Generalized Chaplygin Gas Model: Dark Energy - Dark Matter Unification and CMBR Constraints*

M. C. Bento†, O. Bertolami‡, A. A. Sen§

Instituto Superior Técnico, Departamento de Física,

Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

Abstract

The generalized Chaplygin gas (GCG) model allows for an unified description of the recent accelerated expansion of the Universe and the evolution of energy density perturbations. This dark energy - dark matter unification is achieved through an exotic background fluid whose equation of state is given by $p = -A/\rho^{\alpha}$, where A is a positive constant and $0 < \alpha \leq 1$. Stringent constraints on the model parameters can be obtained from recent WMAP and BOOMERanG bounds on the locations of the first few peaks and troughs of the Cosmic Microwave Background Radiation (CMBR) power spectrum as well as SNe Ia data.

*Essay selected for an honorable mention by the Gravity Research Foundation, 2003.

†Also at CFIF, Instituto Superior Técnico, Lisboa. Email address: bento@sirius.ist.utl.pt

‡Also at CFNUL, Universidade de Lisboa. Email address: orfeu@cosmos.ist.utl.pt

§Also at CENTRA, Instituto Superior Técnico, Lisboa. Email address: anjan@x9.ist.utl.pt

1 Introduction

Cosmology is undergoing a blooming period. Precision measurements and highly predictive theories are coming together to yield a rich lore of data and methods that scrutinize existing models with increasing depth. It is quite remarkable that all available data can be fully harmonized within the *Hot Big Bang Model*, an unifying description in which several branches of physics meet to provide a consistent and testable scenario for the evolution of the Universe. In this picture, a particularly relevant role is played by *Inflation*, a period of accelerGiven the potential of the GCG model as a viable dark energy-dark matter unification scheme, many authors have studied constraints on the model parameters from observational data, particularly those arising from SNe Ia [7] and gravitational lensing statistics [8].

Quite stringent constraints arise also from the study of the position of the acoustic peaks and troughs of the CMBR power spectrum. The CMBR peaks arise from oscillations of the primeval plasma just before the Universe becomes transparent. Driving processes and the ensuing shifts on peak positions [9]ated expansion in the very early Universe that allows for reconciling cosmology with causality and leads to a consistent explanation for the origin of the observed Large Scale Structure of the Universe. However, in order to fully account for the existing observations, one must bring in at least two additional new mysteries: the concept of *Dark Matter*, originally proposed to explain the rotation curves of galaxies and later used to address the issue of structure formation at large scales, and the idea of a smoothly distributed energy that cannot be identified with any form of matter, the so-called *Dark Energy*, needed to explain the recently observed accelerated expansion of the Universe. Even though these concepts are apparently unrelated, a scheme has emerged where an unification of these physical entities is possible through the rather exotic equation of state:

$$p_{ch} = -\frac{A}{\rho_{ch}^{\alpha}} \quad , \tag{1}$$

where A a positive constant and α is a constant in the range $0 < \alpha \leq 1$. This equation of state with $\alpha = 1$ was first put foward in 1904 by the Russian physicist Chaplygin to describe adiabatic processes [1]; its generalization for $\alpha \neq 1$ was originally proposed in Ref. [2] and the ensuing cosmology has been analysed in Ref. [3]. The idea that a cosmological model based on the Chaplygin gas could lead to the unification of dark energy and dark matter, thereby reducing two unknown physical entities into a single one was first advanced for the case $\alpha = 1$ in Refs. [4, 5], and generalized to $\alpha \neq 1$ in Ref. [3].

2 The Model

The interesting behaviour of the equation of state (1) can be better appreciated by inserting it into the relativistic energy-momentum conservation equation, which implies for the evolution of the energy density [3]

$$\rho_{ch} = \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \quad , \tag{2}$$

where a is the scale-factor of the Universe and B an integration constant. Remarkably, this model interpolates between a universe dominated by dust and a De Sitter one with an intermediate phase described by a mixture of vacuum energy density and a "soft" matter equation of state, $p = \alpha\rho$ ($\alpha \neq 1$) [3].

Eq. (1) admits, in principle, a wider range of positive α values; however, the chosen range ensures that the sound velocity ($c_s^2 = \alpha A/\rho_{ch}^{1+\alpha}$) does not exceed, in the "soft" equation of state phase, the velocity of light. Furthermore, as pointed out in Ref. [3], it is only for $0 < \alpha \leq 1$ that the analysis of the evolution of energy density fluctuations is physically meaningful.

More fundamentally, the model can be described, as discussed in Ref. [3], by a complex scalar field whose action can be written as a generalized Born-Infeld action. This can be seen starting with the Lagrangian density for a massive complex scalar field, Φ,

$$\mathcal{L} = g^{\mu\nu}\Phi^*_{,\mu}\Phi_{,\nu} - V(|\Phi|^2) \quad , \tag{3}$$

which can be expressed in terms of its masss, m, as $\Phi = (\frac{\phi}{\sqrt{2m}})\exp(-im\theta)$. Assuming that the scale of the inhomogeneities is set by the spacetime variations of ϕ corresponding to scales greater than m^{-1}, then $\phi_{,\mu} << m\phi$, which, together with Eq.(1), leads to a relationship between ϕ^2 and ρ:

$$\phi^2(\rho_{ch}) = \rho_{ch}^{\alpha}(\rho_{ch}^{1+\alpha} - A)^{\frac{1-\alpha}{1+\alpha}} \quad , \tag{4}$$

and a Lagrangian density that has the form of a *generalized* Born-Infeld action:

$$\mathcal{L}_{GBI} = -A^{\frac{1}{1+\alpha}} \left[1 - (g^{\mu\nu}\theta_{,\mu}\theta_{,\nu})^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \quad . \tag{5}$$

Notice that, for $\alpha = 1$, one recovers the exact Born-Infeld action. It is easy to see that Eq. (2) has a bearing on the observed accelerated expansion of the Universe as it automatically leads to an asymptotic phase where the equation of state is dominated by a cosmological constant, $8\pi G A^{1/1+\alpha}$, while at earlier times the energy density behaves as if dominated by non-relativistic matter. This dual behaviour is at the heart of the unification scheme provided by the GCG model. Figure 1 depicts the way the Universe evolves in the GCG model. It has also been shown that the underlying complex scalar field model admits, under conditions, an inhomogeneous generalization which can be regarded as a unification of dark matter and dark energy [3, 4] without conflict with standard structure formation scenarios [3, 4, 5, 6]. It is clear that the GCG model collapses into the ΛCDM model when $\alpha = 0$.

These remarkable properties make the GCG model an interesting alternative to models where the accelerated expansion of the Universe arises from an uncancelled cosmological constant or a rolling scalar field as in quintessence models.

In what follows, we shall discuss the observational bounds that can be set on the GCG model parameters.

3 Observational Constraints

Given the potential of the GCG model as a viable dark energy-dark matter unification scheme, many authors have studied constraints on the model parameters from observational data, particularly those arising from SNe Ia [7] and gravitational lensing statistics [8].

Quite stringent constraints arise also from the study of the position of the acoustic peaks and troughs of the CMBR power spectrum. The CMBR peaks arise from oscillations of the primeval plasma just before the Universe becomes transparent. Driving processes and the ensuing shifts on peak positions [9]

$$\ell_{p_m} \equiv \ell_A \left(m - \varphi_m\right) \ , \tag{6}$$

where ℓ_A is the acoustic scale

$$l_A = \pi \frac{\tau_0 - \tau_{\rm ls}}{\bar{c}_s \tau_{\rm ls}} \quad , \tag{7}$$

τ_0 and $\tau_{\rm ls}$ being the conformal time ($\tau = \int a^{-1}dt$) today and at last scattering and $\bar{c}_s$ the average sound speed before decoupling, are fairly independent of post recombination physics and hence of the form of the potential and the nature of the late time acceleration mechanism. Hence, the rather accurate fitting formulae of Ref. [10] can be used to compute the phase shifts φ_m for the GCG model. In order to calculate the acoustic scale, we use Eq. (2) and write the Universe expansion rate as

$$H^2 = \frac{8\pi G}{3}\left[\frac{\rho_{r0}}{a^4} + \frac{\rho_{b0}}{a^3} + \rho_{ch0}\left(A_s + \frac{(1-A_s)}{a^{3(1+\alpha)}}\right)^{1/1+\alpha}\right] \quad , \tag{8}$$

where $A_s \equiv A/\rho_{ch0}^{1+\alpha}$, $\rho_{ch0} \equiv (A+B)^{1/1+\alpha}$ and we have included the contribution of radiation and baryons as these are not accounted for by the GCG equation of state. As discussed in Refs. [11, 12], the above set of equations allow for obtaining the value of the fundamental acoustic scale by direct integration, using the fact that $H^2 = a^{-4}\left(\frac{da}{d\tau}\right)^2$.

Comparing results from the above procedure with recent bounds on the location of the first two peaks and the first trough obtained by the WMAP collaboration [13], namely $\ell_{p_1} = 220.1 \pm 0.8$, $\ell_{p_2} = 546 \pm 10$, $\ell_{d_1} = 411.7 \pm 3.5$, together with the bound on the location of the third peak obtained by the BOOMERanG collaboration [14], $l_{p3} = 825^{+10}_{-13}$, leads to quite strong constraints on the model parameters. These constraints can be summarized as follows [12]:

1) The Chaplygin gas model, $\alpha = 1$, is incompatible with the data and so are models with $\alpha \gtrsim 0.6$.

2) For $\alpha = 0.6$, consistency with data requires for the spectral tilt, $n_s > 0.97$ and $h \lesssim 0.68$.

3) The ΛCDM model barely fits the data for values of the spectral tilt

$n_s \simeq 1$ (notice that WMAP data leads to $n_s = 0.99 \pm 0.04$) and for that $h > 0.72$ is required. For low values of n_s, ΛCDM is preferred to the GCG models whereas for intermediate values of n_s, the GCG model is favoured only if $\alpha \simeq 0.2$.

4) Our study of the peak locations in the (A_s, α) plane shows that, varying h within the bounds $h = 0.71^{+0.04}_{-0.03}$ [13], does not lead to very relevant changes in the allowed regions, as compared to the value h=0.71 (see Fig. 3), even though these regions become slightly larger as they shift upwards for $h < 0.71$; the opposite trend is found for $h > 0.71$.

5) Our results are consistent with the bound found in Ref. [11] using BOMERanG data for the third peak and Archeops [15] data for the first peak as well as results from SNe Ia and age bounds, namely $0.81 \lesssim A_s \lesssim 0.85$ and $0.2 \lesssim \alpha \lesssim 0.6$.

Bounds from SNe Ia data, which suggest that $0.6 \lesssim A_s \lesssim 0.85$ [7], are also consistent with our results for $n_s = 1$ and $h = 0.71$, which yield $0.78 \lesssim A_s \lesssim 0.87$.

4 Discussion and Outlook

In this essay, we have described the way the GCG model allows for a consistent description of the accelerated expansion of the Universe and purports a scheme for the unification of dark energy and dark matter. This description is quite detailed and allows for an unambiguous confrontation with observational data. For this purpose, several studies were performed aiming to constrain the parameter space of the model using Supernovae data, the age of distant quasar sources, gravitational lensing statistics and the location of the first few peaks and troughs the CMBR power spectrum, as measured by the WMAP and BOOMERanG collaborations. These studies reveal that a sizeable portion of the parameter space of the GCG model is excluded.

More concretely, our results indicate that the Chaplygin gas model, $\alpha = 1$, is incompatible with the data and so are models with $\alpha \gtrsim 0.6$. For $\alpha = 0.6$, consistency with observations requires that $n_s > 0.97$. We find that the ΛCDM model hardly fits the data for $n_s \simeq 1$ and $h > 0.72$ is required. For lower values of n_s, ΛCDM is preferred to the GCG models whereas for intermediate values of n_s the GCG model is favoured only if $\alpha \simeq 0.2$.

We conclude that the GCG is a viable dark matter - dark energy model in that it is compatible with standard structure formation scenarios. Moreover, although its parameter space is rather constrained, the model is consistent with all the available Supernovae, gravitational lensing and CMBR data. Finally, the model does not suffer from the well-known fine-tuning problems that are present in alternative dark energy candidate theories such as ΛCDM and quintessence models.

Acknowledgments

M.C.B. and O.B. acknowledge the partial support of Fundação para a Ciência e a Tecnologia (Portugal) under the grant POCTI/1999/FIS/36285. The work of A.A.S.

is fully financed by the same grant.

References

[1] S. Chaplygin, *Sci. Mem. Moscow Univ. Math* **21** (1904) 1.

[2] A. Kamenshchik, U. Moschella, V. Pasquier, *Phys. Lett.* **511** (2001) 265.

[3] M. C. Bento, O. Bertolami, A. A. Sen, *Phys. Rev.* **D66** (2002) 043507.

[4] N. Bilić, G.B. Tupper, R.D. Viollier, *Phys. Lett.* **B535** (2002) 17.

[5] J.C. Fabris, S.B.V. Gonçalves, P.E. de Souza, *Gen. Relativity and Gravitation* **34** (2002) 53.

[6] L.M.G. Beça, P.P. Avelino, J.P.M. de Carvalho and C.J.A.P. Martins, astro-ph/0303564.

[7] J.C. Fabris, S.B.V. Gonçalves, P.E. de Souza, astro-ph/0207430; P.P. Avelino, L.M.G. Beça, J.P.M. de Carvalho, C.J.A.P. Martins, P. Pinto, astro-ph/0208528; A. Dev, J.S. Alcaniz, D. Jain, *Phys. Rev.* **D67** (2003) 023515; V. Gorini, A. Kamenshchik, U. Moschella, astro-ph/0209395; M. Makler, S.Q. de Oliveira, I. Waga, astro-ph/0209486; J.S. Alcaniz, D. Jain, A. Dev, *Phys. Rev.* **D67** (2003) 043514 .

[8] P.T. Silva, O. Bertolami, astro-ph/0303353.

[9] W. Hu, M. Fukugita, M. Zaldarriaga, M. Tegmark, *Astroph. J.* **549** (2001) 669.

[10] M. Doran, M. Lilley, C. Wetterich, astro-ph/0105457.

[11] M. C. Bento, O. Bertolami and A. A. Sen, *Phys. Rev.* **D67** (2003) 063003.

[12] M. C. Bento, O. Bertolami and A. A. Sen, astro-ph/0303538.

[13] D.N. Spergel et al., astro-ph/0302207; L. Page et al., astro-ph/0302220.

[14] P. Bernardis et al., *Astroph. J.* **564** (2002) 559.

[15] A. Benoit et al., astro-ph/0210306.

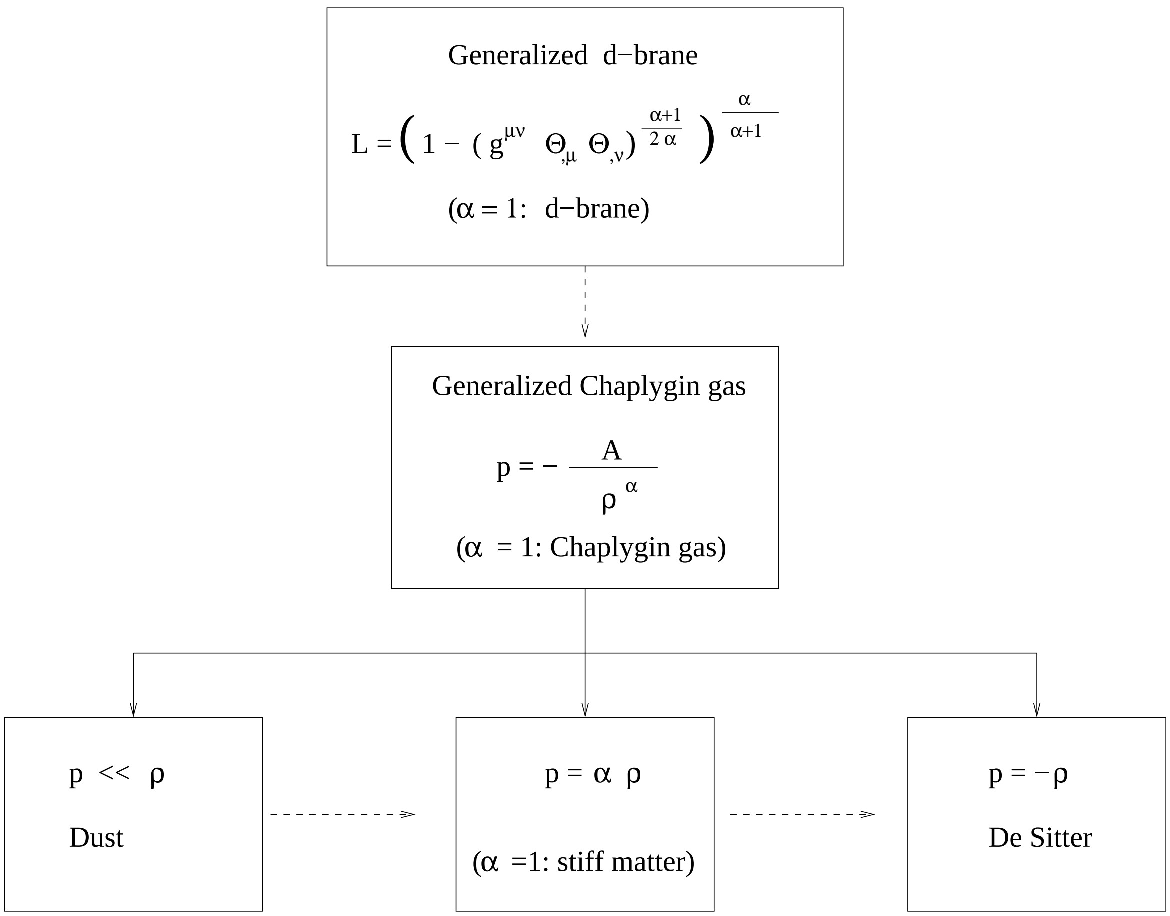

Figure 1: Cosmological evolution of the Universe described by the Generalized Chaplygin Gas model.

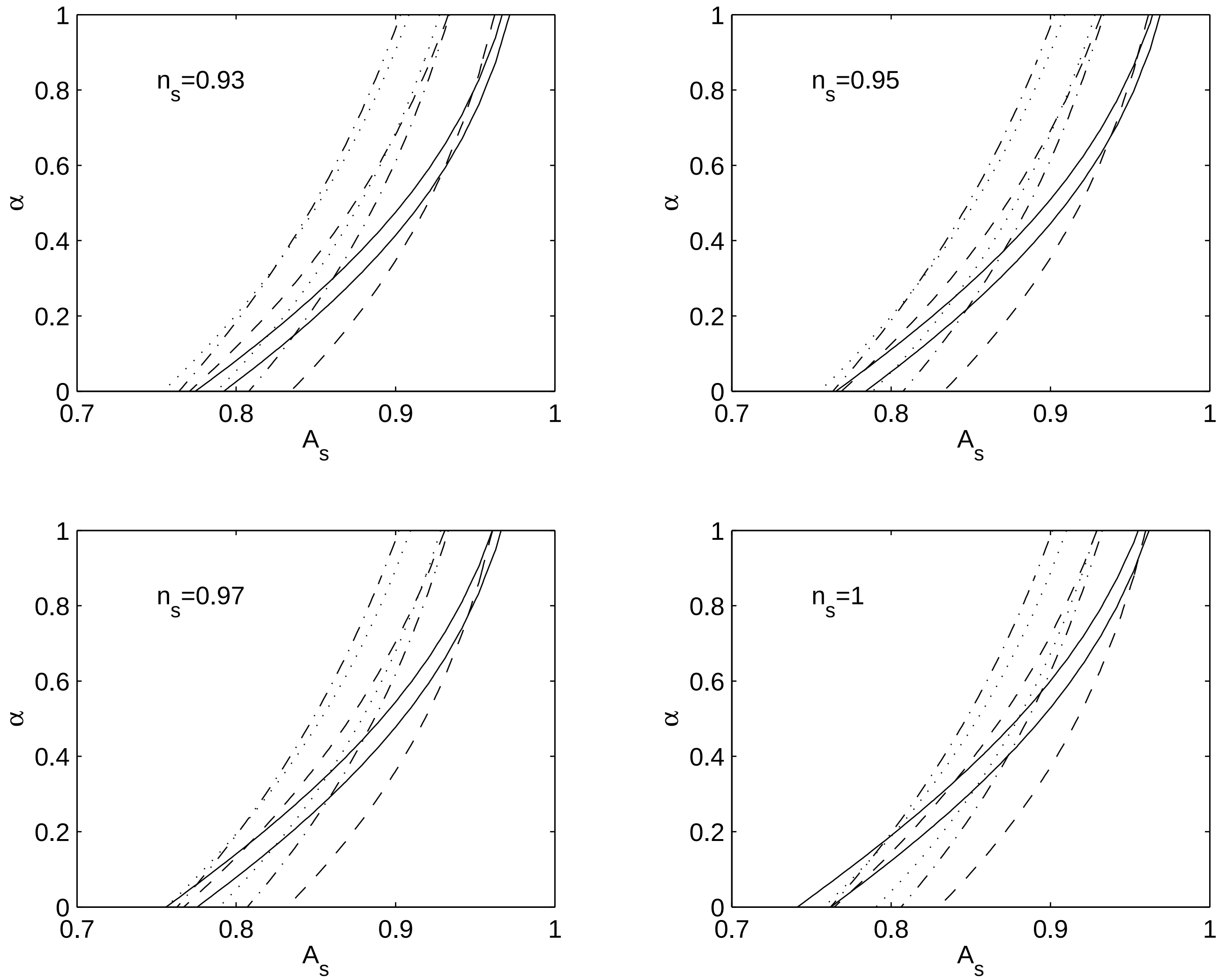

Figure 2: Contour plots of the locations of the first three peaks and first trough of the CMBR power spectrum, in the (A_s, α) plane, for a GCG model, with $h = 0.71$, for different values of n_s. Full, dashed, dot-dashed and dotted contours correspond to observational bounds on ℓ_{p_1}, ℓ_{p_2}, ℓ_{p_3} and ℓ_{d_1}, respectively.