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Letter to the Editor—Comments on a Paper By Romesh Saigal: “A Constrained Shortest Route Problem”

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$$K(\theta) = \int_0^\theta N_Q(t) dt.$$

Consequently, one must have

$$\lambda \bar{w} = \lim_{\theta \rightarrow \infty} (1/\theta) \int_0^\theta N_Q(t) dt = \bar{N}_Q. \quad (7)$$

But the process $N_Q(t)$ is identical in distributions of all order for the three types of service discipline if the systems have a common initial state. The mean waiting time is therefore the same for all three.

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COMMENTS ON A PAPER BY ROMESH SAIGAL: "A CONSTRAINED SHORTEST ROUTE PROBLEM"

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ROMESH SAIGAL presents a zero-one linear program and a dynamic programming algorithm for finding the shortest route containing exactly q arcs from node 1 to node n in a network (N, A) with distances $c(i, j)$. This note shows that the linear programming formulation and his extension based on it are defective, and that the dynamic programming algorithm can lead to suboptimal solutions, but a minor change in the dynamic programming formulation relieves the difficulty.

SAIGAL^[1] gives the following zero-one linear program:

$$\left. \begin{aligned} \sum_{1 \leq n} f(1, j) - \sum_{1 \leq n} f(j, 1) &= 1, \\ \sum_{1 \leq n} f(i, j) - \sum_{1 \leq n} f(j, i) &= 0, \\ \sum_{1 \leq n} f(n, j) - \sum_{1 \leq n} f(j, n) &= -1; \end{aligned} \right\} (i=2, \dots, n-1) \quad (1)$$

$$\sum_{(i,j) \in A} f(i,j) = q, \tag{2}$$

$$f(i,j) = 0 \text{ or } 1, \tag{3}$$

$$\sum_{(i,j) \in A} c(i,j) f(i,j) = z \text{ (min)}. \tag{4}$$

This formulation is incorrect. Criterion (4) and constraints (1) form a well-known linear program for the ordinary shortest route problem. A special feature of this linear program is that there can never be a loop in the basis, because the vectors corresponding to the variables of a loop are linearly dependent. However, by adding constraint (2) the loop-vectors become independent. Thus the optimal solution may have loops, as is shown in the following example.

Suppose we want to find the shortest path containing exactly 4 arcs from node 1 to 5 for the directed network of Fig. 1. The shortest route with 4 arcs and where

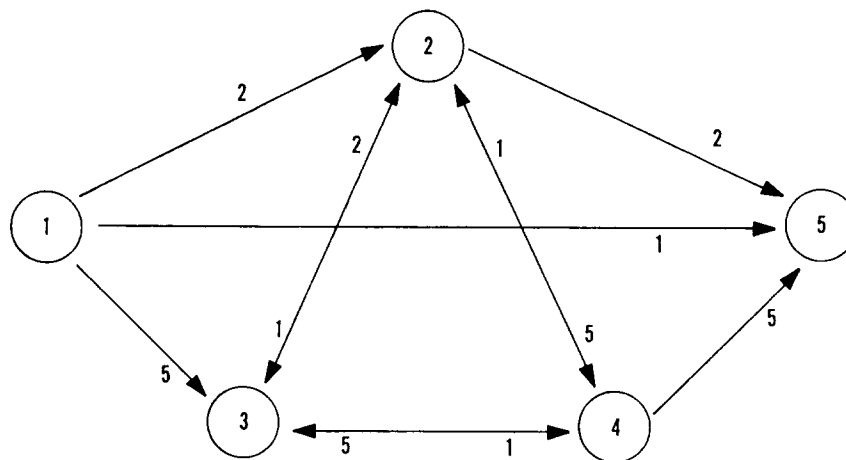


Figure 1

one is allowed to pass through a node more than once is 1-2-3-2-5 with total length 7. But constraints (1), (2), and (3) are also satisfied by the solution 1-5 and loop 2-3-4-2, which is unconnected with arc 1-5. The total length is only 4. This is the optimal solution to the above zero-one linear program as all arcs have length equal to or greater than unity in Fig. 1.

The first extension on page 207 deals with the same zero-one linear program where constraint (2) is replaced by the more general constraint $\sum_{(i,j) \in A} a(i,j) f(i,j) = q$, and where $a(i,j)$ is a positive integer. The proposed method for solving this program consists of converting it into a shortest route problem containing exactly q arcs. This is wrong for the reason outlined before.

Our last comment is related to the dynamic programming algorithm. One is allowed to pass through a node more than once in the shortest route problem of reference 1. But for some arbitrary reason, the dynamic program is formulated in such a way that one can never have starting node 1 more than once in the final

solution. The following simple changes will permit cycles through any node, including node 1 (see the original paper for definitions and the complete algorithm):

$$C_1(k) = [c_{11}(k), c_{12}(k), \dots, c_{1n}(k)],$$

$$P(k) = [p_1(k), p_2(k), \dots, p_n(k)]^T, \text{ an } \times(k+1) \text{ matrix.}$$

It should be noted that $p_j(k)$ does not always exist. Step 0 now becomes:

$$C_1(1) = [c(1, 1), c(1, 2), \dots, c(1, n)], \text{ where } c(1, 1) = \infty,$$

$$p_j(1) = (1, j), j = 1, 2, \dots, n, \text{ where } p_1(1) \text{ does not exist.}$$

Using this modified dynamic program, we find a shorter route from node 1 to 3 and from node 1 to node 4 for the author's numerical example on pages 206-207. The shortest routes are 1-4-1-3 with total length 6 instead of 8, and 1-4-1-4 with total length 5 instead of 7.

Path 1-4-1-4 illustrates that arcs may be repeated in the dynamic programming formulation. As pointed out by the referees, repeated arcs are not allowed in the (defective) zero-one integer programming formulation, unless constraint (3) is replaced by $f(i, j) \geq 0$ and integer.

Finally, two misprints should be corrected.

<i>page</i>	<i>for</i>	<i>read</i>
206	line 3 of step 1	$c_{1s}(k+1)$
207	line 5 from bottom	$i \in (i_1, i_2, \dots, i_k)$

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1. ROMESH SAIGAL, "Constrained Shortest Route Problem," *Opns. Res.* **16**, 205-209 (1968).

A MULTICOMMODITY MAX-FLOW ALGORITHM

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(Received July 10, 1968)

Some simplifications occur when the theory of KORNAL AND LIPTAK is applied to the multicommodity max-flow problem. This note describes the resulting algorithm and comments on some of its properties; it is flexible, easy to code, and involves simple computations. However, the technique is recommended for suboptimization because of its poor convergence properties.

WE SHALL consider a directed network of N nodes and M arcs described by a node-arc incidence matrix E . There are G goods distinguished by their