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LEPTON-HADRON PROCESSES BEYOND LEADING ORDER IN QUANTUM CHROMODYNAMICS

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ABSTRACT

We present a summary of QCD formulae describing the effects of scaling violation in lepton-hadron processes, with the inclusion of recently derived higher order corrections. Deep inelastic leptoproduction, one-hadron inclusive distributions in leptoproduction and in e^+e^- annihilation and Drell-Yan processes are discussed in detail. Higher order corrections to parton densities, fragmentation functions and to lepton-parton cross-sections in the above processes are presented in a common factorization scheme, so that a comparative analysis of various processes as well as an independent analysis of each of them is possible. A discussion of the various scheme dependences at next-to-leading level is also included.

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1. - INTRODUCTION

Recently, a number of higher order QCD calculations for inclusive spectra in lepton-hadron processes have been performed by various groups [1)-9)] : the two-loop corrections to the evolution of parton densities and fragmentation functions as well as the one-loop corrections to all typical lepton-parton cross-sections are available in the literature.

As a consequence, we have now at our disposal all the ingredients necessary for carrying out a precise systematic analysis of the scaling violation effects in the existing data for various hard processes in the framework of perturbative QCD.

The predictive power of perturbative QCD follows from the universality of parton densities : once extracted from the data in one process (or in a set of processes), they can be used to derive an absolute prediction for any other hard process, if only the corresponding lepton-parton cross-section is known.

However, after the next-to-leading effects have been included, one should proceed with care : the parton density extracted in one process can be merged with the partonic cross-section derived for another process only if the QCD formulae for both processes have been derived in the same factorization scheme. This is due to the fact that the distinction between QCD corrections to parton densities and to lepton-parton cross-sections is unique only at the leading log level and is a matter of definition beyond it.

In fact, various higher order calculations quoted above have been performed using different factorization schemes. Usually, the authors define their own favourable scheme which is very general and universal but then the explicit calculations are done for some particular corrections - just because the other effects have been already derived by other groups.

Therefore, even if the results of Refs. 1)-9), when combined together and transformed into one factorization scheme, provide the complete prediction for any process, the present situation is very unsatisfactory for experimentalists : they would have still to do some theoretical work in order to combine in a correct way the results of various groups.

The aim of this paper is to introduce some systematics into the existing set of higher order calculations. As everybody does, we work in our own favourable scheme, defined in Ref. 7), and used in Refs. 7), 9), for the derivation of the two-loop corrections for parton densities and fragmentation functions. However, the one-loop corrections to various hard processes which are available in the literature [3)-6)] are also transformed into our scheme.

Therefore, we present both the general formalism as well as the explicit formulae for all "canonical" hard processes listed in the abstract in a unique factorization scheme. The results presented here can be used both for deriving the absolute prediction for any process as well as for relating various processes, using the universality of parton densities.

The paper is organized as follows. In Section 2 we fix the notation and we present the general structure of the hard lepton-hadron cross-section as predicted by QCD. In Section 3 we present in a pictorial way the main steps of the factorization scheme used in the paper. Section 4 contains a short review of existing higher order results which are relevant for our presentation. The evolution equations for parton densities and fragmentation functions are discussed in Section 5. We present there the explicit solutions for moments and the formal solutions in the x space. The QCD predictions for the processes listed in the abstract are collected in Sections 6 to 9. Section 10 contains a discussion of the nature of various scheme dependences affecting higher order results. The explicit formulae for the relevant lepton-parton cross-sections are given in the Appendix, while for consistently defined evolution probabilities we refer to the results contained in Refs. 7) and 9).

2. - GENERAL STRUCTURE OF THE HARD CROSS-SECTION IN QCD

The QCD prediction for the inclusive cross-section in any hard process can be written, by neglecting power law corrections, in the following form :

$$d\sigma_{\ell h}^{\ell' h'}(\ell,h;\ell',h') = \sum_{pp'} \prod_{i=1}^{n} \prod_{k=1}^{m} \int_0^1 dz_i \int_0^1 dz_k \, F_{h_i}^{p_i}(z_i,Q^2)\, d\hat{\sigma}_{\ell p}^{\ell' p'}(\ell, zh; \ell', \frac{h'}{z'})\, F_{p'_k}^{h'_k}(z'_k,Q^2) \tag{2.1}$$

In this section we explain the notation used in Eq. (2.1). The notational conventions defined here will be used throughout this paper for all particular processes.

The lower (upper) indices of σ, $\hat{\sigma}$ and F refer to the initial state (final state) particles. The indices ℓ (ℓ') and h (h') stand for sets of initial (final) leptons and hadrons, i.e., $\ell = \{\ell_1,\ell_2,\ldots\}$, $\ell' = \{\ell'_1,\ell'_2,\ldots\}$,

$h = \{h_1, h_2, \ldots, h_n\}$, $h' = \{h'_1, h'_2, \ldots, h'_m\}$. Thus $d\sigma_{\ell h}^{\ell' h'}$ is the semi-inclusive differential cross-section for the process :

$$\ell + h \to \ell' + h' + \text{anything} \tag{2.2}$$

Some of the sets ℓ, ℓ', h, h' may be empty for a given hard process. At least one of the leptonic sets ℓ, ℓ' must be present.

The arguments of $d\sigma$ are the four-momenta of particles indicated by the indices ℓ, h, ℓ', h'. For example, the cross-sections for the processes listed in the abstract will be denoted as follows :

$$d\sigma_{ep}^{e}(\ell,h;\ell')\ ,\ d\sigma_{e^+e^-}^{\pi^+}(\ell_1,\ell_2;h')\ ,\ d\sigma_{\nu p}^{\mu^- K^+}(\ell,h;\ell',h')\ ,\ d\sigma_{p\bar{p}}^{\mu^+\mu^-}(h_1,h_2;\ell'_1\ell'_2)$$

The formula (2.1) has the kinematical structure of the parton model : the incoming hadrons h are decomposed into partons p (quarks and gluons), the partons p interact inclusively with leptons :

$$p + \ell \to p' + \ell' + \text{anything} \tag{2.3}$$

and the final state partons p' fragment into hadrons h'. The lepton-parton interaction in Eq. (2.3) is described by a "short-distance" cross-section $d\hat{\sigma}_{\ell p}^{\ell' p'}(\ell,p;\ell' p')$. The notation for $d\hat{\sigma}$ is the same as for $d\sigma$ with all hadrons replaced by partons. As seen in Eq. (2.1), the variables $z(z')$ are parton versus hadron (or vice versa) four-momentum fractions :

$$p = zh \equiv \{p_1, \ldots, p_n\} = \{z_1 h_1, \ldots, z_n h_n\}\ ;\ p' = \frac{h'}{z'} \equiv \{p'_1, \ldots, p'_m\} = \left\{\frac{h'_1}{z'_1}, \ldots, \frac{h'_m}{z'_m}\right\} \tag{2.4}$$

The functions $F_h^p(z,Q^2)$ $(F_{p'}^{h'}(z',Q^2))$ describe the parton densities in hadrons (fragmentation of partons into hadrons). If the hadronic set $h(h')$ is empty, the corresponding set of parton densities (fragmentation functions) is absent in Eq. (2.1).

Let us denote collectively all four-momenta entering Eq. (2.1) by v, i.e., $v = \{v_1, v_2, \ldots\} = \ell, \ell', p, p', h, h'$.

The perturbative QCD can be applied in the kinematical region where all invariants $v_i \cdot v_j$ $(i \neq j)$ are large and all dimensionless fractions $v_i \cdot v_j / v_k \cdot v_\ell$ $(i \neq j,\ k \neq \ell)$ are finite and different from zero. Q^2 in Eq. (2.1) denotes one of the large invariants $v_i \cdot v_j$, in principle no matter which one. In practice it is convenient to construct Q^2 from leptonic momenta. All masses are neglected, i.e., $v^2 = \{v_1^2, v_2^2, \ldots\} = 0$. In this limit, Eq. (2.1) is exact in perturbative QCD. All corrections to Eq. (2.1) are of the form $O(v^2/Q^2)$ and are included in the "higher twist" effects which are neglected in this paper.

Both σ and $\hat{\sigma}$ can be expressed as functions of Q^2 and of appropriate dimensionless fractions. Denoting collectively by x the set of fractions, we can write $d\hat{\sigma}$ in the form

$$\frac{d\hat{\sigma}}{dx} = \frac{1}{Q^2}\, C(x,Q^2) \tag{2.5}$$

In the parton model, the dimensionless functions $F_h^p(z,Q^2)$, $F_{p'}^{h'}(z',Q^2)$, $C_{\ell p}^{\ell' p'}(x,Q^2)$ do not depend on Q^2 at all.

In perturbative QCD, the kinematical structure of the parton model formulae remains valid (up to higher twist effects) but scaling is broken. The Q^2 dependence enters through the effective (running) coupling constant $\alpha(Q^2)$. The functions $C_{\ell p}^{\ell' p'}(x,Q^2)$ are given by a power series in $\alpha(Q^2)$:

$$C_{\ell p}^{\ell' p'}(x,Q^2) = C_{\ell p}^{(0)\ell' p'}(x) + \frac{\alpha(Q^2)}{2\pi}\, C_{\ell p}^{(1)\ell' p'}(x) + \ldots \tag{2.6}$$

where $C_{\ell p}^{(0)\ell' p'}(x)$ is the parton model result. The functions $F(z,Q^2)$ evolve with Q^2 according to the evolution equations :

$$Q^2 \frac{d}{dQ^2} F_h^p(x,Q^2) = \frac{\alpha(Q^2)}{2\pi} \sum_{p'} P^{pp'}(x,\alpha(Q^2)) \otimes F_h^{p'}(x,Q^2) \tag{2.7a}$$

$$Q^2 \frac{d}{dQ^2} F_p^h(x,Q^2) = \frac{\alpha(Q^2)}{2\pi} \sum_{p'} P_{pp'}(x,\alpha(Q^2)) \otimes F_{p'}^h(x,Q^2) \tag{2.7b}$$

where the probabilities $P_{pp'}$, $P^{pp'}$ are given by power series in $\alpha(Q^2)$:

$$P^{pp'}(x,\alpha) = P^{(0)pp'}(x) + \frac{\alpha}{2\pi} P^{(1)pp'}(x) + \ldots \tag{2.8a}$$

$$\mathcal{P}_{pp'}(x,\alpha) = \mathcal{P}^{(0)}_{pp'}(x) + \frac{\alpha}{2\pi}\,\mathcal{P}^{(1)}_{pp'}(x) + \ldots . \tag{2.8b}$$

The symbol $\otimes$ in Eqs. (2.7) denotes the convolution, i.e. $C(x) = A(x) \otimes B(x)$ means

$$C(x) = \int_0^1 dx_1 \int_0^1 dx_2 \; A(x_1)\, B(x_2)\, \delta(x - x_1 x_2) = \int_x^1 \frac{dz}{z} A(z)\, B\left(\frac{x}{z}\right) \tag{2.9}$$

In the following, it will be more convenient to use the notation

$$\mathcal{P}^{(i)\,pp'}(x) = \mathcal{P}^{(i,S)}_{pp'}(x) \quad , \quad \mathcal{P}^{(i)}_{pp'}(x) = \mathcal{P}^{(i,T)}_{pp'}(x) \tag{2.10}$$

where the letters S(T) indicate spacelike (timelike) probability. We will also refer to the functions F^p_h as to the spacelike parton densities and to the functions F^h_p (i.e., to the fragmentation functions) as to the timelike parton densities.

The Q^2 dependence of $\alpha(Q^2)$ is determined by the renormalization group equation [10),11)]

$$Q^2 \frac{d}{dQ^2}\, \alpha(Q^2) = -\alpha(Q^2)\, \bar{\beta}\left[\alpha(Q^2)\right] \tag{2.11}$$

where the β function $\beta(\alpha)$ is given by a power series in α :

$$\bar{\beta}(\alpha) = \beta_0 \frac{\alpha}{4\pi} + \beta_1 \left(\frac{\alpha}{4\pi}\right)^2 + \ldots . \tag{2.12}$$

where

$$\begin{aligned} \beta_0 &= \frac{11}{3} C_G - \frac{2}{3} f \\ \beta_1 &= \frac{34}{3} C_G^2 - \frac{10}{3} C_G \cdot f - 2 C_F \cdot f \end{aligned} \tag{2.13}$$

and our notation for colour factors is as follows :

$$C_F = \frac{N^2-1}{2N} \quad , \quad C_G = N \quad , \quad T_R = \frac{1}{2} f \tag{2.14}$$

with N the number of colours and f the number of flavours. The constants β_0 and β_1 are independent of the factorization scheme.

The solution of Eq. (2.11) can be cast into the form

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0} \frac{1}{\ln Q^2/\Lambda^2} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln \ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2} + O\left(\frac{1}{\ln^2 Q^2/\Lambda^2}\right) \right] \tag{2.15}$$

The value of Λ, fixing the strength of scaling violation effects, should be extracted from the data ; its scheme dependence will be discussed in the last section.

The series in Eqs. (2.6) and (2.8) can be derived order by order in perturbative QCD. In practice, we have to truncate the series rather soon, due to the complexity of higher order calculations. The simplest approximation consists in keeping only the lowest order terms in Eqs. (2.6), (2.8) and is called the leading log approximation. At the leading log level, the partonic cross-sections are given by the parton model expressions and the probabilities $P^{(0)}$ are known as the Parisi-Altarelli probabilities [12]. They are universal for both space and timelike evolutions :

$$P^{(0,S)}_{pp'}(x) = P^{(0,T)}_{p'p}(x) \tag{2.16}$$

[the origin of the transposition in Eq. (2.16) is obvious : as seen from Eqs. (2.7), $P^{(0,S)}_{pp'}$ describes the transition $p \leftarrow p'$ whereas $P^{(0,T)}_{pp'}$ describes the transition $p \rightarrow p'$].

In this paper we present the results for $P^{(1,S)}_{pp'}(x)$, $P^{(1,T)}_{pp'}(x)$ and the corrections $C^{(1)\ell' p'}_{\ell p}(x)$ for the processes listed in the abstract. These functions are referred to collectively as the next-to-leading corrections.

The relevance of the next-to-leading terms in analyzing the data follows immediately from Eq. (2.15) : as seen, the change of Λ by a fixed amount (e.g., by a factor of 4) is equivalent to adding a next-to-leading contribution to the probability of the form :

$$\Delta P^{(1)}(x) = (\beta_0 \ln 4)\, P^{(0)}(x) \tag{2.17}$$

It means that the comparison of values of Λ, extracted from various hard processes by means of the leading log formulae is not very instructive. For example, if we get $\Lambda = 0.5$ GeV for leptoproduction and $\Lambda = 2$ GeV for Drell-Yan with reasonable χ^2 for both processes, it neither supports nor contradicts QCD. It might indicate that, if QCD is right, the difference between the next-to-leading corrections to both processes should be of order $\Delta P^{(1)}(x)$, given by Eq. (2.17).

3. - FACTORIZATION PROCEDURE

In order to calculate the QCD prediction for a given hard process, one has to organize the resummation of Feynman diagrams contributing to the corresponding cross-section in such a way that the parton model structure as in Eq. (2.1) becomes explicit. This procedure is called factorization. It has been proved [13),14)] that the inclusive cross-section for any hard process can be written in a factorized form.

The factorization procedure is not unique beyond the leading log approximation : various schemes provide different intermediate results for the coefficients $C^{(i)\ell'p'}_{\ell p}(x)$ and $P^{(i)}(x)$ in the series (2.6) and (2.8). The final result for $d\sigma^{\ell'p'}_{\ell p}$ is certainly unique but there is a freedom in the definition of parton densities, fragmentation functions and "short-distance" cross-sections. This ambiguity can be used in order to impose some additional constraints on parton spectra or (and) partonic cross-sections.

In this paper we use everywhere the factorization scheme proposed in Ref. 7), based on the general factorization programme of Ref. 13). The parton and fragmentation function densities in this scheme are characterized by the following properties :

a) they are universal (i.e., they are defined without reference to any particular hard process) ;

b) they satisfy the exact sum rules of the parton model :

   i) fermion number conservation

$$\int_0^1 dx \left[P^{(S)}_{qq}(x,\alpha) - P^{(S)}_{q\bar{q}}(x,\alpha) \right] = \int_0^1 dx \left[P^{(T)}_{qq}(x,\alpha) - P^{(T)}_{q\bar{q}}(x,\alpha) \right] = 0 \tag{3.1}$$

ii) momentum conservation :

$$\sum_{p'} \int_0^1 dx\, x\, \mathcal{P}^{(S)}_{p'p}(x,\alpha) = \sum_{p'} \int_0^1 dx\, x\, \mathcal{P}^{(T)}_{pp'}(x,\alpha) = 0 \tag{3.2}$$

In the following, we describe briefly the main steps of the factorization scheme used in the paper. Let us take as an example the deep inelastic electroproduction. First we write the inclusive lepton-hadron cross-section as the modulus squared of the amplitude for the process $\ell + h \to \chi$ (Fig. 1). Next, we perform the decomposition of σ in the hadronic channel into two-particle irreducible blobs $\tilde{F}^p_h$, K and $\tilde{\sigma}$ (Fig. 2). In this way we separate the full process into two well-defined objects : the lepton-parton "cross-section" $\tilde{\sigma}$ and the "parton density" $F' = (1/1-K)\tilde{F}$. We are working in the axial gauge and therefore, as it has been proved in Ref. 13) the blob $\tilde{\sigma}$ contains no mass singularities - all of them are collected in F'.

The decomposition in Fig. 3 already bears some resemblance to the formula in Eq. (2.1) ; however, both objects $\tilde{\sigma}$ and F are still coupled by Lorentz (spinor) indices and four-momentum integrations. Now we make a set of formal manipulations with the "generalized ladder" in Fig. 2. Each kernel K is decomposed

$$K = \mathcal{P}K + (1-\mathcal{P})K \tag{3.3}$$

where P is some suitable defined projection operator, acting on parton lines joining two neighbouring kernels. The action of the projector P is visualized in Fig. 4. The parton lines are disconnected which indicates that P decouples the blob K from the upper part of the ladder in vector (spinor) indices by projecting on the physical polarizations of the propagating parton p. Also the momentum integration is decoupled in the following way : P projects out the logarithmically singular part [pole in ε for $n = 4+\varepsilon$ $(\varepsilon > 0)$ dimensions] of the integral $\int_0 (dp^2/p^2)$ over the virtual mass p^2 of the parton (it is indicated by a dot on the line entering the blob K) and puts the line entering the upper part of the ladder on shell (the round line). Consequently, the momentum of the round line is parallel to the incoming hadron : $p = zh$ and the term PK is coupled to the upper part of the ladder only by one integral over z.

Performing the decomposition (3.3) for each kernel K in the ladder in Fig. 2 we get finally the structure in Fig. 5 : $\hat{\sigma}$ and F are now Lorentz scalars coupled only by one z integral and we get precisely the formula (2.1) :

$$d\sigma_{\ell h}^{\ell'}(\ell,h;\ell') = \sum_{p}\int_0^1 dx\, F_h^p(x,Q^2)\, d\hat{\sigma}_{\ell p}^{\ell'}(\ell,xp;\ell') \qquad (3.4)$$

The formal expressions for $\hat{\sigma}$ and F are

$$\hat{\sigma} = \tilde{\sigma}\,\frac{1}{1-(1-P)K}\;,\quad F = \frac{1}{1-P\,\dfrac{K}{1-(1-P)K}}\,P\tilde{F} \qquad (3.5)$$

Using the renormalization group methods we derive the evolution equation (2.7a) for F_h^p. The coefficients of the series (2.6), (2.8) can be calculated by expanding the expressions in (3.5) in power series in α.

We want to stress the following properties of the formulae (3.5) :

a) the parton density F is independent of the hard process (all details concerning the particular leptonic current are contained in $\tilde{\sigma}$) ;

b) the partonic cross-section $\hat{\sigma}$ does not contain mass singularities [since both $\tilde{\sigma}$ and $(1-P)K$ do not], i.e., it can be obtained by a perturbative expansion in $\alpha(Q^2)$;

c) the ambiguity in the definition of $\hat{\sigma}$ and F is implied by that in the definition of the projector P. In our scheme, P projects out only the singular part of the integral $\int(dp^2/p^2)$ (pole in ε in the dimensional regularization of mass singularities) ; we could, however, change the definition of P by shifting any finite part of $(1-P)K$ into PK in Eq. (3.3) ; this does not spoil the finiteness of the partonic cross-section $\hat{\sigma}$ but both $\hat{\sigma}$ and F are affected by the redefinition of P, as seen in Eqs. (3.5) ; it has been shown in Refs. 7), 9) that our (minimal) definition of P implies the sum rules (3.1)-(3.2).

The presentation of the scheme given above is obviously very simplified. The reader interested in technical details is referred to 7), 9). However, even without going into details it should now be obvious how to proceed for any other hard process : the procedure described above should be just repeated in any hadronic channel independently. In Figs. 6-8 we illustrate the factorization for inclusive leptoproduction, inclusive annihilation and the Drell-Yan process.

The formal expression for the timelike density $F^{(T)}$ in Figs. 6, 7 is very similar to the one given by Eq. (3.5) in the spacelike region. There is, however, a difference in kinematics : the integrals $\int(dp^2/p^2)$ extend now over the positive values of p^2, which introduces some differences at the subleading level as compared with the spacelike case $(k^2 < 0)$. As a consequence :

$$\mathcal{P}^{(i,S)}_{pp'}(x) \neq \mathcal{P}^{(i,T)}_{p'p}(x) \quad \text{for} \quad i>0 \tag{3.6}$$

The pictorial constructions in Figs. 6-8 clearly illustrate that the densities $F^{(S)}$ and $F^{(T)}$ for a specific hadron are identical in all hard processes.

The rule for calculating $d\hat{\sigma}^{\ell'p'}_{\ell p}$ to order α in any hard process is very simple :

a) calculate the appropriate "physical" inclusive cross-section $d\sigma^{\ell'p'}_{\ell p}$ with on-shell massless partons, using the dimensional regularization of ultra-violet divergences in the $\overline{MS}$ scheme in $n = 4-\varepsilon$ dimensions $(\varepsilon > 0)$;

b) regulate the mass singularities using dimensional regularization in $n = 4+\varepsilon$ dimensions $(\varepsilon > 0)$;

c) subtract the mass singularity pole and take $\mu^2 = (1/4\pi)e^{\gamma}Q^2$ (μ is the unit of mass in dimensional regularization and γ is the Euler constant) ; the result is equal to $d\hat{\sigma}^{\ell'p'}_{\ell p}$; the partonic cross-section $d\hat{\sigma}$ defined in this way is gauge invariant, therefore the calculation can be done in any gauge.

4. - EXISTING RESULTS FOR THE NEXT-TO-LEADING CORRECTIONS

4.1. - Probabilities $P^{(1,S)}$, $P^{(1,T)}$

The first calculation of $P^{(1,S)}(x)$ has been performed by means of the OPE technique [1),2)]. In fact, the two-loop contributions to the anomalous dimensions $\gamma^{(1)}(n)$ of the Wilson operators have been calculated. In the meantime, however, it becomes clear that there is a one-to-one correspondence between anomalous dimensions $\gamma^{(1)}(n)$ and moments of $P^{(1,S)}(x)$. The results of Refs. 1), 2) presented originally in a very complicated form were then considerably simplified [15)] and finally inverted to x space [16)].

The diagrammatic calculation of $P^{(1,S)}(x)$ and $P^{(1,T)}(x)$ within the factorization scheme presented in Section 3 has been done in Refs. 7) and 9). It has been proved there that in this scheme the spacelike probabilities $P^{(1,S)}(x)$ coincide exactly with the anomalous dimensions of Wilson operators in the minimal subtraction scheme, inverted into the x space.

In fact, however, the results of Refs. 2) and 9) do not agree, i.e., at least one of them is wrong. There is a difference for the probability $P_{GG}^{(1,S)}(x)$ in the term proportional to the colour factor C_G^2. The simple check of momentum conservation [Eq. (3.2)] cannot help in finding the wrong result since both formulae satisfy the momentum sum rule. For this reason we have done an additional check of our results 9) : using the results for individual diagrams contributing to $P_{pp'}^{(1,S)}$ we have calculated

$$\beta_1\Big|_{qqG} - \beta_1\Big|_{GGG} \tag{4.1}$$

where β_1 is the two-loop contribution to the beta function [Eq. (2.13)] and the subscripts in Eq. (4.1) indicate two three-point vertices in QCD, used for an independent derivation of β_1.

In the $\overline{\mathrm{MS}}$ scheme, the difference in Eq. (4.1) should be zero because of gauge invariance. We have got zero. In the formula (4.1) diagrams contributing to $P_{GG}^{(1,S)}$ are cancelled versus diagrams contributing to other probabilities $P_{pp'}^{(1,S)}$ and for the latter both calculations gave the same result. Therefore vanishing of the difference in Eq. (4.1) provides a severe check of our result for $P_{GG}^{(1,S)}(x)$.

A suggestive check has recently been completed by the authors of Ref. 17), who have shown that our results, when translated into a suitable renormalization scheme, imply the validity, at next-to-leading level, of a simple "supersymmetric" relation among the four probabilities of the singlet matrix.

The calculation of $P^{(1,T)}$ is relatively simple once $P^{(1,S)}$ is known since, as it has been discussed in 7), both probabilities are connected (apart from simple corrections) by analytic continuation. This fact has also been used by the authors of Ref. 18) to derive $P^{(1,T)}$ using $P^{(1,S)}$ derived by the OPE method. In consequence, there is also discrepancy in the formulae for $P_{GG}^{(1,T)}(x)$ given in Refs. 9), 18), which is the analytic continuation of the discrepancy for $P^{(1,S)}(x)$ mentioned above.

To summarize, there are two complete calculations of the two-loop probabilities $P^{(1)}$: the one presented in Refs. 1), 2), 18) and the other one given in Refs. 7), 9). In at least one of the above sets of formulae, both $P_{GG}^{(1,S)}$ and $P_{GG}^{(1,T)}$ are wrong.

4.2. - Cross-sections $d\sigma^{\ell' p'}_{\ell p}$ -

The corrections $C^{(1)}$ for all leptoproduction cross-sections were first derived by using the OPE method [3] (we have done an independent calculation within our scheme [7] and we agree with OPE results).

The corrections $C^{(1)}$ for leptoproduction and for the Drell-Yan process have been derived in Ref. 4) and for one hadron inclusive leptoproduction and one hadron inclusive e^+e^- annihilation in Refs. 5), 6).

The factorization method used in the papers quoted above is different from our approach : instead of analyzing the QCD prediction for each hard process independently, the authors derive only relations between various processes at a given value of Q^2. Nevertheless, since they use the dimensional regularization of ultra-violet and mass singlarities, it is possible to extract the functions $C^{(1)}(x)$ in the form appropriate for our scheme from the intermediate results of the calculation presented in Refs. 3)-6). [We have derived independently the correction to the one-hadron inclusive e^+e^- annihilation by relating it by analytic continuation with the correction to leptoproduction [3] and we have got the same result as in Ref. 5).]

In summary, in the presentation of the results given in this paper, we use :

a) $P^{(1,S)}$ and $P^{(1,T)}$ from Refs. 7), 9) ;

b) $C^{(1)}$ for leptoproduction from Refs. 3), 7) ;

c) $C^{(1)}$ for single hadron inclusive e^+e^- annihilation and inclusive leptoproduction from Refs. 5), 6) ;

d) $C^{(1)}$ for the Drell-Yan process from Refs. 4).

5. - EVOLUTION EQUATION

In this Section we present the solution of the evolution equations (2.7). For this reason we introduce the following simplifications.

a) We drop hadronic indices and space-time indices, i.e., we consider the equations :

$$Q^2 \frac{d}{dQ^2} p(x,Q^2) = \frac{\alpha(Q^2)}{2\pi} \sum_{p'} \mathcal{P}_{pp'}(x,\alpha(Q^2)) \otimes p'(x,Q^2) \qquad (5.1)$$

where

$$p = \{q_1, \dots, q_f, \bar{q}_1, \dots, \bar{q}_f, G\} = \{u, d, c, s, \dots, \bar{u}, \bar{d}, \bar{c}, \bar{s}, \dots, G\}$$

The solutions for F_h^p and F_p^h can be obtained once the solution for p is known by obvious replacements : $p \to F_h^p$, $P \to P^{(S)}$ in the spacelike region and $p \to F_p^h$, $P \to P^{(T)}$ in the timelike region.

b) We do not write explicitly the x dependence of functions appearing in this section [i.e., we write A, B, C, 1 instead of A(x), B(x), C(x), $\delta(1-x)$]. All products like $C = AB$ can then be interpreted either in x space as convolutions [see Eq. (2.9)] or in the n space (n - number of moments) as the product of moments $C(n) = A(n)B(n)$ where

$$\mathcal{Z}(n) = \int_0^1 dx\, x^{n-1} \mathcal{Z}(x) \qquad ; \mathcal{Z} = A, B, C \tag{5.2}$$

c) We will also often neglect the Q^2 and $\alpha(Q^2)$ arguments, i.e., we will write Eq. (5.1) in the form :

$$Q^2 \frac{d}{dQ^2} p = \frac{\alpha}{2\pi} \sum_{p'} P_{pp'} p' \tag{5.3}$$

Writing explicitly all types of partons, we have :

$$Q^2 \frac{d}{dQ^2} q_i = \frac{\alpha}{2\pi} \left[\sum_k \left(P_{q_i q_k} q_k + P_{q_i \bar{q}_k} \bar{q}_k \right) + P_{q_i G} G \right]$$

$$Q^2 \frac{d}{dQ^2} \bar{q}_i = \frac{\alpha}{2\pi} \left[\sum_k \left(P_{\bar{q}_i q_k} q_k + P_{\bar{q}_i \bar{q}_k} \bar{q}_k \right) + P_{\bar{q}_i G} G \right]$$

$$Q^2 \frac{d}{dQ^2} G = \frac{\alpha}{2\pi} \left[\sum_k \left(P_{G q_k} q_k + P_{G \bar{q}_k} \bar{q}_k \right) + P_{GG} G \right] \tag{5.4}$$

The system (5.4) can be considerably simplified after decomposing P_{qq} and $P_{q\bar{q}}$ into "valence" and "sea" parts :

$$\begin{aligned} P_{q_i q_k} &= \delta_{ik} P^V_{qq} + P^S_{qq} \\ P_{q_i \bar{q}_k} &= \delta_{ik} P^V_{q\bar{q}} + P^S_{q\bar{q}} \end{aligned} \tag{5.5}$$

The diagrams contributing to P^V and P^S are shown in Fig. 9 [the diagrams with disconnected lines appear after decomposing the formula (3.5) for F in powers of K]. There is a new qualitative phenomenon in the evolution equation at the subleading level - the probability $P_{q_i\bar{q}_K}$ is different from zero.

From charge conjugation invariance and $SU(N)_{\text{flavour}}$ symmetry we have *)

$$\begin{aligned} &P^V_{qq} = P^V_{\bar{q}\bar{q}} \;, \quad P^V_{q\bar{q}} = P^V_{\bar{q}q} \;; \quad P^S_{qq} = P^S_{q\bar{q}} = P^S_{\bar{q}q} = P^S_{\bar{q}\bar{q}} \\ &P_{q_iG} = P_{\bar{q}_iG} \equiv P_{qG} \;; \quad P_{Gq_i} = P_{G\bar{q}_i} \equiv P_{Gq} \end{aligned} \tag{5.6}$$

Now, we define new probabilities as follows :

$$\begin{aligned} &P_{(+)} = P^V_{qq} + P^V_{q\bar{q}} &\qquad &P_{FG} = 2f\,P_{qG} \\ &P_{(-)} = P^V_{qq} - P^V_{q\bar{q}} & &P_{GF} = P_{Gq} \\ &P_{FF} = P_{(+)} + 2f\,P^S_{qq} & & \end{aligned} \tag{5.7}$$

The difference between $P_{(+)}$ and $P_{(-)}$ appears at the next-to-leading level, as discussed above. Therefore we will use in the following the notation :

$$P_{(\pm)}(x,\alpha) = P^{(0)}_V(x) + \frac{\alpha}{2\pi} P^{(1)}_{(\pm)}(x) + \left(\frac{\alpha}{2\pi}\right)^2 P^{(2)}_{(\pm)}(x) + \ldots \tag{5.8}$$

where

$$P^{(0)}_V(x) = P^{(0)}_{(+)}(x) = P^{(0)}_{(-)}(x) = P^{(0)}_{FF}(x) \tag{5.9}$$

Note that both $P_{(+)}$ and $P_{(-)}$ do not contain gluon intermediate states, i.e., they are diagonal in flavour. Thus the valence (non-singlet) probability $P^{(0)}_V(x)$ splits into two non-singlet probabilities $P_{(+)}$, $P_{(-)}$ beyond leading log level.

It will also be convenient to use the following linear combinations of the quark densities :

$$\begin{aligned} &q^{(+)}_i = q_i + \bar{q}_i \;; \quad q^{(+)} = \sum_{i=1}^{f} q^{(+)}_i \\ &q^{(-)}_i = q_i - \bar{q}_i \;; \quad q^{(-)} = \sum_{i=1}^{f} q^{(-)}_i \end{aligned} \tag{5.10}$$

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*) This is true up to two loops : at higher orders P^S_{qq} may be different from $P^S_{q\bar{q}}$.

After simple algebra, the system (5.4) can be rewritten using the new notation as follows :

$$Q^2 \frac{d}{dQ^2} q^{(+)} = \frac{\alpha}{2\pi}\left[P_{FF}\, q^{(+)} + P_{FG}\, G\right]$$
$$Q^2 \frac{d}{dQ^2} G = \frac{\alpha}{2\pi}\left[P_{GF}\, q^{(+)} + P_{GG}\, G\right] \tag{5.11}$$

$$Q^2 \frac{d}{dQ^2} q_i^{(-)} = \frac{\alpha}{2\pi} P_{(-)}\, q_i^{(-)} \tag{5.12}$$

$$Q^2 \frac{d}{dQ^2}\left[q_i^{(+)} - \frac{1}{f}\, q^{(+)}\right] = \frac{\alpha}{2\pi} P_{(+)}\left[q_i^{(+)} - \frac{1}{f}\, q^{(+)}\right] \tag{5.13}$$

Now only $q^{(+)}$ and G distributions are coupled in a non-trivial way. The non-singlet equation (5.12) is completely decoupled and can be solved independently. The same can be done for Eq. (5.13) : we find first the Q^2 evolution of $q_i^{(+)} - (1/f)q^{(+)}$ and then substitute $q^{(+)}$ derived from the system (5.11).

In the following, we will use the matrix notation

$$P = \begin{pmatrix} P_{FF} & P_{FG} \\ P_{GF} & P_{GG} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \tag{5.14}$$

In order to write the solution of the system (5.11) in a compact form we introduce the evolution matrix

$$E(Q^2,x) = \begin{pmatrix} E_{FF}(Q^2,x) & E_{FG}(Q^2,x) \\ E_{GF}(Q^2,x) & E_{GG}(Q^2,x) \end{pmatrix} = \begin{pmatrix} E_{11}(Q^2,x) & E_{12}(Q^2,x) \\ E_{21}(Q^2,x) & E_{22}(Q^2,x) \end{pmatrix} \tag{5.15}$$

defined as the solution of the equation

$$Q^2 \frac{d}{dQ^2} E = \frac{\alpha}{2\pi} P E \tag{5.16}$$

In Eq. (5.16) and in the following the matrix notation is implicit, i.e.,

$$(PE)_{ik} = \sum_{j=1}^{2} P_{ij}\, E_{jk}$$

In a similar way, we define two non-singlet evolution functions $E_{(\pm)}(Q^2,x)$:

$$Q^2 \frac{d}{dQ^2} E_{(\pm)} = \frac{\alpha}{2\pi} P_{(\pm)} E_{(\pm)} \tag{5.17}$$

The solution of Eqs. (5.11)-(5.13) can be written as follows in terms of the evolution functions :

$$\begin{aligned} q_i^{(-)}(Q^2) &= E_{(-)}(Q^2)\,\tilde{q}_i^{(-)} \\ q^{(+)}(Q^2) &= E_{FF}(Q^2)\,\tilde{q}^{(+)} + E_{FG}(Q^2)\,\tilde{G} \\ G(Q^2) &= E_{GF}(Q^2)\,\tilde{q}^{(+)} + E_{GG}(Q^2)\,\tilde{G} \end{aligned} \tag{5.18}$$

$$q_i^{(+)}(Q^2) = E_{(+)}(Q^2)\,\tilde{q}_i^{(+)} + \frac{1}{f}\left[E_{FF}(Q^2) - E_{(+)}(Q^2)\right]\tilde{q}^{(+)} + \frac{1}{f} E_{FG}(Q^2)\,\tilde{G}$$

Here $\tilde{q}_i^{(\pm)}(x)$, $\tilde{q}^{(+)}(x)$ and $\tilde{G}(x)$ are arbitrary (Q^2 independent) input distributions. They are not determined by perturbative QCD and should be extracted from the data.

In the following, we will use the evolution variable t instead of Q^2 :

$$t = \frac{2}{\beta_0} \ln \frac{\alpha(Q_0^2)}{\alpha(Q^2)} \tag{5.19}$$

The argument of α will be neglected as before, i.e.,

$$\frac{\alpha}{2\pi} = \frac{\alpha_0}{2\pi} e^{-\frac{\beta_0}{2} t} \tag{5.20}$$

Now Eqs. (5.18) can be written as follows :

$$\frac{d}{dt} E(t) = \left[\mathcal{P}^{(0)} + \frac{\alpha}{2\pi} R + O(\alpha^2)\right] E(t) \tag{5.21}$$

where

$$R = \mathcal{P}^{(1)} - \frac{\beta_1}{2\beta_0} \mathcal{P}^{(0)} \tag{5.22}$$

Equation (5.21) is either the matrix singlet equation (5.11) (then the matrix notation is implicit) or one of the non-singlet equations (5.12)-(5.13) [then the subscripts $(\pm)$ are implicit].

Let us denote the leading log solution of Eq. (5.21) by $E^{(0)}(t)$, i.e.,

$$\frac{d}{dt} E^{(0)}(t) = P^{(0)} E^{(0)}(t) \tag{5.23}$$

Now we impose the initial condition

$$E^{(0)}(0) = 1 \tag{5.24}$$

or, more explicitly

$$E^{(0)}_{(\pm)}(t,x)\Big|_{t=0} = E^{(0)}_{V}(t,x)\Big|_{t=0} = \delta(1-x) \tag{5.25}$$

$$E^{(0)}(t,x)\Big|_{t=0} = \begin{pmatrix} \delta(1-x) & 0 \\ 0 & \delta(1-x) \end{pmatrix} \tag{5.26}$$

The full solution of Eq. (5.21) can now be written as a power series in α :

$$E(t) = \left[1 + \frac{\alpha}{2\pi} U + O(\alpha^2)\right] E^{(0)}(t) \tag{5.27}$$

Inserting the formula (5.27) into Eq. (5.21) we obtain the following equation for the functions $U(x)$:

$$[U, P^{(0)}] = \frac{\beta_0}{2} U + R \tag{5.28}$$

where $[U,P^{(0)}] = UP^{(0)} - P^{(0)}U$. For the non-singlet case the commutator vanishes and we simply get :

$$U_{(\pm)} = -\frac{2}{\beta_0} R_{(\pm)} = \frac{\beta_1}{\beta_0^2} P^{(0)}_{V} - \frac{2}{\beta_0} P^{(1)}_{(\pm)} \tag{5.29}$$

The solution of the leading log equation (5.23) is :

$$E_V^{(0)}(t) = e^{P_V^{(0)} t} \tag{5.30}$$

The non-singlet evolution functions are therefore given by the formulae :

$$E_{(\pm)}(t) = \left[1 + \frac{\alpha}{2\pi}\left\{\frac{\beta_1}{\beta_0^2} P_V^{(0)} - \frac{2}{\beta_0} P_{(\pm)}^{(1)}\right\}\right] e^{P_V^{(0)} t} \tag{5.31}$$

For moments, we get :

$$E_{(\pm)}(t,n) = \left[1 + \frac{\alpha}{2\pi}\left(\frac{\beta_1}{\beta_0^2} P_V^{(0)}(n) - \frac{2}{\beta_0} P_{(\pm)}^{(1)}(n)\right)\right] e^{P_V^{(0)}(n) t} \tag{5.32}$$

and the solution in the x variable reads :

$$E_{(\pm)}(t,x) = \left[\delta(1-x) + \frac{\alpha}{2\pi}\left(\frac{\beta_1}{\beta_0^2} P_V^{(0)}(x) - \frac{2}{\beta_0} P_{(\pm)}^{(1)}(x)\right)\right] \otimes e^{P_V^{(0)}(x) t} \tag{5.33}$$

where

$$e^{P_V^{(0)}(x) t} = \delta(1-x) + P_V^{(0)}(x) t + \frac{t^2}{2!} P_V^{(0)}(x) \otimes P_V^{(0)}(x) + \ldots \tag{5.34}$$

In the singlet case, Eq. (5.28) is less trivial since the commutator $[U, P^{(0)}]$ does not vanish in general.

Usually, one solves Eq. (5.28) by going to the frame where the matrix $P^{(0)}$ is diagonal. It appears, however, that it is then rather difficult to write the final solution in a compact matrix form. As a consequence, the formulae are quite complicated [19]. Here we present a rather simple matrix solution of Eq. (5.21) in the singlet case. From now on, all the calculations are done only for moments.

Let us denote by λ_1, λ_2 the eigenvalues of the matrix $P^{(0)}$:

$$\lambda_{1,2} = \frac{1}{2}\left[P_{FF}^{(0)} + P_{GG}^{(0)} \pm \sqrt{\left(P_{FF}^{(0)} - P_{GG}^{(0)}\right)^2 + 4 P_{FG}^{(0)} P_{GF}^{(0)}}\right] \tag{5.35}$$

It is convenient to introduce the projecting matrices e_1, e_2 :

$$e_1 \equiv \frac{1}{\lambda_1-\lambda_2}\left[P^{(0)}-\lambda_2 \mathbb{1}\right] \; ; \quad e_2 \equiv \frac{1}{\lambda_1-\lambda_2}\left[-P^{(0)}+\lambda_1 \mathbb{1}\right] \tag{5.36}$$

where $\mathbb{1}_{ik} = \delta_{ik}$ is the unit 2x2 matrix. We have

$$e_1^2 = e_1 \; , \; e_2^2 = e_2 \; , \; e_1 e_2 = 0 \; , \; e_1 + e_2 = \mathbb{1} \tag{5.37}$$

$$P^{(0)} = \lambda_1 e_1 + \lambda_2 e_2 \tag{5.38}$$

The solution of the leading log equation (5.23) can now be written as

$$E^{(0)}(t) = e^{P^{(0)}t} = e_1 \cdot e^{\lambda_1 t} + e_2 \cdot e^{\lambda_2 t} \tag{5.39}$$

as follows from (5.37)-(5.38).

In order to solve Eq. (5.38), let us note that, since $e_1+e_2 = 1$, we have an obvious identity

$$U = e_1 U e_1 + e_1 U e_2 + e_2 U e_1 + e_2 U e_2 \tag{5.40}$$

Inserting the decomposition (5.38) into Eq. (5.28), we can project out (using matrices e_i) all terms in (5.39). We get

$$U = -\frac{2}{\beta_0}\left[e_1 R e_1 + e_2 R e_2\right] + \frac{e_2 R e_1}{\lambda_1-\lambda_2-\frac{1}{2}\beta_0} + \frac{e_1 R e_2}{\lambda_2-\lambda_1-\frac{1}{2}\beta_0} \tag{5.41}$$

The final solution for $E(t)$ reads :

$$E(t) = \left\{e_1 + \frac{\alpha}{2\pi}\left[-\frac{2}{\beta_0} e_1 R e_1 + \frac{e_2 R e_1}{\lambda_1-\lambda_2-\frac{1}{2}\beta_0}\right]\right\} e^{\lambda_1 t} +$$

$$+ \left\{e_2 + \frac{\alpha}{2\pi}\left[-\frac{2}{\beta_0} e_2 R e_2 + \frac{e_1 R e_2}{\lambda_2-\lambda_1-\frac{1}{2}\beta_0}\right]\right\} e^{\lambda_2 t} \tag{5.42}$$

The direct inversion of the expressions (5.41), (5.42) to the x space is rather complicated since the inverse Mellin transform of the eigenvalues λ_1, λ_2 is quite non-trivial. We can, however, solve the problem by making the following trick : using an obvious identity

$$\frac{1}{\lambda} = \int_0^\infty d\tau\, e^{-\lambda\tau} \tag{5.43}$$

we write Eq. (5.41) as follows :

$$U = -\int_0^\infty d\tau\, e^{-\frac{\beta_0}{2}\tau}\left[e_1 R e_1 + e_2 R e_2 + e^{(\lambda_1-\lambda_2)\tau} e_2 R e_1 + e^{(\lambda_2-\lambda_1)\tau} e_1 R e_2\right] \tag{5.44}$$

Now the eigenvalues λ_i enter only through the exponential factors $e^{\pm\lambda_i\tau}$ and we can try to express the right-hand side of Eq. (5.44) by $E^{(0)}(\pm\tau)$. Indeed, we get

$$U(x) = -\int_0^\infty d\tau\, e^{-\frac{\beta_0}{2}\tau} E^{(0)}(-\tau,x) \otimes R(x) \otimes E^{(0)}(\tau,x) \tag{5.45}$$

The leading log evolution matrix $E^{(0)}(t,x)$ is given by the series

$$E^{(0)}(t,x) = e^{P^{(0)}(x)t} = \delta(1-x)\mathbb{1} + tP^{(0)}(x) + \frac{t^2}{2!}P^{(0)}(x) \otimes P^{(0)}(x) + \dots \tag{5.46}$$

Another derivation of Eq. (5.45) can be obtained by using the experience we have got from quantum mechanics. Identifying t in Eq. (5.21) with an (imaginary) time we reduce the problem to solving the time-dependent Schrödinger equation

$$\frac{d}{dt}|\psi(t)\rangle_S = H(t)\,|\psi(t)\rangle_S \tag{5.47}$$

where

$$H(t) = H^{(0)} + V(t) \tag{5.48}$$

and

$$H^{(0)} = P^{(0)} \quad ; \quad V(t) = \frac{\alpha_0}{2\pi} e^{-\frac{\beta_0}{2}t} R + O(\alpha^2) \tag{5.49}$$

Since we are going to treat the time-dependent "potential" $V(t)$ perturbatively, it is natural to go to the interaction picture :

$$|\psi(t)\rangle_I = e^{-\mathcal{P}^{(0)}t}\,|\psi(t)\rangle_S \tag{5.50}$$

$$\frac{d}{dt}|\psi(t)\rangle_I = V_I(t)\,|\psi(t)\rangle_I \tag{5.51}$$

where

$$V_I(t) = e^{-\mathcal{P}^{(0)}t}\,V(t)\,e^{\mathcal{P}^{(0)}t} = E^{(0)}(-t)V(t)E^{(0)}(t) \tag{5.52}$$

The evolution operator in the interaction picture has the well-known form :

$$E_I(t) = \mathrm{T}\,e^{\int_{-\infty}^{t} d\tau V_I(\tau)} \tag{5.53}$$

where T is the chronological operator. Expanding the exponent in Eq. (5.53) in powers of V and going back to the Schrödinger picture we get

$$\begin{aligned} E_S(t) &= E^{(0)}(t)\,E_I(t) = \\ &= E^{(0)}(t) + \frac{\alpha_0}{2\pi}\int_{-\infty}^{t} d\tau\, e^{-\frac{\beta_0}{2}\tau} E^{(0)}(t-\tau)\,\mathbf{R}\,E^{(0)}(\tau) \end{aligned} \tag{5.54}$$

Now it is only a matter of changing the integration variable $\tau \to t+\tau$ in order to write Eq. (5.54) in the form (5.27) with $U(x)$ given by Eq. (5.45). [A similar method, without making explicit reference to standard methods of quantum mechanics has been used in Ref. 19)].

The decomposition of the evolution matrix into singlet and non-singlet pieces is very useful in the analysis of the deep inelastic leptoproduction and in inclusive e^+e^- annihilation, where it enables one to substantially reduce the number of independent input distributions. In more complicated hard processes, like inclusive leptoproduction or the Drell-Yan process, it may be more useful to work directly with the full evolution matrix, describing the transitions between any two types of partons. We end this section with the presentation of the explicit expressions for the full evolution matrix. We have :

$$\begin{aligned} F_h^p(x,Q^2) &= \sum_{p'} E^{(S)}_{pp'}(x,Q^2)\otimes \tilde{F}_h^{p'}(x) \\ F_p^h(x,Q^2) &= \sum_{p'} E^{(T)}_{pp'}(x,Q^2)\otimes \tilde{F}_{p'}^{h}(x) \end{aligned} \tag{5.55}$$

where the elements $E_{pp'}$ of the full evolution matrix are given by the following linear combinations of the singlet and non-singlet (spacelike or timelike) evolution functions :

$$E_{q_iq_j} = E_{\bar{q}_i\bar{q}_j} = \frac{1}{2}\left[E_{(+)}+E_{(-)}\right]\delta_{ij} + \frac{1}{2f}\left[E_{11}-E_{(+)}\right]$$

$$E_{q_i\bar{q}_j} = E_{\bar{q}_iq_j} = \frac{1}{2}\left[E_{(+)}-E_{(-)}\right]\delta_{ij} + \frac{1}{2f}\left[E_{11}-E_{(+)}\right]$$

$$E_{q_iG} = E_{\bar{q}_iG} = \frac{1}{2f}E_{12}$$

$$E_{Gq_i} = E_{G\bar{q}_i} = E_{21}$$

$$E_{GG} = E_{22} \tag{5.56}$$

Let us present the following important comment. The next-to-leading part of the evolution functions considered above has been determined by the parametrization (5.27) or, equivalently, by the asymptotic condition

$$E\,(t) \xrightarrow[t\to\infty]{} E^{(0)}(t) \tag{5.57}$$

Since the evolution equation (5.21) is homogeneous, there is a freedom in fixing the normalization of the evolution functions. For example, instead of Eq. (5.57) we can impose the initial condition

$$E(t)\big|_{t=0} \equiv 1 \tag{5.58}$$

Solving Eq. (5.21) in the "interaction picture" as before, we get

$$E(t) = E^{(0)}(t) + \frac{\alpha_0}{2\pi}E^{(1)}(t) + O(\alpha_0^2) \tag{5.59}$$

where

$$E^{(1)}(t) = \int_0^t d\tau\, e^{-\frac{\beta_0}{2}\tau} E^{(0)}(t-\tau)\, R\, E^{(0)}(\tau) \tag{5.60}$$

In the non-singlet case, the integral in Eq. (5.60) can be performed in a trivial way and we get, e.g., for $q^{(-)}(t)$:

$$q^{(-)}(t) = \left[1+\frac{\alpha}{2\pi}U^{(-)}\right]E^{(0)}(t)\,\tilde{q}^{(-)} = \left[E^{(0)}(t)+\frac{\alpha_0}{2\pi}E^{(1)}(t)\right]\hat{q}^{(-)}$$

$$= \left[1+\left(\frac{\alpha}{2\pi}-\frac{\alpha_0}{2\pi}\right)U^{(-)}\right]E^{(0)}(t)\,\hat{q}^{(-)} \tag{5.61}$$

As one can see, the solutions with the boundary conditions (5.57) and (5.58) differ only by a t independent part which can be reabsorbed by a redefinition of the input :

$$\tilde{q}^{(-)} \to \hat{q}^{(-)} = \left[1+\frac{\alpha_0}{2\pi}U^{(-)}\right]\tilde{q}^{(-)} \tag{5.62}$$

In other words, any choice of the normalization of the evolution functions corresponds to some particular definition of the input distributions whereas the t dependence of the physical quantities does not depend on this choice.

The same remains true in the singlet case. Technically, however, the situation is slightly more complicated. We discuss the singlet case in the last section, devoted to the detailed analysis of all the typical ambiguities appearing at the subleading level.

Here we report only the following formal rule : in order to switch to any other definition of the input, one should perform the replacement

$$\frac{\alpha}{2\pi}U\,E^{(0)}(t) \longrightarrow \frac{\alpha_0}{2\pi}\left[E^{(0)}(t)\,Y^{(1)} + E^{(1)}(t)\right] \tag{5.63}$$

where $Y^{(1)}$ is an arbitrary 2×2 matrix and $E^{(1)}(t)$ is given by Eq. (5.60). The new parton densities are

$$\begin{pmatrix} q^{(+)} \\ G \end{pmatrix}_Y = \left[1+\frac{\alpha_0}{2\pi}\left(U-Y^{(1)}\right)\right]\begin{pmatrix} \tilde{q}^{(+)} \\ \tilde{G} \end{pmatrix} \tag{5.64}$$

In particular, the parametrization (5.27) of the evolution matrix used in this section (hereafter called the U scheme) corresponds to the choice $Y^{(1)} = U$ [one can easily verify by shifting the integration variables in Eqs. (5.45) and (5.60) that the replacement (5.63) is an identity in this case].

As seen from Eq. (5.63), the U scheme provides the simplest possible parametrization of the evolution : for any $X^{(1)} \neq U$ one has to deal with the complicated integral $E^{(1)}(t)$. Therefore, for notational simplicity, we use the U scheme throughout this paper. Nevertheless, the U matrix representation has serious practical shortcomings : U(x) turns out to be very singular for $x \to 0$. We discuss this point in Section 10, where the reader interested in numerical applications is referred to.

6. - DEEP INELASTIC LEPTOPRODUCTION

In this section we present the QCD prediction for the inclusive cross-section $d\sigma_{\ell h}^{\ell'}$ in the deep inelastic lepton-hadron scattering.

The standard variables for this process are :

$$q = \ell - \ell', \quad Q^2 = -q^2 = 2\ell\ell', \quad x = \frac{Q^2}{2hq}, \quad y = \frac{2hq}{2h\ell}, \quad S = 2h\ell \tag{6.1}$$

The basic formula (2.1) takes the form

$$\frac{d\sigma_{\ell h}^{\ell'}(x,y;Q^2)}{dx\,dy} = \sum_p \int_0^1 d\hat{x}\,dz \; \frac{d\hat{\sigma}_{\ell p}^{\ell'}(\hat{x},y;Q^2)}{d\hat{x}\,dy} \, F_h^p(z,Q^2)\,\delta(x-\hat{x}z) \tag{6.2}$$

Since the variable y does not change after replacing hadron by parton momenta ($y = hq/h\ell = \hat{y} = pq/p\ell$), Eq. (6.2) is diagonal in y. The y dependence of $d\sigma$ and $d\hat{\sigma}$ is entirely determined by the spin structure of a current. In particular, for a vector (or an axial vector) current both $d\sigma$ and $d\hat{\sigma}$ are second order polynomials in y. It follows directly from the standard decomposition of the hadronic tensor

$$\begin{aligned}
\left[W_{\mu\nu}(h,q)\right]_{\ell h}^{\ell'} &= \frac{1}{8\pi}\sum_{SPIN}\int d^4x\, e^{iqx} \langle h|J_\mu^+(x) J_\nu(0)|h\rangle = \\
&= \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] F_{\ell h,1}^{\ell'}(x,Q^2) + \left[h_\mu - \frac{hq}{q^2}q_\mu\right]\left[h_\nu - \frac{hq}{q^2}q_\nu\right]\frac{1}{hq}F_{\ell h,2}^{\ell'}(x,Q^2) \\
&\quad - \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\frac{h^\alpha q^\beta}{hq} F_{\ell h,3}^{\ell'}(x,Q^2)
\end{aligned} \tag{6.3}$$

It is therefore convenient to isolate explicitly the y dependence as follows :

$$\frac{d\hat{\sigma}_{\ell p}^{\ell'}(\hat{x},y;Q^2)}{d\hat{x}\,dy} = \hat{\sigma}_B(S,Q^2)\sum_{r=1}^{3} Y_{B,r}(y)\,\lambda_{Bp,r}\,C_{P,r}(\hat{x},Q^2) \tag{6.4}$$

where $Y_{B,r}(y)$, $r = 1,2,3$ are three linearly independent second order polynomials in y. The subscript B (= vector boson) indicates the type of a current : $B = \gamma$ for the electromagnetic current, $B = W^+$ or W^- for the weak charged current, $B = Z^o$ for the neutral current.

The numbers $\lambda_{Bp,r}$ stand for the vector boson-parton coupling constants. The function $C_{P,r}(\hat{x},Q^2)$ describes the non-trivial part of the short-distance interaction and is given in QCD by a power series in $\alpha(Q^2)$:

$$C_{P,r}(\hat{x},Q^2) = C^{(0)}_{P,r}\,\delta(1-x) + \frac{\alpha(Q^2)}{2\pi}\,C^{(1)}_{P,r}(x,Q^2) \tag{6.5}$$

The values of the coefficients $C^{(1)}_{P,r}$ are given in Appendix I. If the actual current $J_\mu(x)$ has the structure

$$J_\mu(x) = \sum_{i=1}^{f}\sum_{k=1}^{f} \bar{q}_i(x)\,T_{ik}\,\Gamma_\mu\, q_k(x) \tag{6.6}$$

the function $C_{P,r}(\hat{x},Q^2)$ is derived for the current

$$J_\mu(x) = \sum_{i=1}^{f} \bar{q}_i(x)\,\Gamma_\mu\, q_i(x)$$

and all the actual couplings are contained in $\lambda_{Bp,r}$. As a consequence, $C_{P,r}$ is flavour-independent. The meaning of the subscript P is the following : $P = F$ when $p = q_i$ or $p = \overline{q_i}$ and $P = G$ if $p = G$. It follows from the above discussion that :

$$C^{(0)}_{F,r} = 1 \quad , \quad C^{(0)}_{G,r} = 0$$
$$\lambda_{BG,r} = \frac{1}{2f}\sum_{i=1}^{f}\left(\lambda_{Bq_i,r} + \lambda_{B\bar{q}_i,r}\right) \tag{6.7}$$

The lepton-hadron cross-section $d\sigma_{\ell h}^{\ell'}$ has the decomposition :

$$\frac{d\sigma_{\ell h}^{\ell'}(x,y;Q^2)}{dx\,dy} = \hat{\sigma}_B(s,Q^2)\sum_{r=1}^{3} Y_{B,r}(y)\; f_{\ell h,r}^{\ell'}(x,Q^2) \tag{6.8}$$

The basis $Y_{B,r}(y)$ will be chosen in such a way that the functions $f_{\ell h,r}^{\ell'} \equiv f_{Bh,r}$ will be related to the standard structure functions $F_{\ell h,r}^{\ell'}$ in Eq. (6.3) as follows :

$$f_1 = 2F_1 \quad , \quad f_2 = \frac{1}{x} F_2 \quad , \quad f_3 = F_3 \tag{6.9}$$

Inserting Eqs. (6.4) and (6.8) into Eq. (6.2), we get the QCD formulae for the structure functions f_r :

$$\begin{aligned} f_{Bh,r}(x,Q^2) = {} & C_{F,r}(x,Q^2) \otimes \sum_{i=1}^{f} \left\{ \lambda_{Bq_i,r} F_h^{q_i}(x,Q^2) + \lambda_{B\bar{q}_i,r} F_h^{\bar{q}_i}(x,Q^2) \right\} \\ & + C_{G,r}(x,Q^2) \otimes \lambda_{BG,r} F_h^{G}(x,Q^2) \end{aligned} \tag{6.10}$$

In the following, we will neglect the x dependence so that all products are either convolutions in the x variable or usual products of moments, in a similar way as in Section 5. Also, we will use the evolution variable t [Eqs. (5.19), (5.20)]. We then have

$$\begin{aligned} f_{Bh,r}(t) = {} & C_{F,r}(t) \sum_{i=1}^{f} \left\{ \lambda_{Bq_i,r} F_h^{q_i}(t) + \lambda_{B\bar{q}_i,r} F_h^{\bar{q}_i}(t) \right\} + \\ & + C_{G,r}(t)\, \lambda_{BG,r} F_h^{G}(t) \end{aligned} \tag{6.11}$$

Let us now introduce the singlet density

$$F_h^{(+)}(t,x) = \sum_{i=1}^{f} \left\{ F_h^{q_i}(t,x) + F_h^{\bar{q}_i}(t,x) \right\} \tag{6.12}$$

and two non-singlet densities

$$V_{Bh,r}^{(-)}(t,x) = \frac{1}{2} \sum_{i=1}^{f} \left(\lambda_{Bq_i,r} - \lambda_{B\bar{q}_i,r} \right) \left[F_h^{q_i}(t,x) - F_h^{\bar{q}_i}(t,x) \right]$$

$$V^{(+)}_{Bh,r}(t,x) = \frac{1}{2}\sum_{i=1}^{f}\left(\lambda_{Bq_i,r} + \lambda_{B\bar{q}_i,r}\right)\left[F_h^{q_i}(t,x) + F_h^{\bar{q}_i}(t,x) - \frac{1}{f}F_h^{(+)}(t,x)\right] \tag{6.13}$$

Equation (6.11) can now be written as follows :

$$f_{Bh,r}(t) = C_{F,r}(t)\left[V^{(-)}_{Bh,r}(t) + V^{(+)}_{Bh,r}(t)\right] + \lambda_{BG,r}S_{h,r}(t) \tag{6.14}$$

where

$$S_{h,r}(t) = C_{F,r}(t)\,F_h^{(+)}(t) + C_{G,r}(t)\,F_h^{G}(t) \tag{6.15}$$

The densities $V^{(\pm)}$ evolve according to the non-singlet evolution equations (5.12), (5.13) and the densities $F_h^{(+)}$, F_h^{G} according to the singlet equation (5.11).

Combining Eqs. (5.27), (6.5) and (6.14) we get the final result for the QCD prediction for any deep inelastic structure function :

$$\begin{aligned} f_{Bh,r}(t) = {} & \left[E_V^{(0)}(t) + \frac{\alpha}{2\pi}\left(C^{(1)}_{F,r} + U_{(+)}\right)E_V^{(0)}(t)\right]\tilde{V}^{(+)}_{Bh,r} + \\ & + \left[E_V^{(0)}(t) + \frac{\alpha}{2\pi}\left(C^{(1)}_{F,r} + U_{(-)}\right)E_V^{(0)}(t)\right]\tilde{V}^{(-)}_{Bh,r} \\ & + \lambda_{BG,r}\left[E_{11}^{(0)}(t) + \frac{\alpha}{2\pi}\left(\left(C C^{(1)}_{F,r} + U_{11}\right)E_{11}^{(0)}(t) + \left(C^{(1)}_{G,r} + U_{12}\right)E_{21}^{(0)}(t)\right)\right]\tilde{F}_h^{(+)} \\ & + \lambda_{BG,r}\left[E_{12}^{(0)}(t) + \frac{\alpha}{2\pi}\left(C C^{(1)}_{F,r} + U_{11}\right)E_{12}^{(0)}(t) + \left(C^{(1)}_{G,r} + U_{12}\right)E_{22}^{(0)}(t)\right]\tilde{F}_h^{G} \end{aligned} \tag{6.16}$$

As seen in Eq. (6.16), the $O(\alpha)$ corrections to the partonic cross-sections and to the evolution of parton densities always enter in the following combinations :

$$C^{(1)}_{F,r} + U_{(+)}\ ,\ C^{(1)}_{F,r} + U_{(-)}\ ,\ C^{(1)}_{F,r} + U_{11}\ ,\ C^{(1)}_{G,r} + U_{12} \tag{6.17}$$

These functions are scheme independent, whereas the decomposition into a C part and a U part depends on a particular factorization scheme.

For any structure function there are in general two universal (current independent) input distributions $\tilde{F}_h^{(+)}(x)$, $\tilde{F}_h^G(x)$ related with the singlet part of the structure function and a set of non-singlet current dependent input distributions $\tilde{V}_{Bh,r}^{(\pm)}(x)$.

Since all the experimentally relevant currents $(B = \gamma, W^{\pm}, Z^{o})$ couple only to charge or to weak isospin, all non-singlet distributions $V_{Bh,r}^{(\pm)}(t,x)$ can be expressed in practice by the following three functions :

$$V_h^{(+)}(t,x) = 2 \sum_{i=1}^{f} I_3^{q_i} \left[F_h^{q_i}(t,x) + F_h^{\bar{q}_i}(t,x) \right]$$

$$V_h^{(-)}(t,x) = 2 \sum_{i=1}^{f} I_3^{q_i} \left[F_h^{q_i}(t,x) - F_h^{\bar{q}_i}(t,x) \right]$$

$$F_h^{(-)}(t,x) = \sum_{i=1}^{f} \left[F_h^{q_i}(t,x) - F_h^{\bar{q}_i}(t,x) \right] \tag{6.18}$$

where $I_3^{q_i}$ is the third component of the weak isospin of a quark q_i : $I_3^u = I_3^c = \ldots = \frac{1}{2}$, $I_3^d = I_3^s = \ldots = -\frac{1}{2}$.

Therefore, for any hadron we have five input distributions :

$$\tilde{F}_h^{G}(x),\ \tilde{F}_h^{(+)}(x),\ \tilde{F}_h^{(-)}(x),\ \tilde{V}_h^{(+)}(x),\ \tilde{V}_h^{(-)}(x) \tag{6.19}$$

In the following, we present explicit formulae for the electromagnetic current $(B = \gamma)$ and for the weak charged current $(B = W^{\pm})$ *).

a) electromagnetic current $(r = 1,2)$

$$\hat{\sigma}_\gamma(s,Q^2) = \frac{4\pi\alpha_{e.m.}^2\, s}{(Q^2)^2}$$

$$Y_{\gamma,1}(y) = \tfrac{1}{2}y^2 \ ; \ Y_{\gamma,2}(y) = 1-y$$

$$\lambda_{\gamma q_i,r} = \lambda_{\gamma \bar{q}_i,r} = e_i^2 \tag{6.20}$$

---

*) We neglect threshold problems for heavy quark production and full isoweak doublets are supposed to be active in the currents.

$$\lambda_{\gamma G,r} = \frac{5}{18}$$

$$V^{(+)}_{\gamma h,r} = \frac{1}{6} V^{(+)}_h$$

$$V^{(-)}_{\gamma h,r} = 0$$

b) charged weak current ($\omega = 1$ for W^+, $\omega = -1$ for W^-) :

$$\begin{aligned}
&\hat{\sigma}_W(s,Q^2) = \frac{G_F^2}{2\pi} s \\
&Y_{W,1}(y) = \frac{1}{2} y^2, \quad Y_{W,2}(y) = 1-y, \quad Y_{W,3}(y) = \frac{1}{2}\omega\left[1-(1-y)^2\right] \\
&\lambda_{Wq_i,r} = 1-2\omega I_3^{q_i}, \quad \lambda_{W\bar{q}_i,r} = 1+2\omega I_3^{q_i} \qquad (r=1,2) \\
&\lambda_{Wq_i,3} = 1-2\omega I_3^{q_i}, \quad \lambda_{W\bar{q}_i,r} = -1-2\omega I_3^{q_i} \\
&\lambda_{WG,1} = \lambda_{WG,2} = 1, \quad \lambda_{WG,3} = 0 \\
&V^{(+)}_{Wh,r} = 0, \quad V^{(-)}_{Wh,r} = -\omega V^{(-)}_h \qquad (r=1,2) \\
&V^{(+)}_{Wh,3} = -\omega V^{(+)}_h, \quad V^{(-)}_{Wh,3} = F^{(-)}_h
\end{aligned} \qquad (6.21)$$

Finally, we present the formulae for nucleon and nuclear targets. The label A instead of h denotes the nucleus with $A/2[1-\theta]$ protons and $A/2[1+\theta]$ neutrons. We use the simplified notation for parton densities :

$$p \equiv F^p_P \quad , \quad p^{(+)} \equiv F^p_P + F^{\bar{p}}_P \, , \ldots$$

i.e., all densities now refer to proton target. It is convenient to introduce the following densities :

$$v^{(+)} = u^{(+)} - d^{(+)}$$

$$
\begin{aligned}
v^{(-)} &= u^{(-)} - d^{(-)} \\
q^{(-)} &= u^{(-)} + d^{(-)} \\
a &= V^{(+)} - v^{(+)} = c^{(+)} - s^{(+)} + t^{(+)} - b^{(+)} + \ldots\ldots
\end{aligned}
\tag{6.22}
$$

$$S_r = C_{F,r}\, q^{(+)} + C_{G,r}\, G$$

We have then $(r = 1,2)$:

$$
\begin{aligned}
f_{\gamma P,r} &= \frac{1}{6} C_{F,r}\left[v^{(+)} - a\right] + \frac{5}{18} S_r \\
f_{\gamma N,r} &= \frac{1}{6} C_{F,r}\left[-v^{(+)} - a\right] + \frac{5}{18} S_r \\
f_{\gamma A,r} &= -\frac{1}{6} C_{F,r}\left[\theta v^{(+)} + a\right] + \frac{5}{18} S_r
\end{aligned}
$$

$$
\begin{aligned}
f_{WP,r} &= C_{F,r}\left[-\omega v^{(-)}\right] + S_r \\
f_{WN,r} &= C_{F,r}\left[\omega v^{(-)}\right] + S_r \\
f_{WA,r} &= C_{F,r}\left[\omega \theta v^{(-)}\right] + S_r \\
f_{WP,3} &= C_{F,3}\left[\omega\left(a - v^{(+)}\right) + q^{(-)}\right] \\
f_{WN,3} &= C_{F,3}\left[\omega\left(a + v^{(+)}\right) + q^{(-)}\right] \\
f_{WA,3} &= C_{F,3}\left[\omega\left(a + \theta v^{(+)}\right) + q^{(-)}\right]
\end{aligned}
\tag{6.23}
$$

The Q^2 dependence of the structure functions in Eqs. (6.23) is given by the general formula (6.16), i.e.,

$$
\begin{aligned}
S_r(t) = &\left[E_{11}^{(0)}(t) + \frac{\alpha}{2\pi}\left[\left(C_{F,r}^{(1)} + U_{11}\right)E_{11}^{(0)}(t) + \left(C_{G,r}^{(1)} + U_{12}\right)E_{21}^{(0)}(t)\right]\right]\tilde{q}^{(+)} \\
&+ \left[E_{12}^{(0)}(t) + \frac{\alpha}{2\pi}\left[\left(C_{F,r}^{(1)} + U_{11}\right)E_{12}^{(0)}(t) + \left(C_{G,r}^{(1)} + U_{12}\right)E_{22}^{(0)}(t)\right]\right]\tilde{G}
\end{aligned}
\tag{6.24}
$$

$$C_{F,r}\begin{Bmatrix} v^{(+)}(t) \\ a(t) \end{Bmatrix} = \left[1+\frac{\alpha}{2\pi}\left(C^{(1)}_{F,r}+U_{(+)}\right)E^{(0)}_{(+)}(t)\right]\begin{Bmatrix} \tilde{v}^{(+)} \\ \tilde{a} \end{Bmatrix}$$

$$C_{F,r}\begin{Bmatrix} v^{(-)}(t) \\ q^{(-)}(t) \end{Bmatrix} = \left[1+\frac{\alpha}{2\pi}\left(C^{(1)}_{F,r}+U_{(-)}\right)E^{(0)}_{(-)}(t)\right]\begin{Bmatrix} \tilde{v}^{(-)} \\ \tilde{q}^{(-)} \end{Bmatrix}$$

Thus any structure function on the nucleon or the nuclear target can be parametrized by six input distributions : $\tilde{q}^{(+)}$, $\tilde{q}^{(-)}$, $\tilde{v}^{(+)}$, $\tilde{v}^{(-)}$, $\tilde{a}$, $\tilde{G}$.

7. - INCLUSIVE e^+e^- ANNIHILATION

The basic formula (2.1) has the following form for the process $e^+e^- \to h +$ + anything

$$\frac{d\sigma^h_{e^+e^-}(x,y;Q^2)}{dx\,dy} = \sum_p \int d\hat{x}\,dz\, \frac{d\hat{\sigma}^p_{e^+e^-}(\hat{x},y;Q^2)}{d\hat{x}\,dy}\, F^h_p(z,Q^2)\cdot \delta(x-\hat{x}z) \tag{7.1}$$

where

$$q = \ell_1+\ell_2\ ,\quad Q^2=q^2\ ,\quad x=\frac{2hq}{q^2}\ ,\quad y=\frac{h\ell_1}{hq}=\frac{1}{2}\left[1-\cos\theta_{CM}\right] \tag{7.2}$$

As in the case of deep inelastic leptoproduction, Eq. (7.1) is diagonal in y and the y dependence of both $d\sigma$ and $d\hat{\sigma}$ is trivial. We will use the decomposition

$$\frac{d\sigma^h_{e^+e^-}(x,y;Q^2)}{dx\,dy} = N\frac{4\pi\alpha^2_{e.m.}}{3Q^2}\left[\frac{3}{2}f^h_{\gamma,1}(x,Q^2) - 3y(1-y)f^h_{\gamma,2}(x,Q^2)\right] \tag{7.3}$$

and the corresponding one for the partonic cross-section

$$\frac{d\sigma^p_{e^+e^-}(\hat{x},y;Q^2)}{d\hat{x}\,dy} = N\frac{4\pi\alpha^2_{em}}{3Q^2}\cdot\lambda^p_\gamma\left[\frac{3}{2}C^p_1(x,Q^2)-3y(1-y)C^p_2(x,Q^2)\right] \tag{7.4}$$

In Eqs. (7.3)-(7.5) N is the number of colours, $\sigma_o = (4\pi\alpha^2/3Q^2)$ is the total cross-section for Bhabha scattering, P = F if $p = q_i$ or $p = \bar{q}_i$ and P = G if p = G. The couplings λ^p_γ are :

$$\lambda^{q_i}_\gamma = \lambda^{\bar{q}_i}_\gamma = e_i^2 \quad ; \quad \lambda^G_\gamma = \frac{5}{18} \tag{7.5}$$

The structure functions $f^h_{\gamma,r}$ in Eq. (7.3) are related to the standard structure functions in the hadronic tensor $W_{\mu\nu}$ as follows :

$$\left[W_{\mu\nu}\right]^h_{e^+e^-} = \sum_X \langle 0|J_\mu(0)|X,h\rangle\langle X,h|J_\nu(0)|0\rangle =$$

$$= \frac{2hq}{q^2}\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] f^h_{\gamma,1} - \frac{1}{hq}\left[h_\mu - \frac{hq}{q^2}q_\mu\right]\left[h_\nu - \frac{hq}{q^2}q_\nu\right] f^h_{\gamma,2} \tag{7.6}$$

From Eqs. (7.1)-(7.4) we get

$$f^h_{\gamma,r}(x,Q^2) = \sum_p C^P_r(x,Q^2) \otimes \lambda^p_\gamma F^h_p(x,Q^2) \quad . \tag{7.7}$$

As usual, the functions C^P_r are series in $\alpha(Q^2)$:

$$C^F_r(x,Q^2) = \delta(1-x) + \frac{\alpha(Q^2)}{2\pi} C^{F(1)}_r(x) + \dots$$

$$C^G_r(x,Q^2) = \frac{\alpha(Q^2)}{2\pi} C^{G(1)}_r(x) + \dots \tag{7.8}$$

The values of the coefficients are contained in Appendix II. The decomposition of Eq. (7.7) into valence and singlet parts is the same as in the electroproduction. Using the simplified notation introduced in Section 6, we have

$$f^h_{\gamma,r}(t) = \frac{1}{6} C^F_r(t) F^h_v(t) + \frac{5}{18}\left[C^F_r(t) F^h_S(t) + C^G_r(t) F^h_G(t)\right] \tag{7.9}$$

where

$$F^h_v(t) = 2\sum_i I_3^{q_i}\left[F^h_{q_i}(t) + F^h_{\bar{q}_i}(t)\right] = \left(F^h_u + F^h_{\bar{u}}\right) - \left(F^h_d + F^h_{\bar{d}}\right) + \dots$$

$$F^h_S(t) = \sum_i \left(F^h_{q_i} + F^h_{\bar{q}_i}\right) \tag{7.10}$$

The Q^2 dependence of the structure function $f^h_{\gamma,r}$ is the following :

$$f^h_{\gamma,r}(t) = \frac{1}{6}\left[1+\frac{\alpha}{2\pi}\left(C^{F(1)}_r+U_{(+)}\right)\right]E^{(0)}_V(t)\tilde{F}_V^h +$$

$$+\frac{5}{18}\left[E^{(0)}_{11}(t)+\frac{\alpha}{2\pi}\left[\left(C^{F(1)}_r+U_{11}\right)E^{(0)}_{11}(t)+\left(C^{G(1)}_r+U_{12}\right)E^{(0)}_{21}(t)\right]\right]\tilde{F}^h_S$$

$$+\frac{5}{18}\left[E^{(0)}_{12}(t)+\frac{\alpha}{2\pi}\left[\left(C^{F(1)}_r+U_{11}\right)E^{(0)}_{12}(t)+\left(C^{G(1)}_r+U_{12}\right)E^{(0)}_{22}(t)\right]\right]\tilde{F}^h_G \tag{7.11}$$

8. - ONE-HADRON INCLUSIVE LEPTOPRODUCTION

The general formalism for the single hadron inclusive leptoproduction is very similar to the one presented in Section 6 for the totally inclusive case. The cross-section is given by the expression

$$\frac{d\sigma^{\ell' h'}_{\ell h}(x,x',y;Q^2)}{dx\,dx'\,dy} = \sum_{pp'}\int_0^1 dz\,dz'\,d\hat{x}\,d\hat{x}'\,F^p_h(z,Q^2)\,\frac{d\hat{\sigma}^{\ell' p'}_{\ell p}(\hat{x},\hat{x}',y;Q^2)}{d\hat{x}\,d\hat{x}'\,dy}\,F^{h'}_{p'}(z',Q^2)\cdot$$
$$\cdot\,\delta(x-\hat{x}z)\,\delta(x'-\hat{x}'z') \tag{8.1}$$

where

$$q=\ell-\ell'\ ;\ Q^2=-q^2\ ,\quad x=\frac{Q^2}{2hq}\ ,\ x'=\frac{hh'}{hq}\ ,\ y=\frac{hq}{h\ell} \tag{8.2}$$

The structure functions are defined as follows :

$$\frac{d\sigma^{\ell' h'}_{\ell h}(x,x',y;Q^2)}{dx\,dx'\,dy} = \sigma_B(Q^2,s)\sum_{r=1}^{3} Y_{B,r}(y)\,f^{h'}_{Bh,r}(x,x';Q^2) \tag{8.3}$$

$$\frac{d\sigma^{\ell' p'}_{\ell p}(\hat{x},\hat{x}',y;Q^2)}{d\hat{x}\,d\hat{x}'\,dy} = \sigma_B(Q^2,s)\sum_{r=1}^{3} Y_{B,r}(y)\,\lambda^{p'}_{Bp,r}\,C^{\Sigma'}_{\Sigma,r}(\hat{x},\hat{x}';Q^2) \tag{8.4}$$

where the functions $Y_{B,r}(y)$ and the cross-sections σ_B are the same as in Section 6. The couplings $\lambda^{p'}_{Bp,r}$ are as follows $(r = 1,2)$:

$$\lambda^{q_j}_{\gamma q_i,r} = \lambda^{\bar{q}_j}_{\gamma \bar{q}_i,r} = \delta_{ij}\, e_i^2 \quad ; \quad \lambda^{\bar{q}_j}_{\gamma q_i,r} = \lambda^{q_j}_{\gamma \bar{q}_i,r} = 0$$

$$\lambda^{q_i}_{\gamma q,r} = \lambda^{\bar{q}_i}_{\gamma q,r} = \lambda^{G}_{\gamma q_i,r} = \lambda^{G}_{\gamma \bar{q}_i,r} = e_i^2$$

$$\lambda^{u}_{W^+d,r} = \lambda^{\bar{d}}_{W^+\bar{u},r} = \lambda^{c}_{W^+s,r} = \lambda^{\bar{s}}_{W^+\bar{c},r} = 2\cos^2\theta_C \qquad (r=1,2)$$

$$\lambda^{u}_{W^+d,3} = -\lambda^{\bar{d}}_{W^+\bar{u},3} = \lambda^{c}_{W^+s,3} = -\lambda^{\bar{s}}_{W^+\bar{c},3} = 2\cos^2\theta_C \tag{8.5}$$

etc. ....

As in the preceding sections, the indices P, P' = F or G indicate whether the corresponding parton (p,p') is fermion or gluon. The expansion of the functions C in powers of α has the form

$$C^{P'}_{P,r}(x,x';Q^2) = C^{P'(0)}_{P,r}\delta(1-x)\,\delta(1-x') + \frac{\alpha(Q^2)}{2\pi}\, C^{P'(1)}_{P,r}(x,x') + \cdots \tag{8.6}$$

where

$$C^{F(0)}_{F,r} = 1 \quad , \quad C^{G(0)}_{F,r} = C^{F(0)}_{G,r} = C^{G(0)}_{G,r} = 0$$

$$C^{G(1)}_{G,r}(x,x') = 0 \tag{8.7}$$

The functions $C^{F(1)}_{F,r}(x,x')$, $C^{G(1)}_{F,r}(x,x')$, $C^{F(1)}_{G,r}(x,x')$ are given in Appendix III.

The QCD formula for the structure functions f reads :

$$f^{h'}_{Bh,r}(x,x';Q^2) = \sum_{PP'} \int dz\, dz'\, d\hat{x}\, d\hat{x}'\, F^{P}_{h}(z,Q^2)\, \lambda^{P'}_{Bp,r}\, C^{P'}_{P,r}(\hat{x},\hat{x}';Q^2)\, F^{h'}_{P}(z',Q^2) \cdot$$
$$\cdot\, \delta(x-\hat{x}z)\, \delta(x'-\hat{x}'z') \tag{8.8}$$

The convolutions in Eq. (8.8) factorize after taking double moments :

$$f^{h'}_{Bh,r}(n,m;Q^2) = \sum_{PP'} F^{P}_{h}(n,Q^2)\, \lambda^{p'}_{Bp,r}\, C^{P'}_{P,r}(n,m;Q^2)\, F^{h'}_{p'}(m;Q^2) \tag{8.9}$$

where

$$A(n,m) \equiv \int_0^1 dx\, x^{n-1} \int_0^1 dy\, y^{m-1} A(x,y), \quad A(n) \equiv \int_0^1 dx\, x^{n-1} A(x) \tag{8.10}$$

for any function $A(x,y)$, $A(x)$. Therefore we will use as before the simplified notation

$$f^{h'}_{Bh,r}(t) = \sum_{pp'} F_h^p(t)\, \lambda^{p'}_{Bp,r}\, C^{P'}_{P,r}(t)\, F^{h'}_{p'}(t) \tag{8.11}$$

so that Eq. (8.11) can be interpreted either as a double convolution [Eq. (8.8)] or as a product of moments [Eq. (8.9)]. The general expression for the t dependence of the structure functions can be written down in terms of the evolution matrices $E^{(S)}_{pp'}(t)$, $E^{(T)}_{pp'}(t)$, presented in Section 5.

$$f^{h'}_{Bh,r}(t) = \sum_{pp'} \lambda^{p'}_{Bp,r} \sum_{aa'} \tilde{F}^a_h \Big[E^{(0)}_{pa}(t)\, C^{P'(0)}_{P}\, E^{(0)}_{p'a'}(t) + \tag{8.12}$$

$$+ \frac{\alpha}{2\pi} \left(E^{(0)}_{pa}(t)\, C^{P'(1)}_{P,r}\, E^{(0)}_{p'a'}(t) + E^{(1,S)}_{pa}(t)\, C^{P'(0)}_{P,r}\, E^{(0)}_{p'a'}(t) + E^{(0)}_{pa}(t)\, C^{P'(0)}_{P,r}\, E^{(1,T)}_{p'a'}(t) \right) \Big] \tilde{F}^{h'}_{a'}$$

After some algebra, one can decompose the sums over the types of partons in Eqs. (8.12) (p,p',a,a') into the singlet and non-singlet pieces, in a similar way as in the preceding sections.

Finally, let us make a comment about the "factorization breaking" in the inclusive leptoproduction. At the leading log level, Eq. (8.12) for $f(x,x';Q^2)$ factorizes in the variables x and x' :

$$f^{h'}_{Bh}(x,x';Q^2) = \sum_{pp'} F_h^p(x,Q^2)\, \lambda^{p'}_{Bp,r}\, F^{h'}_{p'}(x',Q^2) \tag{8.13}$$

This type of factorization is apparently broken at the subleading level, since the functions $C^{P'(1)}_{P,r}(x,x')$ do not factorize. In fact, however, as seen from the formulae in Appendix III, they can be written in the form of the sum of products :

$$C^{P'(1)}_{P,r}(x,x') = \sum_i A^{P'(1)}_{(i)P,r}(x)\; B^{P'(1)}_{(i)P,r}(x') \tag{8.14}$$

so that the following "generalized factorization" of the structure functions f is valid :

$$f^{h'}_{Bh}(x,x';Q^2) = \sum_{i,p,p'} F_{(i)}{}^{p}_{h}(x,Q^2)\,\lambda^{p'}_{Bp,r}\,F_{(i)}{}^{h'}_{p'}(x',Q^2) \tag{8.15}$$

where

$$F_{(i)}{}^{p}_{h}(x,Q^2) = F^{p}_{h}(x,Q^2) + \frac{\alpha(Q^2)}{2\pi} F^{p}_{h}(x,Q^2) \otimes A_{(i)}{}^{\mathcal{P}'(1)}_{\mathcal{P},r}(x)$$

$$F_{(i)}{}^{h'}_{p'}(x',Q^2) = F^{h'}_{p'}(x',Q^2) + \frac{\alpha(Q^2)}{2\pi} F^{h'}_{p'}(x',Q^2) \otimes B_{(i)}{}^{\mathcal{P}'(1)}_{\mathcal{P},r}(x') \tag{8.16}$$

9. - DRELL-YAN PROCESS

The standard variables in the Drell-Yan process

$$h_1 + h_2 \to \mu^+\mu^- + \text{ anything}$$

are :

$$Q^2 = (\ell'_1 + \ell'_2)^2, \quad s = (h_1 + h_2)^2, \quad \tau = \frac{Q^2}{s} \tag{9.1}$$

The cross-section is given by the expression :

$$\frac{d\sigma^{\mu^+\mu^-}_{h_1h_2}(\tau,Q^2)}{d\tau} = \sum_{P_1P_2} \int dz_1\,dz_2\,d\hat{\tau}\; F^{P_1}_{h_1}(z_1,Q^2)\,F^{P_2}_{h_2}(z_2,Q^2)\,\frac{d\hat{\sigma}^{\mu^+\mu^-}_{P_1P_2}(\hat{\tau},Q^2)}{d\hat{\tau}}\,\delta(\tau-\hat{\tau}z_1z_2) \tag{9.2}$$

where

$$\frac{d\hat{\sigma}^{\mu^+\mu^-}_{P_1P_2}(\hat{\tau},Q^2)}{d\hat{\tau}} = \frac{4\pi\alpha^2_{e.m.}}{3Q^2}\,C_{\mathcal{P}_1\mathcal{P}_2}(\hat{\tau},Q^2)\,\lambda_{P_1P_2}\;; \qquad \frac{d\sigma^{\mu^+\mu^-}_{h_1h_2}}{d\tau} \equiv \frac{4\pi\alpha^2_{e.m.}}{3Q^2}\,f^{\mu^+\mu^-}_{h_1h_2}(\tau)$$

$$C_{\mathcal{P}_1\mathcal{P}_2}(\hat{\tau},Q^2) = C^{(0)}_{\mathcal{P}_1\mathcal{P}_2}\,\delta(1-\hat{\tau}) + \frac{\alpha(Q^2)}{2\pi}\,C^{(1)}_{\mathcal{P}_1\mathcal{P}_2}(\hat{\tau}) + \ldots \tag{9.3}$$

and

$$C^{(0)}_{FF} = 1 \quad , \quad C^{(0)}_{FG} = C^{(0)}_{GF} = C^{(0)}_{GG} = 0 \quad , \quad C^{(1)}_{GG}(x) = 0$$

The functions $C^{(1)}_{FF}(\tau)$, $C^{(1)}_{GF}(\tau)$ and $C^{(1)}_{FG}(\tau)$ are given in Appendix IV.

The couplings $\lambda_{p_1p_2}$ are the following :

$$\lambda_{q_i\bar{q}_j} = \lambda_{\bar{q}_iq_j} = \frac{1}{3}e_i^2\delta_{ij} \quad , \quad \lambda_{q_iq_j} = \lambda_{\bar{q}_i\bar{q}_j} = 0 \tag{9.4}$$

$$\lambda_{q_iG} = \lambda_{\bar{q}_iG} = \lambda_{Gq_i} = \lambda_{G\bar{q}_i} = e_i^2$$

The convolution in Eq. (9.2) factorizes after taking τ moments :

$$f^{\mu^+\mu^-}_{h_1h_2}(n,Q^2) = \sum_{p_1p_2} F^{p_1}_{h_1}(n,Q^2)\,F^{p_2}_{h_2}(n,Q^2)\,\lambda_{p_1p_2}\,C_{P_1P_2}(n,Q^2) \tag{9.5}$$

The t dependence of the function f is the following :

$$f^{\mu^+\mu^-}_{h_1h_2}(t) = \sum_{p_1p_2}\lambda_{p_1p_2}\sum_{a_1a_2}\tilde{F}^{a_1}_{h_1}\tilde{F}^{a_2}_{h_2}\Big[E^{(0)}_{p_1a_1}(t)\,E^{(0)}_{p_2a_2}(t)\,C^{(0)}_{P_1P_2} +$$

$$+\frac{\alpha}{2\pi}\Big[E^{(0)}_{p_1a_1}(t)\,E^{(0)}_{p_2a_2}(t)\,C^{(1)}_{P_1P_2} + \Big(E^{(1,s)}_{p_1a_1}(t)\,E^{(0)}_{p_2a_2}(t) + E^{(0)}_{p_1a_1}(t)\,E^{(1,s)}_{p_2a_2}(t)\Big)C^{(0)}_{P_1P_2}\Big]\Big] \tag{9.6}$$

10. - AMBIGUITIES AT THE NEXT-TO-LEADING LEVEL

Various ingredients of the QCD formulae including non-leading corrections contain a certain amount of ambiguity.

In this section we discuss systematically all the ambiguities appearing at the next-to-leading level. They have three sources :

a) renormalization prescription dependence ;

b) factorization scheme dependence ;

c) freedom in the definition of the input parton densities.

10.a. - Renormalization prescription dependence

There is a well-known freedom in the definition of the set of renormalization conditions fixing the cut-off independent part of the counterterms which make the perturbative expansion finite. We call this the renormalization scheme (prescription) dependence. Any result expressed as a truncated expansion in $\alpha_s(Q^2)$ is affected, beyond leading order, by a change of such a definition, which we assume here universal for all processes.

However, there is in addition an independent ambiguity arising when a parametric form of the running coupling constant is obtained by integrating Eq. (2.11), corresponding to different choices of the imposed boundary conditions. After rescaling Q^2 in Eq. (2.11) by Λ^2 we get

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0\left(\ln Q^2/\Lambda^2 + C\right) + \frac{\beta_1}{\beta_0}\ln\left(1 + \frac{4\pi\beta_0}{\beta_1\alpha(Q^2)}\right)} + O\left(\frac{1}{\ln^3 Q^2/\Lambda^2}\right) \qquad (10.1)$$

where C is a constant, introduced during the integration of Eq. (2.11). The parametrization of $\alpha(Q^2)$ in terms of Λ^2 and C is obviously redundant - C can be reabsorbed by rescaling :

$$\Lambda^2 \to \Lambda^2 e^{-C} \qquad (10.2)$$

As a consequence, we can either keep explicitly the C dependence, which gives, after expanding in powers of $1/\ln(Q^2/\Lambda^2)$,

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0}\frac{1}{\ln Q^2/\Lambda^2}\left[1 - \frac{1}{\ln Q^2/\Lambda^2}\left(C + \frac{\beta_1}{\beta_0^2}\ln\frac{\beta_0^2}{\beta_1} + \frac{\beta_1}{\beta_0^2}\ln\ln Q^2/\Lambda^2\right) + O\left(\frac{1}{\ln^2 Q^2/\Lambda^2}\right)\right] \qquad (10.3)$$

or rescale Λ as follows

$$\Lambda^2 \Rightarrow \Lambda^2 e^{-\left\{C + \frac{\beta_1}{\beta_0^2}\ln\frac{\beta_0^2}{\beta_1}\right\}} \qquad (10.4)$$

which gives

$$\frac{\alpha(Q^2)}{2\pi} = \frac{2}{\beta_0}\frac{1}{\ln Q^2/\Lambda^2}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2} + O\left(\frac{1}{\ln^2 Q^2/\Lambda^2}\right)\right] \qquad (10.5)$$

One can also fix the parameter "C" in Eq. (10.3) to normalize $\alpha_{2\text{-loop}}(Q^2)$ equal to $\alpha_{1\text{-loop}}(Q^2)$ at some $Q^2 = Q_o^2$ [20]. At a two-loop level, any result expanded up to second order in $\alpha_{1\text{-loop}}(Q^2) \equiv 2/(\beta_o \ell n Q^2/\Lambda^2)$ contains ambiguous terms in the coefficient of the order γ_s^2, which both come from the renormalization scheme and from input definition dependence. They can both be reabsorbed in this case by a redefinition of Λ. We will see in subsections 10b and 10c how the same kind of ambiguities arise also from the factorization procedure.

When physical processes are expressed in terms of other physical processes, the above ambiguities drop, but still a last freedom is left : the choice of the "correct" scale which enters in the running coupling constant ("scale definition" dependence). The above freedom can also be seen as a process dependent renormalization prescription, which does not drop when comparing different processes at a given order of perturbation theory ; its definition can be chosen to "optimize" the the convergence of the perturbative expansion by requiring, for example, a "minimal sensitivity" to this choice of the results of the truncated perturbative expansion *).

The formulae we give in this paper are obtained in a universal factorization scheme (the $\overline{\text{MS}}$) and are parametrized by the full $\alpha_s(Q^2)$ of Eq. (10.3). The constant C can be arbitrarily chosen, provided it is kept fixed in the comparison of various processes, where input definition and universal scheme dependences drop. The scale of the running coupling constant is arbitrarily identified with the value of a "typical" large invariant of the process : further optimizations are possible, but legitimate, in our opinion, only if they remain at the level of a small perturbation of the $O(\alpha_s^2)$ result. A warning must, in particular, be made for large next-to-leading corrections which survive the comparison between different processes : these are often of kinematical origin and require resummation techniques (exponentiation) different from those implied by a suitable choice of the running scale.

10.b. - Factorization scheme dependence

The generic QCD formula has the form

$$F(t) = C(t)\,\Gamma(t) \tag{10.6}$$

where $F(t)$ is the experimentally measured distribution, $C(t)$ is the hard lepton-parton cross-section and $\Gamma(t)$ stands for the set of Q^2 dependent parton densities.

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*) In Ref. 21) the generalization of this freedom beyond next-to-leading order is extensively discussed by fully exploiting the renormalization group invariance introduced in Ref. 10).

For simplicity, we will limit the discussion in this section to the deep inelastic current, but all the remarks presented here remain valid for any hard process.

All the formulae written below are valid both for non-singlet and for singlet parts. In the latter case, the matrix notation will be implicit. In the non-singlet case, Eq. (10.6) should be interpreted, e.g., as follows :

$$F(t) = f_{ep,r}(t) - f_{en,r}(t)$$
$$C(t) = \frac{1}{3} C_{F,r}(t)$$
$$\Gamma(t) = v^{(+)}(t) \tag{10.7}$$

with $r = 1,2$ denoting the type of structure function considered, and in the singlet case, e.g., as follows :

$$F(t) = f_{\nu p,r}(t) + f_{\bar{\nu} p,r}(t)$$
$$C(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} C_{F,r}(t) \\ C_{G,r}(t) \end{bmatrix}; \quad \Gamma(t) = \begin{bmatrix} q^{(+)}(t) \\ G(t) \end{bmatrix}$$

$$F(t) = C(t)\,\Gamma(t) = \sum_{i=1}^{2} C_i(t)\,\Gamma_i(t) \tag{10.8}$$

In the following, we will mainly discuss the singlet case which is less trivial - the corresponding non-singlet expressions can be obtained simply by replacing all vectors and matrices by the corresponding scalar quantities.

The parton densities evolve with Q^2 as follows :

$$\frac{d}{dt}\Gamma(t) = \left(P^{(0)} + \frac{\alpha}{2\pi} R\right)\Gamma(t)\ ; \quad R = P^{(1)} - \frac{\beta_1}{2\beta_0} P^{(0)} \tag{10.9}$$

and the cross-sections $C(t)$ are given by power series in α :

$$C(t) = C^{(0)} + \frac{\alpha}{2\pi} C^{(1)} + \dots \tag{10.10}$$

The results for $P^{(1)}$ and $C^{(1)}$ presented in this paper correspond to one particular factorization scheme constructed in Refs. 7),9). Since, as discussed in Section 3, the factorization of F into C and Γ parts is not unique, the functions $P^{(1)}$ and $C^{(1)}$ are different in various factorization schemes. In order to parametrize explicitly this ambiguity, we observe that, given the formula (10.6), we can always redefine $C(t)$ and $\Gamma(t)$ as follows :

$$\begin{aligned} C(t) &\to \hat{C}(t) \equiv C(t) Z^{-1}(t) \\ \Gamma(t) &\to \hat{\Gamma}(t) \equiv Z(t)\, \Gamma(t) \end{aligned} \tag{10.11}$$

where

$$Z(t) = \mathbb{1} + \frac{\alpha}{2\pi} Z^{(1)} + \cdots$$

is a 2x2 matrix.

Various choices of $Z(t)$ correspond to various factorization schemes. We have now

$$\begin{aligned} \hat{C}^{(1)} &= C^{(1)} - C^{(0)} Z^{(1)} \\ \hat{P}^{(1)} &= P^{(1)} + [Z^{(1)}, P^{(0)}] - \frac{\beta_0}{2} Z^{(1)} \end{aligned} \tag{10.12}$$

Equations (10.12) parametrize the ambiguity of $C^{(1)}$ and $P^{(1)}$ due to the freedom in the choice of the factorization scheme. On the other hand, $F(t)$ does not change, as seen from Eqs. (10.11) :

$$F(t) = C(t)\,\Gamma(t) = \hat{C}(t)\,\hat{\Gamma}(t)$$

The cancellation of the scheme dependence must take place order-by-order in α. Let us demonstrate how the $Z^{(i)}$ dependence cancels in the $O(\alpha)$ contribution to $F(t)$.

For this reason, we use the evolution matrix $E(t)$ normalized as in Eq. (5.58). We have now

$$\begin{aligned} F(t) &= C(t)\, E(t)\, \Gamma(0) \equiv \Phi(t)\, \Gamma(0) \\ &= \hat{C}(t)\, \hat{E}(t)\, \Gamma(0) \equiv \hat{\Phi}(t)\, \Gamma(0) \end{aligned} \tag{10.13}$$

where

$$\Phi(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = C^{(0)}E^{(0)}(t) + \frac{\alpha_0}{2\pi}\Big[C^{(1)}E^{(0)}(t)\,e^{-\frac{\beta_0}{2}t} + $$
$$+ C^{(0)}\int_0^t d\tau\, e^{-\frac{\beta_0}{2}\tau}\,E^{(0)}(t-\tau)\,R\,E^{(0)}(\tau)\Big]$$

$$\hat{\Phi}(t) = C^{(0)}E^{(0)}(t) + \frac{\alpha_0}{2\pi}\Big\{C^{(1)}E^{(0)}(t)\,e^{-\frac{\beta_0}{2}t} - C^{(0)}Z^{(1)}E^{(0)}(t)\,e^{-\frac{\beta_0}{2}t}$$
$$+ C^{(0)}E^{(0)}(t)Z^{(1)} +$$
$$+ C^{(0)}\int_0^t d\tau\, e^{-\frac{\beta_0}{2}\tau}\,E^{(0)}(t-\tau)\Big[R + [Z^{(1)}, P^{(0)}] - \frac{\beta_0}{2}Z^{(1)}\Big]E^{(0)}(\tau)\Big\}$$

(10.14)

Now we derive the following identity, which will be used many times in the section ; integrating by parts the expression

$$\int_0^t d\tau\, e^{-\frac{\beta_0}{2}\tau}\,E^{(0)}(t-\tau)\,A\,E^{(0)}(\tau) \qquad (10.15)$$

where A is an arbitrary (t independent) 2×2 matrix, we get

$$e^{-\frac{\beta_0}{2}t} A\,E^{(0)}(t) = E^{(0)}(t)\,A + \int_0^t d\tau\, e^{-\frac{\beta_0}{2}\tau}\,E^{(0)}(t-\tau)\Big\{[A, P^{(0)}] - \frac{\beta_0}{2}A\Big\}E^{(0)}(t)$$

(10.16)

Using the above relation, we get immediately

$$\Phi(t) = \hat{\Phi}(t) \qquad (10.17)$$

10.c. - Freedom in the definition of the input parton densities

The function $F(t)$ depends on two input functions :

$$\Gamma(0) \equiv \Gamma = \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} \qquad (10.18)$$

Again, we can redefine $\Phi(t)$ and Γ as follows :

$$\begin{aligned} \Phi(t) &\rightarrow \Phi_Y(t) \equiv \phi(t) Y \\ \Gamma &\rightarrow \Gamma_Y \equiv Y^{-1}\Gamma \end{aligned} \tag{10.19}$$

where

$$Y = Y^{(0)} + \frac{\alpha_0}{2\pi} Y^{(1)} + \cdots \tag{10.20}$$

is an arbitrary non-singular 2x2 matrix. We have therefore :

$$F(t) = \Phi(t)\Gamma = \Phi_Y(t)\Gamma_Y \tag{10.21}$$

where

$$\Phi_Y(t) = \Phi_Y^{(0)}(t) + \frac{\alpha_0}{2\pi}\Phi_Y^{(1)}(t) + \cdots \tag{10.22}$$

The vector $\Phi_Y(t)$ is independent of the factorization scheme but it contains the ambiguity due to the freedom in the definition of the input densities. Equivalently, we can say that we use the evolution matrix $E_Y(t) = E(t)Y$ normalized as $E_Y(0) = Y$.

In what follows we discuss some particularly interesting choices for the matrix Y.

a) S scheme

We take $Y \equiv 1$, i.e., $Y^{(0)} = 1$, $Y^{(i)} = 0$ $(i > 0)$. In this scheme, the parton densities

$$\Gamma(t) = \begin{bmatrix} q^{(+)}(t) \\ G(t) \end{bmatrix} = \left[E^{(0)}(t) + \frac{\alpha_0}{2\pi}E^{(1)}(t)\right]\Gamma(0) \tag{10.23}$$

satisfy for any value of t the parton model sum rules : momentum conservation for singlet, fermion number conservation for non-singlet. The input densities $\Gamma(0)$ are defined by the formal expressions (3.5) of our scheme *).

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*) This is the scheme we suggest to use : it can be obtained from the formulae used in the paper by means of Eqs. (5.63) and (5.64).

b) U scheme

Taking $Y^{(0)} = 1$ and using the identity (10.16) we can write the expression for $\Phi_Y^{(1)}(t)$ as follows :

$$\Phi_Y^{(1)}(t) = e^{-\frac{\beta_0}{2}t}\left[C^{(1)} + C^{(0)}Y^{(1)}\right]E^{(0)}(t) +$$

$$+\int_0^t d\tau\, e^{-\frac{\beta_0}{2}\tau} E^{(0)}(t-\tau)\left\{R + \left[Y^{(1)},P^{(0)}\right] - \frac{\beta_0}{2}Y^{(1)}\right\}E^{(0)}(\tau) \qquad (10.24)$$

Taking $Y^{(1)} = -U$ we cancel the expression in the bracket { } in Eq. (10.24) [see Eq. (5.28)].

As a consequence, $\Phi_Y(t)$ takes a particularly simple form in the U scheme :

$$\Phi_Y(t) = \left[C^{(0)} + \frac{\alpha}{2\pi}\left(C^{(1)} + C^{(0)}U\right)\right]E^{(0)}(t) \qquad (10.25)$$

In numerical calculations, however, the U scheme is not very useful. If R contains a part which does not commute with $P^{(0)}$, the matrix U(n) has poles on the Mellin plane for positive values of $n > 1$ [factors $1/[\lambda_1-\lambda_2\pm(\beta_0/2)]$ in Eq. (5.41)]. It means that the matrix U(x) behaves like $(1/x)^{\alpha}$ for $x \to 0$ where $\alpha > 1$. For example, for four flavours we have $\alpha \simeq 3.8$. These singularities are completely spurious and should be cancelled by the corresponding zeros of the input densities in the U scheme. However, it will never be the case when we take only the $O(\alpha_0)$ term in the perturbative expansions of both Y and Y^{-1}.

Therefore, the elegant U scheme is unfortunately rather useless in practice.

c) C scheme

We may try to define the parton densities by identifying them with the measurable structure functions in some reference processes. However, in order to define both $q^{(+)}$ and G we have to introduce some hypothetical current which couples to order α^0 to gluon and to order α^i (i > 0) to quark [in a similar way as the electromagnetic current couples to order α^0 to quark and to order α^i (i > 0) to gluon]. For example, we can formally add a term $\phi(\vec{F}^{\mu\nu}\vec{F}_{\mu\nu})$ to the QCD Lagrangian.

Let us introduce the following notation :

$$\sigma(t) = \begin{pmatrix} \sigma_1(t) \\ \sigma_2(t) \end{pmatrix} = \begin{pmatrix} F_\gamma(t) \\ F_\varphi(t) \end{pmatrix} \tag{10.26}$$

where F_γ and F_ϕ are the singlet structure functions for γ and ϕ currents, respectively,

$$\mathbb{C}(t) = \begin{bmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{bmatrix} = \begin{bmatrix} C_{\gamma F}(t) & C_{\gamma G}(t) \\ C_{\varphi F}(t) & C_{\varphi G}(t) \end{bmatrix} \equiv \mathbb{1} + \frac{\alpha}{2\pi}\,\mathbb{C}^{(1)} \tag{10.27}$$

where C_{ij} are the corresponding coefficient functions. We have now

$$\sigma(t) = \mathbb{C}(t)\,E(t)\,\Gamma(0) \tag{10.28}$$

The vector $\sigma(t)$ satisfies the evolution equation

$$\frac{d}{dt}\sigma(t) = \mathbb{P}(t)\,\sigma(t) \tag{10.29}$$

where

$$\mathbb{P}(t) = \mathbb{P}^{(0)} + \frac{\alpha}{2\pi}\mathbb{R}$$

$$\mathbb{R} = R + [\mathbb{C}^{(1)}, \mathbb{P}^{(0)}] - \frac{\beta_0}{2}\mathbb{C}^{(1)}$$

We can now define the $q^{(+)}$ and G densities as being equal to σ_1 and σ_2, respectively. In this way we can relate the singlet parton densities to some "measurable" quantities.

The solution for the γ part reads :

$$\sigma_1(t) = \Phi_1(t)\,\sigma_1(0) + \Phi_2(t)\,\sigma_2(0) \tag{10.30}$$

where

$$\Phi(t) = C^{(0)}E^{(0)}(t) + \frac{\alpha_0}{2\pi}\left\{C^{(0)}E^{(1)}(t) - C^{(0)}E^{(0)}(t)\mathbb{C}^{(1)} + e^{-\frac{\beta_0 t}{2}}C^{(1)}E^{(0)}(t)\right\} \tag{10.31}$$

[in order to derive (10.31) we have used the identity (10.16)]. Comparing Eqs. (10.31) and (10.24) we see that the C scheme corresponds to

$$\mathbb{Y} = \mathbb{1} + \frac{\alpha_0}{2\pi}\,\mathbb{C}^{(1)} \tag{10.31}$$

The C scheme is not very useful in practice - there is no point in doing the explicit calculation of the spurious coefficient functions C_{21}, C_{22} for the ϕ current which does not exist in the real world.

Nevertheless, it provides the physical interpretation of the ambiguity due to the freedom of the choice of Y : in order to define the gluon distribution in terms of some measurement, we get (at the subleading level) also the corrections to this measurement contained in the second row of the matrix $\mathbb{C}^{(1)}$.

Note also that, as seen from Eq. (10.29) there are no coefficient functions in the C scheme at all : the full physical structure function evolves like a parton density. Therefore, from the point of view of the t dependence, we may say that the C scheme corresponds to the change of the factorization structure scheme described by $Z(t) = \mathbb{C}(t)$. Equation (10.31) then follows immediately from (10.12) after substituting $Z^{(1)} = \mathbb{C}^{(1)}$.

d) <u>D scheme</u>

We can completely eliminate the notion of gluon density by writing a second order differential equation for $F(t) = F_\gamma(t)$. Differentiating (10.30) and eliminating $\sigma_2(0)$ we get

$$\ddot{F}(t) = p_1(t)\,\dot{F}(t) + p_0(t)\,F(t) \tag{10.32}$$

where
$$p_i(t) = p_i^{(0)} + \frac{\alpha}{2\pi}\,p_i^{(1)} + \dots$$

$$p_0^{(0)} \equiv \mathcal{P}_{12}^{(0)}\mathcal{P}_{21}^{(0)} - \mathcal{P}_{11}^{(0)}\mathcal{P}_{22}^{(0)} = -\det\left[\mathcal{P}^{(0)}\right]$$

$$p_1^{(0)} \equiv \mathcal{P}_{11}^{(0)} + \mathcal{P}_{22}^{(0)} = \mathrm{Tr}\left[\mathcal{P}^{(0)}\right]$$

$$p_0^{(1)} = \left(R_{11} - \frac{\beta_0}{2}C_1^{(1)}\right)\left(\mathcal{P}_{11}^{(0)} - \mathcal{P}_{22}^{(0)} - \frac{\beta_0}{2}\right) + \\ + \mathcal{P}_{12}^{(0)}R_{21}^{(0)} + \mathcal{P}_{21}^{(0)}R_{12}^{(0)} - \beta_0\,C_2^{(1)}\mathcal{P}_{21}^{(0)} - \mathcal{P}_{11}^{(0)}p_1^{(1)}$$

$$p_1^{(1)} = R_{11} + R_{22} - \frac{\beta_0}{2}\left[2C_1^{(1)} + \frac{1}{\mathcal{P}_{12}^{(0)}}\left[R_{12} + C_2^{(1)}\left(\mathcal{P}_{22}^{(0)} - \mathcal{P}_{11}^{(0)} - \frac{\beta_0}{2}\right)\right]\right] \tag{10.33}$$

The functions $p_i(t)$ are now physical quantities : they are free from any ambiguity discussed so far since they enter the differential equation for a physical quantity. We can write the solution of Eq. (10.32) in the form (10.18) where

$$\Gamma_Y = \begin{bmatrix} F(0) \\ \dot{F}(0) \end{bmatrix} \equiv \begin{bmatrix} F(t) \\ \frac{dF(t)}{dt} \end{bmatrix}_{t=0} \tag{10.34}$$

The matrix Y for the D scheme has the form

$$\begin{aligned} Y^{(0)} &= \frac{1}{P_{12}^{(0)}} \begin{bmatrix} P_{12}^{(0)} & 0 \\ -P_{11}^{(0)} & 1 \end{bmatrix} \\ Y^{(1)} &= \frac{1}{P_{12}^{(0)}} \tilde{Y}^{(1)} \end{aligned} \tag{10.35}$$

where

$$\begin{aligned} \tilde{Y}_{11}^{(1)} &= C_2^{(1)} P_{11}^{(0)} - C_1^{(1)} P_{12}^{(0)} \\ \tilde{Y}_{12}^{(1)} &= -C_2^{(1)} \\ \tilde{Y}_{21}^{(1)} &= \frac{P_{11}^{(0)}}{P_{12}^{(0)}} R_{12} - R_{11} + C_1^{(1)} \left(P_{11}^{(0)} + \frac{\beta_0}{2}\right) + \\ &\quad + C_2^{(1)} \left[\frac{P_{11}^{(0)}}{P_{12}^{(0)}} \left(P_{22}^{(0)} - P_{11}^{(0)} - \frac{\beta_0}{2}\right) - P_{21}^{(0)} \right] \end{aligned} \tag{10.36}$$

$$\tilde{Y}_{22}^{(1)} = -C_1^{(1)} - \frac{1}{P_{12}^{(0)}} \left[R_{12} + C_2^{(1)} \left(P_{22}^{(0)} - P_{11}^{(0)} - \frac{\beta_0}{2}\right)\right]$$

Technically, the D scheme is rather complicated for the analysis of the data in the x variable : the operator $1/P_{12}^{(0)}(x)$ is a complicated object in x space. The virtue of the D scheme is that one is dealing all the time with unambiguous quantities since the input is defined in terms of (in principle) measurable quantities $F(0)$, $\dot{F}(0)$.

Once the vector Γ_Y has been extracted from the data, one can derive the gluon distribution according to [22]

$$\Gamma = \begin{pmatrix} q^{(+)}(0) \\ G(0) \end{pmatrix} = Y \Gamma_Y \tag{10.37}$$

CONCLUSION

The presence of next-to-leading corrections in any quantitative test of QCD in hard lepton-hadron processes is essential. However, the freedom related to the scale definition dependence of theoretical predictions survives the comparison of physical processes in terms of physical processes and always makes it possible to locally adjust, for a better fit, the choice of the scale at which a specific hadron process is "resolved".

In this respect, the "best" theoretical predictions are those where, as in the case of the total Drell-Yan cross-section normalized to totally inclusive leptoproduction, the so-called "next-to-leading" corrections turn out to be as large as the dominant term. A redefinition of the scale cannot mask in these cases the size of the correction which then becomes a potential clearcut test of the theory, provided a systematic resummation method of the large terms is found.

Apart from these peculiar cases, for the remaining ones where the corrections stay perturbative, quantitative indications can be obtained only by an investigation of many processes at the same time, aiming to reach systematic overall agreement with predicted estimates. In order to perform such an analysis we thought it useful to establish a common notation and to transform the existing results for the next-to-leading corrections to hard lepton-hadron processes into a unique renormalization-factorization scheme.

It is now up to the experimentalists to prove (or disprove) the validity of the predictions collected in this paper.

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APPENDIX I

Coefficient functions for inclusive leptoproduction.

$$C^{(1)}_{F,2} = C_F\left[\frac{1+x^2}{1-x}\left(\ln\frac{1-x}{x}-\frac{3}{4}\right)+\frac{1}{4}(9+5x)\right]_+$$

$$C^{(1)}_{F,1} = C^{(1)}_{F,2} - C_F\cdot[2x]$$

$$C^{(1)}_{F,3} = C^{(1)}_{F,2} - C_F\cdot[1+x]$$

$$C^{(1)}_{G,2} = 2f\,T_R\left[(x^2+(1-x)^2)\ln\left(\frac{1-x}{x}\right) - 1 + 8x(1-x)\right]$$

$$C^{(1)}_{G,1} = C^{(1)}_{G,2} - 2f\,T_R\,[4x(1-x)]$$

APPENDIX II

Coefficient functions for inclusive e^+e^- annihilation.

$$C^{F(1)}_{2} = C_F\left\{\left[\frac{1+x^2}{1-x}\left(\ln\frac{1-x}{x}-\frac{3}{4}\right)+\frac{1}{4}(9+5x)\right]_+ + 3\,\frac{1+x^2}{1-x}\ln x - \frac{7}{2}(1+x) + \pi^2\delta(1-x)\right\}$$

$$C^{F(1)}_{1} = C^{F(1)}_{2} + C_F\cdot[2]$$

$$C^{G(1)}_{2} = 2f\cdot C_F\left\{\frac{1+(1-x)^2}{x}\left[\ln(1-x)+2\ln x\right] - 6\,\frac{(1-x)}{x}\right\}$$

$$C^{G(1)}_{1} = C^{G(1)}_{2} + 2f\cdot C_F\left[4\,\frac{(1-x)}{x}\right]$$

APPENDIX III

Coefficient functions for one-particle inclusive leptoproduction.

$$C^{F(1)}_{F,2}(x,z) = C_F\Big\{\left(1-\tfrac{1}{3}\pi^2\right)\delta(1-x)\delta(1-z) + \delta(1-z)\Big[-2-3x+\tfrac{3}{2}\frac{1}{(1-x)_+} +$$

$$+ \left(\tfrac{1}{C_F}\right)\cdot C^{(1)}_{F,2}(x)\Big] + \delta(1-x)\Big[\tfrac{1}{2}(z-1)+\tfrac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ln z + \left(\frac{1}{C_F}\right)\cdot C^{F(1)}_{1}(z)\Big] +$$

$$+ \frac{1+(1-x-z)^2}{(1-x)_+(1-z)_+} + 6xz\Big\}$$

$$C^{F(1)}_{F,3}(x,z) = -C_F\left[2(1-x)(1-z)\right] + C^{F(1)}_{F,1}(x,z)$$

$$C^{F(1)}_{F,1}(x,z) = C^{F(1)}_{F,2}(x,z) - C_F\left[4xz\right]$$

$$C^{G(1)}_{F,2}(x,z) = C_F\Big\{\delta(1-x)\Big[\frac{1+(1-z)^2}{z}(1-\ln z) + \frac{1}{2fC_F}C^{G(1)}_{1}(z)\Big] +$$

$$+ \frac{1+(z-x)^2}{(1-x)_+ z} + 6x(1-z)\Big\}$$

$$C^{G(1)}_{F,3}(x,z) = -C_F\left[2(1-x)z\right] + C^{G(1)}_{F,1}(x,z)$$

$$C^{G(1)}_{F,1}(x,z) = C^{G(1)}_{F,2}(x,z) - C_F\left[4x(1-z)\right]$$

$$C^{F(1)}_{G,2}(x,z) = T_R\cdot\Big\{\delta(1-z)\Big[\left[1-6x(1-x)\right] + \frac{1}{2fT_R}\cdot C^{(1)}_{G,2}(x)\Big] -$$

$$-2\left(1-6x(1-x)\right) + \left[x^2+(1-x)^2\right]\left(\frac{1}{z}+\frac{1}{(1-z)_+}\right)\Big\}$$

$$C^{F(1)}_{G,3}(x,z) = -T_R\left[2\left(x^2+(1-x)^2\right)\cdot\frac{1-z}{z}\right] + C^{F(1)}_{G,1}(x,z)$$

$$C^{F(1)}_{G,1}(x,z) = -T_R\left[8x(1-x)\right] + C^{F(1)}_{G,2}(x,z)$$

APPENDIX IV

Coefficient functions for Drell-Yan process.

$$C_{FF}^{(1)} = C_F \left\{ 4(1+\tau^2)\left[\frac{\ln(1-\tau)}{1-\tau}\right]_+ - 2\,\frac{1+\tau^2}{1-\tau}\ln\tau + \left(\frac{2}{3}\pi^2 - 8\right)\delta(1-\tau) \right\}$$

$$C_{FG}^{(1)} = C_{GF}^{(1)} = T_R \left\{ [\tau^2 + (1-\tau)^2]\left(2\ln(1-\tau) - \ln\tau\right) + \frac{1}{2} + 3\tau - \frac{7}{2}\tau^2 \right\}$$

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FIGURE CAPTIONS

Figure 1 Modulus squared of the amplitude $\ell + h \to \ell' +$ anything.

Figure 2 The squared amplitude, decomposed in a "generalized ladder" ; $\tilde{\sigma}$ and K represent two-particle irreducible kernels.

Figure 3 An incomplete factorized decomposition : the quantity F' contains all the mass singularities of the squared amplitude.

Figure 4 The action of the projector between neighbour kernels.

Figure 5 Representation of the factorized deep inelastic leptoproduction.

Figure 6 Representation of the factorized one-particle inclusive leptoproduction.

Figure 7 Representation of the factorized one-particle inclusive e^+e^- annihilation.

Figure 8 Representation of the factorized Drell-Yan process.

Figure 9 The "valence" (P^V) and "singlet" (P^S) diagrams contributing to the transitions $q_i \to q_j$ and $q_i \to \bar{q}_j$ at two-log level in the evolution probabilities. The black blob denotes fully virtual corrections.

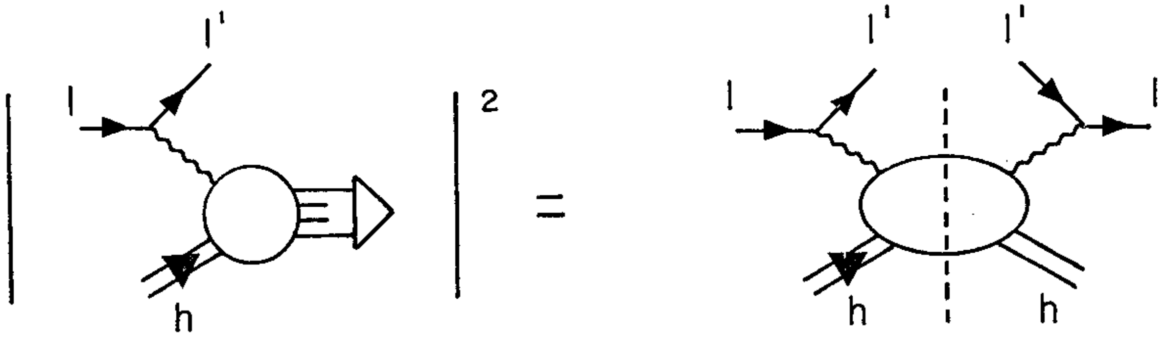


Fig. 1

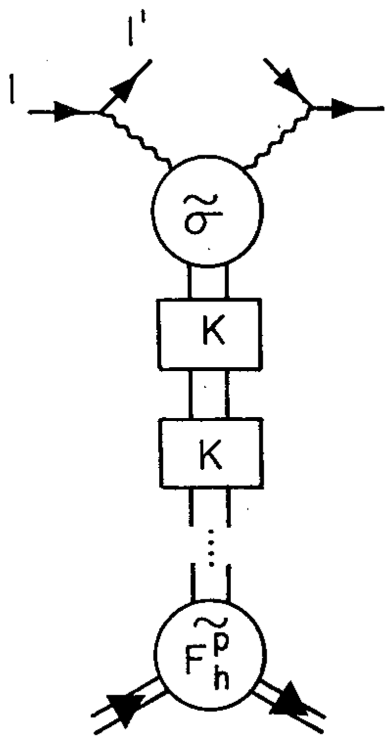


Fig. 2

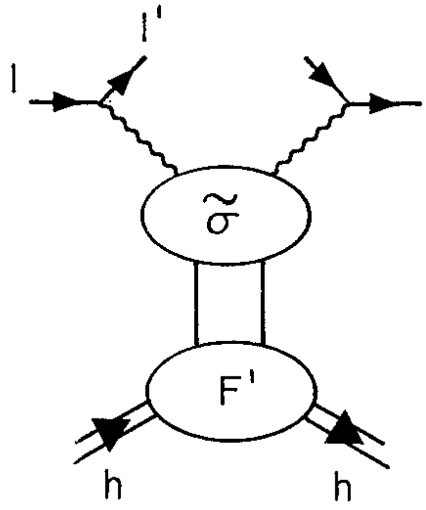
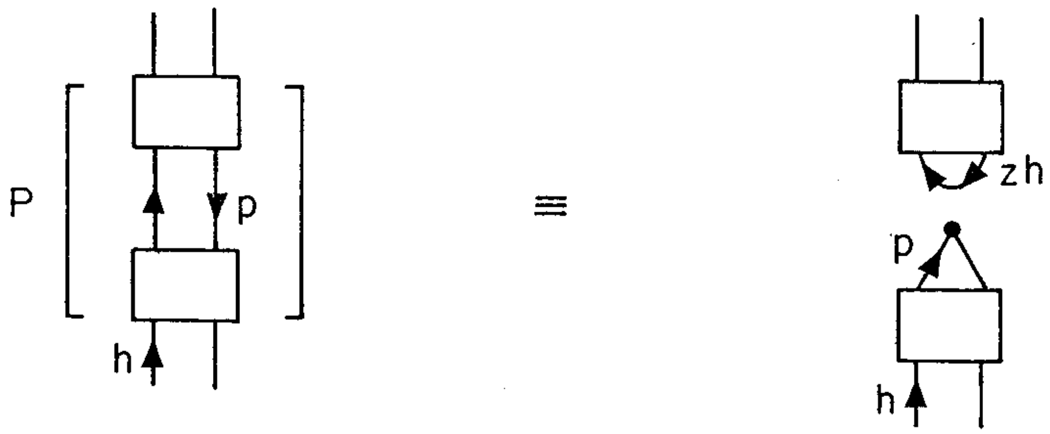


Fig. 3



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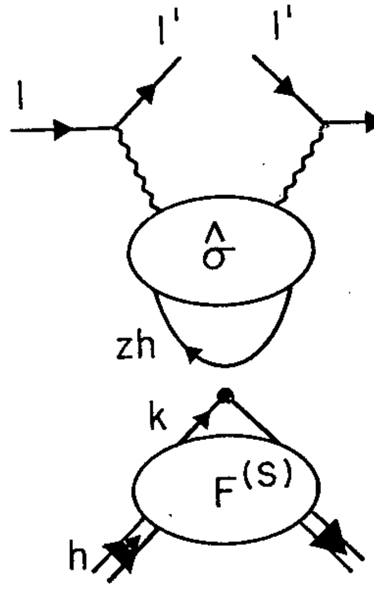
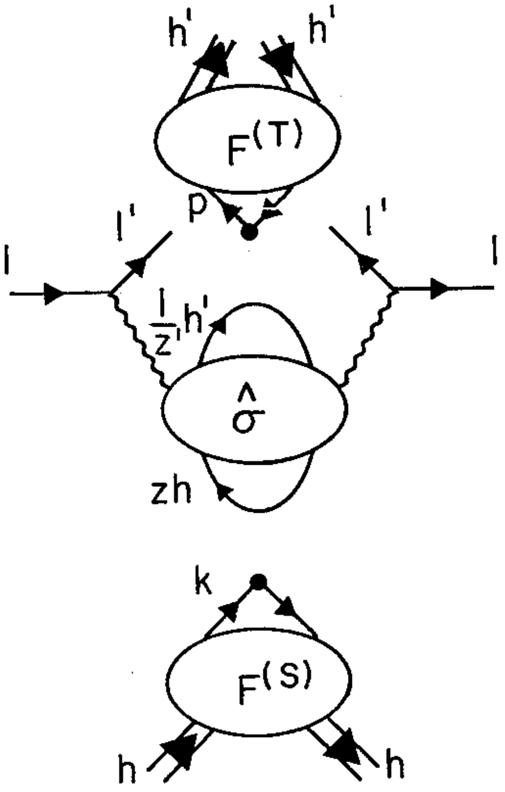


Fig. 5



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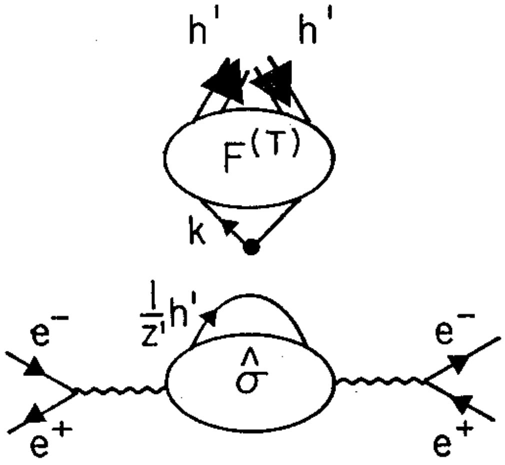

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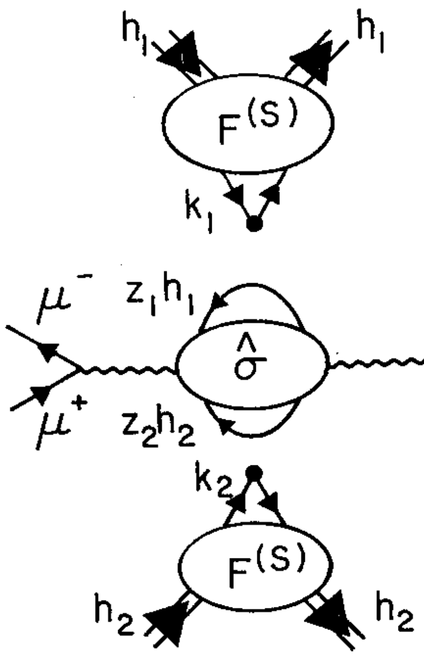

Fig. 8

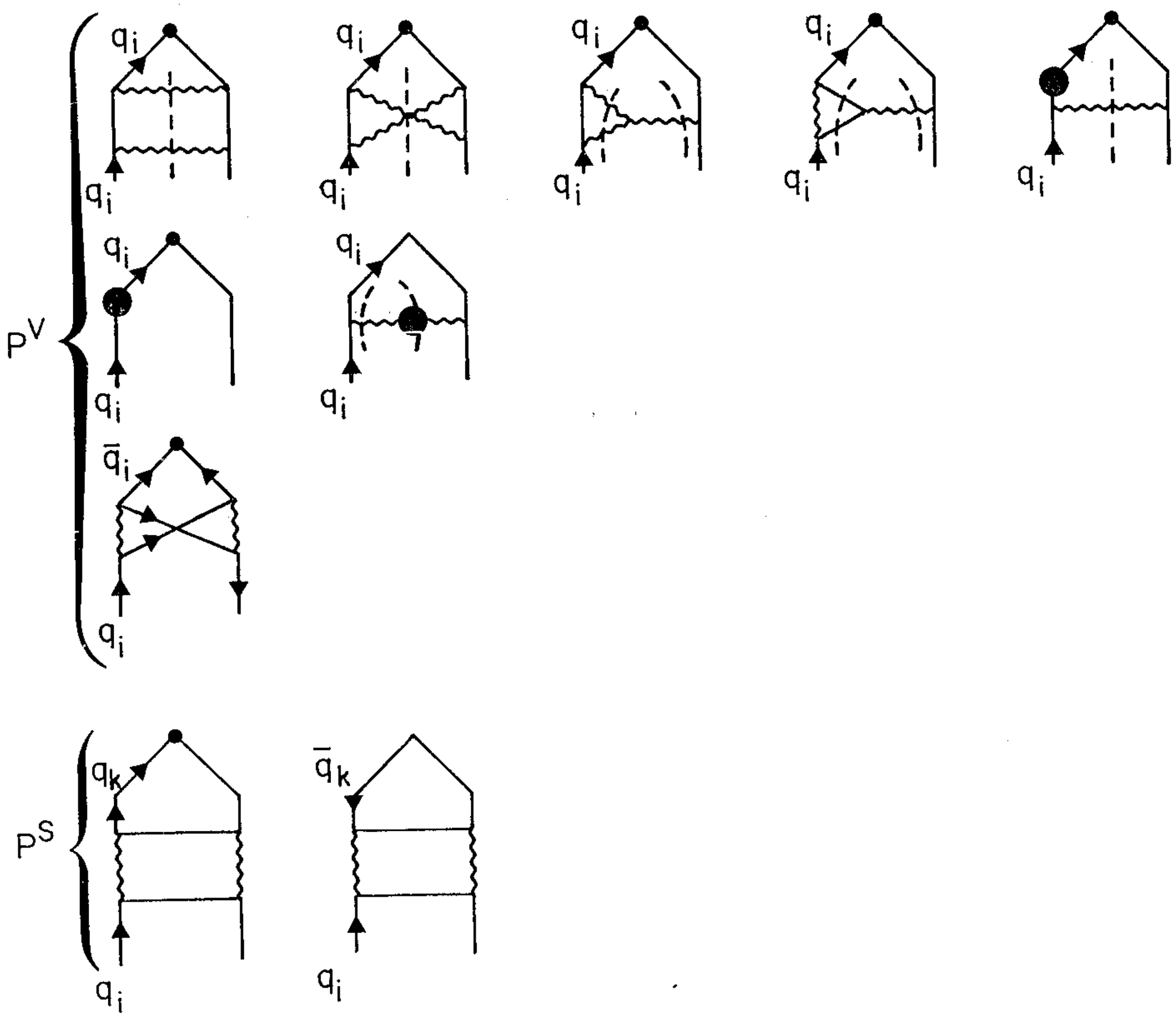

Fig. 9