Lévy Processes and Stochastic Calculus

DAVID APPLEBAUM

University of Sheffield

 $\widetilde{\mathcal{E}}_{1}^{(1)} = \frac{\widetilde{N}_{1}^{(1)}}{\widetilde{N}_{1}^{(1)}} \frac{\widetilde{N}_{1}^{(1)}}{\widetilde{N}_{1}^{(1)}} + \frac{\widetilde{N}_{1}$



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