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Letter to the Editor—A Comparison of Estimates of Nuclear Bomb Casualties from Two Different Urban Models

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4. STEVEN A. LIPPMAN, "On the Set of Optimal Policies in Discrete Dynamic Programming," *J. Math. Analysis and Appl.* **24,** 440–445 (1968).
5. ———, "Criterion Equivalence in Discrete Dynamic Programming," Working Paper No. 142, Western Management Science Insitute, U.C.L.A. (1968).
6. E. C. TITCHMARCH, *The Theory of Functions*, 2nd ed., Oxford University Press, 1939.
7. ARTHUR F. VEINOTT, JR., "On Finding Optimal Policies in Discrete Dynamic Programming with No Discounting," *Ann. Math. Stat.* **37,** 1284–1294 (1966).

A COMPARISON OF ESTIMATES OF NUCLEAR BOMB CASUALTIES FROM TWO DIFFERENT URBAN MODELS

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Expected numbers of human casualties in a single-bomb nuclear attack upon a symmetric city are calculated using SHERRATT'S AND CLARK'S models and for various distances between the city center and ground zero. The calculated results indicate that considerable error may arise in practical cases from the unjustified use of Sherratt's model.

J. J. HUNTER[1] has developed an interesting analytical technique for estimating the casualties in a nuclear attack on an urban center, in which the casualty function is approximated by an exponential or sum of exponential functions, while the population density is taken to be of SHERRATT's form.[2] Hunter has pointed out that some cities are undoubtedly better represented by CLARK's model[3] than by Sherratt's and, in this connection, WEISS[4] has observed that Clark's fitted well to Sydney in 1947.

Thus, it is desirable to compare the number of casualties given by Hunter's approach with those given by Clark's model. For this purpose, we consider a symmetric city of central density A and, as usual with these population models, taken as being infinite in extent, while the attack consists of one bomb dropped at a variable distance R from the city center. For the casualty function, Hunter's single-term approximation is used, and aiming error is neglected.

THE EXPECTED NUMBER OF CASUALTIES AND NUMERICAL COMPARISON

IF THE probability that a person a distance r from ground zero will be a casualty has the form[1] $\exp\{-cr^2\}$, c a constant, and the population density at the point (x, y) is $A\exp\{-(x^2+y^2)b^2/2\}$, b a constant, then the total number of casualities is given by[1]

$$N_s = A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{-(x^2+y^2)b^2/2 - c(x-R)^2 - cy^2\}\, dxdy$$
$$= 2\pi A(b^2+2c)^{-1}\exp\{-cR^2b^2(b^2+2c)^{-1}\}. \quad (1)$$

The origin is taken at the city center and the positive x-axis is through ground zero.

The population density in (1) is the symmetric case of Sherratt's form. How-

TABLE I

R	b = 0.25		b = 0.45		b = 0.75	
	N_s	N_c	N_s	N_c	N_s	N_c
0	25.910	17.963	16.427	11.387	8.462	6.450
0.5	25.760	17.843	16.232	11.267	8.319	6.368
1.0	25.316	17.491	15.662	10.917	7.905	6.100
1.5	24.592	16.927	14.757	10.362	7.259	5.680
2.0	23.614	16.181	13.576	9.641	6.442	5.143
2.5	22.413	15.287	12.196	8.799	5.526	4.532
3.0	21.028	14.274	10.698	7.883	4.581	3.890
3.5	19.501	13.159	9.164	6.934	3.671	3.252
4.0	17.877	11.951	7.664	5.984	2.843	2.652
4.5	16.198	10.652	6.259	5.060	2.128	2.108
5.0	14.508	9.274	4.992	4.181	1.539	1.633
5.5	12.845	7.846	3.887	3.365	1.076	1.231
6.0	11.241	6.417	2.955	2.627	0.727	0.902
6.5	9.724	5.050	2.194	1.983	0.475	0.641
7.0	8.315	3.810	1.591	1.441	0.300	0.440
7.5	7.028	2.747	1.126	1.005	0.183	0.292
8.0	5.871	1.888	0.778	0.671	0.108	0.186
8.5	4.849	1.236	0.525	0.428	0.061	0.114
9.0	3.958	0.769	0.346	0.261	0.034	0.067
9.5	3.194	0.455	0.223	0.151	0.018	0.037
10.0	2.547	0.255	0.140	0.083	0.009	0.020

ever, if Clark's is adopted so that the population density at (x, y) is $\exp\{-b\sqrt{x^2+y^2}\}$, then the number of casualties is given by

$$N_c = A\int_0^{2\pi}\int_0^{\infty} \exp\{-rb - c(r^2 - 2rR\cos\theta + R^2)\ rdrd\theta. \quad (2)$$

This may be expressed as

$$N_c = 2\pi A e^{c(a^2-R^2)} \sum_{n=0}^{n=\infty} (cR)^{2n} I_{2n+1}/(n!)^2, \quad (3)$$

where
$$a=b^2/4c^2, \quad \text{and}$$
$$I_{k+2}=\int_0^\infty e^{-c(r+a)^2}r^{k+2}\,dr=[(k+1)/2c]I_k-aI_{k+1}.$$

I_0 and I_1 are, respectively, $(\frac{1}{2})\text{erfc}(a\sqrt{c})\sqrt{(\pi/c)}$ and $\exp\{-ca^2\}/(2c)-aI_0$, so the I_{k+2} are easily determined.

N_c and N_s depend on the parameters A, b, c, and R. The central density A is essentially just a scaling factor as far as any comparison is concerned, so it was given the value 1 in the computations. Since the total population does not depend on which model is used, b is the same for N_c as for N_s, and was given the values 0.25, 0.45, and 0.75. These values were chosen from Clark's results[3] to give a reasonable coverage of modern cities. The bomb parameter c was fixed at 0.09/mile^2, as Hunter's results[1] indicated this to be fairly representative. The numerical results are shown in Table I.

CONCLUSION

THE CITY is more concentrated according to the Sherratt model than it is according to the Clark model, and the calculated results indicate that the use of expression (1) to derive the estimated number of casualties when a nuclear bomb is exploded often gives results significantly greater than those obtained using expression (2). This difference is greatest when b is small and when the bomb is dropped close to the city center.

As a practical example, the values of A and b for Sydney (1947) have been given[3] as 30 and 0.25, respectively. For a centrally directed attack, and giving A its true value, the estimated number of casualties using (2) is about 539,000 while (1) gives about 777,000.

Similarly, in 1947, the values of A and b for Brisbane are given[3] as 40 and 0.75. The estimated casualities derived from expressions (1) and (2) are 340,000 and 260,000 respectively.

REFERENCES

1a. J. J. HUNTER, "An Analytical Technique for Urban Casualty Estimation from Multiple Nuclear Weapons," *Opns. Res.* **15,** 1096–1108 (1967).

1b. ———, "An Analytical Technique for Estimating Casualties due to Direct Effects of a Multiple Weapon Nuclear Attack on a City," Research Memorandum RM-230-2, R.T.I., Durham, N.C. (1965).

2. G. G. SHERRATT, "A Model for General Urban Growth," *Management Sciences, Models and Techniques*, Vol. 2, pp. 147–159, Pergamon Press, New York, 1960.

3. C. CLARK, "Urban Population Densities," *J. Roy. Stat. Soc.* **114,** Pt. IV, 490–496.

4. H. K. WEISS, "The Distribution of Urban Population and an Application to a Servicing Problem," *Opns. Res.* **9,** 860–874 (1961).

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