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Letter to the Editor

A Note on Fleet Routing Models for Transportation Systems

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This note is a comment on the paper by LEVIN and discusses the problem of fleetsize minimization for transportation systems. A simpler and more compact formulation of this problem results in an Integer Program of considerably reduced size, that can more naturally serve as a basis for finding suitable branch-and-bound methods for its solution.

Several integer programming models for optimal airline fleet routing are presented in two papers of A. Levin. [1,2] The purpose of this note is to show that for many of the problems discussed in his papers, a different formulation results in integer programs of considerably smaller size. This reduction in size can be obtained for problems where the objective function depends only on the flights that are made. As will be shown, this includes the case where the aim is minimizing the fleetsize. The situation we will discuss is that described in reference 1: we will use, where possible, the same notation:

N = the set of services,
K_l = $\{$all alternative flights for service $l\}$,
u_i = 0, 1 according to whether the ith flight is not or is selected, respectively.

In addition, the following notation is introduced:

I = set of flights,
S = set of airports,
T_y = set of all possible departure times from airport y;

and for $y \epsilon S$, $i \epsilon I$ and $t \epsilon T_y$ we define:

$$r_y(i,t) = \begin{cases} 1 \text{ if flight } i \text{ departs from } y \text{ } \textit{at } t, \\ -1 \text{ if flight } i \text{ arrives at } y \text{ } \textit{after } t-1, \text{ but } \textit{on} \text{ or } \textit{before } t, \\ 0 \text{ otherwise;} \end{cases}$$

$$R_y(i,t) = \textstyle\sum_{s \leq t} r_y(i,s).$$

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It is then clear that for a given set of decisions $\{u_i\}$ the expression,

$$\sum_{i\in I} R_y(i,\, t)u_i \,,$$

denotes the number of departures from y minus the number of arrivals at y, before or at t.

It has been shown[4] that the number of aircraft that has to be available at airport y at the beginning of the schedule is given by:

$$F_y = \max_{t\in T_y} \sum_{i\in I} R_y(i,\, t)u_i \,; \tag{1}$$

and the total fleetsize that is needed is equal to:

$$F = \sum_{y\in S} F_y \,. \tag{2}$$

Here it is assumed that flight connections are made whenever possible. The problem of minimizing the fleetsize can now be formulated as follows:

$$\min \sum_{y\in S} \mathrm{F}_y \,,$$

under the constraints;

$$W_y \geqq \sum_{i\in I} R_y(i,\, t)u_i \,,\ (y\in S,\ t\in T_y). \tag{3}$$

$$\sum_{i\in K_l} u_i = 1, \qquad (l\in N)$$

This is a more compact formulation of the problem of fleetsize minimization than that presented by Levin for the case of a variable schedule. In fact his variables x_{ij}, indicating if flight i is connected to flight j, do not appear at all.

In the case of a fixed schedule there is no problem left to be solved; formulas (1) and (2) give the answer. The connections can, for instance, be made on a first-come, first-served basis.

It is also possible to include in the cost function of program (3) the costs of the flights. However, if costs of connections between flights i and j have to be considered also, as suggested in reference 1 on page 240, in which case the variables x_{ij} have to be introduced, then Levin's approach is to be preferred. The number of variables in the program presented here is equal to $|I| + |S|$. The number of constraints is equal to $|I| + |N|$. Some of these constraints, however, can be eliminated by a simple dominance rule: if for some y, t, and s,

$$\sum_{i\in I} R_y(i,\, t)u_i \leqq \sum_{i\in I} R_y(i,\, s)u_i \,,$$

for all u_i's satisfying

$$\sum_{i\in K_l} u_i = 1, \qquad (l\in N)$$

then the constraint corresponding to t is redundant. This happens, for instance, when $s > t$ and there are *no* arrivals at y between t and s.

ILP (3), the smallest program presented by Levin in reference 1, after pass 1 and pass 2 have been applied to reduce its size, contains many more variables and at least the number of constraints of the program presented here. The formulation for fleetsize minimization presented in this note is essentially that used in references 3 and 5 for suburban railway systems.

REFERENCES

1. A. Levin, "Scheduling and Fleet Routing Models for Transportation Systems," *Trans. Sci.* **5,** 232–255 (1971).
2. ———, "Some Fleet Routing and Scheduling Problems for Air Transportation Systems," FTL Report R-68-5, M.I.T., January, 1969.
3. F. J. M. Salzborn, "The Minimum Fleetsize for a Suburban Railway System," *Trans. Sci.* **4,** 383–402 (1970).
4. ———, "Optimum Bus Scheduling," *Trans. Sci.* **6,** 137–148 (1972).
5. L. G. Proll, "A Note on the Minimum Fleetsize for a Suburban Railway System," *Trans. Sci.* **6,** 204–207 (1972).

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