Geophys. J. R. astr. Soc. (1975) **40**, 475–487.

Lg, Li and Rg from Rayleigh Modes

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(Received 1974 May 29)

Summary

We demonstrate in terms of group velocities and surface amplitude displacements, that the vertical and longitudinal component of the Lg wave can be identified with higher Rayleigh modes for crustal sources. Low velocity layers in the crust are not required for the existence of Lg waves. Li could be used as a sufficient but not necessary discriminant for structures containing a low velocity channel in the upper mantle.

Analogous group velocity and amplitude reasoning applies to Rg waves generated by crustal shocks; such waves are associated with the fundamental mode. Also, in this case, the existence of low velocity zones in the crust is not required. Lg, Rg propagate in structures with and without a low velocity channel in the upper mantle.

Introduction

Press & Ewing (1952) identified two types of slow surface waves, Lg and Rg, associated with continental structures. Later, Båth (1956) identified another type of wave '. . . denoted Li, where i stands for the intermediate layer in the same way as g, in Lg refers to the granitic layer'. Both Li and Lg wave particle motion has a dominant transverse horizontal component mixed with longitudinal and vertical components, while the Rg wave has a typical Rayleigh wave motion.

Båth (1954) demonstrated experimentally that the Lg event is composed of several arrivals. Both he and Gutenberg (1955) confirmed the existence of a significant vertical component of motion.

Several different suggestions have been put forward to explain the propagation of Lg waves. The two usual explanations were that they were channel waves trapped in low velocity layers in the crust or mantle (Gutenberg 1955), or that they were Airy phases associated with the maxima and minima of the higher mode dispersion curves (Oliver & Ewing 1957, 1958a, 1958b; Kovach & Anderson 1964).

Concerning Rg, it is widely accepted that this phase is the result of propagation of the fundamental Rayleigh mode through surface layers (crustal) including a low-velocity layer (Båth 1954; Gutenberg 1955). An analogous explanation is given by Båth (1956) concerning Li propagation.

Recently Knopoff, Schwab & Kausel (1973) presented the complete demonstration today of the identification of the horizontal transverse component of Lg with higher-mode Love wave propagation, and demonstrated that a crustal low velocity zone is not required for its existence.

The purpose of this paper is to investigate whether the observed vertical and longitudinal components of Lg and Li can be identified with higher Rayleigh modes, and to verify the identification of Rg with the fundamental Rayleigh mode.

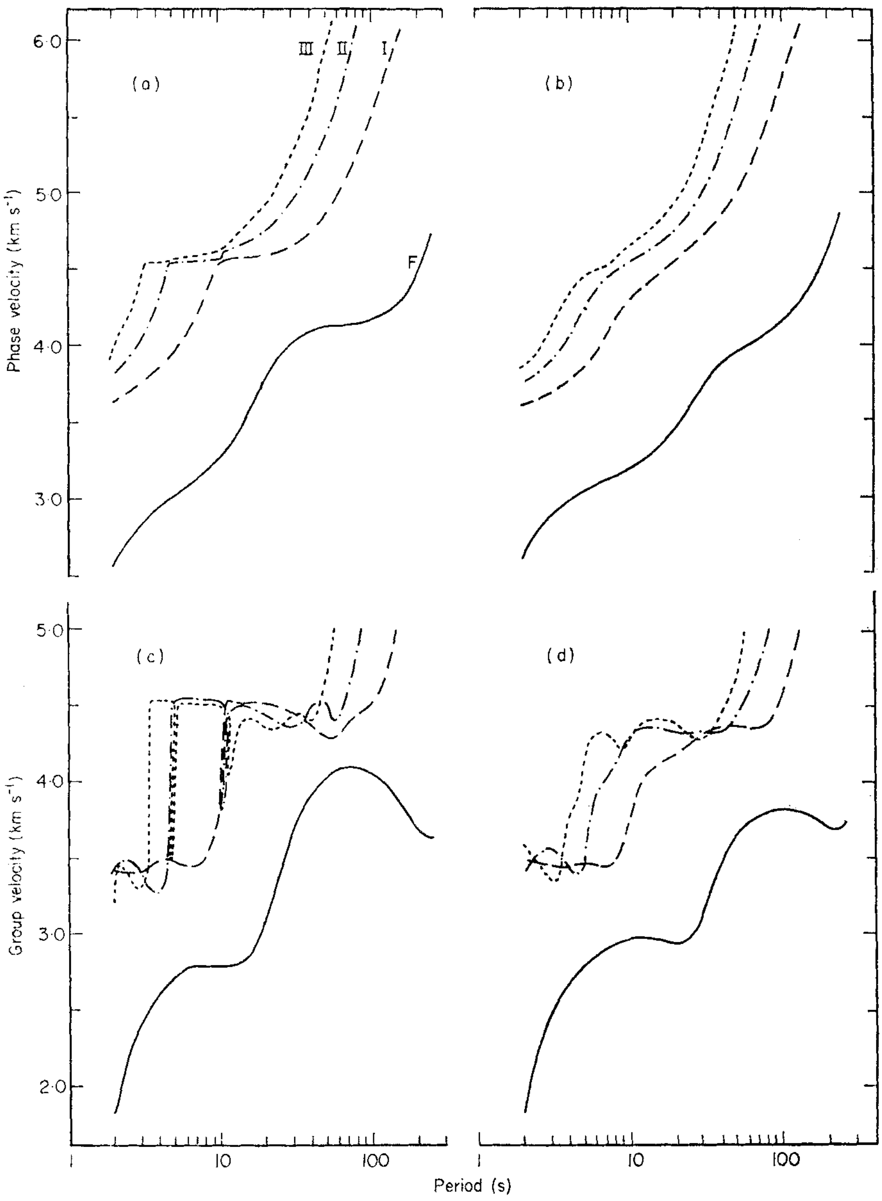

FIG. 1. Phase and group velocities of the fundamental and first three higher Rayleigh wave modes; the fundamental mode is denoted by F and the higher mode numbers are denoted by Roman numerals. (a) and (c) refer to the continental structure with LVC, (b) and (d) refer to the continental structure without LVC.

Basic computations

We have considered two standard continental structures: one containing a low velocity channel (LVC) in the upper mantle (Panza, Schwab & Knopoff 1972) and a second of the Jeffreys–Bullen type (without LVC) (Panza & Calcagnile 1974), which also differs in crustal thickness (see Fig. 7).

The phase and group velocity for the fundamental and first three higher Rayleigh modes, for the two structures considered, are given in Fig. 1.

Complete details concerning the methods of computation used to obtain the dispersion curves are given by Schwab & Knopoff (1972).

The mode-to-mode continuation of the members of the Rayleigh-type channel and crustal waves family (Panza *et al.* 1972) is clearly evident from both the phase and group velocity curves shown in Fig. 1(a) and (c). As demonstrated by the results of Panza *et al.* (1972), only the fundamental mode and crustal waves need be considered as exciting Rayleigh waves, since the channel waves do not generate significant amplitudes at the free surface. The situation in Fig. 1(b) and (d) is completely different: in this case where no LVC exists, the fundamental and higher modes need to be considered in their entirety as exciting Rayleigh waves.

A feature common to both Fig. 1(c) and (d) is the presence of stationary phases and well-pronounced minima in the short-period range of the higher mode dispersion curves, and a significant minimum in the curve referring to the fundamental mode (around 10 s in Fig. 1(c) and around 19 s in Fig. 1(d)).

The fault model of an earthquake, the geometry, and co-ordinate system at the focus, and the technique we have used to compute the Rayleigh wave response are all given by Panza *et al.* (1973).

As Knopoff *et al.* (1973), we have chosen as source mechanisms for this study, strike-slip motion along a vertical fault plane and dip-slip motion along a fault plane with dip $\delta = 45°$ in order to be able to make a direct estimation of the relative importance of Love and Rayleigh waves in the formation of Lg. The Rayleigh wave radiation pattern function $\chi(\theta)$, where θ is the station azimuth, for the two mechanisms, are shown in Figs 2 and 3. For the strike-slip mechanism, the form of the radiation pattern is independent of mode number; the amplitudes differ only by a constant scale factor at each period, and the phases are either 0 or π radians (Fig. 2).

In Fig. 3 we have plotted the amplitude and phase of $\chi(\theta)$ for the dip-slip mechanism in order to show its more complex azimuthal dependence; namely the amplitude ratio for the modes depends significantly upon the azimuth, and the π-radian phase jump occurs at different periods for the different modes and for different azimuths.

The difference in the radiation patterns for the two mechanisms is due to the fact that for the former it depends only upon the vertical displacement-depth function $u^*(h)$ while the later depends upon both $u^*(h)$ and the stress-depth function $\sigma^*(h)$ (Panza *et al.* 1973). For $\theta = 30°$ the amplitude spectra for the dip-slip source are given in Figs 4, 5 and 6, for the three source depths used in this study: 7, 16·5, 35 km. Part (a) of each figure refers to the structure containing a LVC, while part (b) refers to the no LVC case. All of our results are given for U_z^{DC}, the vertical component of the response.

The most general observation is that the higher-mode contribution becomes more and more significant as period decreases; for $h = 16{\cdot}5$ km it dominates up to about 4·5 s and for $h = 35$ km up to about 8 s. Moreover, part (a) of the figures shows clearly that the higher-mode energy contribution is given by the different members of the crustal waves family, while from part (b) we can see that, in absence of the LVC, the individual higher-modes contribute to the energy content in their entirety. Another interesting observation is that, for $h = 16{\cdot}5$ km, the maximum amplitude of the higher-modes is about one-third to one-quarter that of the funda-

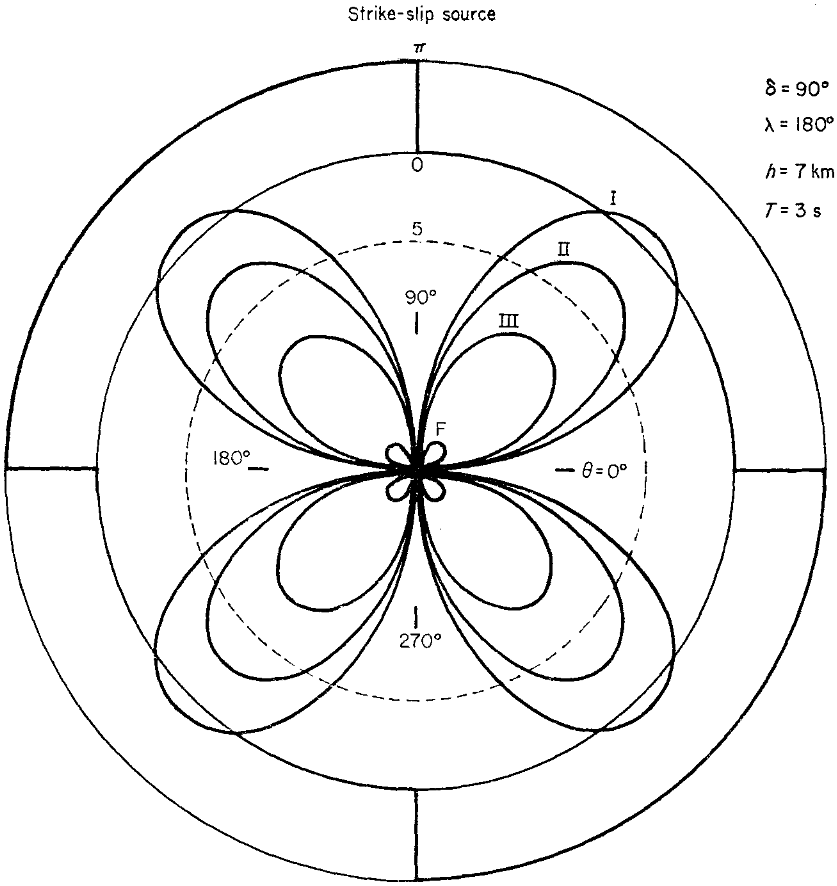

FIG. 2. Amplitude and phase radiation pattern functions for a strike-slip source. The phases are given in radians, the amplitudes in arbitrary units, the geometrical spreading factor is removed.

mental. Comparison of the amplitude spectra given in these figures with the group velocity curves of Fig. 1 shows the striking correspondence between the amplitude maxima of the higher-modes spectra and the stationary portions of the corresponding group velocity curves for periods shorter than 9 s. This correspondence suggests that the records of Lg on vertical seismographs (Båth 1959) be interpreted as higher-mode Rayleigh waves, so that, in a complete study of Lg, it is necessary to consider Rayleigh as well as Love waves.

A comparison of our Fig. 7 with Fig. 9 of Knopoff *et al.* (1973) shows that in the case of Rayleigh waves it is also possible to identify different Lg stationary phases; thus according to Knopoff *et al.* (1973), it is justified the use of the notation Lg (i, j) where i refers to the higher-mode number and j denotes the stationary portion of the crustal wave group velocity curve; j begins at the longest period stationary phase. In Fig. 7, beside the structures we have given the vertical displacement- and the normal stress-depth functions for each Lg(i, j). These curves are normalized to the maximum value at each period. A general feature is that as the mode number increases the sampling of the crust becomes more uniform. As is the case for Love waves, the Rayleigh wave Lg$(i$, 1) are the only phases to sample a significant portion of the structure below the Moho, in both cases (LVC and no LVC). The phases

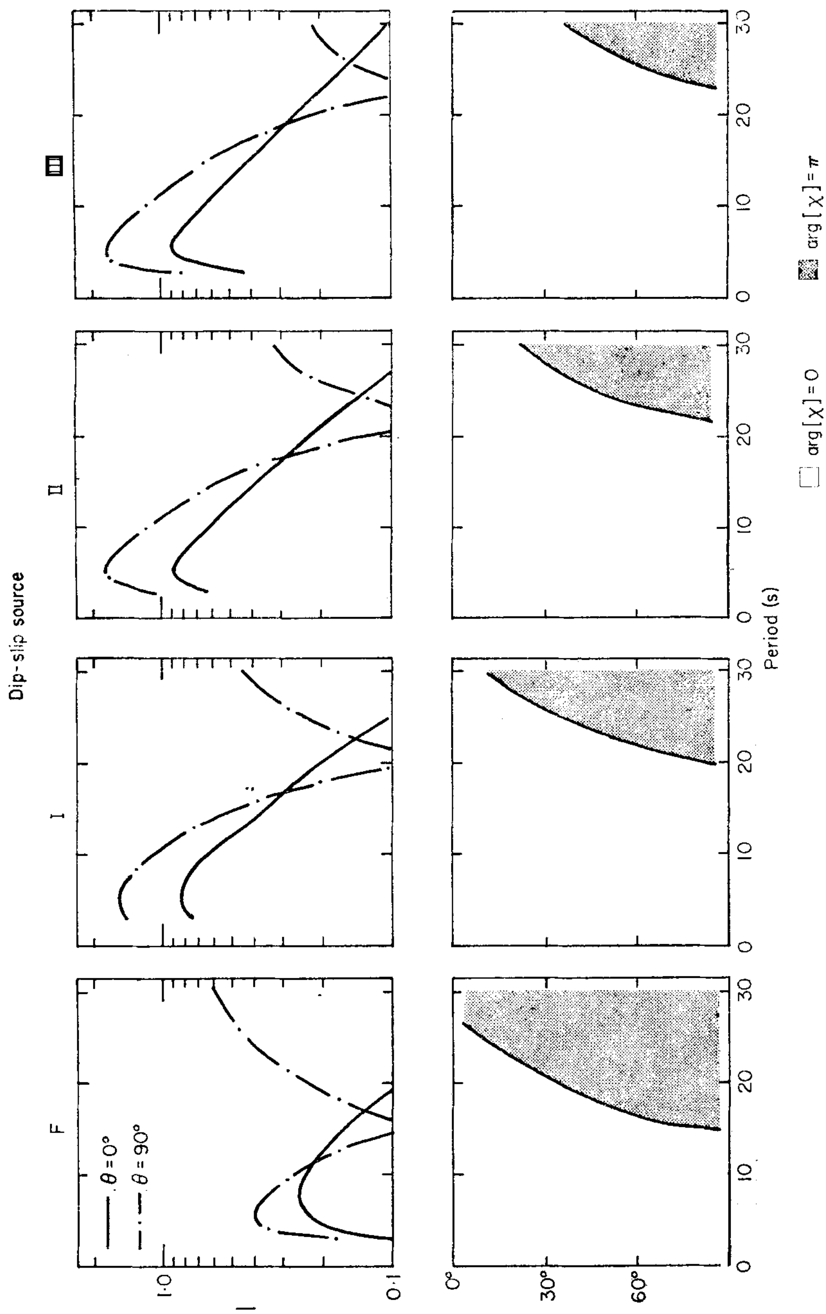

FIG. 3. Amplitude and phase radiation pattern functions for a dip-slip source dipping at 45°.

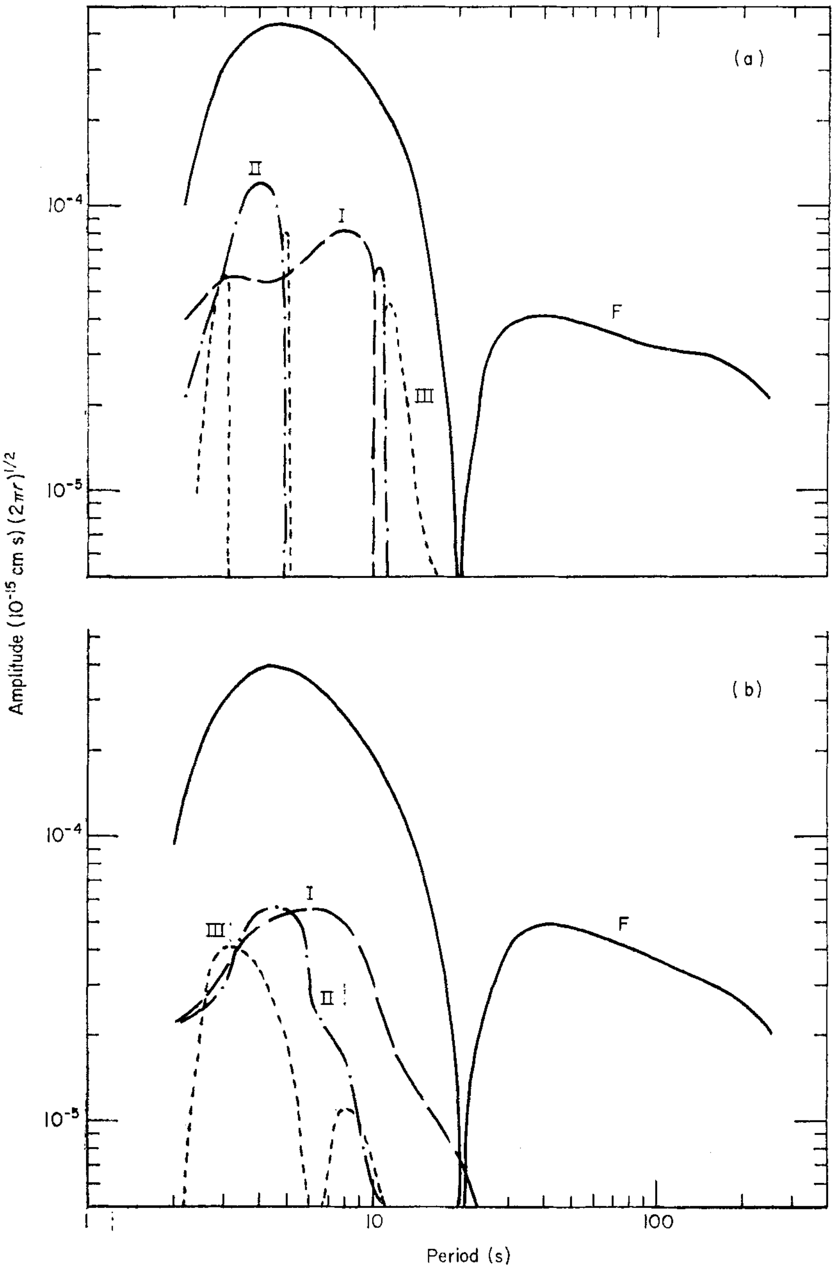

FIG. 4. Rayleigh wave amplitude spectra for a dip-slip source, with $\delta = 45°$ and $h = 7$ km. (a) refers to the structure containing a LVC, (b) refers to the other structure.

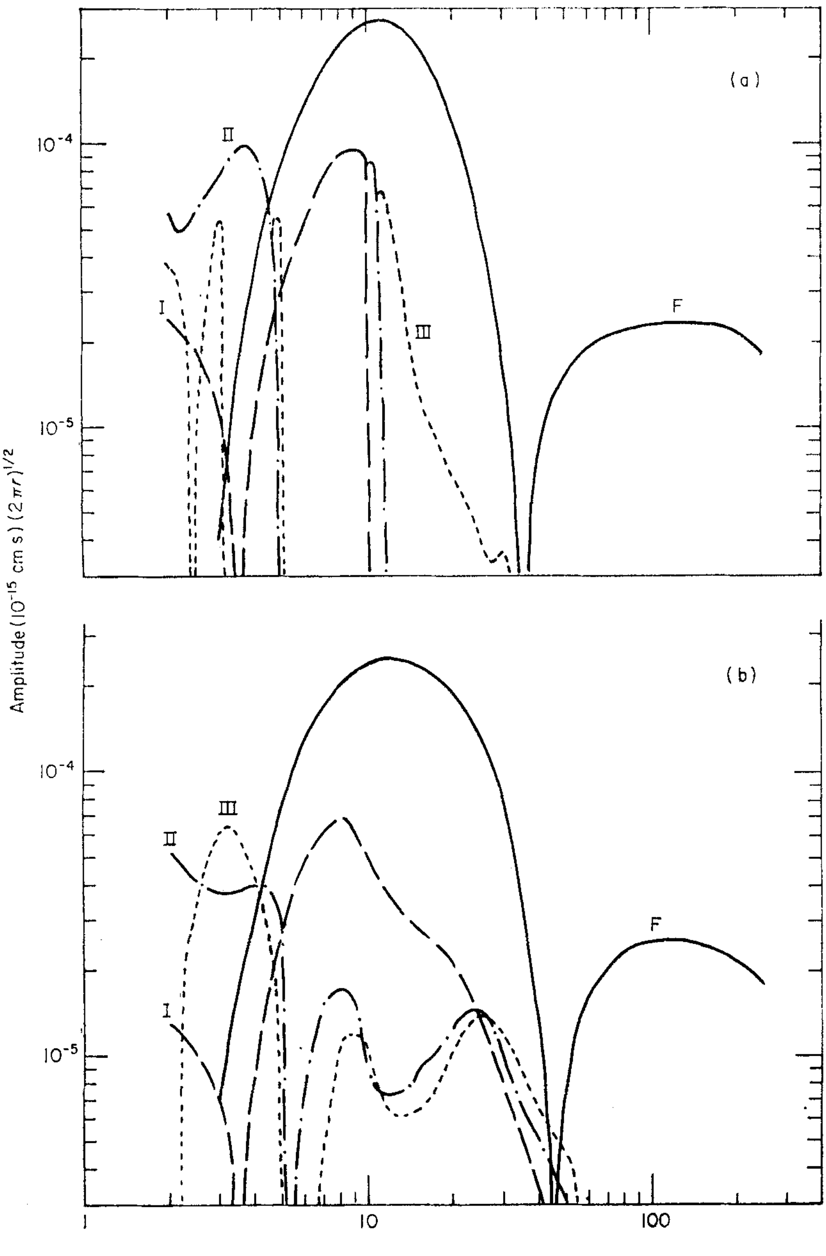

FIG. 5. Rayleigh wave amplitude spectra for a dip-slip source, with $\delta = 45°$ and $h = 16{\cdot}5$ km. (a) refers to the structure containing a LVC, (b) refers to the other structure.

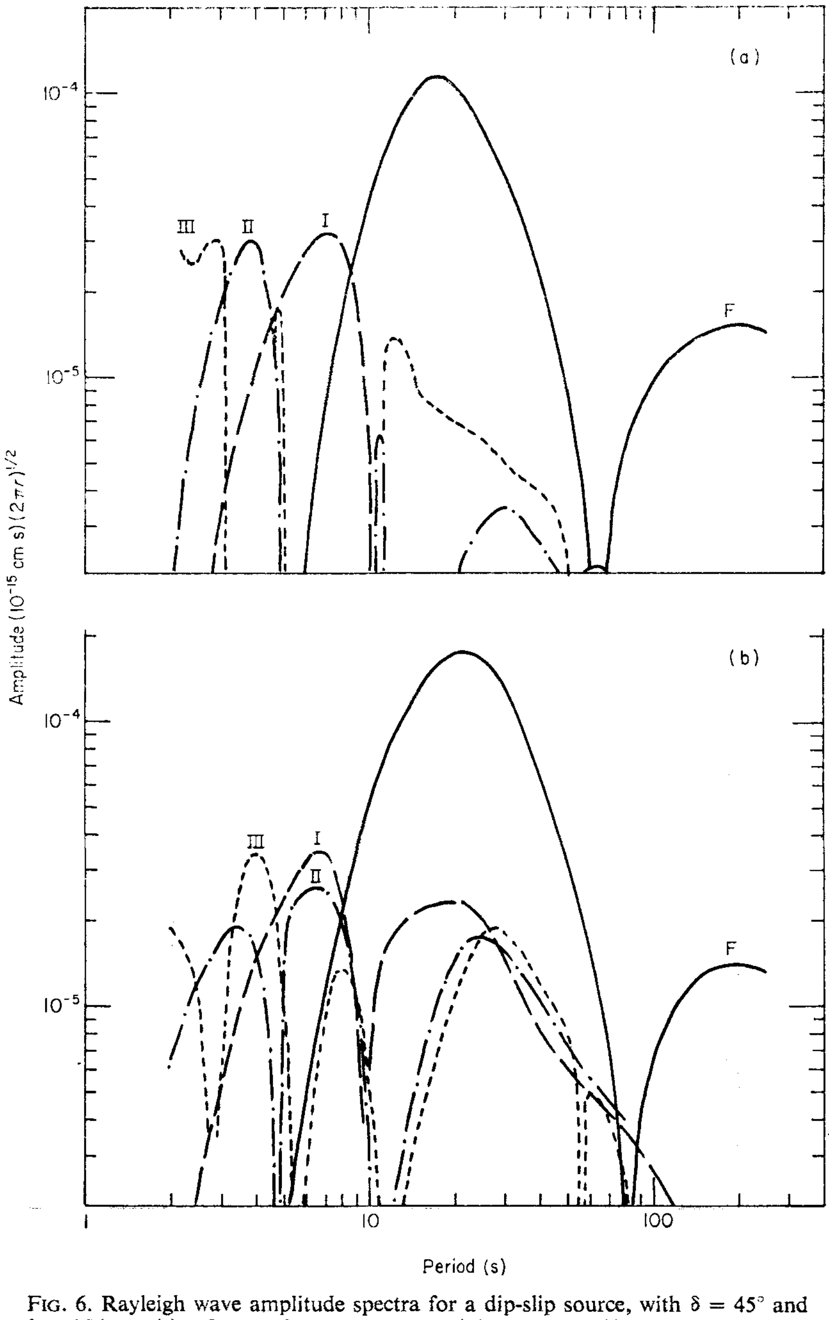

FIG. 6. Rayleigh wave amplitude spectra for a dip-slip source, with $\delta = 45^\circ$ and $h = 35$ km. (a) refers to the structure containing a LVC, (b) refers to the other structure.

Lg(i, 2) are confined almost entirely to the crust; Lg(i, 3) grazes the Moho and Lg(I, 4) samples the crust only above the Conrad discontinuity.

Comparison of Fig. 7 with the amplitude spectra in Figs 4, 5 and 6 will show which Lg(i, j) should be recognizable in the seismogram, for a given source depth. Moreover it is possible to see which portions of the crust most affect a given Lg phase. The identification of the vertical and longitudinal component of Lg with higher-mode Rayleigh wave propagation is thus consistent not only with group velocity evidence, but also with amplitude spectra from realistic source mechanisms in continental structures with and without a LVC. For instance the phase Lg (I, 1), which is significant only for $h = 16 \cdot 5$ and 35 km, can be easily identified with the Lg reported by Båth (1954) for periods greater than 6 s, while the Lg he reports with periods of about 5 s can be explained in terms of Lg (I, 2) and Lg (II, 1), as can be seen from an analysis of Fig. 6 together with Fig. 1.

Furthermore Lg (II, 1) which has a group velocity of about $3 \cdot 3$ km s^{-1}, in the structure with LVC, can explain the low group velocity value, $3 \cdot 22$ km s^{-1}, observed by Herrin & Minton (1960).

The deep minima in Fig. 1(c) around 11 and $4 \cdot 5$ s correspond to the long-period portions of the first and second member of the crustal wave family as defined by Panza *et al.* (1972). A comparison of this figure with part (a) of Figs 4, 5 and 6 shows that Li waves recorded on vertical instruments could be identified with the long-period contribution of each member of the Rayleigh crustal waves. In fact these stationary phases (Fig. 1(c)) and amplitude maxima (Figs 4(a), 5(a), 6(a)) occur at periods in good agreement with the observed period distribution (Båth 1956).

To this interpretation of Li two objections can be made: (a) the energy is concentrated in too narrow a band to be significant on the seismogram; (b) even where sufficient energy is present to register on the seismogram, the delta-function nature of the amplitude excitation would spread the energy out in the time domain and it is not possible to get the pulse-like events from which Båth (1956) determines group velocities for Li. However interference phenomena can occur among the different components of a member of the crustal wave family and this can result in pulse like events (Grudeva, Levshin & Frantsuzova, 1971; Frantsuzova, Levshin & Shkadinskaya, 1972). This implies that Li is a very unstable phenomenon, but may allow the conclusion that the presence of Li on a seismogram is a sufficient but not necessary condition for the existence of a LVC along the wave path. This can be easily seen from comparison of Fig. 1(d) with part (b) of Figs 4, 5 and 6 which shows that the presence of a LVC is a requirement for the existence of Li.

Båth (1954) gives also several examples of Rg waves defined as a short-period Rayleigh wave with a group velocity of about $3 \cdot 0$ km s^{-1} noticing that the observed group velocities scatter more than those for Lg waves, and that Rg may be identified with a minimum in the fundamental mode group velocity (Båth 1962).

An analysis of the group velocity curves of the fundamental mode given in Fig. 1(c) and 1(d) together with the corresponding amplitude spectra given in Figs 4, 5 and 6 shows that the identification of Rg with the stationary phases of the fundamental Rayleigh mode is correct also in terms of energy and explains the observed scattering in the Rg periods and group velocities, also as a consequence of path variations. In fact, in Fig. 1(c) the stationary portion goes from 6 to 13 s with a group velocity of about $2 \cdot 8$ km s^{-1} and in Fig. 1(d) it goes from 8 to 24 s with a group velocity of about 3 km s^{-1}.

Discussion

In this paper we have directly compared our theoretical computations, made considering a perfectly elastic flat earth, with experimental Lg and Rg data.

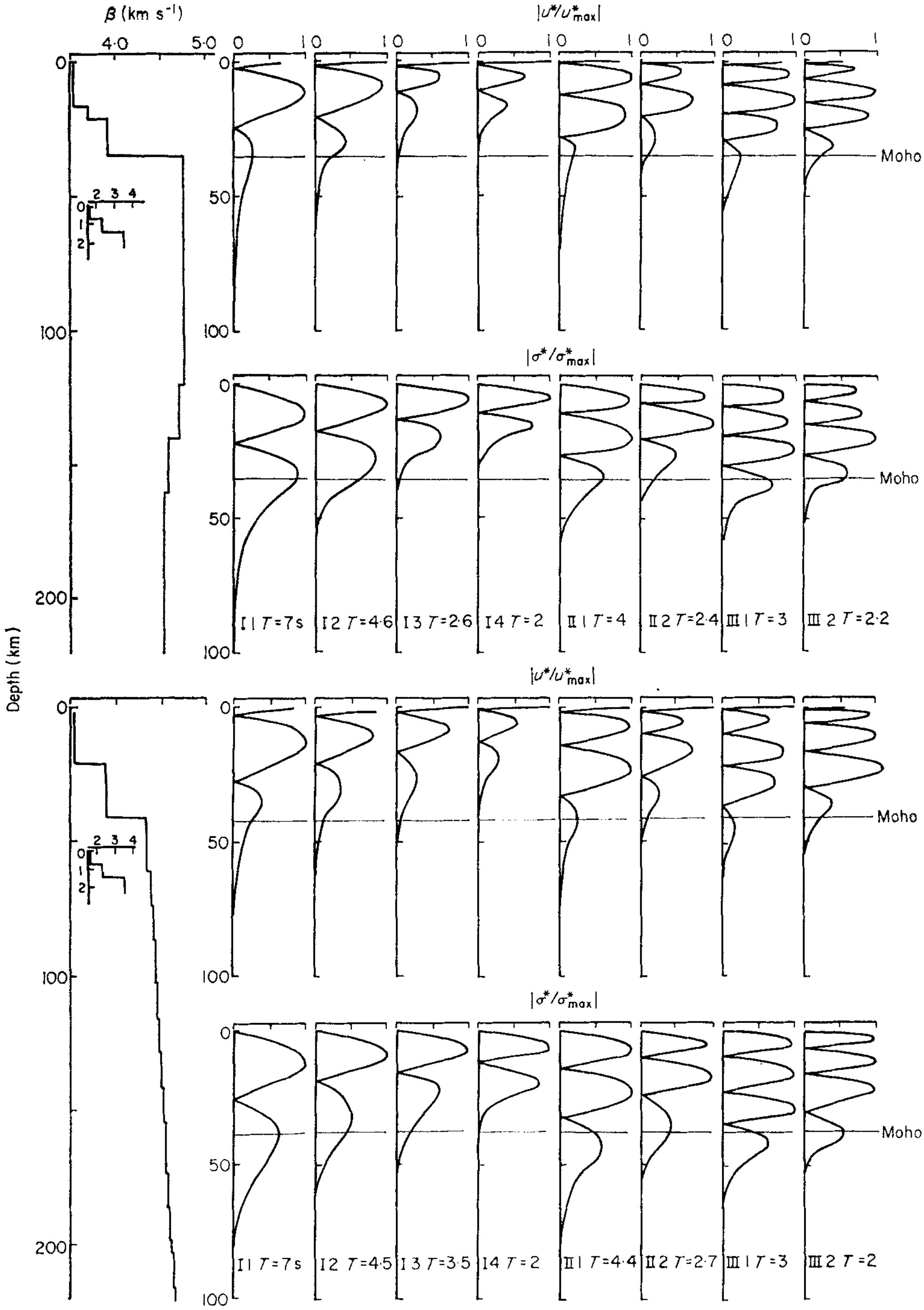

FIG. 7. Structures used in computations, displacement- and stress-depth functions associated with Lg stationary phases. The quantity β is the S-wave velocity.

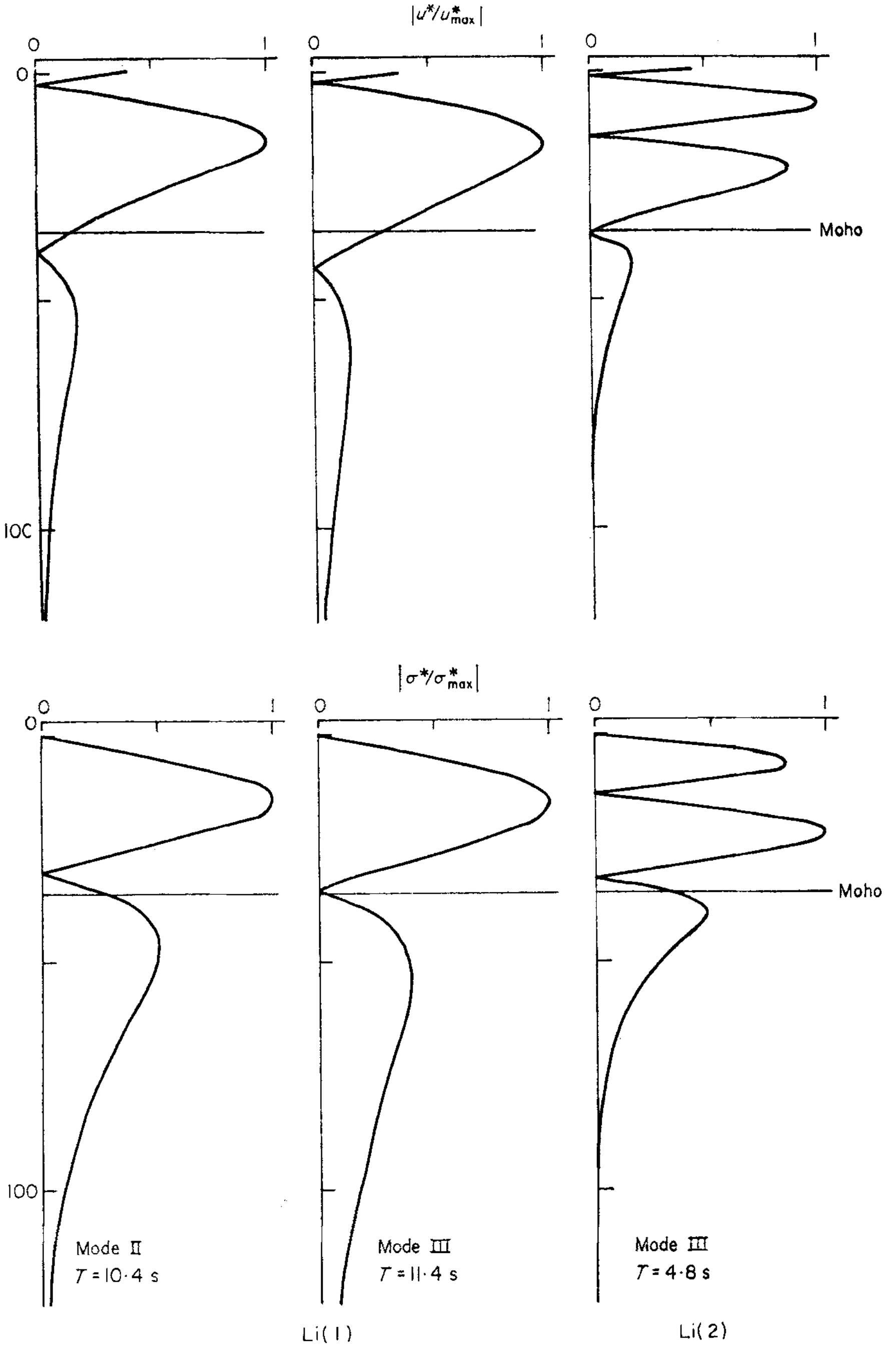

FIG. 8. Displacement- and stress-depth functions associated with Li stationary phases.

This limitation of a flat earth will not influence significantly our conclusions since we are interested in periods shorter than 20 s (Bolt & Dorman 1961; Kausel & Schwab 1973).

Moreover as Knopoff *et al.* (1973) have shown, the introduction of anelasticity into our results improves the identification of Lg on the seismograms.

Conclusions

The identification of Lg, recorded by vertical instruments, with higher-mode Rayleigh wave propagation is consistent not only with group velocity evidence, but with amplitude spectra from realistic source mechanisms. The presence of low-velocity zones in the crust is not a requirement for the existence of Lg. This is true for continental structures both with and without a LVC in the upper mantle. Li(*j*) identification as the contribution of the long-period portion of the crustal wave family members, is possible but their existence requires the presence of an LVC in the upper mantle.

A comparison of Fig. 5 with Fig. 4 of Knopoff *et al.* (1973) explains the considerably smaller number of observed Lg and Li waves on records obtained with vertical instruments, since, in the period range of interest, the amplitude ratio of the higher-modes to the fundamental is about unity for Love waves and about one-third for Rayleigh waves. Finally the identification of Rg with the fundamental Rayleigh mode is consistent both with group velocity evidence and amplitude spectra. In this case also, neither a low-velocity zone in the crust nor one in the upper mantle is a requirement for the existence of Rg.

Acknowledgments

The authors are very grateful to Fred Schwab for critically reading the manuscript and to Professor Leon Knopoff for valuable suggestions. We wish also to thank Professor Anatoly Levshin who made available the Soviet Bibliography.

This research was partly supported by the National Research Council's (CNR) Committee for Physical Sciences.

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