The predictions for the rates of lepton-flavour changing processes $\mu\to e\gamma$, $\mu\to ee\bar{e}$, $Z\to\mu\bar{e}$, $\mu N\to eN$, $K_L\to\mu\bar{e}$ and $K^+\to\pi^+\mu e^-$ in the present gauge model are compared with similar predictions in other types of gauge models with different types of neutrino masses.

## § 1. Introduction

The $SU(2)_L\times U(1)$ gauge model[1] appears to have been established experimentally as the theory of electroweak interactions at low energies. However, we do not know much about physics at higher energies. It is therefore natural to look for (gauge) theories which depart from the standard model at energy scales much larger than the weak-boson mass, but reduce to it at low energies. At low energies, the properties of such theories can be probed by studying rare processes, such as baryon number nonconserving processes. In particular, if neutrinos have nonvanishing masses, they induce lepton-flavour changing processes which are absent in the standard model.

In the gauge theories with left-right symmetry, neutrinos naturally have masses, because of the presence of right-handed neutrinos. The question will then be raised as to why neutrinos are so light, if they are not exactly massless. A possible answer has recently been suggested to this question.[2] Since neutrinos are electrically neutral, they may have Majorana masses besides ordinary Dirac masses. In gauge theories with such generalized mass terms for neutrinos, the small neutrino masses follow naturally from large Majorana masses of conjectured right-handed neutrinos $\nu_R$, through the formula

$$m_{\nu L}\sim m_D{}^2/m_{\nu R}\ , \qquad (1\cdot1)$$

where $m_D$ is the typical value of possible Dirac masses of neutrinos.

In this paper we shall study lepton-flavour changing processes in the left-right symmetric gauge theory with both Majorana and Dirac masses of neutrinos. We take the minimal model with $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry.[3] However, our results hold in a more general class of left-right symmetric gauge models, including the $SO(10)$ model.[4]

In the left-right symmetric model the small masses of light neutrinos are related to the large energy scale at which spontaneous breakdown of the left-right symmetry occurs.[3] Roughly speaking, defining the hierarchy of symmetry breaking by $\eta \equiv (M_{WL}/M_{WR})^2$, we have

$$m_{\nu L}^2/m_D{}^2 \sim \eta \cdot (\omega_D/\omega_M), \tag{1·2}$$

where $\omega$'s are the ratios of the Yukawa couplings to the gauge couplings, $\omega_D \equiv (m_D/M_{WL})^2$ and $\omega_M \equiv (m_{\nu R}/M_{WR})^2$. Therefore, the examination of lepton-flavour changing processes should provide an important clue to the "physics" at energy scale much larger than that of the breakdown of $SU(2)_L \times U(1)$ symmetry.

Heavy neutrinos decouple from low energy processes at the tree level, as is the case in neutrinoless double $\beta$-decays. It will be shown that in lepton-flavour changing processes such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$, $K_L \to \mu\bar{e}$, which are induced by one-loop graphs involving massive neutrinos, heavy Majorana neutrinos cannot be neglected as being decoupled from light particle processes. In fact they give more important contributions than light neutrinos.

The predicted decay rates for such processes turn out to be very different depending on the type of neutrino masses (pure Dirac, or Majorana plus Dirac). In models with only Dirac masses of neutrinos the rates are suppressed by powers of light neutrino masses $m_{\nu L}$, and are expected to be outrageously small. In our model with both Majorana and Dirac masses of neutrinos, the decay rates are sensitive to the value of $\eta$ and are not necessarily very small (even if we set $m_{\nu L}$ to be very small, e.g., $m_{\nu L} \simeq 100\,\mathrm{eV}$). This prediction is derived from general properties of gauge models independently of their details, and should provide a useful test of the mass type for neutrinos.

Lepton-flavour changing processes were previously discussed by several authors.[5] In particular, Mohapatra and Senjanovic[6] have recently studied the same subject in the same type of model as we consider here. However, the estimate of the effect of heavy Majorana neutrinos in the rare processes, which is the main purpose of the present paper, has not previously been possible because of the following two reasons: i) Previously the approximation $m_\nu \ll M_{WL}$ was used in the computation of one-loop graphs. In the model with heavy Majorana neutrinos, it is necessary to evaluate the graphs for arbitrary values of intermediate fermion masses. ii) As a consequence of including Majorana mass terms, the theory possesses its characteristic interactions, such as lepton number violating ($\Delta L \neq 0$) interactions and lepton-flavour nonconserving Yukawa couplings and neutral currents. Processes induced by these interactions also play important roles in the lepton-flavour changing processes and have to be evaluated.

The organization of this paper is as follows. In § 2 we describe the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory with both Majorana and Dirac masses of neutrinos. The interaction vertices of scalar bosons as well as those of gauge bosons, which were not constructed in the previous works, are worked out here. We refer the readers to Ref. 6) for other details of the model. In the subsequent two sections we study phenomenological consequences of this model. In § 3 we consider purely leptonic processes $\mu \to e\gamma$ and $\mu \to ee\bar{e}$. Section 4 deals with other lepton-flavour changing processes $Z \to \mu\bar{e}$, $\mu N \to eN$, $K_L \to \mu\bar{e}$ and $K^+ \to \pi^+\mu\bar{e}$. In §§ 3 and 4, we compare the predictions of our model with those of other models with different types of neutrino masses. Finally we summarize our work in § 5. In Appendix A we construct the Higgs potential. The minimization of the potential and the diagonalization of the mass matrices of scalar fields are also performed. In Appendices B, C and D we present the results of our calculation of the one-loop graphs contributing to the effective Lagrangian for lepton-flavour changing processes, i.e., the induced $\mu\bar{e}\gamma$, $\mu\bar{e}Z$ and $\mu\bar{e}Z'$ vertices and the box type graphs. The calculation is made by taking the limit $\xi \to 0$ (unitary gauge limit) in $R_\xi$ gauge after the loop integration.[7)]

## § 2. $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory

### 2.1. *Hierarchy of symmetry breaking*

The theory contains the following minimum set of fields: gauge bosons $W_L^{i\mu}$, $W_R^{i\mu}$ and $B^\mu$ with gauge coupling constants $g_L = g_R = g$ and $g_{B-L}$; $n$ generations of leptons,

$$\psi_{iL} = \begin{pmatrix} \nu_{0i} \\ l_{0i} \end{pmatrix}_L \quad \text{and} \quad \psi_{iR} = \begin{pmatrix} \nu_{0i} \\ l_{0i} \end{pmatrix}_R, \qquad (i = 1, \cdots, n) \tag{2·1}$$

three multiplets of scalar fields,

$$\phi = \begin{pmatrix} \phi_1{}^0 & \phi_2{}^- \\ \phi_1{}^+ & \phi_2{}^0 \end{pmatrix}, \qquad \left[\left(\frac{1}{2}, \frac{1}{2}, 0\right)\right],$$

$$\Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix}, \qquad \begin{bmatrix} (1, 0, 2) \\ (0, 1, 2) \end{bmatrix}. \tag{2·2}$$

The quark sector will not be considered in this section. In Eq. (2·1) the lower index 0 refers to the weak-current eigenstates.

The vacuum expectation values (VEV's) of the scalar fields

$$\langle \phi^0_{1,2} \rangle = k_{1,2}/\sqrt{2}\,, \qquad \langle \delta^0_{L,R} \rangle = v_{L,R}/\sqrt{2}\,, \tag{2·3}$$

can be shown to satisfy the hierarchy,[6),8)]

$$v_L \ll k_1\,, \qquad k_2 \ll v_R\,, \tag{2·4}$$

$$v_L v_R \sim k_1 k_2 \tag{2·5}$$

for an appropriate range of the parameters. The first relation means that the breakdown $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}\to SU(2)_L\times U(1)$ occurs at the mass scale $v_R$ and $SU(2)_L\times U(1)\to U(1)_{\text{e.m.}}$ at $k\equiv\sqrt{k_1{}^2+k_2{}^2}$. The second relation follows naturally from the first.[6),8)] Hereafter, to avoid calculational complication we shall work in the theory with a simpler form of VEV's,

$$v_L = k_2 = 0\,, \qquad v_R,\ k_1 \neq 0\,. \tag{2·6}$$

In Appendix A it is shown at the tree level that there exists a solution of the form (2·6) to the minimum of the effective potential.

### 2.2. *Fermion masses*

The Yukawa coupling term of the Lagrangian is given by

$$\mathcal{L}_Y = -\sum_{i,j}(f_{ij}^*\bar{\psi}_{iL}^C i\tau_2\Delta_L\psi_{jL} + g_{ij}\bar{\psi}_{iR}^C i\tau_2\Delta_R\psi_{jR}$$

$$+ h_{ij}\bar{\psi}_{iL}\phi\psi_{jR} + h'_{ij}\bar{\psi}_{iL}\tau_2\phi^*\tau_2\psi_{jR}) + \text{h.c.}, \tag{2·7}$$

where $\psi_{L,R}^C$ denotes $(\psi_{L,R})^C (=(\psi^C)_{R,L})$. The lepton mass terms are written as

$$l:\quad -\bar{l}_{0L}H' l_{0R} + \text{h.c.}, \tag{2·8}$$

$$\nu:\quad -\frac{1}{2}(\bar{\nu}_{0L},\ \bar{\nu}_{0R}^c)K\begin{pmatrix}\nu_{0L}^c\\ \nu_{0R}\end{pmatrix} + \text{h.c.},\quad K=\begin{pmatrix}0 & H\\ H^T & G\end{pmatrix}. \tag{2·9}$$

Here, $l_{0L,R}$ and $\nu_{0L,R}$ are $n$-component column vectors, and

$$H'_{ij} = k_1 h'_{ij}/\sqrt{2}\,, \qquad H_{ij} = k_1 h_{ij}/\sqrt{2}\,,$$

$$G_{ij} = v_R(g_{ij}+g_{ji})/\sqrt{2}\,. \tag{2·10}$$

The $n\times n$ matrix $H'$ can be diagonalized as usual with two unitary matrices,

$$l_L = U_L l_{0L}\,, \qquad l_R = U_R l_{0R}\,. \tag{2·11}$$

The charged lepton mass term then becomes

$$-\bar{l}_L M_l l_R + \text{h.c.} \tag{2·12}$$

The $2n\times 2n$ neutrino mass matrix $K$ is symmetric and can be diagonalized by a unitary matrix as follows. Set

$$\begin{pmatrix}\nu_L\\ \nu_R{}^c\end{pmatrix} = U_\nu{}^*\begin{pmatrix}\nu_{0L}\\ \nu_{0R}^c\end{pmatrix}, \qquad \begin{pmatrix}\nu_L{}^c\\ \nu_R\end{pmatrix} = U_\nu\begin{pmatrix}\nu_{0L}^c\\ \nu_{0R}\end{pmatrix}, \tag{2·13}$$

and define light and heavy Majorana neutrinos $N$ and $N'$ by

$$\mathcal{N} \equiv \begin{pmatrix} N \\ N' \end{pmatrix} = \begin{pmatrix} \nu_L + \nu_L{}^c \\ \nu_R + \nu_R{}^c \end{pmatrix}. \tag{2·14}$$

Then, the neutrino mass term takes the form

$$-\frac{1}{2}\bar{\mathcal{N}} M_\nu \mathcal{N} = \frac{1}{2}\mathcal{N}^T C^{-1} M_\nu \mathcal{N}\ , \qquad (C = i\gamma_2\gamma_0) \tag{2·15}$$

where $M_\nu((M_\nu)_{ij} = m_{\nu i}\delta_{ij})$ is composed of two $n\times n$ diagonal sub-matrices $M_N$ and $M_{N'}$. Note that neutrino masses satisfy the following relation:

$$(\text{Det } M_N)\cdot(\text{Det } M_{N'}) = |\text{Det } H|^2\ . \tag{2·16}$$

Under the condition of (2·4), it follows from Eq. (2·16) that

$$m_{N'} = O(f_M v_R) \gg m_D\ , \qquad m_N = O(f_D{}^2 k^2/f_M v_R) \ll m_D\ ,$$
$$(m_D \sim f_D k) \tag{2·17}$$

where $f_D$ and $f_M$ are typical values of the Yukawa coupling constants $h_{ij}$ and $g_{ij}$ respectively. In this way, the smallness of light neutrino masses, $m_N \ll m_D$, can be understood without assuming unnaturally small Yukawa coupling constants, once the iso-doublet dominance in the $SU(2)_L$ breaking is implemented in the theory.[8]

Write

$$U_\nu = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{2·18}$$

Then, in connection with the hierarchy of the VEV, we have

$$AA^\dagger \sim DD^\dagger \simeq 1\ , \qquad B,\ C = O(m_N/m_D). \tag{2·19}$$

The second equation means that, besides the usual mixing among generations, small left-right neutrino mixing $\nu_L \to \nu_R{}^c$ is brought about. This mixing is one of the characteristic features of our model, not shared with other theories with different mass types for neutrinos.

### 2.3. *Gauge boson and scalar masses*

As a result of the hierarchy of symmetry breaking, mass eigenstates of gauge bosons separate into three classes:
(i) photon

$$A_\mu = \sin\theta_W(W^0_{L\mu} + W^0_{R\mu}) + \sqrt{\cos 2\theta_W}\, B_\mu\ , \tag{2·20}$$

(ii) light gauge bosons

$$W^{\pm}_{L\mu}\,,\qquad M^2_{WL}\simeq g^2k^2/4\,,$$

$$Z_\mu{}^0=\cos\varepsilon^0 Z^0_{L\mu}+\sin\varepsilon^0 Z^0_{R\mu}\,,\qquad M_Z{}^2\simeq M^2_{WL}/\cos^2\theta_W\,,\tag{2·21}$$

(iii) heavy gauge bosons

$$W^{\pm}_{R\mu}\,,\qquad M^2_{WR}\simeq g^2 v_R{}^2/2\,,$$

$$Z'^0_\mu=-\sin\varepsilon^0 Z^0_{L\mu}+\cos\varepsilon^0 Z^0_{R\mu}\,,\quad M^2_{Z'}\simeq(\cos^2\theta_W/\cos 2\theta_W)\cdot g^2 v_R{}^2\,,\tag{2·22}$$

where $\theta_W$ is the Weinberg angle, $\sin^2\theta_W=g^2_{B\text{-}L}/(2g^2_{B\text{-}L}+g^2)$, and

$$Z^0_{L\mu}=(W^0_{L\mu}-\sin\theta_W A_\mu)/\cos\theta_W\,,\quad Z^0_{R\mu}=(\sqrt{\cos 2\theta_W}/\cos\theta_W)W^0_{R\mu}-\tan\theta_W B_\mu\,.\tag{2·23}$$

The mixing angle $\varepsilon^0$ is very small, $O(k^2/v_R{}^2)$. In the simplified case $v_L=k_2=0$, there is no $W_L{}^\pm\leftrightarrow W_R{}^\pm$ mixing. It can be shown that the $W_L{}^\pm\leftrightarrow W_R{}^\pm$ mixing can be safely neglected in the estimation of the rare decay rates.

The neutrino masses are related to gauge boson masses as

$$m_{N'}=O(\sqrt{\omega_M}M_{WR}),\qquad m_N=O(\omega_D M^2_{WL}/\sqrt{\omega_M}M_{WR}),\tag{2·24}$$

where $\omega_D=f_D{}^2/g^2$ and $\omega_M=f_M{}^2/g^2$. In the second relation, small neutrino masses are easily recognized as a consequence of the large mass scale of the left-right symmetry breaking.[3)]

As for scalar fields, of their 20 degrees of freedom, six become Nambu-Goldstone (N-G) modes $\chi_L{}^\pm$, $\chi_R{}^\pm$, $\chi^0$ and $\chi'^0$ furnishing the longitudinal components to $W_L{}^\pm$, $W_R{}^\pm$, $Z^0$ and $Z'^0$. The remaining 14 become the physical Higgs scalars

$$\delta_L^{\pm\pm},\quad \delta_R^{\pm\pm},\quad h^\pm,\quad h'^\pm,\quad h^0,\quad h'^0,\quad \phi_r{}^0,\quad \phi_i{}^0,\quad \delta_r{}^0,\quad \delta_i{}^0\,,$$

of which only $h^0$ is light $(\sim M_{W_L})$ while the rest are heavy $(\sim M_{W_R})$. The expression of these fields in terms of the shifted scalar fields is given in Appendix A. All light particles mentioned above, $A$, $W_L{}^\pm$, $Z^0$ and $h^0$, coincide with the fields of the standard model in the limit $v_R\to\infty$, where the symmetry of the theory reduces to $SU(2)_L\times U(1)$.

## 2.4. *Interaction vertices*

The interaction vertices can now be expressed in terms of the fields corresponding to the mass eigenstates obtained above.

The charged and neutral weak-currents are given by

$$J^\dagger_{L\mu}=\sqrt{1/2}\,g\,\bar{\mathcal{N}}\,V_L\gamma_\mu l_L=-\sqrt{1/2}\,g\,\bar{l}_L{}^c\gamma_\mu V_L{}^T\mathcal{N}\,,$$

$$J^\dagger_{R\mu}=\sqrt{1/2}\,g\,\bar{\mathcal{N}}\,V_R\gamma_\mu l_R=-\sqrt{1/2}\,g\,\bar{l}_R{}^c\gamma_\mu V_R{}^T\mathcal{N}\,,$$

$$J^0_{L\mu}=\frac{1}{2}g(\bar{\mathcal{N}}\, T_L\gamma_\mu L\,\mathcal{N}-\bar{l}_L\gamma_\mu l_L),$$

$$J^0_{R\mu}=\frac{1}{2}g(\bar{\mathcal{N}}\, T_R\gamma_\mu R\,\mathcal{N}-\bar{l}_R\gamma_\mu l_R), \tag{2·25}$$

where $L,\ R\equiv(1\mp\gamma_5)/2$. The $2n\times n$ mixing matrices $V_L$ and $V_R$ are defined by

$$V_L{}^*\equiv\begin{pmatrix}A'\\C'\end{pmatrix}=\begin{pmatrix}A\\C\end{pmatrix}U_L{}^T\,,$$

$$V_R\equiv\begin{pmatrix}B'\\D'\end{pmatrix}=\begin{pmatrix}B\\D\end{pmatrix}U_R{}^\dagger\,. \tag{2·26}$$

Because of the small but non-zero matrices $B'$ and $C'$ due to $\nu_L\to\nu_R{}^C$ mixing, the generalized GIM mechanism is not exact in each sector of the left and right-handed neutrinos,

$$A'^\dagger A'+C'^\dagger C'=1\,,\qquad B'^\dagger B'+D'^\dagger D'=1\,,$$

$$B',\ C'=O(m_N/m_D)\ll 1\,. \tag{2·27}$$

The $2n\times 2n$ matrices $T_{L,R}$ appearing in the neutral currents for neutrinos are

$$T_L=V_L V_L{}^\dagger=\begin{pmatrix}AA^\dagger & AC^\dagger\\ CA^\dagger & CC^\dagger\end{pmatrix}^*,$$

$$T_R=V_R V_R{}^\dagger=\begin{pmatrix}BB^\dagger & BD^\dagger\\ DB^\dagger & DD^\dagger\end{pmatrix}, \tag{2·28}$$

with $T_L{}^*+T_R=1$. They contain tiny off-diagonal ($N_i\leftrightarrow N_j$, $N_i'\leftrightarrow N_j'$ and $N_i\leftrightarrow N_j'$) transition elements, while the diagonal components of $AA^\dagger$ and $DD^\dagger$ differ slightly from unity. All of these effects are caused by $\nu_L\leftrightarrow\nu_R{}^C$ mixing and are in contrast to the case with either pure Dirac or Majorana mass terms of neutrinos, where $T_L$ and $T_R$ take the form

$$T_L=\begin{pmatrix}I & 0\\ 0 & 0\end{pmatrix},\qquad T_R=\begin{pmatrix}0 & 0\\ 0 & I\end{pmatrix}. \tag{2·29}$$

The interactions of scalars (including N-G bosons) with leptons are given by

$$\mathcal{L}_{\delta_L^{++}}=\delta_L^{++}\,\bar{l}_L{}^c U_L{}^*\left(\frac{f+f^T}{2}\right)^* U_L{}^\dagger l_L+\text{h.c.},$$

$$\mathcal{L}_{\delta_R^{++}}=(g/2M_{WR}\sqrt{1-\eta})\delta_R^{++}\,\bar{l}_R{}^c V_R{}^T M_\nu V_R l_R+\text{h.c.},$$

$$\mathcal{L}_{\phi_r{}^0}=-(g/2M_{WL})\phi_r{}^0\,\bar{l}_L V_L{}^\dagger M_\nu V_R l_R+\text{h.c.},$$

$$\mathcal{L}_{\phi_i{}^0}=-i(g/2M_{WL})\phi_i{}^0\,\bar{l}_L V_L{}^\dagger M_\nu V_R l_R+\text{h.c.},$$

$$\mathcal{L}_{\chi_L^+}=(g/\sqrt{2}M_{WL})\chi_L^+\bar{\mathcal{N}}[-M_\nu V_L L+V_L M_l R]l+\text{h.c.},$$
$$\mathcal{L}_{\chi_R^+}=(g/\sqrt{2}M_{WR})\chi_R^+\bar{\mathcal{N}}[-M_\nu V_R R+V_R M_l L]l+\text{h.c.} \qquad (2\cdot30)$$

Here we have restricted ourselves to the couplings which participate in lepton-flavour changing processes to be discussed later. In the $\delta_L^{++}$ coupling, $f$ is the matrix composed of $f_{ij}$. In the case $v_L=0$, neutrinos get no left-handed Majorana masses, and Yukawa coupling constants $f_{ij}$ are left undetermined from physical masses and mixings. The Yukawa couplings of $\delta_{L,R}^{++}$ and $\phi_{r,i}^0$ are flavour-changing and bring about $\mu\to ee\bar{e}$ decay at the tree level.

The self-interaction of gauge bosons and the interactions of gauge bosons with N-G bosons will also have to be known for the computation of the induced $\mu\bar{e}Z$ and $\mu\bar{e}Z'$ vertices. They are given by

$$\mathcal{L}_{WWW}=ig[W_{L\nu}^0\{(\partial_\nu W_{L\mu}^-)W_{L\mu}^+-(\partial_\nu W_{L\mu}^+)W_{L\mu}^-\}$$
$$+(\text{cyclic permutation of } 0,-,+)]+\begin{pmatrix}W_L^\pm\to W_R^\pm\\ W_L^0\to W_R^0\end{pmatrix},$$

$$\mathcal{L}_{WW\chi}=-gM_{WL}\sqrt{\eta}\,W_{R\mu}^-W_{L\mu}^0\chi_R^+-gM_{WL}W_{L\mu}^-W_{R\mu}^0\chi_L^+$$
$$+gM_{WR}\sqrt{1-\eta}\,W_{R\mu}^-W_{R\mu}^0\chi_R^+-2\frac{\sin\theta_W}{\sqrt{\cos2\theta_W}}gM_{WR}\sqrt{1-\eta}\,W_{R\mu}^-B_\mu\chi_R^++\text{h.c.},$$

$$\mathcal{L}_{W\chi\chi}=\frac{i}{2}g(\partial_\mu\chi_L^+)\chi_L^-(W_{L\mu}^0+W_{R\mu}^0)+\frac{i}{2}g\eta(\partial_\mu\chi_R^+)\chi_R^-(W_{L\mu}^0+W_{R\mu}^0)$$
$$+i\frac{\sin\theta_W}{\sqrt{\cos2\theta_W}}g(1-\eta)(\partial_\mu\chi_R^+)\chi_R^-B_\mu+\text{h.c.} \qquad (2\cdot31)$$

## § 3. $\mu\to e\gamma$ and $\mu\to ee\bar{e}$

In this section we consider pure leptonic lepton-flavour changing processes, $\mu\to e\gamma$ and $\mu\to ee\bar{e}$.

### 3.1. *Effective Lagrangian and branching ratios*

A. $\mu\to e\gamma$

The effective $\mu\bar{e}\gamma$ vertex is induced by the one-loop graphs illustrated in Fig. 1. It is expressed in the following form:

$$\Gamma_\kappa^\gamma(q)=\frac{1}{(4\pi)^2}e\frac{g^2}{M_{WL}^2}\bar{e}[(q^2\gamma_\kappa-q_\kappa\not{q})(\Gamma_E^L L+\Gamma_E^R R)$$
$$+im_\mu\sigma_{\kappa\lambda}q^\lambda(\Gamma_M^L L+\Gamma_M^R R)]\mu\,. \qquad (3\cdot1)$$

(The coupling constant $e$ should not be confused with the Dirac spinor $e$.)

The coefficients $\Gamma_E^L$, $\Gamma_M^L$, etc. are functions of neutrino masses

$$x_i \equiv m_{\nu i}^2 / M_{WL}^2 \, , \qquad y_i \equiv m_{\nu i}^2 / M_{WR}^2 \, , \qquad (i=1, \cdots, 2n) \tag{3·2}$$

and lepton mixing angles. The result of our computation is given in Appendix B. Only the term $\Gamma_M$ contributes to the $\mu \to e\gamma$ decay. We have the branching ratio[5)]

$$B(\mu \to e\gamma) = 6(\alpha/\pi) \cdot (|\Gamma_M{}^L|^2 + |\Gamma_M{}^R|^2). \tag{3·3}$$

B. $\mu \to e e \bar{e}$

The effective four-Fermi interactions for $\mu \to e e \bar{e}$ get contributions from three classes of graphs:

(a) $Z$, $Z'$ and $\gamma$ exchanges (Fig. 2)

The one-loop graphs for the induced $\mu \bar{e} Z$ and $\mu \bar{e} Z'$ vertices are shown in Fig. 3.

(b) Box graphs with $W_L^{\pm}$ and $W_R^{\pm}$ (and N-G bosons) exchanges (Fig. 4)

Though the total lepton number is conserved in $\mu \to e e \bar{e}$, this process involves lepton number violating interactions (Figs. 4(e)~(h)), the same as those which appear in neutrinoless double $\beta$-decays.

(c) Tree graphs of physical Higgs exchange (Fig. 5)

The computation of such graphs is trivial and is not discussed in this sub-section.

The contribution of Higgs exchange contains a large uncertainty due to that of the Higgs masses. Here we consider only the one-loop graphs of gauge boson

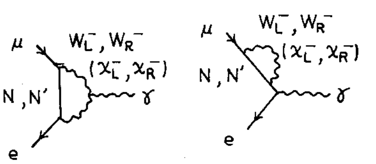


Fig. 1. One-loop graphs contributing to the induced $\mu \bar{e} \gamma$ vertex.

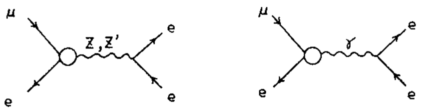


Fig. 2. The $Z$, $Z'$ and $\gamma$-exchange graphs contributing to $\mu \to e e \bar{e}$. The blobs denote the induced $\mu \bar{e} Z$, $\mu \bar{e} Z'$ and $\mu \bar{e} \gamma$ vertices.

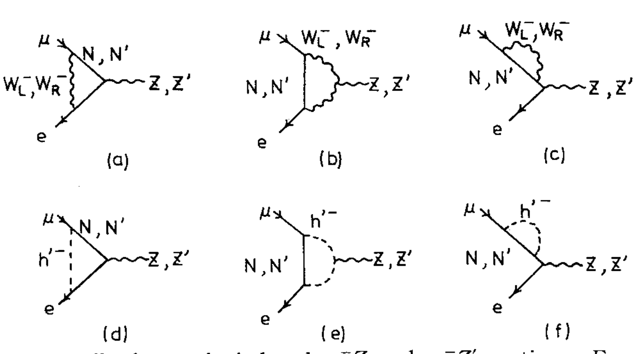


Fig. 3. The one-loop contribution to the induced $\mu \bar{e} Z$ and $\mu \bar{e} Z'$ vertices. For (a), (b) and (c), the same type of graphs with $W_L^{\pm}$ and $W_R^{\pm}$ replaced by $\chi_L^{\pm}$, $\chi_R^{\pm}$ respectively also contribute.

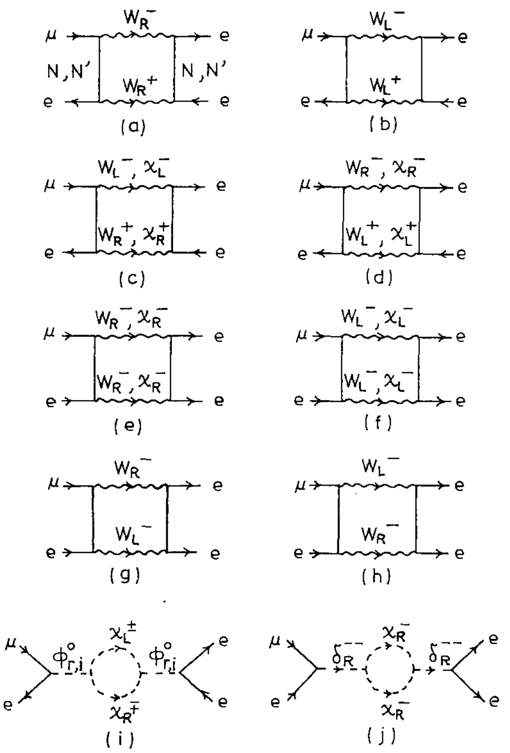


Fig. 4. The box graphs for $\mu\to ee\bar{e}$. The one-loop graphs involving physical Higgs exchange, (i) and (j), are needed to cancel out the gauge ($\xi$) dependence of graphs (c), (d) and (e). For (a), (b), (g) and (h), the same type of graphs with $W_L{}^{\pm}$ and $W_R{}^{\pm}$ replaced by $\chi_L{}^{\pm}$ and $\chi_R{}^{\pm}$ do not contribute in the limit $\xi_L$, $\xi_R\to 0$.

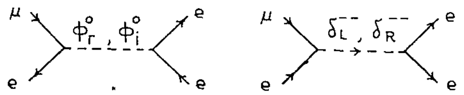

Fig. 5. The tree graphs with physical Higgs exchange contributing to $\mu\to ee\bar{e}$.

(and N-G boson) exchange. However, certain types of one-loop graphs involving physical Higgs exchange (Figs. 3(d)~(f) and Figs. 4(i), (j)) have also to be taken into account to cancel out ultraviolet divergences and $\xi$ dependence in $R_\xi$ gauge. They are included in the result presented below.

The effective four-Fermi Lagrangian turns out to be of the form

$$\begin{aligned}\mathcal{L}_{\rm eff}=&\frac{1}{(4\pi)^2}\frac{g^2}{M_{WL}^2}\{g^2[C_1(\bar{e}\gamma_\kappa L\mu)(\bar{e}\gamma^\kappa Le)\\&+C_2(\bar{e}\gamma_\kappa R\mu)(\bar{e}\gamma^\kappa Re)+D_1(\bar{e}\gamma_\kappa L\mu)(\bar{e}\gamma^\kappa Re)\\&+D_2(\bar{e}\gamma_\kappa R\mu)(\bar{e}\gamma^\kappa Le)+S_1(\bar{e}L\mu)(\bar{e}Re)\\&+S_2(\bar{e}R\mu)(\bar{e}Le)]+e^2[E_1(\bar{e}\gamma_\kappa L\mu)+E_2(\bar{e}\gamma_\kappa R\mu)\\&+im_\mu(q^\lambda/q^2)(M_1(\bar{e}\sigma_{\kappa\lambda}R\mu)+M_2(\bar{e}\sigma_{\kappa\lambda}L\mu))](\bar{e}\gamma^\kappa e).\end{aligned}\qquad(3\cdot4)$$

Denote the $\gamma$-exchange, $Z$-exchange and box graph contributions to the coefficients by suffices $\gamma$, $Z$ and $\square$ respectively. The result of our lengthy computation can be summarized as follows:

$$C_1=C_1{}^Z+C_1{}^\square=\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{\eta}{1-\eta}\right)f_{(-),L}+\frac{1}{2}\frac{\eta}{1-\eta}f_{(-),R}\right]+b^{(b)}+b^{(f)},$$

$$C_2=C_2{}^Z+C_2{}^{\Box}=\frac{1}{8}\frac{\eta}{1-\eta}[f_{(+),L}+f_{(+),R}]+b^{(a)}+b^{(e)}\,,$$

$$D_1=D_1{}^Z+D_1{}^{\Box}=\frac{1}{8}\frac{\eta}{1-\eta}[f_{(-),L}+f_{(-),R}]+b^{(h)}\,,$$

$$D_2=D_2{}^Z+D_2{}^{\Box}=\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{\eta}{1-\eta}\right)f_{(+),L}+\frac{1}{2}\frac{\eta}{1-\eta}f_{(+),R}\right]+b^{(g)}\,,$$

$$E_1=E_1{}^\gamma+E_1{}^Z=-\frac{1}{2}\left[\frac{1}{1-\eta}f_{(-),L}+\frac{\eta}{1-\eta}f_{(-),R}\right]-\gamma_E{}^L\,,$$

$$E_2=E_2{}^\gamma+E_2{}^Z=-\frac{1}{2}\left[\frac{1}{1-\eta}f_{(+),L}+\frac{\eta}{1-\eta}f_{(+),R}\right]-\gamma_E{}^R\,,$$

$$M_1=M_1{}^\gamma=-\gamma_M{}^R\,,\qquad M_2=M_2{}^\gamma=-\gamma_M{}^L\,,$$

$$S_1=S_1{}^{\Box}=b^{(c)}\,,\qquad S_2=S_2{}^{\Box}=b^{(d)}\,. \tag{3·5}$$

where $\gamma$, $f$ and $b$ as functions of $x_i$ or $y_i$ are given in Appendices B, C and D respectively. The results are $\xi$-independent, as they should be.

The branching ratio for $\mu\to ee\bar{e}$ is expressed in terms of the coefficients appearing in (3·4) as follows:

$$\begin{aligned}B(\mu\to ee\bar{e})=\frac{\alpha^2}{(4\pi)^2S_W{}^4}&[8|C_1|^2+4|D_1|^2+|S_1|^2\\&+4S_W{}^4\left\{3|E_1|^2+\left(-7+16\ln\frac{m_\mu}{2m_e}\right)|M_1|^2\right\}\\&-4\,\mathrm{Re}(D_1+S_W{}^2E_1)S_1{}^*+8S_W{}^2\,\mathrm{Re}(2C_1+D_1)^*(E_1-2M_1)\\&-48S_W{}^4\,\mathrm{Re}\,E_1{}^*M_1+(1\to2)],\end{aligned} \tag{3·6}$$

where $S_W\equiv\sin\theta_W$.

### 3.2. *Analysis of the branching ratios*

The symmetry breakdown hierarchy of the $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}$ model is determined by $M_{W_L}$ and $M_{W_R}$. In addition, the model contains three typical mass scales related to neutrino masses, $m_N$, $m_D$ and $m_{N'}$ with $m_Nm_{N'}\sim m_D{}^2$ in the simplified case $v_L=0$.

If we regard the $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}$ model as a gauge theory for quantum flavourdynamics, $M_{W_R}$ is an arbitrary parameter of the theory. The same is true for partially unified gauge theories such as $SU(4)_{\text{P.S.}}\times SU(2)_L\times SU(2)_R$ model.[10] The previously derived experimental lower bound on $M_{W_R}$ is rather weak, $M_{W_R}\gtrsim 3M_{W_L}$.[11] As for the neutrino mass parameters, the following experimental and theoretical constraints on their orders of magnitudes can be used. The experimental upper bounds on $m_N$ are known to be[12)]

$$m_{Ne} < 60\ \text{eV}\ , \qquad m_{N\mu} < 0.57\ \text{MeV} \quad \text{and} \quad m_{N\tau} < 250\ \text{MeV}\ . \tag{3·7}$$

In addition we have the following bounds from cosmological considerations[13)]

$$m_N < 70\ \text{eV} \qquad \text{or} \qquad m_N > 23\ \text{MeV}\ . \tag{3·8}$$

The allowed range of $m_{N\mu}$ is therefore $m_{N\mu} < 70$ eV. We take two representative mass values of charged fermions as the orders of magnitudes of neutrino Dirac masses $m_D$

$$m_D \sim m_e\ , \quad m_u,\ m_d \sim 1\ \text{MeV}$$

or

$$m_D \sim m_\mu,\ m_s \sim 100\ \text{MeV}\ . \tag{3·9}$$

The typical values of $m_N$, $m_D$ and $m_{N'}$ to be used in the following analysis can be summarized as follows:

| $m_N$ | $m_D$ | $m_{N'}$ |
|---|---|---|
| 1 eV | 1 MeV | 1 TeV |
| | 100 MeV | $10^4$ TeV |
| 100 eV | 1 MeV | 10 GeV |
| | 100 MeV | 100 TeV . |

(3·10)

The hierarchy of the mass parameters is thus assumed to be

$$m_N \ll m_D \ll M_{WL} \ll m_{N'} \lesssim M_{WR}\ . \tag{3·11}$$

We have excluded the possibility of $m_{N'} \simeq 10$ GeV by assuming $M_{W_L} \ll m_{N'}$. Under this condition of the mass parameters, the orders of magnitudes of $B(\mu \to e\gamma)$ and $B(\mu \to ee\bar{e})$ can be analyzed without specifying the values of $m_N$, $m_D$ and $M_{W_R}$.

The rates for the flavour changing processes also depend on the generation mixing angles. The mixing angles are related to the Yukawa couplings of the Higgs scalar fields and cannot be determined within the present scheme of gauge theories. Since the lepton-flavour mixing angles are not constrained either theoretically or empirically, the generation mixing factors will be assumed to be of the order unity in the following analysis,

$$(V_L^\dagger)_{1i}(V_L)_{i2}(i \leq n) = O(1), \qquad (V_R^\dagger)_{1i}(V_R)_{i2}(i \geq n+1) = O(1). \tag{3·12}$$

A. $\mu \to e\gamma$

Note the following orders of magnitudes

$$x_i = O(m_N{}^2/M_{WL}^2) \ll 1\ , \qquad y_i = O(m_N{}^2/M_{WR}^2) \ll 1 \qquad \text{for } i \leq n\ ,$$

$$x_i = O(m_{N'}^2/M_{WL}^2) \gg 1\,, \qquad y_i = O(m_{N'}^2/M_{WR}^2) \lesssim 1 \qquad \text{for } i \geq n+1\,. \tag{3·13}$$

From the result of Appendix B, Eqs. (B·2) and (B·4), we find

$$\Gamma_M{}^R \simeq \Gamma^{(a)} + \Gamma^{(b)}\,, \qquad \Gamma_M{}^L \simeq \Gamma^{(C)}\,, \tag{3·14}$$

with

$$\Gamma^{(a)} = \frac{1}{8}\sum_{i=1}^{n}(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\,, \quad \Gamma^{(b)} = \frac{1}{4}\sum_{i=n+1}^{2n}(V_L{}^\dagger)_{1i}(V_L)_{i2}\,,$$

$$\Gamma^{(c)} = \frac{\eta}{2}\sum_{i=n+1}^{2n}(V_R{}^\dagger)_{1i}(V_R)_{i2}g_2(y_i), \tag{3·15}$$

where $g_2(x)$ is a smooth function of $x$ behaving as $g_2(x) \simeq \frac{1}{4}x$ for $x \ll 1$ and $g_2(x) \simeq \frac{1}{2}$ for $x \gg 1$.[14),15)] We set $g_2(y_i) = O(m_{N'}^2/M_{W_R}^2)$ for $i \geq n+1$. $\Gamma^{(a)}$ is the contribution from intermediate light neutrinos. $\Gamma^{(b)}$ is the contribution from heavy Majorana neutrinos in the $W_L{}^\pm$-exchange processes, through $\nu_L \leftrightarrow \nu_R{}^C$ mixings. $\Gamma^{(c)}$ is the contribution from heavy Majorana neutrinos in the $W_R{}^\pm$-exchange processes. The orders of the $\nu_L \leftrightarrow \nu_R{}^C$ mixing factors are

$$(V_L{}^\dagger)_{1i}(V_L)_{i2}(i \geq n+1) = O(m_N{}^2/m_D{}^2), \quad (V_R{}^\dagger)_{1i}(V_R)_{i2}(i \leq n) = O(m_N{}^2/m_D{}^2). \tag{3·16}$$

Thus the orders of magnitudes of $\Gamma^{(i)}$ are

$$\Gamma^{(a)} = O(m_N{}^2/M_{WL}^2), \qquad \Gamma^{(b)} = O(m_N{}^2/m_D{}^2),$$
$$\Gamma^{(c)} = O(M_{WL}^2 m_{N'}^2/M_{WR}^4). \tag{3·17}$$

The contribution from $\Gamma^{(a)}$ can be neglected because of the mass hierarchy (3·11). Hence we consider the following two cases.

(i) Dominance of the $W_R{}^\pm$-exchange term $\Gamma^{(c)}(\omega_M \gtrsim \sqrt{\omega_D})$

$$B(\mu \to e\gamma) = O(\alpha M_{WL}^4 m_{N'}^4/M_{WR}^8) = O(\alpha\eta^2\omega_M{}^2), \tag{3·18}$$

(ii) Dominance of the $\nu_L \leftrightarrow \nu_R{}^C$ mixing term $\Gamma^{(b)}(\omega_M \lesssim \sqrt{\omega_D})$

$$B(\mu \to e\gamma) = O(\alpha m_N{}^4/m_D{}^4) = O(\alpha\eta^2\omega_D{}^2/\omega_M{}^2), \tag{3·19}$$

where the parameters $\eta$ and $\omega$ were previously defined to be $\eta \equiv (M_{W_L}/M_{W_R})^2$, $\omega_D \equiv (m_D/M_{W_L})^2$ and $\omega_M \equiv (m_{N'}/M_{W_R})^2$.

Interestingly enough, both $\Gamma^{(b)}$ and $\Gamma^{(c)}$ terms are the contribution from heavy Majorana neutrinos $N'$. Thus we have obtained the result that heavy Majorana neutrinos do not decouple from the low-energy process $\mu \to e\gamma$, as advertised in the Introduction. It should be noticed that the term $\Gamma^{(c)}$ is independent of small masses of light neutrinos, and may yield relatively large $B(\mu \to e\gamma)$ with rather small value of $M_{W_R}$.

The eventual measurement of $B(\mu \to e\gamma)$ in the future will be used to predict, through the relation of Eq. (3·18), the value of $M_{W_R}$. In fact, the presently known upper bound[12)]

$$B(\mu\to e\gamma)<1.9\times10^{-10}, \tag{3·20}$$

provides

$$\eta=(M_{WL}/M_{WR})^2<\frac{1}{\omega_M}\times10^{-4}. \tag{3·21}$$

We get $M_{W_R}>10$ TeV for $\omega_M\simeq1$.

We have so far considered the $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}$ model as a gauge theory for QFD. On the other hand, if we assume that the left-right symmetry is a subgroup of $SO(10)$ GUT,[4)] $M_{W_R}$ is related to the presently known values of the gauge coupling constants by renormalization group method. Assuming the usual breaking pattern of $SO(10)$ through $SU(3)\times SU(2)\times SU(2)\times U(1)$ we have $M_{W_R}\simeq10^{10}$ GeV for $\sin^2\theta_W\simeq0.23$ and $\alpha_s(\mu=10\text{ GeV})\simeq0.1$.[16)] In this case the term $\Gamma^{(b)}$ dominates the decay rate (case (ii)); the contribution from $\Gamma^{(c)}$ to $B(\mu\to e\gamma)$ is bounded by $B(\mu\to e\gamma)\lesssim\alpha\eta^2\simeq10^{-34}$. Through Eq. (3·19) we obtain hopelessly small branching ratio. For example, with $m_N\simeq100$ eV and $m_D\simeq100$ MeV we get

$$B(\mu\to e\gamma)\simeq10^{-26}. \tag{3·22}$$

B. $\mu\to ee\bar{e}$

The expression for $B(\mu\to ee\bar{e})$ is quite complicated because of the presence of many types of effective couplings. We shall not go into the details of the expression but we only estimate the order of magnitude of $B(\mu\to ee\bar{e})$.

Write the effective four-Fermi couplings as

$$\mathcal{L}_{\text{eff}}=\frac{g^4}{(4\pi)^2}\frac{1}{M_{WL}^2}\sum_{ij}C^{ij}[\bar{e}\,\mathcal{O}_i\mu][\bar{e}\,\mathcal{O}_je]. \tag{3·23}$$

The contribution of the gauge boson exchange graphs, denoted by $C^G$, can be shown to be of the same order as that in $\mu\to e\gamma$. As for the contribution from the physical Higgs exchange at the tree level, denoted by $C^H$, we have

$$C^H=O((\eta\omega/\alpha)\cdot(M_{WR}/M_H)^2), \tag{3·24}$$

where $M_H$ is a typical mass scale of Higgs scalars $\delta_{L,R}^{++}$ and $\phi_{r,i}^0$ and $\omega$ is $\omega_M$ or $\omega_D$. Masses of physical Higgs scalars are undetermined from gauge boson masses. We may imagine two typical situations:

(i) Dominance of the Higgs-exchange term ($M_H\ll M_{W_R}/\sqrt{\alpha}$)

$$B(\mu\to3e)=O(\eta^2\omega_M{}^2\xi^2)\gg\alpha B(\mu\to e\gamma), \tag{3·25}$$

where $\xi\equiv(M_{W_R}/M_H)^2\gg\alpha$.

(ii) Dominance of the $W_R{}^{\pm}$-exchange term ($M_H\gg M_{WR}/\sqrt{\alpha}$)

$$B(\mu\to 3e)=O(\alpha^2\eta^2\omega_M{}^2)\simeq\alpha B(\mu\to e\gamma). \qquad (3\cdot 26)$$

The results obtained above are those for $\omega_M\gtrsim\sqrt{\omega_D}$. So long as $M_H$ is not exceedingly small, the following relation holds

$$B(\mu\to 3e)\lesssim B(\mu\to e\gamma). \qquad (3\cdot 27)$$

3.3. *Dependence of the decay rates on the type of neutrino masses*

In this subsection we study the phenomenological consequences of gauge theories with distinct types of neutrino masses, taking the rare process $\mu\to e\gamma$ as an example.

We classify gauge models into three types by their types of neutrino masses:

(A) Majorana plus Dirac (the model considered in this paper)
(B) pure Majorana,
(C) pure Dirac. (3·28)

As for gauge models of type (A), the analysis in the preceding subsection tells that $B(\mu\to e\gamma)$ is sensitive to the value of $\eta$, and is not necessarily very small (even for the small value of $m_N$). In the case of type (B), the $\nu_L\leftrightarrow\nu_R{}^C$ mixing term $\Gamma^{(b)}$ is absent in Eq. (3·14). Of the remaining two terms in Eq. (3·14), the term $\Gamma^{(c)}$ may become significant for rather small $M_{W_R}$, while $\Gamma^{(a)}$ is always very small. As in the case of type (A), the experimental value of $B(\mu\to e\gamma)$ is expected to provide an important information about the order of $M_{W_R}$. In the case of type (C), the branching ratio is obtained by retaining only the contribution from light neutrinos $\Gamma^{(a)}$ in Eq. (3·14),[5]

$$B(\mu\to e\gamma)=(3\alpha/32\pi)\cdot\Big|\sum_{i=1}^{n}(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\Big|^2, \qquad (3\cdot 29)$$

where $x_i=(m_{Ni}/M_{W_L})^2$ and $m_{Ni}$ should be identified with the masses of light Dirac neutrinos. We obtain

$$B(\mu\to e\gamma)=O(\alpha m_N{}^4/M^4_{WL}). \qquad (3\cdot 30)$$

Thus the predicted rate for $\mu\to e\gamma$ receives an overwelmingly strong suppression by powers of light neutrino masses independently of the value of $M_{W_R}$. We have for $m_N\sim 100$ eV.

$$B(\mu\to e\gamma)\sim 10^{-38}. \qquad (3\cdot 31)$$

It seems appropriate to make a remark on the $SU(2)_L\times U(1)$ model with Majorana neutrinos. In this model, heavy Majorana neutrinos can participate in low energy processes only through $\nu_L\leftrightarrow\nu_R{}^C$ mixing induced by Dirac mass terms for neutrinos, i.e., the term $\Gamma^{(c)}$ is absent in Eq. (3·14). The branching ratio is then strongly suppressed by powers of small $\nu_L\leftrightarrow\nu_R{}^C$ mixings (the case of type

(A)) or by powers of light neutrino masses $m_N{}^2$ (the case of type (B)).[15)]

### 3.4. *Non-decoupling effects of heavy Majorana neutrinos*

We have previously studied the effects of heavy Dirac fermions (quarks and charged leptons) in the rare processes $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$.[14)] It has been demonstrated by explicit computations that heavy Dirac fermions do not decouple from such low energy processes,[17)] unlike in QED and QCD.[18)] Furthermore, the degree of the non-decoupling of heavy fermions is found to be different depending on the processes. In particular, in the case of large Dirac fermion mass $m_F \gg M_{W_L}$, a heavy Dirac fermion gives an effect proportional to $m_F{}^2$ in the induced $d\bar{s}z$ vertex, while the effect is a constant independent of $m_F$ (modulo ln $m_F$) in the induced $d\bar{s}\gamma$ vertex.

One may expect that analogous non-decoupling of heavy Majorana neutrinos brings about distinct effects in $\mu \to e\gamma$ and $\mu \to ee\bar{e}$, leading to vastly different branching ratios $B(\mu \to ee\bar{e})_{M+D} \gg B(\mu \to e\gamma)_{M+D}$. However, this expectation does not hold true, as is seen from Eq. (3·27) of § 3.2.B.

In gauge theories with only Majorana masses of neutrinos, heavy Majorana neutrinos $N' = \nu_R + \nu_R{}^C$ completely decouple from effective low energy theory $SU(2)_L \times U(1)$. Contrarily, heavy Majorana neutrinos take part in $W_L$-exchange processes through small $\nu_L \leftrightarrow \nu_R{}^C$ mixing in gauge theories with both Majorana and Dirac masses of neutrinos. In the latter case, the terms proportional to $m_{N'}^2$ in effective Lagrangian for $\mu \to ee\bar{e}$ are suppressed by $\varepsilon^4$($\varepsilon$ : $\nu_L \leftrightarrow \nu_R{}^C$ mixing), while the constant terms in the induced vertex for $\mu \to e\gamma$ are suppressed by $\varepsilon^2$. As a result, the contribution of $N'$ to $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ are of the same order of magnitude,

$$B(\mu \to ee\bar{e})_{M+D} \lesssim B(\mu \to e\gamma)_{M+D} . \qquad (3\cdot32)$$

Of course, if we assume the existence of heavy Dirac neutrinos ($m_\nu \gg M_{W_L}$) and that they give dominant contributions to $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ even after suppression by generation mixing angles, we will get $B(\mu \to ee\bar{e}) \gg B(\mu \to e\gamma)$.

## § 4. Other processes

The reaction rates for other lepton-flavour changing processes can be estimated by using the result of our computation of the (one-loop) graphs for $\mu \to ee\bar{e}$; no essentially new types of graphs are involved. Here we shall briefly discuss the rates for the following four processes, ignoring the details which depend on the spin structure of the interactions:

$$\text{(a)}\ Z \to \mu\bar{e}\,, \quad \text{(b)}\ \mu + (A, Z) \to e + (A, Z), \quad \text{(c)}\ K_L \to \mu\bar{e}\,,$$
$$\text{(d)}\ K^+ \to \pi^+ \mu\bar{e}\,, \qquad (4\cdot1)$$

where (b) is "$\mu$-$e$ conversion" process. Quark-flavour also changes in processes (c) and (d), to which only the box type graphs contribute (physical Higgs-exchange processes at the tree level are not considered here for simplicity).

Let us write the effective couplings for these processes (including those for $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ for later comparison) as follows:

$$\text{(a)} \quad : \quad \Gamma_{\text{eff}}^{Z} = \frac{g^3}{(4\pi)^2} \sum_i \zeta^i J_{i\kappa} Z^\kappa \,,$$

$$\mu \to e\gamma \,: \quad \Gamma_{\text{eff}}^{\gamma} = \frac{g^3}{(4\pi)^2} \frac{i m_\mu q^\kappa}{M_{W_L}^2} \sum_i \zeta^i J_{i\kappa} \cdot A^\kappa \,,$$

$$\text{(b)} \sim \text{(d)}, \mu \to ee\bar{e} \,: \quad \mathcal{L}_{\text{eff}} = \frac{g^4}{(4\pi)^2} \frac{1}{M_{W_L}^2} \sum_{i,j} \zeta^{ij} J_i^\dagger J_j \tag{4·2}$$

with $J_{i\kappa}$, $J_i$ and $J_j$ the appropriate types of current composed of $\mu$, $e$ or quarks. Here $\zeta$'s are functions of $x_i$ (or $y_i$) and represent how strongly the induced interactions are suppressed compared with the ordinary Fermi interaction. We shall not give their explicit forms, which can be obtained without too much difficulty from our computations in the preceding section. Instead, we shall give in Table I the predicted orders of magnitudes for $\zeta$ (the largest one of $\zeta^i$ or $\zeta^{ij}$) in each of three types of gauge models mentioned previously (recall (3·28)). As for gauge models of types (B) and (C), it seems appropriate to classify them further by their gauge symmetries ($SU(2)_L \times U(1)$ or left-right symmetric). The predictions of the left-right symmetric models of types (B) and (C) are those for "small" $M_{W_R}$. Cahn and Harari have recently made an analysis of the reaction

Table I. The predicted orders of magnitudes for $\zeta$, in each type of gauge theories of Eq. (3·28). As for the gauge theories of type (A), we have supposed three cases; (a) $\omega_M \lesssim \omega_D$, (b) $\omega_D \lesssim \omega_M \lesssim \sqrt{\omega_D}$ and (c) $\sqrt{\omega_D} \lesssim \omega_M \lesssim 1$.

<table>
<tr><th rowspan="2">model / process</th><th>$SU(2)_L \times U(1)$</th><th colspan="3">left-right symmetric</th><th rowspan="2">experiment</th></tr>
<tr><th>$B$, $C$</th><th>$A$</th><th>$B$</th><th>$C$</th></tr>
<tr><td>$\mu \to e\gamma$</td><td rowspan="6">$\left(\frac{m_N}{M_{W_L}}\right)^2$</td><td rowspan="4">$\left(\frac{m_N}{m_D}\right)^{2\ \text{(a),(b)}}$, $\left(\frac{M_{W_L} m_{N'}}{M_{W_R}^2}\right)^{2\ \text{(c)}}$</td><td rowspan="4">$\left(\frac{M_{W_L} m_{N'}}{M_{W_R}^2}\right)^2$</td><td rowspan="3">$\left(\frac{m_N}{M_{W_L}}\right)^2$</td><td>$<1\times10^{-4}$</td></tr>
<tr><td>$\mu \to ee\bar{e}$</td><td>$<2\times10^{-3}$</td></tr>
<tr><td>$Z \to \mu\bar{e}$</td><td></td></tr>
<tr><td>$\mu + N \to e + N$</td><td rowspan="3">$\frac{m_N m_D}{M_{W_R}^2}$</td><td>$<2\times10^{-4}$</td></tr>
<tr><td>$K_L \to \mu\bar{e}$</td><td rowspan="2">$\left(\frac{m_N}{M_{W_L}}\right)^{2\ \text{(a)}}$, $\left(\frac{m_D}{M_{W_R}}\right)^{2\ \text{(b),(c)}}$</td><td rowspan="2">$\left(\frac{M_{W_L} m_D}{M_{W_R}^2}\right)^2$</td><td>$<1\times10^{-3}$</td></tr>
<tr><td>$K^+ \to \pi^+\mu\bar{e}$</td><td>$<2\times10^{-2}$</td></tr>
</table>

rates of various rare processes assuming the $V-A$ type effective couplings for simplicity and deduced experimental upper bounds for $\zeta$.[19] We have not repeated the same type of analysis but quote the upper bound obtained in Ref. 19) for comparison with our prediction.

As is seen in Table I, in the case of pure Dirac mass (type (C)), the predicted values of $\zeta$ are always insignificant. In gauge theories of type (A) or (B), the predicted $\zeta$ can be relatively large for rather small $M_{W_R}$. It is worth noticing that predicted rates for the first two processes (a) and (b) in Eq. (4·1) are larger than those for the latter two (c) and (d).

At present the most stringent experimental bounds on $\zeta$ appear to come from the rates for $\mu\to e\gamma$ and $\mu+N\to e+N$. However, uncertainty present in the theoretical analysis of the $\mu$-$e$ conversion process due to strong interaction effects and complicated structure of nuclei will have to be reduced before an eventual accurate measurement of this process may give an important clue to neutrino masses.

## § 5. Summary

We have investigated the lepton-flavour nonconservation in gauge theories with heavy (right-handed) neutrinos, taking the $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}$ gauge model as a model for quantum flavour-dynamics. The value of $M_{W_R}$ is considered to be an arbitrary parameter of the theory.

The rates for lepton-flavour changing processes are shown to depend crucially on the type of neutrino masses; pure Dirac, pure Majorana or Majorana plus Dirac. Only light neutrinos appear in the intermediate states of one-loop graphs in the model with pure Dirac masses. Hence the rates are strongly suppressed by powers of light neutrino masses $m_N$ and are hopelessly small, e.g., $B(\mu\to e\gamma)\simeq O(10^{-38})$. In the model with Majorana plus Dirac masses, on the other hand, the loop graphs with heavy Majorana neutrinos in the intermediate states are shown to give dominant contributions. The rates are suppressed by powers of $\eta=M_{W_L}^2/M_{W_R}^2$ instead of powers of $m_N$, and hence are not necessarily outrageously small, e.g., $B(\mu\to e\gamma)\simeq O(10^{-14})$ for $M_{W_R}=O(100\text{ TeV})$. It appears possible that leptonflavour nonconservation occurs at a rate of experimentally detectable level in the future if the breakdown of the left-right symmetry occurs at a relatively small mass scale.

If the left-right gauge symmetry is to be grand unified as dictated by the $SO(10)$ model, the parameter $M_{W_R}$ is no longer free but receives a constraint that $M_{W_L}/M_{W_R}\simeq O(10^{-8})$ to be consistent with the value $\sin^2\theta_W\simeq 0.23$ at low energies. Though the dominance of heavy Majorana neutrinos in lepton-flavour nonconserving processes still holds, the rates are uninterestingly small, e.g., $B(\mu\to e\gamma)\simeq O(10^{-26})$.

The value of $M_{W_R}$ has recently been discussed from a somewhat different viewpoint, i.e., in relation to the baryon number nonconservation. It has been argued that the mass scale of the breakdown of left-right symmetry is bounded (order of magnitude wise) from below to be able to explain the cosmological baryon number[20] and the stability of nucleus against $n$-$\bar{n}$ oscillation.[21]

However, the analyses are model dependent and it has not been settled yet how small $M_{W_R}$ can be.


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## Appendix A

*—— Higgs Potential ——*

Here we consider a simplified case of the potential term $V$ possessing the left-right symmetry ($\Delta_L \leftrightarrow \Delta_R$ and $\phi \leftrightarrow \phi^+$) and the discrete symmetry[22] for

$$\Delta_L \to \Delta_L\,, \qquad \Delta_R \to -\Delta_R\,, \qquad \phi \to i\phi\,. \tag{A·1}$$

We shall show that the potential has the minimum for the scalar fields of the form

$$\langle \Delta_L \rangle = 0\,, \qquad \langle \Delta_R \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \qquad \langle \phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} k_1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{A·2}$$

The general form of $V$ consistent with the symmetries mentioned above is given by

$$\begin{aligned} V = & -\mu_1^2 \operatorname{Tr} \phi^\dagger \phi + \lambda_1 (\operatorname{Tr} \phi^\dagger \phi)^2 + \lambda_2 \operatorname{Tr} \phi^\dagger \phi \phi^\dagger \phi \\ & + \frac{1}{2}\lambda_3 (\operatorname{Tr} \phi^\dagger \tilde{\phi} + \operatorname{Tr} \tilde{\phi}^\dagger \phi)^2 + \frac{1}{2}\lambda_4 (\operatorname{Tr} \phi^\dagger \tilde{\phi} - \operatorname{Tr} \tilde{\phi}^\dagger \phi)^2 \\ & + \lambda_5 \operatorname{Tr} \phi^\dagger \phi \tilde{\phi}^\dagger \tilde{\phi} + \frac{1}{2}\lambda_6 (\operatorname{Tr} \phi^\dagger \tilde{\phi} \phi^\dagger \tilde{\phi} + \text{h.c.}) \\ & - \mu_2^2 (\operatorname{Tr} \Delta_L^\dagger \Delta_L + \operatorname{Tr} \Delta_R^\dagger \Delta_R) + \rho_1 [(\operatorname{Tr} \Delta_L^\dagger \Delta_L)^2 + (\operatorname{Tr} \Delta_R^\dagger \Delta_R)^2] \\ & + \rho_2 (\operatorname{Tr} \Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L + \operatorname{Tr} \Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R) \\ & + \rho_3 (\operatorname{Tr} \Delta_L^\dagger \Delta_L)(\operatorname{Tr} \Delta_R^\dagger \Delta_R) \\ & + \alpha_1 \operatorname{Tr} \phi^\dagger \phi (\operatorname{Tr} \Delta_L^\dagger \Delta_L + \operatorname{Tr} \Delta_R^\dagger \Delta_R) + \alpha_2 (\operatorname{Tr} \Delta_R^\dagger \phi^\dagger \phi \Delta_R \\ & + \operatorname{Tr} \Delta_L^\dagger \phi \phi^\dagger \Delta_L) + \alpha_2' (\operatorname{Tr} \Delta_R^\dagger \tilde{\phi}^\dagger \tilde{\phi} \Delta_R + \operatorname{Tr} \Delta_L^\dagger \tilde{\phi} \tilde{\phi}^\dagger \Delta_L). \end{aligned} \tag{A·3}$$

The VEV's of the scalar fields which minimize the potential preserving $U(1)_{\text{e.m.}}$ symmetry are written as

$$\langle\Delta_L\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\ v_L & 0\end{pmatrix},\qquad \langle\Delta_R\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix},\qquad \langle\phi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}k_1 & 0\\ 0 & k_2\end{pmatrix}. \tag{A·4}$$

We obtain the following conditions for the extremum

$$\begin{aligned}
\frac{\partial V}{\partial v_L}&=[-2\mu_2{}^2+\rho_3 v_R{}^2+\alpha_1(k_1{}^2+k_2{}^2)+(\alpha_2 k_2{}^2+\alpha_2' k_1{}^2)]\\
&\quad\times v_L+2(\rho_1+\rho_2)v_L{}^3=0\,,\\
\frac{\partial V}{\partial v_R}&=[-2\mu_2{}^2+\rho_3 v_L{}^2+\alpha_1(k_1{}^2+k_2{}^2)+(\alpha_2 k_2{}^2+\alpha_2' k_1{}^2)]\\
&\quad\times v_R+2(\rho_1+\rho_2)v_R{}^3=0\,,\\
\frac{\partial V}{\partial k_1}&=[-2\mu_1{}^2+2(\lambda_1+16\lambda_3+\lambda_5+\lambda_6)k_2{}^2+(\alpha_1+\alpha_2')\\
&\quad\times(v_R{}^2+v_L{}^2)]k_1+2(\lambda_1+\lambda_2)k_1{}^3=0\,,\\
\frac{\partial V}{\partial k_2}&=[-2\mu_1{}^2+2(\lambda_1+16\lambda_3+\lambda_5+\lambda_6)k_1{}^2+(\alpha_1+\alpha_2)\\
&\quad\times(v_R{}^2+v_L{}^2)]k_2+2(\lambda_1+\lambda_2)k_2{}^3=0\,.
\end{aligned}\tag{A·5}$$

We easily see that these extremum conditions are met by the VEV of the form (A·2) with $v_R$ and $k_1$ satisfying the simpler equations,

$$\begin{aligned}
2\mu_1{}^2&=(\alpha_1+\alpha_2')v_R{}^2+2(\lambda_1+\lambda_2)k_1{}^2\,,\\
2\mu_2{}^2&=2(\rho_1+\rho_2)v_R{}^2+(\alpha_1+\alpha_2')k_1{}^2\,.
\end{aligned}\tag{A·6}$$

We shall next show that the solution to these conditions actually gives a minimum by demonstrating that the mass-squared matrix of scalar fields is positive definite for certain ranges of potential parameters.

(i) Doubly charged scalars

The physical Higgs scalars $\delta_L^{++}$, $\delta_R^{++}$ have the masses

$$\begin{aligned}
M^2_{\delta_L^{++}}&=\frac{1}{2}(\rho_3-2\rho_1-2\rho_2)v_R{}^2+(\Delta\alpha)k_1{}^2\,,\\
M^2_{\delta_R^{++}}&=-\rho_2 v_R{}^2+(\Delta\alpha)k_1{}^2\,,
\end{aligned}\tag{A·7}$$

where $\Delta\alpha\equiv(\alpha_2-\alpha_2')/2$.

(ii) Charged scalars

The mass-squared matrix for charged scalars is given by

$$\begin{array}{c} \phi_2{}^+ \\ \delta_L{}^+ \\ \phi_1{}^+ \\ \delta_R{}^+ \end{array}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \left(\frac{1}{2}\rho_3-\rho_1-\rho_2\right)v_R{}^2+\frac{1}{2}(\varDelta\alpha)k_1{}^2 & 0 & 0 \\ 0 & 0 & (\varDelta\alpha)v_R{}^2 & \frac{1}{\sqrt{2}}(\varDelta\alpha)k_1 v_R \\ 0 & 0 & \frac{1}{\sqrt{2}}(\varDelta d)k_1 v_R & \frac{1}{2}(\varDelta\alpha)k_1{}^2 \end{bmatrix} . \tag{A·8}$$

The diagonalization of this matrix gives two charged Higgs scalars

$$h^+=\delta_L{}^+\ , \quad M^2_{h^+}=\left(\frac{1}{2}\rho_3-\rho_1-\rho_2\right)v_R{}^2+\frac{1}{2}(\varDelta\alpha)k_1{}^2\ ,$$

$$h'^+=\frac{g}{2}\frac{1}{M_{W_R}}(\sqrt{2}v_R\phi_1{}^+ + k_1\delta_R{}^+), \quad M^2_{h'^+}=\frac{1}{2}(\varDelta\alpha)(k_1{}^2+2v_R{}^2). \tag{A·9}$$

and two N-G bosons $\chi_L{}^+$, $\chi_R{}^+$ furnishing longitudinal components to $W_L{}^+$, $W_R{}^+$ respectively,

$$\chi_L{}^+=\phi_2{}^+\ , \qquad \chi_R{}^+=\frac{g}{2}\frac{1}{M_{W_R}}(k_1\phi_1{}^+ - \sqrt{2}v_R\delta_R{}^+). \tag{A·10}$$

(iii) Neutral scalars

The mass-squared matrix for the real components of shifted neutral scalar fields $\delta^0_{Lr}$, $\delta^0_{Rr}$, $\phi^0_{1r}$ and $\phi^0_{2r}$ is

$$\begin{array}{c} \phi^0_{2r} \\ \delta^0_{Lr} \\ \phi^0_{1r} \\ \delta^0_{Rr} \end{array}\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 2(\lambda_1+\lambda_2)k_1{}^2 & (\alpha_1+\alpha_2{}')k_1 v_R \\ 0 & 0 & (\alpha_1+\alpha_2{}')k_1 v_R & 2(\rho_1+\rho_2)v_R{}^2 \end{bmatrix} , \tag{A·11}$$

where

$$A=(-\lambda_2+4\lambda_3+\lambda_5+\lambda_6)k_1{}^2+(\varDelta\alpha)v_R{}^2\ ,$$

$$B=(\rho_3-2\rho_1-2\rho_2)v_R{}^2/2\ . \tag{A·12}$$

We have then four physical Higgs scalars

$$\phi_r{}^0=\phi^0_{2r}\ , \qquad M^2_{\phi_r{}^0}=A\ ,$$

$$\delta_r{}^0=\delta^0_{Lr}\ , \qquad M^2_{\delta_r{}^0}=B\ ,$$

$$h^0=\cos\theta^0\phi^0_{1r}+\sin\theta^0\delta^0_{Rr}\ , \quad M^2_{h^0}\simeq\left\{2(\lambda_1+\lambda_2)-\frac{(\alpha_1+\alpha_2{}')^2}{2(\rho_1+\rho_2)}\right\}k_1{}^2\ ,$$

$$h'^0=-\sin\theta^0\phi^0_{1r}+\cos\theta^0\delta^0_{Rr}\ , \quad M^2_{h'^0}\simeq 2(\rho_1+\rho_2)v_R{}^2\ , \tag{A·13}$$

where

$$\tan 2\theta^0 = -\frac{(\alpha_1+\alpha_2')k_1 v_R}{(\rho_1+\rho_2)v_R^2-(\lambda_1+\lambda_2)k_1^2}, \qquad |\theta^0| \ll 1 . \tag{A·14}$$

The mass-squared matrix for the imaginary components $\delta^0_{Li}$, $\delta^0_{Ri}$, $\phi^0_{1i}$ and $\phi^0_{2i}$ is

$$\begin{array}{c} \phi^0_{2i} \\ \delta^0_{Li} \\ \phi^0_{1i} \\ \delta^0_{Ri} \end{array}\begin{pmatrix} C & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A·15}$$

where

$$C = (-\lambda_2 - 4\lambda_4 + \lambda_5 - \lambda_6)k_1^2 + (\Delta\alpha)v_R^2 . \tag{A·16}$$

We obtain two physical Higgs scalars

$$\begin{aligned} \phi_i{}^0 &= \phi^0_{2i}, \qquad M^2_{\phi_i{}^0} = C, \\ \delta_i{}^0 &= \delta^0_{Li}, \qquad M_{\delta_i{}^0} = M_{\delta_r{}^0}, \end{aligned} \tag{A·17}$$

and two N-G bosons $\chi^0$ and $\chi'^0$ corresponding to $Z^0$, $Z'^0$ respectively

$$\begin{aligned} \chi^0 &= \frac{g}{\sqrt{2}}\frac{1}{M_Z}(\cos\varepsilon^0 \chi_L{}^0 + \sin\varepsilon^0 \chi_R{}^0), \\ \chi'^0 &= \frac{g}{\sqrt{2}}\frac{1}{M_{Z'}}(-\sin\varepsilon^0 \chi_L{}^0 + \cos\varepsilon^0 \chi_R{}^0), \end{aligned} \tag{A·18}$$

where

$$\chi_L{}^0 = \frac{k_1}{\sqrt{2}C_W}\phi^0_{1i}, \quad \chi_R{}^0 = \frac{-\sqrt{C_{2W}}}{\sqrt{2}C_W}k_1\phi^0_{1i} - \frac{\sqrt{2}C_W}{\sqrt{C_{2W}}}v_R\delta^0_{Ri}, \tag{A·19}$$

and the $Z_L \leftrightarrow Z_R$ mixing angle $\varepsilon^0$ is

$$\varepsilon^0 \simeq \frac{C_{2W}^{3/2}}{4C_W{}^4}\frac{k_1{}^2}{v_R{}^2} \ll 1, \tag{A·20}$$

with $C_W \equiv \cos\theta_W$ and $C_{2W} \equiv \cos 2\theta_W$.

Summarizing the above result, the condition of minimum is found to be

$$\begin{aligned} &\alpha_2 - \alpha_2' > 0, \quad \rho_2 < 0, \quad \rho_1 + \rho_2 > 0, \quad \rho_3 - 2(\rho_1+\rho_2) > 0, \\ &-\lambda_2 + 4\lambda_3 + \lambda_5 + \lambda_6 > 0, \quad -\lambda_2 - 4\lambda_4 + \lambda_5 - \lambda_6 > 0, \\ &4(\lambda_1+\lambda_2)(\rho_1+\rho_2) - (\alpha_1+\alpha_2')^2 > 0 . \end{aligned} \tag{A·21}$$

There indeed exists a range of parameters for which the above constraints are satisfied.

## Appendix B

—— *Induced $\mu\bar{e}\gamma$ Vertex* ——

The coefficients $\Gamma$ appearing in the induced $\mu\bar{e}\gamma$ vertex (3·1) can be obtained with some modification from similar quantities for $d\bar{s}\gamma$ vertex computed previously.[14] To this end, express $\Gamma$'s as

$$\Gamma_E{}^L=\gamma_E{}^L-\frac{1}{8}(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\ln\xi_L\,,$$

$$\Gamma_E{}^R=\gamma_E{}^R-\frac{1}{8}\eta(V_R{}^\dagger)_{1i}(V_R)_{i2}y_i\ln\xi_R\,,$$

$$\Gamma_M{}^L=\gamma_M{}^L\,,\qquad \Gamma_M{}^R=\gamma_M{}^R\,,\tag{B·1}$$

where $\xi_L$, $\xi_R$ are gauge parameters associated with $W_L$, $W_R$ respectively, and $\gamma$'s are given by

$$\gamma_E{}^L=\frac{1}{2}(V_L{}^\dagger)_{1i}(V_L)_{i2}g_1(x_i),\quad \gamma_E{}^R=\frac{1}{2}\eta(V_R{}^\dagger)_{1i}(V_R)_{i2}g_1(y_i),$$

$$\gamma_M{}^R=\frac{1}{2}(V_L{}^\dagger)_{1i}(V_L)_{i2}g_2(x_i),\quad \gamma_M{}^L=\frac{1}{2}\eta(V_R{}^\dagger)_{1i}(V_R)_{i2}g_2(y_i).\tag{B·2}$$

Here the functions $g_1(x)$ and $g_2(x)$ are defined by

$$g_1(x)=\left[-\frac{1}{4}-\frac{7}{3}\frac{1}{x-1}-\frac{13}{12}\frac{1}{(x-1)^2}+\frac{1}{2}\frac{1}{(x-1)^3}\right]x$$
$$+\left[-\frac{1}{4}+\frac{4}{3}\frac{1}{x-1}+\frac{35}{12}\frac{1}{(x-1)^2}+\frac{5}{6}\frac{1}{(x-1)^3}-\frac{1}{2}\frac{1}{(x-1)^4}\right]x\ln x\,,\tag{B·3}$$

$$g_2(x)=\left[\frac{1}{2}\frac{1}{x-1}+\frac{9}{4}\frac{1}{(x-1)^2}+\frac{3}{2}\frac{1}{(x-1)^3}\right]x-\frac{3}{2}\frac{x^3}{(x-1)^4}\ln x\,.\tag{B·4}$$

The contribution of the $\gamma$-exchange graph to the effective four-Fermi Lagrangian for $\mu\to ee\bar{e}$ is easily found to be

$$E_1{}^\gamma=-\Gamma_E{}^L\,,\quad E_2{}^\gamma=-\Gamma_E{}^R\,,\quad M_1{}^\gamma=-\Gamma_M{}^R\,,\quad M_2{}^\gamma=-\Gamma_M{}^L\,.\tag{B·5}$$

## Appendix C

—— *Induced $\mu\bar{e}Z$ and $\mu\bar{e}Z'$ Vertices* ——

Here we calculate the induced $\mu\bar{e}W_L{}^0$, $\mu\bar{e}W_R{}^0$ vertices instead of the induced $\mu\bar{e}Z$, $\mu\bar{e}Z'$ vertices. The effective four-Fermi interaction for $\mu\to ee\bar{e}$ induced by $Z$, $Z'$-exchange can then be found by the use of the method of Georgi and Weinberg.[23]

Write the induced neutral currents coupled to $W^0_{L\mu}$ and $W^0_{R\mu}$ as follows:

$$\Gamma_{L\kappa}=\frac{g^3}{2}\frac{1}{(4\pi)^2}\bar{e}\gamma_\kappa\left[F_{(-),L}\frac{1-\gamma_5}{2}+F_{(+),L}\frac{1+\gamma_5}{2}\right]\mu\,,$$

$$\Gamma_{R\kappa}=\frac{g^3}{2}\frac{1}{(4\pi)^2}\bar{e}\gamma_\kappa\left[F_{(-),R}\frac{1-\gamma_5}{2}+F_{(+),R}\frac{1+\gamma_5}{2}\right]\mu\,. \qquad (\text{C}\cdot 1)$$

We give below the result of our calculation (in the limit $\xi\to 0$ as stated before) of $F^{(i)}_{(\pm),L}$ and $F^{(i)}_{(\pm),R}$ ($i=\text{a}\sim\text{f}$), the contribution of graphs (a)$\sim$(f) in Fig. 3 to $F_{(\pm),L}$ and $F_{(\pm),R}$.

(i) $F_{(-),L}$

$$F^{(\text{a})}_{(-),L}=T_1{}^L(x)+(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\ln\xi_L\,,$$

$$F^{(\text{b})}_{(-),L}=T_3{}^L(x)-(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\left[\frac{1}{2}d(M_{WL})+\frac{7}{4}\ln\xi_L\right],$$

$$F^{(\text{c})}_{(-),L}=T_4{}^L(x)+(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\left[\frac{1}{2}d(M_{WL})+\ln\xi_L\right], \qquad (\text{C}\cdot 2)$$

(ii) $F_{(+),R}$

$$F^{(\text{a})}_{(+),R}=T_1{}^R(y)-\frac{1}{2}(V_R{}^\dagger)_{1i}(T_R{}^*)_{ij}(V_R)_{j2}\sqrt{y_iy_j}\,d(M_{WR})$$
$$+(V_R{}^\dagger)_{1i}(V_R)_{i2}y_i\ln\xi_R\,,$$

$$F^{(\text{b})}_{(+),R}=T_3{}^R(y)-(V_R{}^\dagger)_{1i}(V_R)_{i2}y_i\left[-\frac{1-\eta}{2}+\frac{\eta}{2}d(M_{WR})+\left(\frac{3}{2}+\frac{\eta}{4}\right)\ln\xi_R\right],$$

$$F^{(\text{c})}_{(+),R}=T_4{}^R(y)+(V_R{}^\dagger)_{1i}(V_R)_{i2}y_i\left[\frac{1}{2}d(M_{WR})+\ln\xi_R\right].$$

$$F^{(\text{d})\sim f)}_{(+),R}=\left[\frac{1}{2}(V_R{}^\dagger)_{1i}(T_R{}^*)_{ij}(V_R)_{j2}\sqrt{y_iy_j}-\frac{1}{2}(1-\eta)(V_R{}^\dagger)_{1i}(V_R)_{i2}y_i\right]d(M_{WR}),$$
$$(\text{C}\cdot 3)$$

(iii) $F_{(-),R}$

$$F^{(\text{a})}_{(-),R}=T_2^{LR}(x)+\frac{1}{2}(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i d(M_{WL}),$$

$$F^{(\text{b})}_{(-),R}=-(V_L{}^\dagger)_{1i}(V_L)_{i2}x_i\left[\frac{1}{2}+\frac{1}{2}d(M_{WL})+\frac{1}{4}\ln\xi_L\right],$$

$$F^{(\text{c})}_{(-),R}=0\,, \qquad (\text{C}\cdot 4)$$

(iv) $F_{(+),L}$

$$F^{(a)}_{(+),L}=T_2^{RL}(y)+\frac{1}{2}(V_R^\dagger)_{1i}(T_L)_{ij}(V_R)_{j2}\sqrt{y_i y_j}\,d(M_{WR}),$$

$$F^{(b)}_{(+),L}=-\eta(V_R^\dagger)_{1i}(V_R)_{i2}y_i\left[\frac{1}{2}+\frac{1}{2}d(M_{WR})+\frac{1}{4}\ln\xi_R\right],$$

$$F^{(c)}_{(+),L}=0\,,$$

$$F^{(d\sim f)}_{(+),L}=\left[-\frac{1}{2}(V_R^\dagger)_{1i}(T_L)_{ij}(V_R)_{j2}\sqrt{y_i y_j}+\frac{\eta}{2}(V_R^\dagger)_{1i}(V_R)_{i2}y_i\right]d(M_{WR}),\qquad (\text{C}\cdot 5)$$

where the ultraviolet divergent term $d(M)$ is

$$d(M)=\frac{1}{4-n}-\frac{1}{2}\gamma_E+\frac{1}{2}\ln 4\pi+\frac{1}{4}-\frac{1}{2}\ln\frac{M^2}{\mu^2}\,,\qquad (\text{C}\cdot 6)$$

in the dimensional method, and the functions $T_i$ are defined by

$$T_1^A(x)=-\frac{1}{2}(V_A^\dagger)_{1i}(T_A)_{ij}(V_A)_{j2}\left[x_i x_j\frac{\ln x_i-\ln x_j}{x_i-x_j}-(x_i+x_j)\frac{x_i\ln x_i-x_j\ln x_j}{x_i-x_j}\right]$$
$$+\frac{1}{4}(V_A^\dagger)_{1i}(T_A^*)_{ij}(V_A)_{j2}\frac{\sqrt{x_i x_j}}{x_i-x_j}\left[\left(1+3\frac{1}{1-x_i}\right)x_i\ln x_i-(x_i\to x_j)\right],$$

$$T_2^{AB}(x)=\frac{1}{2}(V_A^\dagger)_{1i}(T_B^*)_{ij}(V_A)_{j2}\left[x_i x_j\frac{\ln x_i-\ln x_j}{x_i-x_j}-(x_i+x_j)\frac{x_i\ln x_i-x_j\ln x_j}{x_i-x_j}\right]$$
$$-\frac{1}{4}(V_A^\dagger)_{1i}(T_B)_{ij}(V_A)_{j2}\frac{\sqrt{x_i x_j}}{x_i-x_j}\left[\left(1+3\frac{1}{1-x_i}\right)x_i\ln x_i-(x_i\to x_j)\right],$$
$$(\text{C}\cdot 7)$$

$$T_3^A(x)=\frac{3}{2}(V_A^\dagger)_{1i}(V_A)_{i2}x_i\left[\frac{1}{1-x_i}\left(\frac{x_i}{1-x_i}+\frac{x_i}{(1-x_i)^2}\right)\ln x_i\right],$$

$$T_4^A(x)=\frac{1}{4}(V_A^\dagger)_{1i}(V_A)_{i2}x_i\left[1-\frac{3}{1-x_i}-3\left(\frac{x_i}{1-x_i}+\frac{x_i}{(1-x_i)^2}\right)\ln x_i\right].$$

As remarked in the text, the ultraviolet divergences present in the gauge boson exchange graphs are cancelled out by the contribution from the Higgs exchange graphs, $F^{(d\sim f)}_{(\pm),R}$.

Performing the summation of graphs (a)~(f), we obtain

$$F_{(-),L}=f_{(-),L}+\frac{1}{4}(V_L^\dagger)_{1i}(V_L)_{i2}x_i\ln\xi_L\,,$$

$$F_{(+),R}=f_{(+),R}+\left(\frac{1}{2}-\frac{\eta}{4}\right)(V_R^\dagger)_{1i}(V_R)_{i2}y_i\ln\xi_R\,,$$

$$F_{(-),R}=f_{(-),R}-\frac{1}{4}(V_L^\dagger)_{1i}(V_L)_{i2}x_i\ln\xi_L\,,$$

$$F_{(+),L}=f_{(+),L}-\frac{\eta}{4}(V_R^\dagger)_{1i}(V_R)_{i2}y_i\ln\xi_R\,. \qquad \text{(C·8)}$$

Here $\xi$-independent (in the limit $\xi\to 0$) terms are given by

$$f_{(-),L}=T_1^L(x)+\frac{1}{4}(V_L^\dagger)_{1i}(V_L)_{i2}x_i$$
$$\times\left[1+3\left\{\frac{1}{1-x_i}+\left(\frac{1}{1-x_i}+\frac{1}{(1-x_i)^2}\right)x_i\ln x_i\right\}\right],$$

$$f_{(+),R}=T_1^R(y)+\frac{1}{4}(V_R^\dagger)_{1i}(V_R)_{i2}y_i$$
$$\times\left[3-2\eta+3\left\{\frac{1}{1-y_i}+\left(\frac{1}{1-y_i}+\frac{1}{(1-y_i)^2}\right)y_i\ln y_i\right\}\right],$$

$$f_{(-),R}=T_2^{LR}(x)-\frac{1}{2}(V_L^\dagger)_{1i}(V_L)_{i2}x_i\,,$$

$$f_{(+),L}=T_2^{RL}(y)-\frac{\eta}{2}(V_R^\dagger)_{1i}(V_R)_{i2}y_i\,. \qquad \text{(C·9)}$$

Extending the method of Georgi and Weinberg[23] to neutral current processes induced by one-loop effective vertices, we find that the contribution of $Z$ and $Z'$ exchange graphs to $\mu\to ee\bar{e}$ is

$$C_1{}^Z=\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{\eta}{1-\eta}\right)f_{(-),L}+\frac{1}{2}\frac{\eta}{1-\eta}f_{(-),R}\right]+\frac{1}{16}(V_L^\dagger)_{1i}(V_L)_{i2}x_i\ln\xi_L\,,$$

$$C_2{}^Z=\frac{1}{8}\frac{\eta}{1-\eta}[f_{(+),L}+f_{(+),R}]+\frac{1}{16}\eta(V_R^\dagger)_{1i}(V_R)_{i2}y_i\ln\xi_R\,,$$

$$D_1{}^Z=\frac{1}{8}\frac{\eta}{1-\eta}[f_{(-),L}+f_{(-),R}],$$

$$D_2{}^Z=\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{\eta}{1-\eta}\right)f_{(+),L}+\frac{1}{2}\frac{\eta}{1-\eta}f_{(+),R}\right],$$

$$E_1{}^Z=-\frac{1}{2}\frac{1}{1-\eta}[f_{(-),L}+\eta f_{(-),R}]-\frac{1}{8}(V_L^\dagger)_{1i}(V_L)_{i2}x_i\ln\xi_L\,,$$

$$E_2{}^Z=-\frac{1}{2}\frac{1}{1-\eta}[f_{(+),L}+\eta f_{(+),R}]-\frac{1}{8}\eta(V_R^\dagger)_{1i}(V_R)_{i2}y_i\ln\xi_R\,. \qquad \text{(C·10)}$$

## Appendix D

—— *Box Type Graphs* ——

There are many types of box graphs as shown in Fig. 4. The computation of the graphs (c), (d), (e) and (f) involves some technical complication because of the presence of gauge dependent terms. These terms are found to be cancelled out by the contribution from one-loop graphs with physical Higgs exchange (of the types (i) and (j) in Fig. 4). Such contributions from physical Higgs exchange graphs are taken into account in the result presented below.

Denote the contribution from each graph in Fig. 4 by $B^{(i)}$. Then the box graph contribution to the effective Lagrangian for $\mu \to ee\bar{e}$ is found to be

$$C_1{}^{\square} = B^{(b)} + B^{(f)}\,, \qquad C_2{}^{\square} = B^{(a)} + B^{(e)}\,,$$
$$D_1{}^{\square} = B^{(h)}\,, \qquad D_2{}^{\square} = B^{(g)}\,, \qquad S_1{}^{\square} = B^{(c)}\,, \qquad S_2{}^{\square} = B^{(d)}\,. \qquad (D\cdot 1)$$

The result of our calculation of $B^{(i)}$ is summarized as follows:

$$B^{(a)} = b^{(a)} - \frac{1}{16}\eta (V_R{}^{\dagger})_{1i}(V_R)_{i2} y_i \ln \xi_R\,,$$
$$b^{(a)} = -\frac{1}{4}\eta \Lambda_{ij}^{(a)} I_1(y_i, y_j),$$
$$B^{(b)} = b^{(b)} - \frac{1}{16}(V_L{}^{\dagger})_{1i}(V_L)_{i2} x_i \ln \xi_L\,,$$
$$b^{(b)} = -\frac{1}{4}\Lambda_{ij}^{(b)} I_1(x_i, x_j),$$
$$B^{(c)} = b^{(c)} = \frac{1}{4}\Lambda_{ij}^{(c)} \frac{m_i m_j}{M_{WR}^2} I_2(M_{WL}, M_{WR}, m_i, m_j),$$
$$B^{(d)} = b^{(d)} = \frac{1}{4}\Lambda_{ij}^{(d)} \frac{m_i m_j}{M_{WR}^2} I_2(M_{WL}, M_{WR}, m_i, m_j),$$
$$B^{(e)} = b^{(e)} = \frac{1}{8}\eta \Lambda_{ij}^{(e)} \sqrt{y_i y_j}\, I_3(y_i, y_j),$$
$$B^{(f)} = b^{(f)} = \frac{1}{8}\Lambda_{ij}^{(f)} \sqrt{x_i x_j}\, I_3(x_i, x_j),$$
$$B^{(g)} = b^{(g)} = \frac{1}{16}\Lambda_{ij}^{(g)} I_4(M_{WL}, M_{WR}, m_i, m_j),$$
$$B^{(h)} = b^{(h)} = \frac{1}{16}\Lambda_{ij}^{(h)} I_4(M_{WL}, M_{WR}, m_i, m_j), \qquad (D\cdot 2)$$

where $m_i \equiv m_{\nu_i}$ and the mixing factor $\Lambda_{ij}$ are given by

$$\Lambda_{ij}^{(a)}=(V_R^\dagger)_{1i}(V_R)_{i2}(V_R^\dagger)_{1j}(V_R)_{j1}\,,\qquad \Lambda_{ij}^{(b)}=(V_L^\dagger)_{1i}(V_L)_{i2}(V_L^\dagger)_{1j}(V_L)_{j1}\,,$$
$$\Lambda_{ij}^{(c)}=(V_R^\dagger)_{1i}(V_L)_{i2}(V_L^\dagger)_{1j}(V_R)_{j1}\,,\qquad \Lambda_{ij}^{(d)}=(V_L^\dagger)_{1i}(V_R)_{i2}(V_R^\dagger)_{1j}(V_L)_{j1}\,,$$
$$\Lambda_{ij}^{(e)}=(V_R^T)_{1i}(V_R)_{i2}(V_R^\dagger)_{1j}(V_R^*)_{j1}\,,\qquad \Lambda_{ij}^{(f)}=(V_L^T)_{1i}(V_L)_{i2}(V_L^\dagger)_{1j}(V_L^*)_{j1}\,,$$
$$\Lambda_{ij}^{(g)}=(V_L^T)_{1i}(V_R)_{i2}(V_R^\dagger)_{1j}(V_L^*)_{j1}\,,\qquad \Lambda_{ij}^{(h)}=(V_R^T)_{1i}(V_L)_{i2}(V_L^\dagger)_{1j}(V_R^*)_{j1}\,. \tag{D·3}$$

The functions $I_i$ are defined by

$$I_1(x,x')=x\left[\frac{1}{4}-\frac{3}{4}\frac{1}{(1-x)(1-x')}+\left\{\frac{1}{x-x'}\left(\frac{1}{4}+\frac{3}{2}\frac{1}{1-x}-\frac{3}{4}\frac{1}{(1-x)^2}\right)x\ln x+(x\leftrightarrow x')\right\}\right],$$

$$I_2(M,M',m,m')=1-3\frac{M^2M'^2}{M^2-M'^2}\left\{\frac{M^2\ln M^2}{(M^2-m^2)(M^2-m'^2)}-(M\to M')\right\}$$
$$-\left\{\frac{1}{m^2-m'^2}\left(1+3\frac{M^2M'^2}{(M^2-m^2)(M'^2-m^2)}\right)m^2\ln m^2+(m\leftrightarrow m')\right\},$$

$$I_3(x,x')=1+\left[\frac{1}{x-x'}\left\{-3\frac{1}{1-x}-\left(1+3\frac{1}{(1-x)^2}\right)x\ln x\right\}+(x\leftrightarrow x')\right], \tag{D·4}$$

$$I_4(M,M',m,m')=\frac{m^2}{M'^2(m^2-m'^2)}\left[\{-3(M^2+M'^2)+m^2\}\ln m^2\right.$$
$$\left.+3\frac{1}{M^2-M'^2}\left(\frac{M^6}{M^2-m^2}\ln\frac{m^2}{M^2}-\frac{M'^6}{M'^2-m^2}\ln\frac{m^2}{M'^2}\right)\right]+(m\leftrightarrow m').$$

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