

Lessons in Estimation Theory for Signal Processing, Communications, and Control

Jerry M. Mendel

*Department of Electrical Engineering
University of Southern California
Los Angeles, California*



PRENTICE HALL PTR
Englewood Cliffs, New Jersey 07632

Contents

	Preface	xvii
LESSON 1	Introduction, Coverage, Philosophy, and Computation	1
	Summary 1	
	Introduction 2	
	Coverage 3	
	Philosophy 6	
	Computation 7	
	Summary Questions 8	
LESSON 2	The Linear Model	9
	Summary 9	
	Introduction 9	
	Examples 10	
	Notational Preliminaries 18	
	Computation 20	
	Supplementary Material: Convolutional Model in Reflection Seismology 21	
	Summary Questions 23	
	Problems 24	

LESSON 3	Least-squares Estimation: Batch Processing	27
	Summary 27	
	Introduction 27	
	Number of Measurements 29	
	Objective Function and Problem Statement 29	
	Derivation of Estimator 30	
	Fixed and Expanding Memory Estimators 36	
	Scale Changes and Normalization of Data 36	
	Computation 37	
	Supplementary Material: Least Squares, Total Least Squares, and Constrained Total Least Squares 38	
	Summary Questions 39	
	Problems 40	
LESSON 4	Least-squares Estimation: Singular-value Decomposition	44
	Summary 44	
	Introduction 44	
	Some Facts from Linear Algebra 45	
	Singular-value Decomposition 45	
	Using SVD to Calculate $\hat{\theta}_{LS}(k)$ 49	
	Computation 51	
	Supplementary Material: Pseudoinverse 51	
	Summary Questions 53	
	Problems 54	
LESSON 5	Least-squares Estimation: Recursive Processing	58
	Summary 58	
	Introduction 58	
	Recursive Least Squares: Information Form 59	
	Matrix Inversion Lemma 62	
	Recursive Least Squares: Covariance Form 63	
	Which Form to Use 64	
	Generalization to Vector Measurements 66	
	Computation 66	
	Supplementary Material: Derivation of Start-up Conditions for Recursive Algorithms 67	
	Summary Questions 69	
	Problems 70	
LESSON 6	Small-sample Properties of Estimators	74
	Summary 74	
	Introduction 75	

Unbiasedness	76
Efficiency	77
Supplementary Material: Proof of Theorem 6-4	86
Summary Questions	87
Problems	88

LESSON 7 Large-sample Properties of Estimators 91

Summary	91
Introduction	92
Stochastic Convergence	92
Asymptotic Unbiasedness	94
Consistency	95
Asymptotic Distributions	99
Asymptotic Normality	101
Asymptotic Efficiency	103
Summary Questions	104
Problems	105

LESSON 8 Properties of Least-squares Estimators 108

Summary	108
Introduction	109
Small-sample Properties of Least-squares Estimators	109
Large-sample Properties of Least-squares Estimators	115
Supplementary Material: Expansion of Total Expectation	117
Summary Questions	117
Problems	118

LESSON 9 Best Linear Unbiased Estimation 121

Summary	121
Introduction	122
Problem Statement and Objective Function	122
Derivation of Estimator	124
Comparison of $\hat{\theta}_{BLU}(k)$ and $\hat{\theta}_{WLS}(k)$	125
Some Properties of $\hat{\theta}_{BLU}(k)$	126
Recursive BLUEs	130
Computation	131
Supplementary Material: Lagrange's Method	131
Summary Questions	132
Problems	133

LESSON 10 Likelihood 137

Summary	137
Likelihood Defined	137

Likelihood Ratio	140
Results Described by Continuous Distributions	141
Multiple Hypotheses	141
Decision Making Using Likelihood Ratio	142
Supplementary Material: Transformation of Variables and Probability	144
Summary Questions	145
Problems	145

LESSON 11 Maximum-likelihood Estimation 147

Summary	147
Likelihood	148
Maximum-likelihood Method and Estimates	148
Properties of Maximum-likelihood Estimates	151
The Linear Model [$\mathbf{H}(k)$ Deterministic]	152
A Log-likelihood Function for an Important Dynamical System	154
Computation	156
Summary Questions	157
Problems	158

LESSON 12 Multivariate Gaussian Random Variables 164

Summary	164
Introduction	164
Univariate Gaussian Density Function	165
Multivariate Gaussian Density Function	165
Jointly Gaussian Random Vectors	165
The Conditional Density Function	166
Properties of Multivariate Gaussian Random Variables	168
Properties of Conditional Mean	169
Summary Questions	171
Problems	171

LESSON 13 Mean-squared Estimation of Random Parameters 173

Summary	173
Introduction	174
Objective Function and Problem Statement	174
Derivation of Estimator	175
Properties of Mean-squared Estimators when θ and $\mathbf{Z}(k)$ Are Jointly Gaussian	178
Generalizations	179
Mean-squared Estimator for the Generic Linear and Gaussian Model	179
Best Linear Unbiased Estimation, Revisited	181
Computation	184

Supplementary Material: The Conditional Mean Estimator	184
A Nonlinear Estimator	185
Summary Questions	188
Problems	189

LESSON 14	Maximum a Posteriori Estimation of Random Parameters	192
	Summary	192
	Introduction	192
	General Results	193
	The Generic Linear and Gaussian Model	195
	Computation	199
	Supplementary Material: Elements of Binary Detection Theory	200
	Summary Questions	204
	Problems	205
LESSON 15	Elements of Discrete-time Gauss–Markov Random Sequences	211
	Summary	211
	Introduction	212
	Definition and Properties of Discrete-time Gauss–Markov Random Sequences	212
	The Basic State-variable Model	215
	Properties of the Basic State-variable Model	216
	Signal-to-Noise Ratio	220
	Computation	222
	Summary Questions	223
	Problems	224
LESSON 16	State Estimation: Prediction	227
	Summary	227
	Introduction	228
	Single-stage Predictor	228
	A General State Predictor	229
	The Innovations Process	233
	Computation	235
	Supplementary Material: Linear Prediction	235
	Summary Questions	238
	Problems	239
LESSON 17	State Estimation: Filtering (the Kalman Filter)	242
	Summary	242
	Introduction	243

A Preliminary Result	245
The Kalman Filter	246
Observations about the Kalman Filter	248
Computation	253
Supplementary Material: MAP Derivation of the Kalman Filter	253
Summary Questions	255
Problems	256

LESSON 18 State Estimation: Filtering Examples 259

Summary	259
Introduction	260
Examples	260
Supplementary Material: Applications of Kalman Filtering	271
Summary Questions	276
Problems	277

LESSON 19 State Estimation: Steady-state Kalman Filter and Its Relationship to a Digital Wiener Filter 279

Summary	279
Introduction	280
Steady-state Kalman Filter	280
Single-channel Steady-state Kalman Filter	282
Relationships between the Steady-state Kalman Filter and a Finite Impulse Response Digital Wiener Filter	286
Comparisons of Kalman and Wiener Filters	293
Computation	294
Supplementary Material: Some Linear System Concepts	294
The Levinson Algorithm	295
Summary Questions	300
Problems	301

LESSON 20 State Estimation: Smoothing 304

Summary	304
Three Types of Smoothers	305
Approaches for Deriving Smoothers	306
A Summary of Important Formulas	306
Single-stage Smoother	306
Double-stage Smoother	309
Single- and Double-stage Smoothers as General Smoothers	312
Computation	314
Summary Questions	314
Problems	315

LESSON 21 State Estimation: Smoothing (General Results)**317**

Summary	317
Introduction	318
Fixed-interval Smoothing	318
Fixed-point Smoothing	323
Fixed-lag Smoothing	325
Computation	328
Supplementary Material:	
Second-order Gauss–Markov Random Sequences	328
Minimum-variance Deconvolution (MVD)	329
Steady-state MVD Filter	332
Relationship between Steady-state MVD Filter and an Infinite Impulse Response Digital Wiener Deconvolution Filter	338
Maximum-likelihood Deconvolution	340
Summary Questions	341
Problems	342

**LESSON 22 State Estimation for the Not-so-basic
State-variable Model****345**

Summary	345
Introduction	346
Biases	346
Correlated Noises	347
Colored Noises	350
Perfect Measurements: Reduced-order Estimators	354
Final Remark	357
Computation	357
Supplementary Material: Derivation of Equation (22-14)	359
Summary Questions	360
Problems	361

LESSON 23 Linearization and Discretization of Nonlinear Systems**364**

Summary	364
Introduction	365
A Dynamical Model	365
Linear Perturbation Equations	367
Discretization of a Linear Time-varying State-variable Model	371
Discretized Perturbation State-variable Model	374
Computation	374
Supplementary Material: Proof of Theorem 23-1	375
Summary Questions	376
Problems	377

LESSON 24 Iterated Least Squares and Extended Kalman Filtering 384

Summary 384
Introduction 385
Iterated Least Squares 385
Extended Kalman Filter 386
Application to Parameter Estimation 391
Computation 392
Supplementary Material: EKF for a Nonlinear
Discrete-time System 393
Summary Questions 394
Problems 394

LESSON 25 Maximum-likelihood State and Parameter Estimation 397

Summary 397
Introduction 398
A Log-likelihood Function for the Basic State-variable Model 398
On Computing $\hat{\theta}_{ML}$ 400
A Steady-state Approximation 404
Computation 408
Summary Questions 409
Problems 410

LESSON 26 Kalman–Bucy Filtering 413

Summary 413
Introduction 413
System Description 414
Statistics of the State Vector 415
Notation and Problem Statement 416
The Kalman–Bucy Filter 417
Derivation of KBF Using a Formal Limiting Procedure 418
Steady-state KBF 421
An Important Application for the KBF 423
Computation 426
Supplementary Material: Proof of Theorem 26-1 427
Derivation of the KBF when the Structure
of the Filter Is Prespecified 428
Summary Questions 431
Problems 432

LESSON A Sufficient Statistics and Statistical Estimation of Parameters 436

Summary 436
Introduction 437

	Concept of Sufficient Statistics	437
	Exponential Families of Distributions	439
	Exponential Families and Maximum-likelihood Estimation	441
	Sufficient Statistics and Uniformly Minimum-variance Unbiased Estimation	444
	Summary Questions	448
	Problems	449
LESSON B	Introduction to Higher-order Statistics	450
	Summary	450
	Introduction	451
	Definitions of Higher-order Statistics	452
	Properties of Cumulants	464
	Supplementary Material:	
	Relationships between Cumulants and Moments	466
	Proof of Theorem B-3	466
	Summary Questions	469
	Problems	469
LESSON C	Estimation and Applications of Higher-order Statistics	473
	Summary	473
	Estimation of Cumulants	474
	Estimation of Bispectrum	476
	Applications of Higher-order Statistics to Linear Systems	478
	Computation	490
	Summary Questions	491
	Problems	492
LESSON D	Introduction to State-variable Models and Methods	499
	Summary	499
	Notions of State, State Variables, and State Space	500
	Constructing State-variable Representations	503
	Solutions of State Equations for Time-invariant Systems	509
	Miscellaneous Properties	512
	Computation	514
	Summary Questions	514
	Problems	515
APPENDIX A	Glossary of Major Results	518
APPENDIX B	Estimation Algorithm M-Files	524
	Introduction	524
	Recursive Weighted Least-squares Estimator	525

Kalman Filter	526
Kalman Predictor	529
Suboptimal Filter	532
Suboptimal Predictor	534
Fixed-interval Smoother	536

APPENDIX C	Answers to Summary Questions	539
	References	542
	Index	553