Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Letter to the Editor—Approximation to the Value of the Objective Function in Linear Programming by the Method of Partitions

Thomas L. Saaty,

hensive illumination of the problem at hand, one must set up many models, each of which take into account only certain specific factors in the problem The results of these model studies have then to be taken together in their entirety, at which point one has to weigh the significance of each in relation to the importance and the accuracy of the formal assumptions on which each model rests

In this phase one encounters the third of the above-mentioned types of factors in the problem at hand In the foreseeable future it is not certain that all the conditions that are significant in evaluating a military situation can be described quantitatively This is especially true as to conditions concerning human reactions The function of integrating all these elements can still best be performed by the human intellect, and the goal of operations analysis is to create such a foundation that these special qualities of intellect can be put to actual use

APPROXIMATION TO THE VALUE OF THE OBJECTIVE FUNCTION IN LINEAR PROGRAMMING BY THE METHOD OF PARTITIONS

THOMAS L. SAATY

Operations Evaluation Group, Massachusetts Institute of Technology, P O Box 2176, Potomac Station, Alexandria, Virginia

(Received January 6, 1956)

A LINEAR-PROGRAMMING PROBLEM with a large number of inequalities and unknowns generally requires a considerable amount of expense and time for its solution In fact, a solution may not be obtainable at all if a digital computer is not available Further, the precise value of the minimum of the objective function is often not required That is to say, lower and upper bounds on this value may be sufficient to answer many questions as effectively as the complete solution, and may be obtained with much less effort

For example, a knowledge of the *order of magnitude* of the minimum value of the cost function may permit a decision *not* to undertake a complete solution because the minimum cost will be prohibitively large

Conversely, an approximate solution of the sort enunciated here may show management that the situation presently prevailing may be very far from optimum and a detailed solution is therefore justified

Alternatively when computing facilities are not available, the approximate solution suggested may be all that is feasible in a reasonable time A justification of the method follows

It is clear that increasing the number of constraints in a linear-programming problem generally entails an increase in the minimum value of the objective function This follows from the fact that the convex hull of the larger set of inequalities is contained in the convex hull of the smaller set, and thus would place the objective function at a greater distance from the origin The approximation problem is one of solving linear-programming problems with the same objective function relative to a subset of the constraints

From the above statement one has the important relation $\min P \geq \max(\min P_i)$, where the quantity on the left is the minimum value of the cost function for the entire problem and the quantities in parentheses on the right are minima of the several partition problems solved separately with the original objective function One also has $\min(\max P_i) \geq \max P$

ALGORITHM *When minimizing, solve the linear-programming problems corresponding to subsets (partitions) of the constraints with the objective function of the original problem For a lower bound to the minimum of the original problem take the maximum of the minima obtained*

Note that to obtain the minimum value in the case of single partitions the trial vertices may be obtained by setting all the variables but one equal to zero in the constraint The value of the remaining variable is obtained from the constraint written as an equality This gives the intersection vertices of the constraining hyperplane with the coordinate planes The value of the objective function is obtained for each vertex and the minimum of these selected When this is done for every constraint individually taken with the objective function, the maximum of the several minima provides a simple lower bound to the minimum value of the entire problem This result may be further refined by taking the constraints two at a time and solving the corresponding simple linear-programming problems, and so on

Suppose the dual problem is formulated Then if the problem was one of maximizing the objective function of the primal, it now becomes one of minimizing the objective function of the dual Thus the method of partitions may be applied to obtain lower bounds for the minimum of the dual which are in turn lower bounds for the maximum of the primal Using both the problem and its dual provides upper and lower bounds to the maximum (or minimum) value of the objective function of the primal

Note that if two (or three) inequalities are taken at a time with the cost function, and the dual formed, it may then be solved geometrically Hence, through the dual finer upper bounds to the minimum and lower bounds to the maximum of the primal are readily obtainable

This rapid method of placing a lower and upper bound on the minimum (maximum) is very useful, and it may give sufficient information for making a decision as to whether the cost thus obtained makes feasible the adoption of certain strategies or whether the ofttimes costly solution of the entire problem is warranted Finally, this may be the only feasible procedure in the absence of a computing machine