International Journal of Technology (2014) 3: 242-250
ISSN 2086-9614

LESSER SUNDA ISLANDS EARTHQUAKE INTER-OCCURRENCE TIMES DISTRIBUTION MODELING

Indira Puteri Kinasih*[1], Giri Wahyu Wiriasto[2], Bulkis Kanata[2], Teti Zubaidah[2]

[1] *Mathematical Education Department, IKIP Mataram, Mataram 83125, Indonesia*
[2] *Electrical Engineering Department, Faculty of Engineering, Mataram University, Jl. Majapahit 62, Mataram 83125, Indonesia*

(Received: July 2014 / Revised: September 2014 / Accepted: November 2014)

ABSTRACT
This paper is aimed towards analyzing and modeling earthquake interoccurence times in the Lesser Sunda Islands region using Weibull distribution. The data were classified into three categories, based on their magnitude; i.e. weak, medium, and strong earthquakes. Cumulative distribution functions and hazard rates are also explored in order to obtain the characteristics of earthquake inter-occurrences time data. Empirical results indicate the probability and rate of an earthquake recurrence time with a certain magnitude and in a certain time. Medium and weaker earthquakes have a higher chance of occurrence, reaching up to a 100% probability for the following 60 months. Meanwhile, the stronger earthquake has a 75.80% probability of occurrence. It can be seen that the earthquake occurrence probability increases together with the time increment factor.

Keywords: Earthquake; Interoccurrence times; Lesser Sunda Islands; Magnitude; Weibull distribution

1. INTRODUCTION
The Lesser Sunda Islands (LSI) or Nusa Tenggara are a group of islands in the southern Maritime Southeast Asia. They consist of many islands, most of which are part of Indonesia and are administered as the provinces of Bali, West Nusa Tenggara, East Nusa Tenggara, and southern part of Maluku,namely Kepulauan Tanimbar and also Kepulauan Barat Daya. The LSI region lies at the collision of three tectonic plates, where a nearly perpendicular sub-duction of the Indo-Australian plates along the Java trench is well known (Zubaidah et al., 2014). The LSI region comprises some of the most geologically complex and active regions in the world. Therefore, one of the most threatening events for these kinds of regions is a damaging earthquake.

However, there are many statistical studies about earthquake predictions. Chen et al. (2012) proposed the best distribution for the cumulative probability of interoccurence periods for Taiwan. They also analyzed different behaviors of the IOP probability for the two magnitude ranges. They suggested that Gamma distribution was the best fit among them. Hasumi et al. (2009) tested some catalogs in Japan, Southern California, and Taiwan from 2001 to 2007. They suggested that distribution of the interoccurence time can be described clearly by the superposition of Weibull and Log-Weibull distribution. Talbi and Yamazaki (2009) constructed the interevent time distribution by mixing the distribution of clustered seismicity, with a suitable distribution of background seismicity.

---
* Corresponding author's email: indiraputeri@gmail.com, Tel. +62-81--907067674, Fax. +62-370-636629
Permalink/DOI: http://dx.doi.org/10.14716/ijtech.v5i3.610

For southern California, Japan, and Turkey, the best fit is found when a Weibull distribution is implemented as a model for background seismicity. Cheng et al. (2007) conducted a review of readily available information on tectonic setting, geology, and seismicity, and the attenuation of peak ground acceleration (PGA) in Taiwan for completing the revised probabilistic seismic hazard maps by the state-of-the-art probabilistic seismic hazard analysis (PSHA) method. They evaluate the earthquake recurrence rates for regional sources and subduction. The revised PSHA takes into consideration the fact that subduction plate sources induce higher ground motion levels than crustal sources, and active faults induce the hanging-wall effect in attenuation relationships. Here, we elaborate on the nature of LSI earthquakes through modeling the earthquake inter-occurrence time data and calculating the earthquake occurrence rates for weak, medium, and strong earthquakes and for several ranges of time periods.

2. METHODOLOGY

Mathematical models have been used in solving real-world problems from many different disciplines (Murthy et al., 2004). The Weibull model is one of such class of models. Here, the data used in this study were obtained from earthquake catalogs owned by the United States Geological Survey and these became the basis for earthquake model building.

2.1. Data Set

The data examined are earthquakes that occurred in the LSI region at coordinates $9^\circ S$ and $120^\circ E$, during the period from 1900 to 2014. There were some attributes described for the earthquake data, such as date, time, longitude, latitude, magnitude, place, and earthquake type. The data was customized in that the minimum threshold magnitude was set at 4 and the maximum depth was 40 km. Thereafter, 2081 earthquakes were obtained based on all these attributes. They consist of 1709 earthquakes, with a 4 to 4.9 magnitude, 336 earthquakes with a 5 to 5.9 magnitude, and 36 earthquakes with a 6 to 8 magnitude. Earthquakes in the LSI region have been visualized in Figure 1.

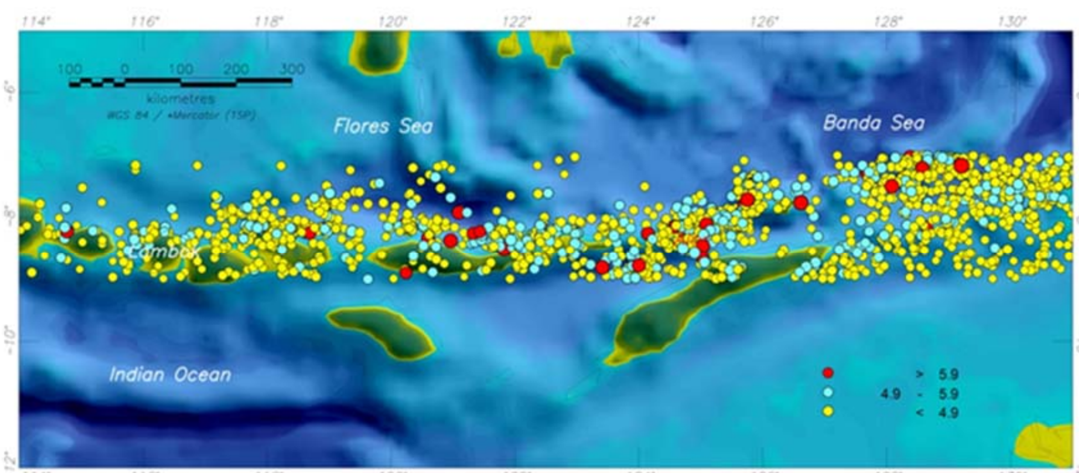

Figure 1 Earthquakes visualization in the Lesser Sunda Islands from 1900 to 2014. Earthquake events were assigned with a 3 colour dot symbol. Yellow dots for a $4 \le m_3 < 5$, Cyan dots for a $5 \le m_2 < 6$ and Red dots for a $m_1 \ge 6$

The earthquakes' interoccurence time is obtained by calculating the absolute difference between the time occurrences of the earthquake, sequentially, by first classifying events into three categories, based on the 3 class range of magnitude levels mentioned above. In this case, we only analyzed events that have a time difference of more than 3 days and a difference in location of more than 5 km. Determination of those minimum values were undertaken to avoid aftershocks events (Chen et al., 2012), so that by filtering the data based on these spatiotemporal parameters, there were only main shocks data included for analysis. After this spatiotemporal filtering, the remaining data are 701 earthquakes, with a 4 to 4.9 magnitude, 219 earthquakes with a 5 to 5.9 magnitude, and 29 earthquakes with a 6 to 8 magnitude.

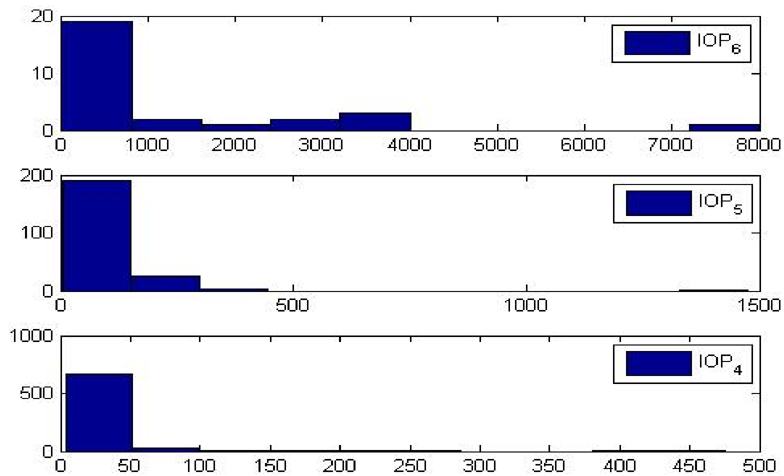

Figure 2 Histogram Plot of Earthquake Inter-occurrence Times data for each range of magnitude. The plots show that each data set has a tendency to skew to the left, so that they can be classified into a skewed distribution family, such as Weibull, exponential, Gamma, Beta, etc.

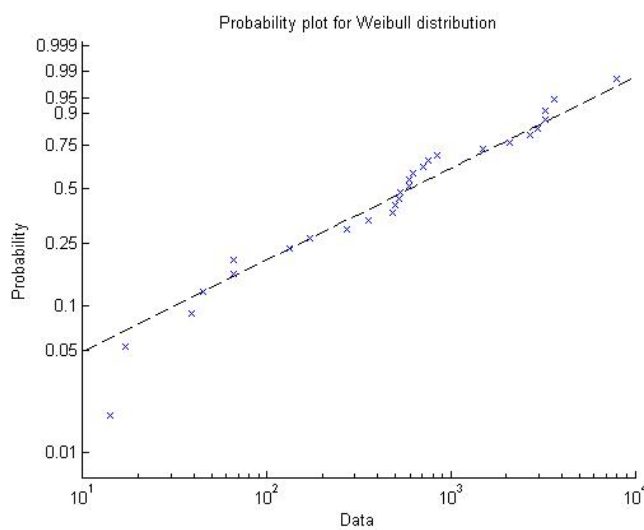

Figure 3(a) Probability Plot of Earthquake interoccurrence times, with $m_1 \geq 6$ and $n = 29$. This plot shows that almost all actual data fall near the Weibull reference line, except one data in the first order. However, this plot is strong evidence that the underlying distribution is Weibull

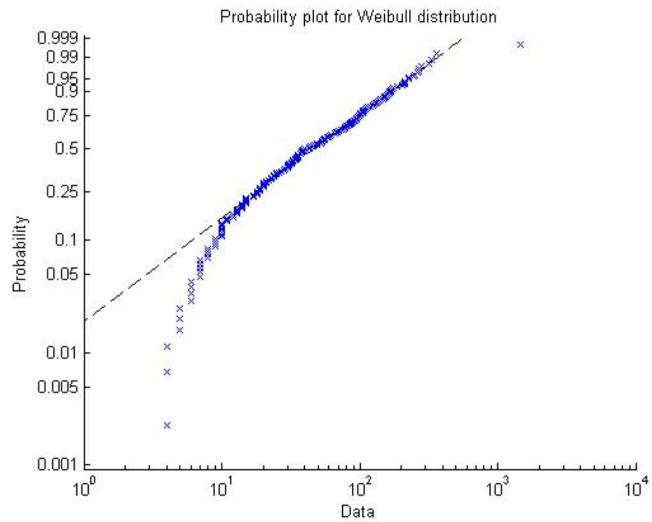

Figure 3(b) Probability Plot of Earthquake interoccurrence times, with $5 \le m_2 < 6$ and $n = 219$. This plot shows that some actual data fall near the Weibull reference line. There were also some data on the range $x \le 10$ that failed to fall near the Weibull reference line

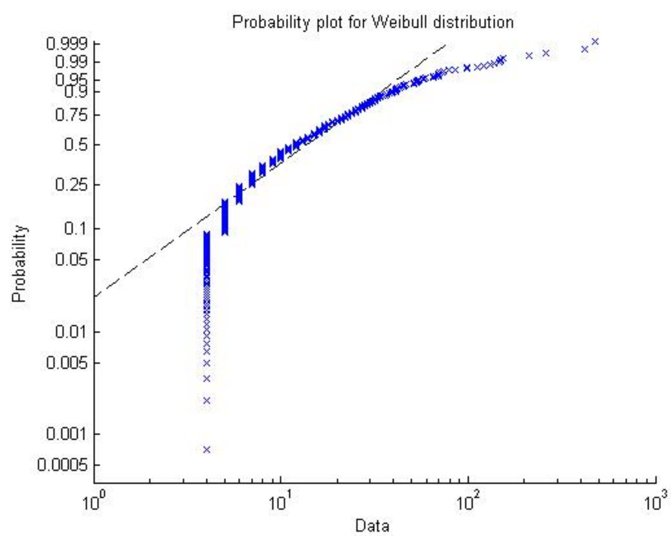

Figure 3(c) Probability Plot of Earthquake interoccurrence times, with $4 \le m_3 < 5$ and $n = 701$. This plot shows that a lot of actual data failed to fall near the Weibull reference line, especially for those in the $x \le 10$ range

Prior knowledge about the distribution of time between events was obtained through observation of a histogram plot of each data set. Based on the histogram (Figure 2), it can be seen that each of the data sets have a tendency to skew to the left. This fact shows that the distribution of time between the occurrence of earthquakes followed the family of skewed distribution, such as Weibull, exponential, gamma, beta, etc. (Hasumi et al., 2009). Another justification for earthquake data distribution also can be seen on the probability plot given in Figure 3. The probability plot produces a comparation between the earthquake data distribution and the Weibull distribution. Each plot includes a reference line useful for judging whether the data follow a Weibull distribution or not. The probability plot of earthquake inter-occurrence times data for $m_1 \geq 6$, show a strong evidence of Weibull distribution, because most of its actual data were distributed near the reference line. However, we could not gain such strong evidence on identifying the probability plot of earthquake inter-occurrence times data with $5 \leq m_2 < 6$ and $4 \leq m_3 < 5$.

2.2. Weibull Distribution Model for Earthquake Inter-Occurrence Times

Now, let X be viewed as a non-negative random variable representing the time between the occurrence of earthquakes. X is said to have a two-parameter Weibull distribution or $X \sim Weibull(\alpha.\beta)$ if it has a cumulative distribution function (CDF) which satisfies the equation:

$$F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} \tag{1}$$

The cumulative distribution function of X is defined as the opportunity of time between earthquake occurrence that is at an interval $(0, x]$. Meanwhile, the hazard function stating the rate of change between the time opportunities for earthquakes are not on a time interval $(0, x]$, but is in the interval time $(x, x + dx]$. Mathematically, the hazard function follows the concept of conditional probability (Murthy et.al., 2004), ie. $\lim_{dx \to 0} \frac{P(x \leq X < x + dx | X \geq x)}{dx}$, and also simply it can be expressed as the quotient between the density function of opportunities with the cumulative distribution function (Nakagawa, 2005), and it satisfies the following equation:

$$h(x) = \left(\frac{\beta}{\alpha^{\beta}}\right) x^{\beta - 1} \tag{2}$$

The scale parameter, α is called the characteristic values of the Weibull distribution, since 63.212% of the time between earthquakes totaled α, while the shape parameter, β is a measure of the data spread (Kinasih, 2012). Parameter estimation is done by numerical computation, using the Newton-Raphson algorithm computation. It is also included a 95% parameter confidence interval.

The Anderson-Darling test was employed for goodness of fit or testing the performance of the model (Romeu, 2003). The Anderson-Darling test assesses whether a sample comes from a specified distribution. The Anderson-Darling statistics test satisfies this following equation:

$$AD = \left[\sum_{i=1}^{n} \frac{1 - 2i}{n} \left(\ln\left(1 - e^{-Y_i}\right) - Y_{n+1-i}\right)\right] - n \tag{3}$$

where $Y_i = \left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}$. The null hypothesis proposed on this distribution fitting is *"The underlined distribution of the population from where these data were obtained is Weibull"*. The null hypothesis will be accepted if *observed significance level OSL<0.05*. The observed significance level is computed from this following equation (Romeu, 2003):

$$OSL = 1 \Big/ \left(1 + e^{\left(-0.1 + 1.24 \ln(AD^*) + 4.48(AD^*)\right)}\right) \tag{4}$$

where $AD^* = \left(1 + \frac{0.2}{\sqrt{n}}\right) AD$. Besides this, empirical cumulative distribution functions were also plotted, to show the proportion failing up to each possible survival time. The dotted curves give 95% confidence intervals for these probabilities. After computing parameter estimates, the CDF for fitted Weibull model were evaluated using those estimates. The plots of empirical CDF and the fitted CDF were superimposed, to judge how well the Weibull distribution models the earthquake inter-occurrence times data (Chen et al., 2012). The earthquake occurrence rate can be obtained by the following equation (Nakagawa, 2011).

$$\lambda(t,x)=\frac{F(t,x)-F(t)}{\overline{F}(t)}, \quad x>0 \tag{5}$$

where $\overline{F}(t)$ is a survival function or a probability that there will not be an earthquake before time t. Based on Equation 5, the probability of earthquake occurrence between interval t and $t+x$ under a condition that there is no earthquake occurrence until time t can be obtained by calculating $\lambda(t,x)$.

3. RESULTS AND DISCUSSION

The earthquake inter-occurrence times characteristic for each magnitude range can be inferred from Table 1; it is based on the definition of Weibull distribution in Equation 1, that α is the scale parameter that represents the mean life time since about 63.2% of earthquakes have occurred until time $x=\alpha$ (Nakagawa, 2005).

Table 1 The estimate parameters and 95% confidence intervals of earthquake inter-occurrence times data

EQ's Magnitude	$\hat{\alpha}$	$\hat{\alpha}$ Confidence Interval	$\hat{\beta}$	$\hat{\beta}$ Confidence Interval
$m_1 \geq 6$	990.7	(527.6,1714.4)	0.7	(0.5,1.0)
$5 \leq m_2 < 6$	72.4	(62.1,84.3)	0.9	(0.8,1.0)
$4 \leq m_3 < 5$	20.2	(18.7,21.8)	1.0	(0.9,1.0)

While β is the shape parameter, its contribution can be gained clearly on the behavior of the hazard rate function, as seen in Figure 4.

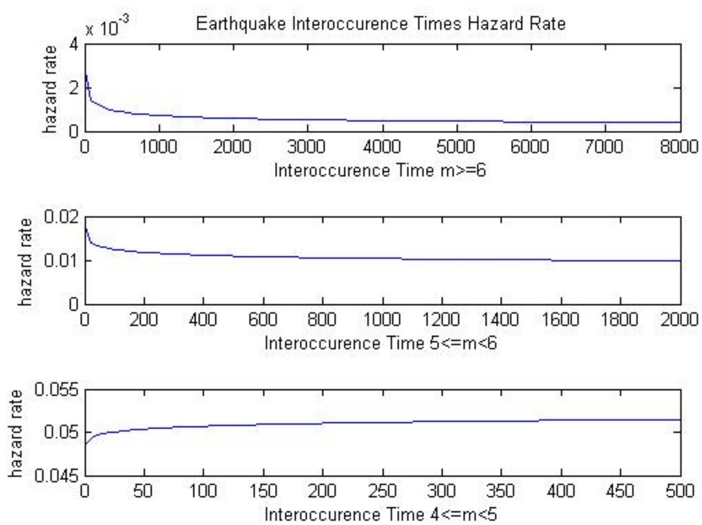

Figure 4 Plots of hazard rate function for each earthquake inter-occurrence times data set. This plot shows how β, as the shape parameter, controls the behavior of the hazard rate function. It can be decreased, increased, or constant

The IOP with $m_1 \geq 6$ and $5 \leq m_2 < 6$ have $\beta < 1$. It means that these IOP will have a decreased hazard rate. In other words, the rate of earthquake occurence in those magnitude ranges was decreasing along with the enlargement of time inter-occurrence. However, this behavior did not find on IOP with $4 \leq m_3 < 5$, since it has $\beta = 1$. From this fact, it can be concluded that the occurring rate of small magnitude earthquake ($4 \leq m_3 < 5$) is constant.

Unfortunately, the accuracy of fitting the Weibull distribution can be seen in Table 2, gives us an empirical result about how the Weibull distribution is not actually suitable for earthquakes with $4 \leq m_3 < 5$ and $5 \leq m_2 < 6$ parameters; therefore, the null hypothesis has been rejected. The comparison between empirical CDF and Weibull model plotting are shown in Figure 5.

Table 2 The estimate parameters and 95% confidence intervals of earthquake inter-occurrence times data

EQ's Magnitude	AD	AD^*	OSL	Decision
$m_1 \geq 6$	0.74	0.75	0.0519	Accepted H_0
$5 \leq m_2 < 6$	1.67	1.67	0.0003	Rejected H_0
$4 \leq m_3 < 5$	23.0	23.0	3×10^{-47}	Rejected H_0

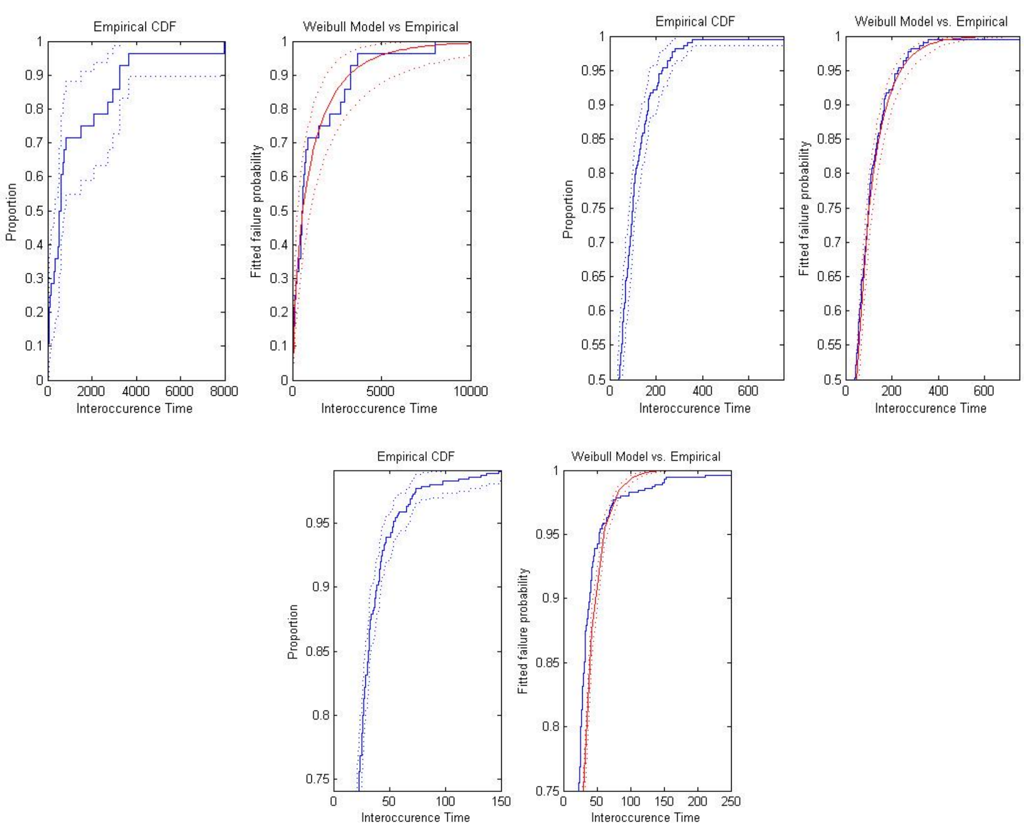

Figure 5 Plots of empirical CDF and comparation plots of empirical CDF and Weibull models for each earthquake inter-occurrence times data set. The Weibull model is plotted by a red line, and a dashed line indicates its interval confidence, while the empirical data are plotted by a solid blue line

This is also consistent with the Goodness-of-fit testing, which implies that a small magnitude earthquake could not be represented accurately by Weibull distribution.

Table 3 The earthquake occurrence rate for each EQ magnitude categories

EQ's Magnitude	*t (days)*	Rate of EQ Occurrence in future $\lambda(t,x)$			
		1 month	6 months	24 months	60 months
$m_1 \geq 6$	90	4.03%	19.34%	49.99%	75.80%
$5 \leq m_2 < 6$	30	32.76%	89.18%	99.97%	100%
$4 \leq m_3 < 5$	30	77.89%	99.99%	100%	100%

Note: this table shows the earthquake occurrence probability from 1 month until 5 years

The earthquake occurrence rate or the probability for each magnitude interval can be observed in Table 3. Based on Equation 4, it can be concluded that the occurrence rate for earthquake with $m_1 \geq 6$ for $t = 90$ days, is 4.03%, 19.34%, 49.99%, and 75.80% for 1, 6, 24, and 60 month(s), respectively. In other words, the probability that an earthquake will occur within the next 1 month, falls under the condition that it did not occur in the past 90 days, is 4.03%. While, for earthquake with a magnitude $5 \leq m_2 < 6$, there is a 32.76% probability of occurrence in the next month, without occurrences during the last 30 days, conditionally. Whereas, the probability of earthquake occurrences in the next month where $4 \leq m_3 < 5$ is 77.89%.

4. CONCLUSION

We elaborated on the nature of LSI earthquakes through modeling the earthquake inter-occurrence time data. Parameter estimation and Goodness-of-fit (Anderson-Darling) testing were obtained in order to evaluate the appropriateness of selected probability models for simulating earthquake inter-occurrence times data. Table 1 summarizes the estimated parameters for the model; Table 2 summarizes the Anderson-Darling testing for each data set and Table 3 summarizes the rate of earthquake occurrence for each magnitude range from 1 month to 5 years. According to empirical results, it can be concluded that medium and weaker earthquakes have a bigger chance of reoccurrence and this reaches a 100% probability for the next 60 months (5 years). It can be seen that the earthquake occurrence probability will increase together with time increment.

For further research, based on LSI earthquake mapping in Figure 1, it is important to gain a different perspective about LSI earthquake characteristics by dividing the LSI region into some distinct sectors and modeling their earthquake behavior. It is also recommended to examine another type of lifetime distribution such as log Weibull, exponential, or gamma distribution to gain a better result for modeling the earthquake inter-occurrence times in the Lesser Sunda Island region.

5. ACKNOWLEDGEMENT

This research was supported by The Applied Electromagnetic Technology Research Group, University of Mataram. We would like to thank Dr. Monika Korte for reviewing this paper and suggesting to us with some constructive comments. The computation and graph plotting was done using MATLAB (License Code: 863485). LSI Earthquake mapping was done using Oasis Montaj/Geosoft Software (License Code: F5CNLP1Q).

6. REFERENCES

Abaimov, S.G., Turcotte, D.L., Shcherbakov, R., Rundle, J.B., Yakovlev, G., Goltz, C., Newman, W.I., 2008. *Earthquake: Recurrence and Interoccurence Times*. Earthquakes, Simulations, Sources and Tsunamis, pp. 777–795. Springer eBook. Birkhauser Basel.

Charpenter, A., Durand, M., Boudreault, M., 2013. *Modeling Earthquake Dynamics*.

Cheng, C.T., Chiou, S.J., Lee, C.T., Tsai, Y.B., 2007. Study on Probabilistic Seismic Hazard Maps of Taiwan after Chi-Chi Earthquake. *Journal of GeoEngineering*, Volume 2(1), pp.19–28

Chen, C.H., Wang, J.P., Wu, Y.M., Chan, C.H., Chang, C.H., 2012. *A Study of Earthquake Inter-occurrence Times Distribution Models in Taiwan*. Springer Science+Business Media Dordrecht.

Godinho, J., 2007. Probabilistic Seismic Hazard Analysis-Introduction to Theoretical Basis and Applied Methodology. *A Dissertation Report*. University of Patras, Greece

Hasumi T., Akimoto T., Aizawa Y., 2009. The Weibull-Log Weibull Distribution for Interoccurence Times of Earthquakes. *Phys A*, Volume 388, pp. 491–498

Konşuk, H., Aktaş, S., 2013. Estimating the Recurrence Periods of Earthquake Data in Turkey. *Open Journal of Earthquake Research*, Volume 2(1), pp. 21–25

Kinasih, I.P., Pasaribu, U.S., 2012. Two Dimensional Weibull Failure Modeling. In: *Proceedings of the 6th SEAMS-GMU International Conference on Mathematics and Its Applications*. Department of Mathematics, Faculty of Mathematics and Natural Sciences 2011, Universitas Gajah Mada, Yogyakarta, Indonesia

Langenbruch, C., Shapiro, S.A., 2010. Inter Event Times of Fluid Induced Seismicity. *72nd EAGE Conference & Exhibition Incorporating SPE EUROPEC 2010*, Barcelona, Spain

Matthews, M.V., Ellsworth, W.L., Reasenberg, P.A., 2002. A Brownian Model for Recurrent Earthquakes. *Bulletin of the Seismological Society of America*, Volume 92(6), pp. 2233–2250

Murthy, D.N.P., Xie, M., Jiang, R., 2004. *Weibull Models*. John Wile & Sons, Inc. Hoboken, New Jersey

Nakagawa, T., 2011. *Stochastic Process, with Application to Reliability Theory*. Springer London Dordrecht Heidelberg New York

Nakagawa, T., 2005. *Maintenance Theory of Reliability*. Springer-Verlag London Limited

Romeu, J.L., 2003. *Anderson-Darling: A Goodness of Fit Test for Small Samples Assumptions*. Selected Topics in Assurance Related Technologies.

Zubaidah, T., Korte, M., Mandea, M., Hamoudi, M., 2014. New Insights into Regional Tectonics of the Sunda-Banda Arcs Region from Integrated Magnetic and Gravity Modeling. *Journal of Asian Earth Sciences*, Volume 80, pp. 172–184