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LEVEL POPULATIONS OF HYDROGEN IN GASEOUS NEBULAE

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SUMMARY

The departures, b_n, of the populations of excited levels of hydrogen, from those under conditions of thermodynamic equilibrium, are calculated. Collisions between all levels, n and n', are allowed for. Solutions are obtained using the matrix condensation technique of Burgess and Summers. Tables of b_n and $\log_{10}(d \ln b_n/d_n)$ are given for $n \leqslant 300$ and for a wide range of electron temperatures and densities. The results differ considerably from those obtained allowing only for collisional transitions in which the change in principal quantum number is small.

1. INTRODUCTION

Kardashev (1959) first suggested the possibility of observing radio frequency recombination lines emitted in H II regions. Since the populations of discrete hydrogen levels of high quantum number differ by only a few per cent from their thermodynamic populations, Kardashev argued that such differences could be neglected in computing line intensities. However, Goldberg (1966) has shown that very slight differential departures of the equilibrium populations of the lower and upper levels of a transition may lead to population inversion and amplification of the lines by stimulated emission. The differential departure from equilibrium is extremely sensitive to the electron density (Dyson 1967). Most of the previous treatments of the problem have assumed that collisional transitions between two levels, n and n', need only be allowed for when $s = |n-n'|$ is small. Seaton (1964), Hayler (1967), Hoang-Binh (1968) and Dyson (1969) performed computations where $s \leqslant 4$. The effects of increasing s are well illustrated by Hoang-Binh's results. Sejnowski & Hjellming (1969) and Brocklehurst (1970) have made calculations allowing for all collisional transitions, but using different estimates of cross-sections and different methods of solution. The present paper extends the calculations, using improved atomic data. The departures from equilibrium are very dependent upon the form of the electron–hydrogen atom cross-sections used. Many of the previous treatments have used very approximate cross-sections, whose validity is doubtful. We consider the cross-sections used in this paper to be the most accurate available.

2. FORMULATION

The most important processes affecting the level populations in gaseous nebulae are:

(i) Radiative capture and cascade;
(ii) Collisional excitations and de-excitations induced by electrons and protons;
(iii) Collisional ionization and three-body recombination;
(iv) Redistribution of angular momentum by collisions;

(v) Transitions induced by the free–free continuum within the nebula; and
(vi) Radiative excitation from the ground level.

Van Blerkom (1969) and Seaton (1968, 1969) have shown that radiative excitation of hydrogen from the ground level by stellar radiation is negligible. Transitions induced by the free–free continuum do not affect the level populations for any reasonable emission measure (Dyson 1969).

It is assumed that collisional processes of the type

$$\mathrm{H}(n, l)+\mathrm{H}^{+}\rightarrow\mathrm{H}(n, l\pm 1)+\mathrm{H}^{+} \tag{2.1}$$

are much faster than radiative processes. Pengelly & Seaton (1964) discuss the validity of this assumption, which results in the individual (n, l) states being statistically populated,

$$N_{n,l} = N_n(2l+1)/n^2. \tag{2.2}$$

Negligible error in the populations of highly excited levels ($n>40$) will result from assuming (2.2) holds true for all n. (This problem will be treated in greater detail in a later paper.) The cross-sections for excitation and de-excitation by collisions with protons have been computed using a modification of the impact parameter theory for electrons and atoms (see Section 3). It is found, below an electron temperature $T_e \leqslant 10^5\,^{\circ}\mathrm{K}$ and for levels n which are not completely determined by collisional processes, that electron collisions are much more effective than proton collisions in redistributing energy, and proton collisions may therefore be neglected.

Allowing for the relevant processes mentioned above, the equation of statistical equilibrium for a level n may be written

$$N_e N_+(\alpha_n+C_{i,n})+\sum_{n'=n+1}^{\infty} N_{n'}A_{n',n}+\sum_{n'=n_0}^{\infty} N_{n'}C_{n',n} = N_n\sum_{n'=n_0}^{n-1} A_{n,n'}+N_nC_{n,i}+N_n\sum_{n'=n_0}^{\infty} C_{n,n'} \tag{2.3}$$

where α_n is the radiative recombination coefficient, $A_{n',n}$ the $n'\rightarrow n$ radiative transition probability, $C_{n',n}$ the $n'\rightarrow n$ collisional transition probability, $C_{n,i}$ the collisional ionization probability and $C_{i,n}$ the coefficient for three-body recombination. From a consideration of detailed balancing in thermodynamic equilibrium, we obtain the relations

$$\omega_n\, \mathrm{e}^{x_n}\, C_{n,n'} = \omega_{n'}\, \mathrm{e}^{x_{n'}}\, C_{n',n} \tag{2.4}$$

and

$$C_{i,n} = \left(\frac{h^2}{2\pi mkTe}\right)^{3/2}\frac{\omega_n}{2}\,\mathrm{e}^{x_n}\,C_{n,i} \tag{2.5}$$

where $\omega_n = 2n^2$ and $x_n = I_n/kTe$. In equation (2.3), $n_0 = 1$ for Case A and $n_0 = 2$ for Case B (Baker & Menzel 1938). In the following discussion we shall consider only Case B, which assumes the nebula to be optically thick in the Lyman lines. Osterbrock (1962) considers this to be the most likely case for an isolated H II region.

The populations are conveniently described in terms of the b_n factors. Thus, for N_n, the number density of atoms in a state n;

$$N_n = N_eN_+\left(\frac{h^2}{2\pi mkTe}\right)^{3/2}\frac{\omega_n}{2}\,\mathrm{e}^{x_n}\,b_n. \tag{2.6}$$

Hence b_n provides a measure of the departure from thermodynamic equilibrium. Using (2.3) and (2.6) and allowing for detailed balancing relations (2.4) and (2.5), we obtain:

$$\sum_{n'=n+1}^{\infty}\left(A_{n',n}\frac{n'^2}{n^2}\,e^{x_{n'}-x_n}+C_{n,n'}\right)b_{n'}+\sum_{n'=2}^{n-1}C_{n,n'}b_{n'}-\left(\sum_{n'=2}^{n-1}A_{n,n'}+\sum_{n'=2}^{\infty}C_{n,n'}+C_{n,i}\right)b_n=-\frac{\alpha_n}{n^2}\left(\frac{h^2}{2\pi mkTe}\right)^{-3/2}e^{-x_n}-C_{n,i} \quad (2.7)$$

or more conveniently,

$$\sum_{n'=2}^{\infty}K(n,n')b(n')=R(n). \quad (2.8)$$

3. ATOMIC DATA

Radiative transition probabilities are given by Baker & Menzel (1938)

$$A_{n,n'}=\frac{\gamma g_{n,n'}}{n'n^3(n^2-n'^2)} \quad (3.1)$$

$$\gamma=\left(\frac{16\alpha^4 c}{3\pi\sqrt{3}\,a_0}\right)=1{\cdot}574\ 10^{10}\ s^{-1}. \quad (3.2)$$

Gaunt factors, $g_{n,n'}$, are computed using hypergeometric functions (Menzel & Pekeris 1935). For certain ranges of n and n', simplified expressions for $g_{n,n'}$ may be used without loss of accuracy. Thus, for $n\gg n'$, we use the asymptotic expansions derived by Menzel and Pekeris and corrected by Burgess (1959),

$$g_{n,n'}=1-\frac{0{\cdot}1728(1+n'^2/n^2)}{(1-n'^2/n^2)^{2/3}(n')^{2/3}}-\frac{0{\cdot}0496(1-\frac{4}{3}n'^2/n^2+n'^4/n^4)}{(1-n'^2/n^2)^{4/3}(n')^{4/3}}+\ldots. \quad (3.3)$$

For $|n-n'|\ll n$, Menzel (1968) has derived an expression for the oscillator strength, $f_{n,n'}$, in terms of Bessel functions of the first kind.

$$f_{n,n'}=\frac{4J_s(s)J_s'(s)n}{3s^2}(1+1{\cdot}5s/n) \quad (3.4)$$

where the prime denotes differentiation with respect to the argument, and $s=|n-n'|$. The Gaunt factors $g_{n,n'}$ are obtained using the relationship:

$$f_{n,n'}=\frac{2^5}{3\sqrt{3}\,\pi}\frac{1}{n^3n'^5}\frac{g_{n,n'}}{\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)^3}. \quad (3.5)$$

Radiative recombination rates, α_n, may be computed to better than 1 per cent using the theory of Seaton (1959) and the computer program of Flower & Seaton (1969). The collisional excitation, de-excitation and ionization rates are obtained by integrating the relevant cross-sections over a Maxwellian energy distribution corresponding to an electron temperature Te,

$$C_{n,n'}=Ne\overline{(v_{n'}Q_{n\to n'})}. \quad (3.6)$$

The cross-sections for electron impact excitation and ionization may be calculated in three different approximations:

(i) *Classical theory*

Using classical theory, one first calculates a cross-section $Q(E, E')\,dE'$ for transitions from an initial state in which the atomic electron has an energy E to a final state in which the electron has an energy in the range E' to $E'+dE'$. The cross-section for ionization is then

$$Q_{n\to i} = \int_0^{E_{\max}} Q(E, E')\,dE' \tag{3.7}$$

where $E_{\max}$ is the maximum energy for the ejected electron consistent with energy conservation. The cross-section for transitions between two quantized energy levels, E_n and $E_{n'}$, is calculated as

$$Q_{n\to n'} = \int_{E_{n'}}^{E_{n'+1}} Q(E_n, E')\,dE' \tag{3.8}$$

involving integration over an energy range $\delta E = (E_{n'+1}-E_{n'})$. The exact form adopted for the range of integration in (3.8) is unimportant so long as δE is small compared with the transition energy difference, $\Delta E = (E_{n'}-E_n)$. It may be expected that the largest uncertainties will arise for transitions of the type $n\to n+1$.

The classical cross-sections $Q(E, E')$ may be calculated using the symmetrized rate-coefficient form of the binary encounter theory (Burgess & Percival 1968). This will be referred to as the B.E. approximation. In classical theory, the energy transfer $\Delta E = (E'-E)$, may be arbitrarily small. In the limit of small ΔE, $Q(E, E')$ behaves like $1/\Delta E$. On the other hand, in quantum theory one has a definite minimum energy transfer, $\Delta E_{\min} = (E_{n+1}-E_n)$. In using (3.8) one neglects the classical energy transfers with $\Delta E < \Delta E_{\min}$, and it may therefore be expected that (3.8) will underestimate $Q_{n\to n'}$ for small values of $s = |n'-n|$.

(ii) *Impact parameter theory*

In the impact parameter theory one assumes that the colliding electron follows a rectilinear classical path with impact parameter ρ. The probability $P_{n,n'}(\rho)$ that the $n\to n'$ transition occurs in the atom is calculated using the first-order quantum perturbation theory. The cross-section is

$$Q_{n\to n'} = \int_0^{\infty} P_{n,n'}(\rho) 2\pi\rho\,d\rho. \tag{3.9}$$

From the definition of $P_{n,n'}$ it follows that $P_{n,n'}$ should not be greater than unity, but using perturbation theory one may obtain $P_{n,n'}(\rho) > 1$ for small values of ρ. In the procedure introduced by Seaton (1962) and used by Saraph (1964) for $n\to n+1$ transitions, one calculates a value ρ_1 of ρ such that perturbation theory gives $P_{n,n+1}(\rho_1)+P_{n,n-1}(\rho_1) = \frac{1}{2}$. One then uses perturbation theory for $\rho > \rho_1$ and takes $P_{n,n+1}(\rho) = \frac{1}{2}$ for $\rho < \rho_1$. It is found that a large contribution to (3.9) comes from values of ρ greater than ρ_1, for which perturbation theory should be reliable. This method, referred to as the I.P. approximation, should give good results for $n\to n+1$ transitions. The I.P. cross-sections for these transitions are a good deal larger than the B.E. cross-sections.

It is difficult to extend the I.P. method for transitions with $s > 1$. For these transitions relatively larger contributions to the cross-sections come from values of ρ less than ρ_1, and for these values perturbation theory is not valid.

In the author's earlier calculation (Brocklehurst 1970), the I.P. method was used for $s = 1$, and the B.E. method was used for $s \geqslant 2$. The largest uncertainty was in the cross-section for $s = 2$, but comparison of I.P. and B.E. results for $s = 1$ suggested that the uncertainty was not too serious. Sejnowski & Hjellming (1969) use cross-sections based on I.P. theory for all values of s. For large values of s their cross-sections are a good deal smaller than B.E. results, and are almost certainly underestimated.

(iii) *The correspondence principle*

An important recent advance, by Percival & Richards (1969), has been to obtain cross-sections using classical theory together with the quantum correspondence principle. This will be referred to as the C.P. approximation. It is found that the C.P. method gives good agreement with I.P. results for $s = 1$ and with B.E. results for $s \gg 1$. It appears that, for $s = 2$, the B.E. method underestimates the cross-sections by factors of about 2.

In the calculations reported in the present paper we have used the I.P. method for $s = 1$, the C.P. method for $2 \leqslant s \leqslant 6$, and the B.E. method for $s > 6$, and for ionization.

4. NUMERICAL METHODS

It is required to solve the infinite set of equations

$$\sum_{n'=n_0}^{\infty} K(n, n')b(n') = R(n) \tag{4.1}$$

where $n_0 = 2$ for Case B. The differential equation method introduced by Seaton (1964) may be used if, from each initial level, one considers only a finite number of collisional transitions. The differential equations are obtained on substituting in (4.1)

$$b(n') = b(n) + (n'-n)\frac{db(n)}{dn} + \tfrac{1}{2}(n'-n)^2\frac{d^2b(n)}{dn^2}. \tag{4.2}$$

Certain subsidiary approximations are made in the cascade terms and in the boundary conditions imposed for small n. The method is unsatisfactory if one includes an infinite number of collisional transitions, since in this case substitution of (4.2) in (4.1) leads to divergent summations.

We solve the equations (4.1) using a matrix condensation technique similar to that described by Burgess & Summers (1969). Let us first consider the case of a finite system of equations,

$$\sum_{j=1}^{N} c(i, j)X(j) = Y(i). \tag{4.3}$$

We introduce an integer variable $t = 1, 2, \ldots, M$ where $M < N$, and a dimensioned array $m(t)$ such that

$$m(1) = 1, \quad m(t+1) > m(t), \quad m(M) = N. \tag{4.4}$$

Let $\bar{X}(t)$, $\bar{Y}(t)$ be condensed arrays such that

$$\bar{X}(t) = X(m(t)), \quad \bar{Y}(t) = Y(m(t)). \tag{4.5}$$

It is assumed that $X(j)$ in (4.3) can be interpolated from $\bar{X}(t)$ at the points $t = t_1, t_1+1, \ldots, t_2$, where, in general, t_1 and t_2 depend on j. We use the Lagrange interpolation formula

$$X(j) = \sum_{t=1}^{M} L(j, t)\bar{X}(t) \tag{4.6}$$

where

$$\begin{aligned} L(j, t) &= 0 \quad \text{if} \quad t < t_1 \\ L(j, t) &= \frac{\prod_{\substack{\tau=t_1 \\ (\tau\neq t)}}^{t_2} (\tau - j)}{\prod_{\substack{\tau=t_1 \\ (\tau\neq t)}}^{t_2} (\tau - m(t))} \quad \text{if} \quad t_1 \leqslant t \leqslant t_2 \\ L(j, t) &= 0 \quad \text{if} \quad t > t_2. \end{aligned} \tag{4.7}$$

We substitute (4.7) in the equation (4.3) for $i = m(s)$, $s = 1$ to M, to obtain

$$\sum_{t=1}^{M} \bar{c}(s, t)\bar{X}(t) = \bar{Y}(t) \tag{4.8}$$

where

$$\bar{c}(s, t) = \sum_{j=1}^{N} c(m(s), j)L(j, t). \tag{4.9}$$

We write (4.1) as

$$\sum_{n'=n_0}^{n_1} K(n, n')b(n') + \sum_{n'=n_1+1}^{\infty} K(n, n')b(n') = R(n) \tag{4.10}$$

where n_1 is large (in practice we take $n_1 > 300$). For $n' \leqslant n_1$ we use the matrix condensation technique with pivotal points $n = m(t)$, $t = 1, 2, \ldots, M$ where $M < N$. For $n' > n_1$ we put

$$b(n') = 1 + \frac{\epsilon_{n'}{}^{\mu}}{\ln \epsilon_{n'}} \sum_{l=1}^{P} \epsilon_{n'}{}^{l-1} A_l \tag{4.11}$$

where $\mu = 5/2$, and $\epsilon_{n'} = -1/n'^2$. This expansion is based upon the analytic asymptotic solutions of the differential equation method (Seaton 1964). We use the approximation

$$\begin{aligned} \sum_{n'=n_1+1}^{\infty} K(n, n')b(n') = \sum_{n'=n_1+1}^{n_2-1} K(n, n')b(n') + \tfrac{1}{2}K(n, n_2)b(n_2) \\ + \int_{-\epsilon_{n_2}}^{0} K(n, n')b(n') \frac{n'^3}{2} d\epsilon_{n'}. \end{aligned} \tag{4.12}$$

The P coefficients A_l in (4.11) are expressed in terms of the values of $b(n')$ at the P pivotal points, $n' = m(t)$. Normally we take the P points for which

$$\begin{aligned} \epsilon_{m(t=M)} &= -1/N^2 \\ \epsilon_{m(t_{s-1})} &\approx 2\epsilon_{m(t_s)} \\ t_{s-1} &< t_s. \end{aligned} \tag{4.13}$$

We thus finally obtain a system of M equations in M unknowns. Solutions accurate to 0·1 per cent are obtained with $P = 4$, $n_2 = n_1 + 200$ and $M = 75$. The interval $(t_2 - t_1)$ increases with increasing j such that $1 \leqslant t_2 - t_1 \leqslant 10$. Having obtained $b(n)$ at the pivotal points $n = m(t)$, values of $b(n)$ and $d \ln b(n)/dn$ for all n are calculated using the Lagrange interpolation formula.

5. RESULTS

A convenient quantity for tabulation is

$$C_n = -\log_{10}\left(\frac{d}{dn}\ln b_n\right). \tag{5.1}$$

Table I gives values of b_n and C_n for a wide range of parameters. Results are given down to $n = 40$, in view of the detection of recombination lines in the millimetre band (Sorochenko *et al.* 1969). (Results for $n < 40$, allowing for collisional redistribution of angular momentum will be presented in a later paper.) The tables are sufficient to permit Lagrangian interpolation in n, Te and Ne for any observable nebula.

Table II shows the effects of including collisions of progressively higher s in the calculations. I.P. and B.E. cross-sections were used at $Te = 10^4$°K and $Ne = 10^4$ cm^{-3}. Inclusion of all collisions may reduce the gradient of b_n by a factor of two compared with results obtained assuming $n \to n+1$ collisions only. The gradients are considerably smaller than any others previously published. This is due to the inclusion of all collisions, and because the larger C.P. cross-sections have been used. Both effects decrease the departure from thermodynamic equilibrium. Comparing the results with Brocklehurst (1970) the use of C.P. cross-sections has decreased the gradients by 10 per cent.

We have stated that the numerical error in the computations should not exceed about 0·1 per cent. There are also uncertainties in the atomic data, particularly in the collision cross-sections. Percival and Richards estimate that the errors in the C.P. cross-sections should not exceed 10 per cent. Since the cross-sections enter the computations as products of cross-sections and electron density, we may estimate the errors in b_n or C_n due to errors in cross-sections on considering the variation of b_n or C_n with Ne. Thus, for example, at $T = 10^4$°K and $Ne = 10^4$ cm^{-3} we obtain from Table I

$$C_{100} = 3{\cdot}458.$$

Allowing for an increase in Ne of 10 per cent we obtain

$$C_{100} = 3{\cdot}486.$$

It is seen that an error of 10 per cent in cross-sections would imply an error of less than 1 per cent in C_{100}.

6. CONCLUSION

The importance of using accurate cross-sections, and of including collisions to any level has been shown. Collisions excluding $n \to n+1$ can be as effective as the $n \to n+1$ collisions in redistributing energy and establishing thermodynamic equilibrium. The tables cover a sufficient range of principal quantum number, electron temperature and electron density to permit interpolation to any required parameters.

TABLE I

n	b_n	C_n	n	b_n	C_n
T=2.5(3)	LOG(NE)=1.0		T=2.5(3)	LOG(NE)=2.0	
40	0.4757	2.030	40	0.4847	1.995
45	0.4961	2.117	45	0.5078	2.062
50	0.5139	2.184	50	0.5292	2.095
55	0.5300	2.242	55	0.5509	2.096
60	0.5444	2.294	60	0.5742	2.058
70	0.5710	2.340	70	0.6353	1.944
80	0.5983	2.303	80	0.7141	1.942
90	0.6329	2.199	90	0.7916	2.047
100	0.6786	2.127	100	0.8539	2.204
110	0.7317	2.130	110	0.8990	2.381
120	0.7846	2.191	120	0.9300	2.563
130	0.8314	2.289	130	0.9511	2.743
140	0.8699	2.405	140	0.9654	2.916
150	0.9001	2.531	150	0.9751	3.084
160	0.9233	2.660	160	0.9818	3.244
170	0.9409	2.790	170	0.9866	3.397
180	0.9543	2.918	180	0.9899	3.544
190	0.9643	3.043	190	0.9923	3.684
200	0.9720	3.165	200	0.9941	3.818
220	0.9823	3.398	220	0.9964	4.070
240	0.9885	3.616	240	0.9977	4.302
260	0.9923	3.821	260	0.9985	4.517
280	0.9947	4.014	280	0.9990	4.717
300	0.9963	4.195	300	0.9993	4.904
T=2.5(3)	LOG(NE)=2.5		T=2.5(3)	LOG(NE)=3.0	
40	0.4933	1.953	40	0.5092	1.857
45	0.5203	1.983	45	0.5475	1.809
50	0.5487	1.954	50	0.5955	1.744
55	0.5825	1.891	55	0.6537	1.727
60	0.6239	1.839	60	0.7153	1.771
70	0.7217	1.863	70	0.8223	1.964
80	0.8119	2.014	80	0.8937	2.212
90	0.8777	2.215	90	0.9362	2.466
100	0.9209	2.430	100	0.9608	2.713
110	0.9483	2.644	110	0.9753	2.947
120	0.9656	2.849	120	0.9839	3.166
130	0.9766	3.045	130	0.9893	3.373
140	0.9838	3.231	140	0.9927	3.566
150	0.9885	3.408	150	0.9949	3.749
160	0.9917	3.575	160	0.9963	3.920
170	0.9940	3.734	170	0.9973	4.083
180	0.9955	3.885	180	0.9980	4.237
190	0.9966	4.029	190	0.9985	4.383
200	0.9974	4.166	200	0.9989	4.523
220	0.9984	4.423	220	0.9993	4.782
240	0.9990	4.658	240	0.9996	5.020
260	0.9993	4.876	260	0.9997	5.240
280	0.9996	5.078	280	0.9998	5.443
300	0.9997	5.267	300	0.9999	5.633

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=2.5(3)	LOG(NE)=3.5		T=2.5(3)	LOG(NE)=4.0	
40	0.5450	1.670	40	0.6236	1.523
45	0.6110	1.624	45	0.7165	1.605
50	0.6865	1.655	50	0.7970	1.750
55	0.7589	1.751	55	0.8578	1.922
60	0.8196	1.881	60	0.9009	2.102
70	0.9018	2.179	70	0.9507	2.456
80	0.9459	2.479	80	0.9742	2.785
90	0.9691	2.763	90	0.9857	3.087
100	0.9816	3.028	100	0.9917	3.363
110	0.9887	3.274	110	0.9949	3.617
120	0.9928	3.503	120	0.9968	3.851
130	0.9952	3.715	130	0.9979	4.068
140	0.9968	3.914	140	0.9986	4.270
150	0.9977	4.100	150	0.9990	4.458
160	0.9984	4.275	160	0.9993	4.635
170	0.9988	4.440	170	0.9995	4.802
180	0.9991	4.596	180	0.9996	4.959
190	0.9994	4.744	190	0.9997	5.109
200	0.9995	4.884	200	0.9998	5.250
220	0.9997	5.146	220	0.9999	5.514
240	0.9998	5.386	240	0.9999	5.754
260	0.9999	5.607	260	0.9999	5.976
280	0.9999	5.811	280	1.0000	6.181
300	0.9999	6.002	300	1.0000	6.371
T=2.5(3)	LOG(NE)=4.5		T=2.5(3)	LOG(NE)=5.0	
40	0.7387	1.558	40	0.8467	1.738
45	0.8267	1.751	45	0.9073	1.993
50	0.8868	1.964	50	0.9430	2.241
55	0.9257	2.177	55	0.9641	2.476
60	0.9505	2.383	60	0.9767	2.698
70	0.9768	2.768	70	0.9894	3.101
80	0.9882	3.115	80	0.9948	3.459
90	0.9936	3.427	90	0.9972	3.779
100	0.9963	3.711	100	0.9984	4.067
110	0.9978	3.970	110	0.9990	4.329
120	0.9986	4.207	120	0.9994	4.569
130	0.9991	4.427	130	0.9996	4.791
140	0.9994	4.631	140	0.9997	4.997
150	0.9996	4.822	150	0.9998	5.188
160	0.9997	5.000	160	0.9999	5.367
170	0.9998	5.168	170	0.9999	5.536
180	0.9998	5.326	180	0.9999	5.695
190	0.9999	5.476	190	0.9999	5.845
200	0.9999	5.618	200	1.0000	5.988
220	0.9999	5.883	220	1.0000	6.253
240	1.0000	6.124	240	1.0000	6.495
260	1.0000	6.346	260	1.0000	6.717
280	1.0000	6.551	280	1.0000	6.922
300	1.0000	6.741	300	1.0000	7.110

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=5.0(3)	LOG(NE)=1.0		T=5.0(3)	LOG(NE)=2.0	
40	0.5871	2.223	40	0.5936	2.190
45	0.6033	2.305	45	0.6116	2.254
50	0.6172	2.365	50	0.6281	2.288
55	0.6300	2.426	55	0.6443	2.300
60	0.6412	2.477	60	0.6610	2.273
70	0.6613	2.531	70	0.7032	2.145
80	0.6812	2.512	80	0.7601	2.096
90	0.7049	2.412	90	0.8199	2.161
100	0.7365	2.315	100	0.8708	2.291
110	0.7749	2.283	110	0.9091	2.450
120	0.8153	2.315	120	0.9362	2.620
130	0.8526	2.389	130	0.9549	2.791
140	0.8844	2.488	140	0.9678	2.959
150	0.9102	2.601	150	0.9767	3.121
160	0.9303	2.721	160	0.9829	3.278
170	0.9458	2.843	170	0.9873	3.429
180	0.9578	2.965	180	0.9905	3.573
190	0.9669	3.086	190	0.9927	3.711
200	0.9738	3.204	200	0.9944	3.843
220	0.9833	3.431	220	0.9966	4.092
240	0.9891	3.646	240	0.9978	4.322
260	0.9927	3.848	260	0.9985	4.536
280	0.9949	4.038	280	0.9990	4.734
300	0.9964	4.217	300	0.9993	4.920
T=5.0(3)	LOG(NE)=2.5		T=5.0(3)	LOG(NE)=3.0	
40	0.5997	2.152	40	0.6106	2.069
45	0.6202	2.186	45	0.6382	2.028
50	0.6410	2.167	50	0.6718	1.948
55	0.6648	2.107	55	0.7134	1.903
60	0.6936	2.039	60	0.7593	1.914
70	0.7650	2.007	70	0.8443	2.060
80	0.8360	2.116	80	0.9044	2.281
90	0.8908	2.292	90	0.9416	2.520
100	0.9281	2.490	100	0.9637	2.757
110	0.9524	2.693	110	0.9769	2.984
120	0.9680	2.891	120	0.9849	3.199
130	0.9781	3.082	130	0.9899	3.402
140	0.9847	3.264	140	0.9930	3.593
150	0.9892	3.438	150	0.9951	3.774
160	0.9922	3.603	160	0.9965	3.944
170	0.9942	3.760	170	0.9974	4.105
180	0.9957	3.909	180	0.9981	4.258
190	0.9967	4.052	190	0.9986	4.403
200	0.9975	4.188	200	0.9989	4.542
220	0.9985	4.442	220	0.9993	4.800
240	0.9990	4.676	240	0.9996	5.036
260	0.9994	4.892	260	0.9997	5.255
280	0.9996	5.094	280	0.9998	5.457
300	0.9997	5.281	300	0.9999	5.646

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=5.0(3)	LOG(NE)=3.5		T=5.0(3)	LOG(NE)=4.0	
40	0.6343	1.689	40	0.6886	1.701
45	0.6809	1.814	45	0.7588	1.739
50	0.7365	1.807	50	0.8230	1.851
55	0.7926	1.870	55	0.8737	2.002
60	0.8418	1.976	60	0.9106	2.168
70	0.9114	2.244	70	0.9547	2.504
80	0.9503	2.529	80	0.9760	2.824
90	0.9713	2.804	90	0.9866	3.120
100	0.9828	3.063	100	0.9922	3.392
110	0.9893	3.305	110	0.9952	3.643
120	0.9931	3.530	120	0.9970	3.875
130	0.9955	3.740	130	0.9980	4.090
140	0.9969	3.937	140	0.9986	4.291
150	0.9978	4.122	150	0.9991	4.478
160	0.9985	4.296	160	0.9993	4.654
170	0.9989	4.459	170	0.9995	4.820
180	0.9992	4.615	180	0.9996	4.977
190	0.9994	4.762	190	0.9997	5.125
200	0.9995	4.901	200	0.9998	5.266
220	0.9997	5.162	220	0.9999	5.528
240	0.9998	5.401	240	0.9999	5.768
260	0.9999	5.620	260	0.9999	5.988
280	0.9999	5.824	280	1.0000	6.193
300	0.9999	6.014	300	1.0000	6.383
T=5.0(3)	LOG(NE)=4.5		T=5.0(3)	LOG(NE)=5.0	
40	0.7758	1.679	40	0.8643	1.820
45	0.8474	1.840	45	0.9163	2.056
50	0.8985	2.033	50	0.9479	2.292
55	0.9325	2.234	55	0.9669	2.520
60	0.9545	2.432	60	0.9784	2.737
70	0.9784	2.806	70	0.9901	3.134
80	0.9889	3.147	80	0.9951	3.488
90	0.9940	3.456	90	0.9973	3.805
100	0.9965	3.737	100	0.9985	4.091
110	0.9979	3.994	110	0.9991	4.352
120	0.9987	4.230	120	0.9994	4.590
130	0.9991	4.448	130	0.9996	4.811
140	0.9994	4.651	140	0.9997	5.016
150	0.9996	4.840	150	0.9998	5.206
160	0.9997	5.018	160	0.9999	5.385
170	0.9998	5.185	170	0.9999	5.553
180	0.9998	5.342	180	0.9999	5.711
190	0.9999	5.492	190	1.0000	5.861
200	0.9999	5.633	200	1.0000	6.003
220	0.9999	5.897	220	1.0000	6.267
240	1.0000	6.137	240	1.0000	6.508
260	1.0000	6.359	260	1.0000	6.730
280	1.0000	6.563	280	1.0000	6.934
300	1.0000	6.753	300	1.0000	7.122

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=1.0(4)	LOG(NE)=1.0		T=1.0(4)	LOG(NE)=2.0	
40	0.7006	2.438	40	0.7050	2.407
45	0.7124	2.518	45	0.7179	2.472
50	0.7225	2.570	50	0.7296	2.503
55	0.7318	2.632	55	0.7409	2.525
60	0.7398	2.688	60	0.7521	2.513
70	0.1540	2.748	70	0.7791	2.385
80	0.7674	2.745	80	0.8163	2.298
90	0.7826	2.659	90	0.8581	2.323
100	0.8025	2.548	100	0.8957	2.424
110	0.8277	2.485	110	0.9251	2.563
120	0.8555	2.487	120	0.9467	2.719
130	0.8824	2.537	130	0.9619	2.880
140	0.9063	2.617	140	0.9726	3.041
150	0.9261	2.716	150	0.9800	3.198
160	0.9420	2.824	160	0.9853	3.351
170	0.9545	2.938	170	0.9890	3.498
180	0.9642	3.053	180	0.9917	3.639
190	0.9718	3.169	190	0.9936	3.775
200	0.9776	3.282	200	0.9951	3.906
220	0.9856	3.503	220	0.9970	4.152
240	0.9905	3.713	240	0.9981	4.379
260	0.9936	3.912	260	0.9987	4.591
280	0.9956	4.099	280	0.9991	4.788
300	0.9969	4.276	300	0.9994	4.973
T=1.0(4)	LOG(NE)=2.5		T=1.0(4)	LOG(NE)=3.0	
40	0.7090	2.374	40	0.7159	2.302
45	0.7234	2.413	45	0.7344	2.274
50	0.7375	2.401	50	0.7562	2.190
55	0.7532	2.351	55	0.7833	2.122
60	0.7717	2.278	60	0.8144	2.105
70	0.8190	2.200	70	0.8754	2.203
80	0.8695	2.266	80	0.9214	2.394
90	0.9108	2.413	90	0.9510	2.616
100	0.9402	2.593	100	0.9692	2.841
110	0.9599	2.783	110	0.9802	3.061
120	0.9727	2.973	120	0.9869	3.271
130	0.9812	3.158	130	0.9912	3.470
140	0.9868	3.336	140	0.9939	3.658
150	0.9906	3.506	150	0.9957	3.835
160	0.9932	3.668	160	0.9969	4.004
170	0.9950	3.822	170	0.9977	4.163
180	0.9962	3.970	180	0.9983	4.315
190	0.9971	4.111	190	0.9987	4.459
200	0.9978	4.245	200	0.9990	4.596
220	0.9987	4.498	220	0.9994	4.852
240	0.9991	4.730	240	0.9996	5.087
260	0.9994	4.945	260	0.9998	5.305
280	0.9996	5.145	280	0.9998	5.506
300	0.9997	5.331	300	0.9999	5.694

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=1.0(4)	LOG(NE)=3.5		T=1.0(4)	LOG(NE)=4.0	
40	0.7304	2.137	40	0.7645	1.922
45	0.7607	2.047	45	0.8126	1.920
50	0.7979	2.006	50	0.8591	2.000
55	0.8373	2.037	55	0.8974	2.127
60	0.8733	2.118	60	0.9262	2.276
70	0.9269	2.354	70	0.9617	2.593
80	0.9581	2.620	80	0.9794	2.901
90	0.9755	2.884	90	0.9884	3.190
100	0.9852	3.136	100	0.9932	3.458
110	0.9907	3.373	110	0.9958	3.706
120	0.9940	3.594	120	0.9973	3.935
130	0.9960	3.802	130	0.9982	4.148
140	0.9973	3.997	140	0.9988	4.347
150	0.9981	4.179	150	0.9992	4.533
160	0.9986	4.352	160	0.9994	4.708
170	0.9990	4.514	170	0.9996	4.872
180	0.9993	4.668	180	0.9997	5.028
190	0.9994	4.814	190	0.9998	5.175
200	0.9996	4.954	200	0.9998	5.316
220	0.9997	5.212	220	0.9999	5.577
240	0.9998	5.450	240	0.9999	5.815
260	0.9999	5.668	260	1.0000	6.035
280	0.9999	5.871	280	1.0000	6.238
300	1.0000	6.060	300	1.0000	6.427
T=1.0(4)	LOG(NE)=4.5		T=1.0(4)	LOG(NE)=5.0	
40	0.8242	1.848	40	0.8900	1.946
45	0.8772	1.976	45	0.9308	2.161
50	0.9166	2.146	50	0.9563	2.384
55	0.9437	2.332	55	0.9719	2.603
60	0.9617	2.520	60	0.9815	2.814
70	0.9814	2.883	70	0.9914	3.203
80	0.9904	3.216	80	0.9957	3.551
90	0.9947	3.520	90	0.9977	3.865
100	0.9969	3.798	100	0.9987	4.148
110	0.9981	4.052	110	0.9992	4.407
120	0.9988	4.286	120	0.9995	4.645
130	0.9992	4.503	130	0.9997	4.864
140	0.9995	4.704	140	0.9998	5.067
150	0.9996	4.892	150	0.9998	5.257
160	0.9997	5.069	160	0.9999	5.434
170	0.9998	5.235	170	0.9999	5.602
180	0.9999	5.392	180	0.9999	5.759
190	0.9999	5.540	190	1.0000	5.908
200	0.9999	5.682	200	1.0000	6.050
220	1.0000	5.943	220	1.0000	6.313
240	1.0000	6.183	240	1.0000	6.553
260	1.0000	6.403	260	1.0000	6.773
280	1.0000	6.607	280	1.0000	6.977
300	1.0000	6.796	300	1.0000	7.164

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=1.5(4)	LOG(NE)=1.0		T=1.5(4)	LOG(NE)=2.0	
40	0.7680	2.589	40	0.7712	2.560
45	0.7771	2.667	45	0.7811	2.624
50	0.7849	2.713	50	0.7901	2.651
55	0.7923	2.778	55	0.7988	2.680
60	0.7984	2.837	60	0.8071	2.678
70	0.8092	2.899	70	0.8266	2.556
80	0.8192	2.906	80	0.8535	2.450
90	0.8303	2.831	90	0.8849	2.452
100	0.8444	2.716	100	0.9142	2.535
110	0.8626	2.638	110	0.9377	2.662
120	0.8833	2.622	120	0.9552	2.809
130	0.9040	2.657	130	0.9678	2.964
140	0.9227	2.726	140	0.9767	3.120
150	0.9385	2.816	150	0.9829	3.273
160	0.9514	2.917	160	0.9874	3.423
170	0.9617	3.025	170	0.9905	3.568
180	0.9697	3.136	180	0.9928	3.708
190	0.9760	3.248	190	0.9945	3.842
200	0.9809	3.359	200	0.9957	3.971
220	0.9876	3.575	220	0.9974	4.216
240	0.9918	3.783	240	0.9983	4.442
260	0.9944	3.979	260	0.9989	4.652
280	0.9961	4.164	280	0.9992	4.848
300	0.9973	4.340	300	0.9995	5.032
T=1.5(4)	LOG(NE)=2.5		T=1.5(4)	LOG(NE)=3.0	
40	0.7742	2.528	40	0.7791	2.463
45	0.7851	2.570	45	0.7928	2.442
50	0.7957	2.560	50	0.8087	2.358
55	0.8073	2.520	55	0.8285	2.282
60	0.8206	2.448	60	0.8514	2.251
70	0.8552	2.347	70	0.8980	2.321
80	0.8936	2.388	80	0.9346	2.493
90	0.9262	2.517	90	0.9588	2.703
100	0.9499	2.685	100	0.9738	2.922
110	0.9661	2.867	110	0.9831	3.136
120	0.9768	3.052	120	0.9888	3.342
130	0.9839	3.233	130	0.9924	3.539
140	0.9887	3.407	140	0.9947	3.725
150	0.9919	3.575	150	0.9963	3.901
160	0.9941	3.735	160	0.9973	4.068
170	0.9956	3.889	170	0.9980	4.227
180	0.9967	4.035	180	0.9985	4.377
190	0.9975	4.175	190	0.9989	4.520
200	0.9981	4.309	200	0.9992	4.657
220	0.9988	4.559	220	0.9995	4.912
240	0.9993	4.790	240	0.9997	5.146
260	0.9995	5.004	260	0.9998	5.363
280	0.9997	5.203	280	0.9999	5.563
300	0.9998	5.389	300	0.9999	5.751

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=1.5(4)	LOG(NE)=3.5		T=1.5(4)	LOG(NE)=4.0	
40	0.7893	2.308	40	0.8134	2.084
45	0.8112	2.213	45	0.8493	2.060
50	0.8386	2.155	50	0.8850	2.120
55	0.8684	2.169	55	0.9152	2.233
60	0.8963	2.235	60	0.9385	2.372
70	0.9390	2.450	70	0.9677	2.676
80	0.9647	2.704	80	0.9825	2.977
90	0.9791	2.961	90	0.9901	3.262
100	0.9873	3.209	100	0.9941	3.526
110	0.9920	3.442	110	0.9964	3.771
120	0.9948	3.662	120	0.9977	3.999
130	0.9965	3.868	130	0.9985	4.211
140	0.9976	4.061	140	0.9990	4.408
150	0.9983	4.242	150	0.9993	4.594
160	0.9988	4.414	160	0.9995	4.768
170	0.9991	4.575	170	0.9996	4.931
180	0.9994	4.729	180	0.9997	5.087
190	0.9995	4.874	190	0.9998	5.234
200	0.9996	5.013	200	0.9998	5.373
220	0.9998	5.271	220	0.9999	5.634
240	0.9999	5.507	240	0.9999	5.871
260	0.9999	5.725	260	1.0000	6.090
280	0.9999	5.927	280	1.0000	6.293
300	1.0000	6.115	300	1.0000	6.482
T=1.5(4)	LOG(NE)=4.5		T=1.5(4)	LOG(NE)=5.0	
40	0.8578	1.980	40	0.9092	2.053
45	0.8991	2.088	45	0.9421	2.254
50	0.9306	2.245	50	0.9631	2.469
55	0.9527	2.421	55	0.9761	2.682
60	0.9676	2.603	60	0.9842	2.888
70	0.9842	2.957	70	0.9926	3.272
80	0.9918	3.286	80	0.9963	3.617
90	0.9954	3.587	90	0.9980	3.929
100	0.9973	3.862	100	0.9988	4.211
110	0.9984	4.115	110	0.9993	4.468
120	0.9990	4.348	120	0.9996	4.704
130	0.9993	4.563	130	0.9997	4.923
140	0.9995	4.764	140	0.9998	5.125
150	0.9997	4.952	150	0.9999	5.314
160	0.9998	5.127	160	0.9999	5.492
170	0.9998	5.293	170	0.9999	5.658
180	0.9999	5.449	180	0.9999	5.815
190	0.9999	5.597	190	1.0000	5.964
200	0.9999	5.738	200	1.0000	6.105
220	1.0000	6.000	220	1.0000	6.368
240	1.0000	6.238	240	1.0000	6.607
260	1.0000	6.458	260	1.0000	6.827
280	1.0000	6.661	280	1.0000	7.030
300	1.0000	6.849	300	1.0000	7.216

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=2.0(4)	LOG(NE)=1.0		T=2.0(4)	LOG(NE)=2.0	
40	0.8162	2.718	40	0.8186	2.689
45	0.8234	2.795	45	0.8265	2.753
50	0.8296	2.833	50	0.8335	2.775
55	0.8355	2.900	55	0.8404	2.809
60	0.8403	2.964	60	0.8468	2.816
70	0.8488	3.028	70	0.8615	2.700
80	0.8566	3.040	80	0.8818	2.582
90	0.8649	2.974	90	0.9060	2.569
100	0.8755	2.859	100	0.9291	2.639
110	0.8891	2.771	110	0.9481	2.756
120	0.9050	2.744	120	0.9625	2.897
130	0.9211	2.768	130	0.9729	3.047
140	0.9360	2.828	140	0.9803	3.199
150	0.9488	2.911	150	0.9855	3.350
160	0.9593	3.007	160	0.9892	3.498
170	0.9678	3.111	170	0.9919	3.641
180	0.9745	3.218	180	0.9939	3.779
190	0.9797	3.327	190	0.9953	3.913
200	0.9838	3.436	200	0.9963	4.041
220	0.9895	3.650	220	0.9977	4.284
240	0.9930	3.854	240	0.9985	4.509
260	0.9952	4.049	260	0.9990	4.718
280	0.9967	4.233	280	0.9993	4.914
300	0.9976	4.408	300	0.9995	5.097
T=2.0(4)	LOG(NE)=2.5		T=2.0(4)	LOG(NE)=3.0	
40	0.8209	2.659	40	0.8246	2.597
45	0.8295	2.702	45	0.8352	2.582
50	0.8377	2.692	50	0.8472	2.499
55	0.8467	2.659	55	0.8622	2.420
60	0.8567	2.591	60	0.8797	2.379
70	0.8828	2.477	70	0.9162	2.429
80	0.9127	2.499	80	0.9456	2.588
90	0.9387	2.615	90	0.9654	2.789
100	0.9581	2.775	100	0.9779	3.002
110	0.9715	2.951	110	0.9856	3.213
120	0.9804	3.131	120	0.9905	3.416
130	0.9864	3.309	130	0.9935	3.611
140	0.9904	3.481	140	0.9955	3.795
150	0.9931	3.647	150	0.9968	3.970
160	0.9949	3.806	160	0.9977	4.136
170	0.9963	3.959	170	0.9983	4.294
180	0.9972	4.104	180	0.9987	4.444
190	0.9979	4.243	190	0.9990	4.587
200	0.9984	4.376	200	0.9993	4.723
220	0.9990	4.626	220	0.9996	4.977
240	0.9994	4.856	240	0.9997	5.210
260	0.9996	5.069	260	0.9998	5.426
280	0.9997	5.268	280	0.9999	5.627
300	0.9998	5.453	300	0.9999	5.813

TABLE I—*continued*

n	b_n	C_n	n	b_n	C_n
T=2.0(4)	LOG(NE)=3.5		T=2.0(4)	LOG(NE)=4.0	
40	0.8321	2.451	40	0.8500	2.222
45	0.8487	2.354	45	0.8775	2.184
50	0.8695	2.285	50	0.9056	2.230
55	0.8927	2.287	55	0.9298	2.333
60	0.9147	2.343	60	0.9487	2.464
70	0.9492	2.543	70	0.9728	2.758
80	0.9703	2.788	80	0.9851	3.054
90	0.9823	3.040	90	0.9915	3.335
100	0.9892	3.283	100	0.9950	3.597
110	0.9932	3.515	110	0.9969	3.841
120	0.9956	3.732	120	0.9980	4.067
130	0.9970	3.937	130	0.9987	4.278
140	0.9980	4.129	140	0.9991	4.475
150	0.9986	4.310	150	0.9994	4.659
160	0.9990	4.480	160	0.9996	4.833
170	0.9993	4.641	170	0.9997	4.996
180	0.9994	4.794	180	0.9998	5.151
190	0.9996	4.939	190	0.9998	5.297
200	0.9997	5.077	200	0.9999	5.437
220	0.9998	5.335	220	0.9999	5.696
240	0.9999	5.570	240	0.9999	5.934
260	0.9999	5.788	260	1.0000	6.152
280	0.9999	5.989	280	1.0000	6.355
300	1.0000	6.177	300	1.0000	6.543
T=2.0(4)	LOG(NE)=4.5		T=2.0(4)	LOG(NE)=5.0	
40	0.8840	2.099	40	0.9249	2.154
45	0.9167	2.192	45	0.9517	2.344
50	0.9423	2.339	50	0.9690	2.552
55	0.9604	2.508	55	0.9798	2.761
60	0.9727	2.685	60	0.9866	2.964
70	0.9865	3.033	70	0.9937	3.344
80	0.9930	3.359	80	0.9968	3.687
90	0.9961	3.657	90	0.9983	3.997
100	0.9977	3.931	100	0.9990	4.277
110	0.9986	4.182	110	0.9994	4.534
120	0.9991	4.414	120	0.9996	4.769
130	0.9994	4.629	130	0.9997	4.987
140	0.9996	4.829	140	0.9998	5.189
150	0.9997	5.016	150	0.9999	5.378
160	0.9998	5.191	160	0.9999	5.554
170	0.9999	5.356	170	0.9999	5.721
180	0.9999	5.512	180	1.0000	5.877
190	0.9999	5.660	190	1.0000	6.026
200	0.9999	5.800	200	1.0000	6.167
220	1.0000	6.061	220	1.0000	6.429
240	1.0000	6.300	240	1.0000	6.668
260	1.0000	6.519	260	1.0000	6.887
280	1.0000	6.721	280	1.0000	7.089
300	1.0000	6.909	300	1.0000	7.275

TABLE II

Dependence of d ln *bn/dn on s*

Te = 10^4°K *Ne* = 10^4 cm^{-3}

	s= 1	2	3	25	100	∞
n						
60	7·83 (−3)	7·42	7·14	5·79	5·55	5·51 (−3)
80	2·49 (−3)	2·27	2·13	1·52	1·36	1·33 (−3)
100	7·32 (−4)	6·72	6·33	4·50	3·84	3·69 (−4)
120	2·43 (−4)	2·27	2·15	1·56	1·30	1·23 (−4)
150	5·89 (−5)	5·61	5·40	4·10	3·38	3·11 (−5)
200	8·94 (−6)	8·78	8·60	6·96	5·80	5·13 (−6)

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