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Letter to the Editor—Savings Approach to the Multiple Terminal Delivery Problem

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Letter to the Editor

Savings Approach to the Multiple Terminal Delivery Problem

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For the solution of the multiple terminal delivery problem a formula for the actual savings is presented and a method of solution based on these actual savings is described. This method is simple both conceptually and computationally and gives results that appear to be 'good.'

For the solution of the multiple terminal delivery problem TILLMAN[3] suggested an extension of CLARKE AND WRIGHT's[1] savings approach to the single terminal problem. UEBE[7] criticized that Tillman's savings are not the true savings at each stage of the iteration. TILLMAN[4] replied that his savings are true relative to the rules of his algorithm. LAM[2] gave further comments on the limitations of Tillman's algorithm. TILLMAN AND HERING[6] tried to improve the algorithm by implementing a look-ahead procedure. A further algorithm based on Tillman's savings was given by TILLMAN AND CAIN.[5]

There are a lot of questions about Tillman's algorithm as there are about Clarke and Wright's algorithm. A basic question is whether the savings approach is an adequate method to get a 'good' initial solution with regard to its limitations and whether there are other methods that produce better results within the same computer time. For this question some answers are given in the literature and it is not discussed here further.

One point of Uebe's criticism was that Tillman's savings are not the true savings at each stage of the iteration. Tillman[3,4] presented the following formula for the savings (other notation):

$$S_{jk}^{i} = 2\cdot(c_j + c_k) - (d_j{}^i + d_k{}^i) - d_{jk}, \qquad (1)$$

with S_{jk}^{i} = saving if points j and k are linked and the new route with points j, k is assigned to terminal i,

c_l = distance of point l to the nearest terminal,

d_l^s = distance between terminal s and point l, and

d_{jk} = distance between points j and k.

For example see Fig. 1. If points j and k are to be linked and the new route is to be assigned to terminal $i = 1$, then the linkages 1, 2, 3, 4 must be eliminated in the solution and the linkages 5, 6, 7 must be added to it. As the equivalences given in Table I exist, the savings of Tillman are obviously true for this case. In the generalized case, however, where not only two single points but also two routes are to be linked, Tillman's savings are indeed not true. For example see Fig. 2. If points j and k are to be linked and the new route is to be assigned to terminal $i = 1$, then the linkages 1, 2, 3, 4 must be eliminated in the solution and the linkages 5, 6, 7 must be added to it. The equivalences given in Table II exist with

$\bar{j}$ = other endpoint of the route with point j,

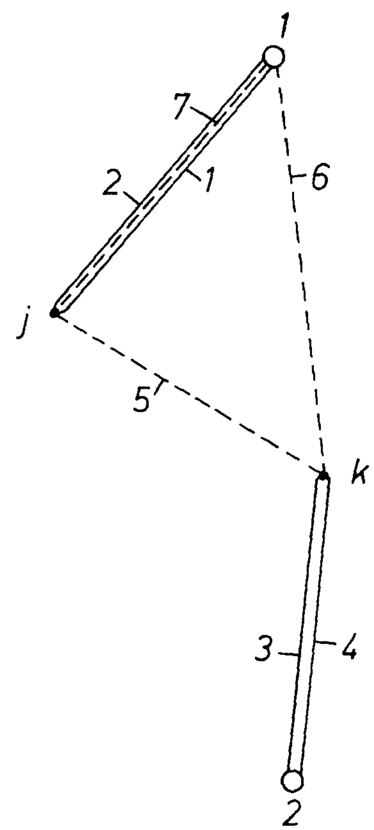

Fig. 1. Two single points are to be linked.

TABLE I

link	distance
1	c_j
2	c_j
3	c_k
4	c_k
5	d_{jk}
6	$d_k{}^i$
7	$d_j{}^i$

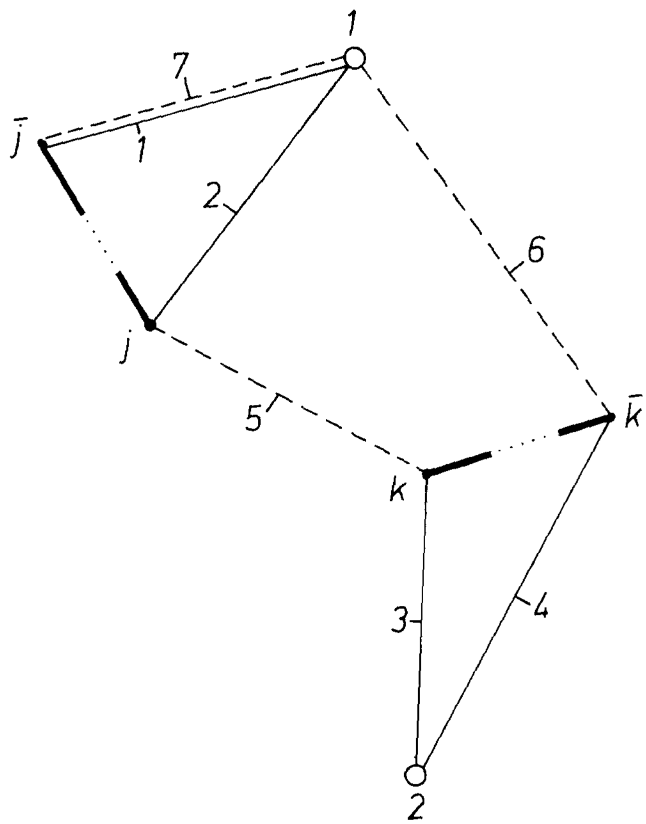

Fig. 2. Two routes are to be linked.

$\bar{k}$ = other endpoint of the route with point k, and

$\bar{c}_l$ = distance of the endpoint l of a route to the *assigned* terminal.

For the case shown in Fig. 2, which is a generalized case to calculate the savings

TABLE II

link	distance
1	$\bar{c}_{\bar{j}}$
2	$\bar{c}_j$
3	$\bar{c}_k$
4	$\bar{c}_{\bar{k}}$
5	d_{jk}
6	$d_{\bar{k}}^i$
7	$d_{\bar{j}}^i$

for a multiple terminal delivery problem, one finds:

$$S_{jk}^i = (\bar{c}_j + \bar{c}_{\bar{j}}) + (\bar{c}_k + \bar{c}_{\bar{k}}) - (d_{\bar{j}}^i + d_{\bar{k}}^i) - d_{jk}. \qquad (2)$$

If only two single points j and k are to be linked then $j \equiv \bar{j}$ and $k \equiv \bar{k}$ and the saving is identical with Tillman's for this special case. In the case of the single terminal problem equation (2) is identical with Clarke and Wright's formula given in [1] (other notation):

$$S_{jk} = \bar{c}_j + \bar{c}_k - d_{jk}. \qquad (3)$$

As for Clarke and Wright's savings in the case of the single terminal problem the following holds true also for equation (2): in an optimal solution the sum of the savings corresponding to the selected links must be maximal with regard to any restrictions in the problem. This property can be used to find a near-optimal solution. Tillman as well as Clarke and Wright try to improve iteratively an initial solution in which each point is the only point of a route coming from and returning to the closest terminal. This solution is improved by selecting such a link at each stage of the iteration that gives the highest saving and that is permissible with regard to any restrictions. To find such a solution the following procedure may be suggested (see WEBB,[8] Chap. 37).

For the calculation of the highest permissible saving at each stage of the iteration it is useful to modify equation (2) somewhat:

$$S_{jk}^i = (\bar{c}_j + \bar{c}_{\bar{j}} - c_{\bar{j}}) + (\bar{c}_k + \bar{c}_{\bar{k}} - c_{\bar{k}}) - (d_{\bar{j}}^i - c_{\bar{j}}) - (d_{\bar{k}}^i - c_{\bar{k}}) - d_{jk}. \qquad (4)$$

It is assumed that for each endpoint j of a route in the solution (a route can contain only one point) the value $a^j = (\bar{c}_j + \bar{c}_{\bar{j}} - c_{\bar{j}})$ is calculated and that all a-values are in descending order so that $a_i \geqq a_{i+1}$.

1. The highest saving found so far is $S_0 = -\infty$. Set $l = 0$ to indicate that no link with $S > -\infty$ has been found so far.
2. Vary index i: $i = 1, \cdots, (N - 1)$. N is the number of points.
3. If $a_i + a_{i+1} \leqq S_0$ then go to step 9.
4. If point j_1 corresponding to index i can not permissibly be linked with any other point j_2 then go to step 2.

5. Vary index $j: j = (i + 1), \cdots, N$.
6. If $a_i + a_j \leqq S_0$ then go to step 2.
7. If point j_2 corresponding to index j can not permissibly be linked with point j_1, e.g., because both points belong to the same route, then go to step 5.
8. Find terminal k_b so that

$$b = \min_k (d^k_{\bar{j}_1} - c_{\bar{j}_1} + d^k_{\bar{j}_2} - c_{\bar{j}_2})$$

and calculate

$$d = [(x_{j1} - x_{j2})^2 + (y_{j1} - y_{j2})^2]^{1/2}$$

with x_p, y_p = coordinates of a point p. If $S = a_i + a_j - b - d > S_0$ then set $S_0 = S$, $k_1 = j_1$, $k_2 = j_2$, $k_3 = k_b$, and $l = i$. Go to step 5.

9. The highest saving is found. If $l = 0$ then no further links can be selected and the solution is complete. If $l = 1$ then the points k_1 and k_2 are linked and a new route is formed and assigned to terminal k_3. For the endpoints j and $\bar{j}$ of the new route the a-values are calculated and sorted in. The a-values of the endpoints of both old routes are sorted out. Go to step 1.

This procedure has been programmed and for three sample problems of different size solutions have been calculated on an IBM/370–168 computer. It was assumed that an infinite number of vehicles is available at each terminal and that each vehicle has a capacity C that must not be exceeded. The sum of the distances travelled should be minimal. The results given in Table III were found.

The sample problems are of the same scheme. In a square of (100 × 100) the coordinates of the points are pseudo-randomly generated and nearly uniformly distributed. The coordinates of the depots are fixed to (25, 25), (25, 75), (75, 25), and (75, 75). The demands Q_i of all points are positive and not greater than 50

TABLE III

Results of Computation for Three Sample Problems

Capacity C	Sample 1 50 points 4 Terminals	Sample 2 100 Points 4 Terminals	Sample 3 200 Points 4 Terminals
50	1263/25/0.27	2384/53/1.21	4873/111/ 7.44
100	904/15/0.44	1514/26/1.96	2847/ 53/10.23
200	710/ 8/0.50	1145/13/2.41	1925/ 27/10.34
400	638/ 4/0.52	975/ 7/2.57	1558/ 15/10.96
800	—	943/ 4/2.61	1426/ 8/10.97
1600	—	—	1370/ 5/11.13
SUM(Q_i)	1069	2234	4704

Legend: a/b/c with a = sum of distances travelled,
b = number of routes, and
c = computer execution time for link-selection (in seconds)

each. They are also pseudo-randomly generated and nearly uniformly distributed.

The results given in Table III cannot be expected to be optimal because the procedure of their evaluation is a heuristic one and there are limitations of the procedure as pointed out by Lam.[2] However, the procedure suggested is simple both conceptually and computationally and gives results that appear to be 'good.' This may justify the approach.

The author did not find in the literature a formula that is equivalent to (2) to calculate the savings in a multiple terminal delivery problem.

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