Operations Research

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Letter to the Editor—Comments on a Paper By Romesh Saigal: "A Constrained Shortest Route Problem"

Marc Rosseel,

$$K(\theta)=\int_0^\theta N_Q(t)\ dt.$$

Consequently, one must have

$$\lambda \bar{T}_W=\lim_{\theta\to\infty}(1/\theta)\int_0^\theta N_Q(t)\ dt=\bar{N}_Q. \qquad (7)$$

But the process $N_Q(t)$ is identical in distributions of all order for the three types of service discipline if the systems have a common initial state. The mean waiting time is therefore the same for all three.

Acknowledgment. This research is a product of the System Science Center of the University of Rochester, which is supported by a grant from the Center for Naval Analyses. Such support does not imply endorsement of the content by the Navy.

REFERENCES

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COMMENTS ON A PAPER BY ROMESH SAIGAL: "A CONSTRAINED SHORTEST ROUTE PROBLEM"

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(Received April 22, 1968)

Romesh Saigal presents a zero-one linear program and a dynamic programming algorithm for finding the shortest route containing exactly q arcs from node 1 to node n in a network (N, A) with distances $c(i, j)$. This note shows that the linear programming formulation and his extension based on it are defective, and that the dynamic programming algorithm can lead to suboptimal solutions, but a minor change in the dynamic programming formulation relieves the difficulty.

Saigal[1] gives the following zero-one linear program:

$$\left.\begin{aligned}&\textstyle\sum_{j\epsilon N} f(1, j)-\sum_{j\epsilon N} f(j, 1)=1,\\ &\textstyle\sum_{j\epsilon N} f(i, j)-\sum_{j\epsilon N} f(j, i)=0, \qquad (i=2, \cdots, n-1)\\ &\textstyle\sum_{j\epsilon N} f(n, j)-\sum_{j\epsilon N} f(j, n)=-1;\end{aligned}\right\} \quad (1)$$

$$\sum_{(i,j)\epsilon A} f(i,j)=q, \tag{2}$$

$$f(i,j)=0 \text{ or } 1, \tag{3}$$

$$\sum_{(i,j)\epsilon A} c(i,j)\, f(i,j)=z \text{ (min)}. \tag{4}$$

This formulation is incorrect. Criterion (4) and constraints (1) form a well-known linear program for the ordinary shortest route problem. A special feature of this linear program is that there can never be a loop in the basis, because the vectors corresponding to the variables of a loop are linearly dependent. However, by adding constraint (2) the loop-vectors become independent. Thus the optimal solution may have loops, as is shown in the following example.

Suppose we want to find the shortest path containing exactly 4 arcs from node 1 to 5 for the directed network of Fig. 1. The shortest route with 4 arcs and where

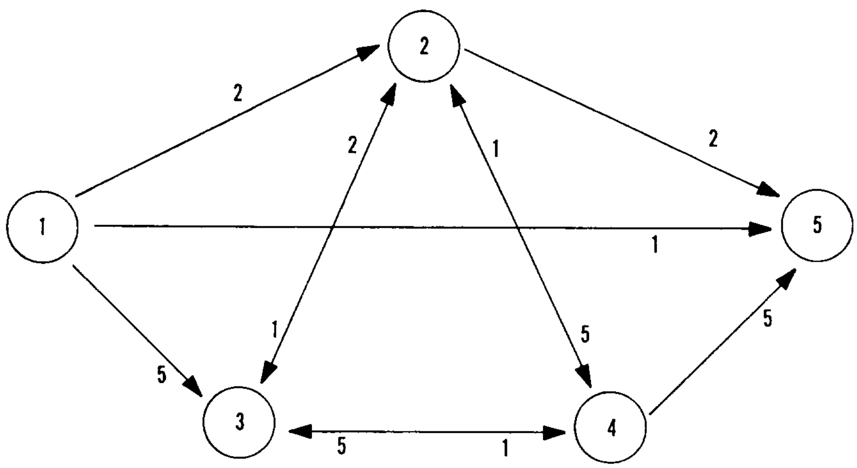

Figure 1

one is allowed to pass through a node more than once is 1-2-3-2-5 with total length 7. But constraints (1), (2), and (3) are also satisfied by the solution 1-5 and loop 2-3-4-2, which is unconnected with arc 1-5. The total length is only 4. This is the optimal solution to the above zero-one linear program as all arcs have length equal to or greater than unity in Fig. 1.

The first extension on page 207 deals with the same zero-one linear program where constraint (2) is replaced by the more general constraint $\sum_{(i,j)\epsilon A} a(i,j)\, f(i,j)=q$, and where $a(i,j)$ is a positive integer. The proposed method for solving this program consists of converting it into a shortest route problem containing exactly q arcs. This is wrong for the reason outlined before.

Our last comment is related to the dynamic programming algorithm. One is allowed to pass through a node more than once in the shortest route problem of reference 1. But for some arbitrary reason, the dynamic program is formulated in such a way that one can never have starting node 1 more than once in the final solution. The following simple changes will permit cycles through any node, including node 1 (*see* the original paper for definitions and the complete algorithm):

$$C_1(k)=[c_{11}(k),\ c_{12}(k),\ \cdots,\ c_{1n}(k)],$$

$$P(k)=[p_1(k),\ p_2(k),\ \cdots,\ p_n(k)]^T, \text{ an } \times(k+1) \text{ matrix.}$$

It should be noted that $p_j(k)$ does not always exist. Step 0 now becomes:

$$C_1(1)=[c(1,1),\ c(1,2),\ \cdots,\ c(1,n)], \text{ where } c(1,1)=\infty,$$

$$p_j(1)=(1,j),\ j=1.\ 2,\ \cdots,\ n, \text{ where } p_1(1) \text{ does not exist.}$$

Using this modified dynamic program, we find a shorter route from node 1 to 3 and from node 1 to node 4 for the author's numerical example on pages 206–207. The shortest routes are 1-4-1-3 with total length 6 instead of 8, and 1-4-1-4 with total length 5 instead of 7.

Path 1-4-1-4 illustrates that arcs may be repeated in the dynamic programming formulation. As pointed out by the referees, repeated arcs are not allowed in the (defective) zero-one integer programming formulation, unless constraint (3) is replaced by $f(i,j)\geqq 0$ and integer.

Finally, two misprints should be corrected.

page		*for*	*read*
206	line 3 of step 1	$c_{1s}(k+1)$	$c_{1j}(k+1)$
207	line 5 from bottom	$i\epsilon(i_2, i_3, \cdots, i_k)$	$i\epsilon(i_1, i_2, \cdots, i_k)$

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1. Romesh Saigal, "Constrained Shortest Route Problem," *Opns. Res.* **16,** 205–209 (1968).

A MULTICOMMODITY MAX-FLOW ALGORITHM

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(Received July 10, 1968)

Some simplifications occur when the theory of Kornai and Liptak is applied to the multicommodity max-flow problem. This note describes the resulting algorithm and comments on some of its properties; it is flexible, easy to code, and involves simple computations. However, the technique is recommended for suboptimization because of its poor convergence properties.

WE SHALL consider a directed network of N nodes and M arcs described by a node-arc incidence matrix E. There are G goods distinquished by their