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Lift and Measurements in an Aerofoil in Unsteady Flow

D. W. HOLMES

S.R.C. Turbomachinery Laboratory,
Cambridge, England

A wind tunnel is described which is capable of producing both "transverse" and "streamwise" gusts. An account is given of the lift and pressure fluctuations measured on an isolated aerofoil tested in the tunnel. The response to a transverse gust compares well with Kemp's (1) [1] theory although the pressure distribution is not as predicted. The results suggest that the wake behavior and in particular the existence of a separation region can in practice seriously affect the validity of applying the now classical unsteady vortex theory.

[1]Underlined numbers in parentheses designate References at end of paper.

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D. W. HOLMES

INTRODUCTION

The need to understand the behavior of an aerofoil in an unsteady flow environment is well recognized, and, to this end, there exist comprehensive theories by Kemp and Sears (2), Whitehead (3), Giesing (4), Henderson and Daneshyar (5) among many others. All these theories make the familiar, small perturbation approximations, and even the most general of them ignores viscous effects. The author has, therefore, attempted experimentally to test the validity of this theoretical approach.

Two flows have been examined: (a) "transverse gust" flow and (b) "streamwise gust" flow. These two flows are illustrated in Fig. 1.

In the "transverse gust" flow, a constant mean flow acts in the direction of the chord line. Perpendicular to this, there exists a transverse velocity field, which instantaneously has a magnitude that varies sinusoidally along the chord line. At a point, the velocity varies sinusoidally in time. The transverse velocity field then propagates downstream as a frozen pattern.

In the "streamwise gust" flow, a constant mean flow acts at an incidence to the aerofoil chord line. Superimposed upon the mean flow and acting in the same direction is the streamwise gust velocity field. Instantaneously, the magnitude of this varies sinusoidally in the mean flow direction while at a point varying sinusoidally in time. This field also propagates downstream as a frozen pattern.

THE WIND TUNNEL

For the purpose of the experiments, it was necessary to design a tunnel capable of consistently producing the required gust flows. In particular, it was required to be able to produce either flow over a range of frequencies.

The initial design work was directed toward producing a gust which convected at the mean flow velocity. It was decided that any design based upon a closed working section and a rotating throttle system, or similar device to periodically restrict the flow, would only produce a pulsating flow which instantaneously would

NOMENCLATURE

U = mean free stream velocity

Up = perturbation

Uo = amplitude of the perturbation in the mean flow direction

Vo = amplitude of the perturbation perpendicular to the mean flow direction

$\not{U}$ = velocity of propagation of the gust

Uw = velocity of propagation of the wave on the tunnel wall

ω = reduced frequency parameter based upon U and the half chord, fc/2U

$\not{\omega}$ = reduced frequency parameter based upon $\not{U}$ and the half chord, fc/2U

f = frequency

x = position parameter in the streamwise direction

α = aerofoil incidence

t = time

c = chord length

λ = gust wave length

ρ = air density

R = wake propagation velocity divided by U

$C(\omega)$ = Theodorsen's function

$S(\omega)$ = Sears' function

$S(\omega,\not{\omega})$ = Kemp's function

$H(\omega)$ = Horlock's function

$H(\omega,\not{\omega})$ = Holmes function

I_1, I_2, I_3 = terms within Kemp's function

K_o, K_1 = modified Bessel functions of the second kind

Re = Reynolds' number based upon U and C

Cp_2 = unsteady pressure coefficient, Pressure/ρ U Uo or Pressure/ρ U Vo

Phase: The phase in all cases is referred to the phase of the gust at the mid-chord position.

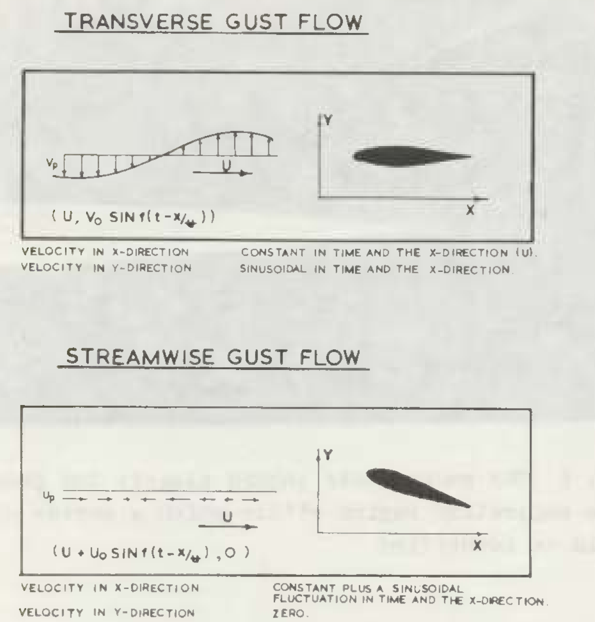

Fig. 1

THE WIND TUNNEL

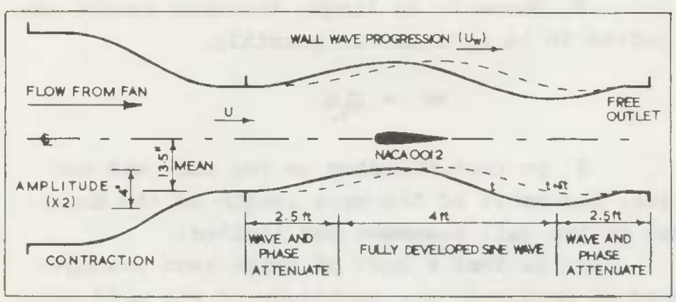

Fig. 2 Wind tunnel set to produce a transverse gust

have no spatial velocity variation.

$$U + U_0 \sin[ft]$$

The required flow was to have the form

$$U + U_0 \sin f\left[t - \frac{x}{U}\right]$$

Three design possibilities existed.

1 To abandon the closed jet approach and design a free jet working section
2 To design a system where the aerofoil moved within the tunnel through streamlines of different strengths
3 To design a closed jet system where the tunnel geometry was time varying.

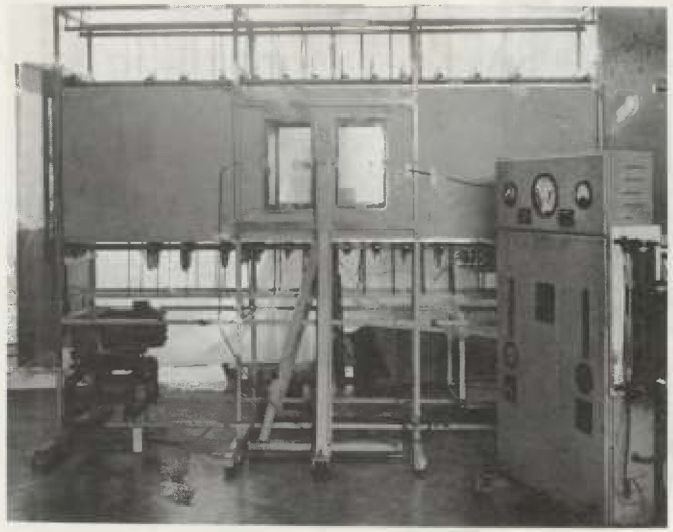

Fig. 3 A side view of the wind tunnel. The cams that drive the upper and lower surfaces are just visible

Fig. 4 A view of the springs and cams used to constrain the tunnel wall into a sinusoidal shape

Approaches 1 and 3 seemed to be the simplest and, since others had experienced difficulties with the first approach, the third approach was adopted.

The rig developed is illustrated in Fig. 2, and in the photographs, Figs. 3 through 6 inclusive. The gust is generated in a specially

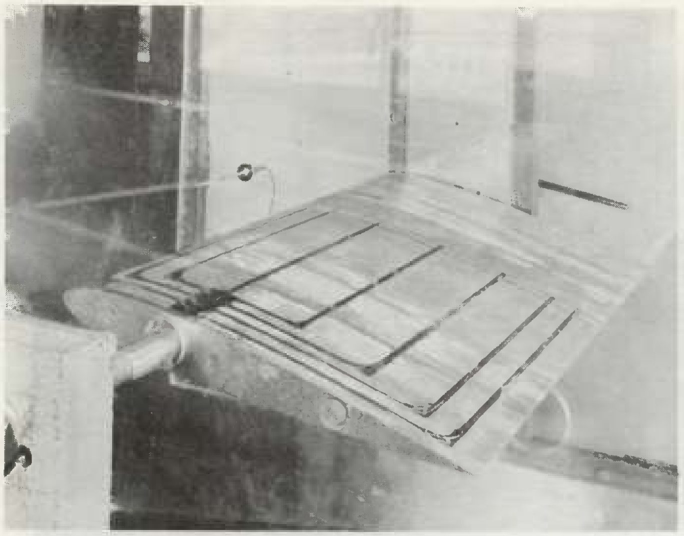

Fig. 5 The NACA 0012 aerofoil showing the 32 pressure tappings, four for each of the eight positions across the chord

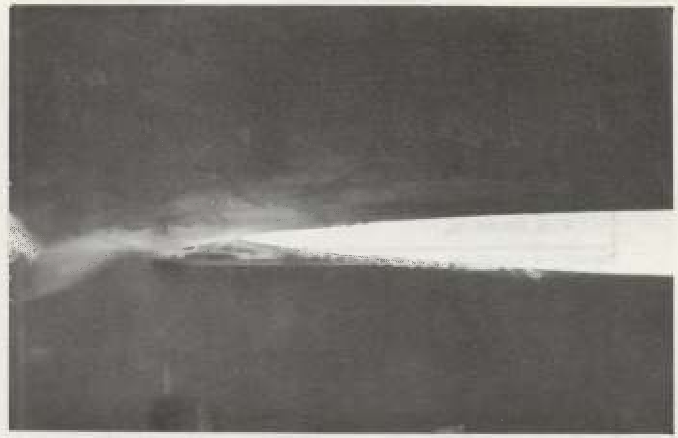

Fig. 6 The smoke tests showed clearly the presence of a separation region within which a vortex system could be identified

constructed 9-ft section which can be attached to a conventional subsonic wind tunnel. A steady airflow enters from a settling chamber via a contraction and passes out through a fixed geometry outlet.

The generation section is rectangular with rigid side walls. The upper and lower surfaces consist of flexible metal sheets. The metal sheets are constrained to form a sine wave by a system of cams and springs. A full sine wave form is maintained over the mid-section and allowed to attenuate to zero amplitude at either end. A pair of camshafts run the whole 9-ft length of each wall. The rotation of the shafts causes the sine wave to move along the tunnel. Fig. 2 illustrates a subsequent position of the wall following the rotation of the shaft.

In the configuration illustrated, the upper and lower surfaces are in phase, and a rotation of the shafts produces a transverse gust pattern of the form

$$U + V_0 \sin f\left[t - \frac{x}{\vartheta}\right]$$

where ϑ is the propagation velocity of the wave down the tunnel.

A streamwise gust is generated by altering the phase of one wall by 180 deg.

The working section has a width of 18 in. and in the transverse gust configuration, a height of 27 in. In the streamwise configuration, the height varies between 23 and 31 in. The wave length of the wave maintained on the wall is 6 ft, although only a 4 foot section is ever fully generated at a time.

The value of 6 ft was dictated by the following considerations:

1 For ease of pressure measurement, a chord of at least 10 in. was required.

2 For ω to be large, the wave length was required to be as small as possible.

$$\omega = \frac{\pi c}{\lambda}$$

3 So that the flow on the wall did not stall, the ratio of the wave length to the amplitude of the wall movement was limited.

4 So that a gust of sufficient strength could be produced, the amplitude of the wall movement had to be large compared to the tunnel height.

5 So that any tunnel interference effects would be minimized, the ratio of tunnel height to chord length had to be large.

6 The relationship between the tunnel section and the settling chamber size had to be such as to give a steady airflow at entry.

The gust propagation velocity, at the time of the experiments, was limited to 12 fps. Subsequent development by others has raised this figure. The system was operated successfully over the range of mean flow velocities from 27 to 54 fps. The lower velocity represented a practical minimum below which the pressure fluctuations, induced on the aerofoil, were difficult to measure, and the maintenance of an even flow became a problem.

THE TUNNEL PERFORMANCE

A transverse gust of good quality could

Table 1 Transverse Gust Calibration Results

ω	$\bar{\omega}$	$\bar{v}/U$	Uw/U	Vo/U%,	Re.10^5	Gust wave length, ft	Typical velocities U, fps	Typical velocities Uw, fps
0.016	0.361	0.045	0.037	9.5	2.75	7.26	54	2.0
0.068	0.357	0.191	0.156	8.3	2.75	7.33	54	8.4
0.136	0.334	0.407	0.312	6.7	1.38	7.84	27	8.4
0.190	0.296	0.642	0.436	5.6	1.38	8.83	27	11.8

Table 2 Streamwise Gust Calibration Results

ω	$\bar{\omega}$	$\bar{v}/U$	Uw/U	Uo/U%,	Re.10^5	Gust wave length, ft	Typical velocities U, fps	Typical velocities Uw, fps
0.016	0.445	0.036	0.037	9.7	2.75	5.88	54	2.0
0.068	0.663	0.102	0.156	6.3	2.75	3.85	54	8.4

be generated over the full operating range of the tunnel.

The amplitude was substantially constant along the working section varying, in the worst case, by 7 percent between the leading edge and the trailing edge. The movement of the gust was not, however, synchronous with the wall movement. The gust propagated at a faster velocity than the wave on the tunnel wall and had a shorter wave length. At the lowest operating speed, the error was about 20 percent and at the highest speed, about 50 percent. Four flow conditions were calibrated, and the results are summarized in Table 1.

It was not possible to produce a good quality streamwise gust at low air velocities. Although a large fluctuation could be produced upstream of the working section, the amplitude and form degenerated rapidly in the downstream direction; after various unsuccessful attempts to improve the flow, the calibrations were carried out at the higher flow velocity. Different flow conditions were obtained by varying the gust frequency. Two flow conditions were calibrated, and the results are summarized in Table 2.

Once the calibration work was finished, the experiments with the aerofoil were begun. A NACA 0012 aerofoil with a 10-in. chord was chosen, it being a well-documented profile of significant thickness. Pressure measurement was achieved by using a pressure transducer system, located outside the tunnel, connected to a network of pressure tappings across the aerofoil surface as illustrated in Fig. 5.

With the aerofoil at zero incidence, measurements were taken of the aerofoil response to a transverse gust. With the tunnel set to produce a streamwise gust, measurements were taken at incidences of 0, ±7 and ±9 deg. Below 7 deg, the pressure fluctuations were too small for the effect of incidence to be accurately distinguished from the effect due to thickness. At incidences greater than 9 deg, the results became confused by the onset of stall.

The results obtained were analyzed in two ways. The first method assumed that the pressure fluctuations were sinusoidal and could thus be defined in terms of an amplitude and phase. On this basis, all the results obtained for a particular flow condition were correlated to give the best estimate of the amplitude and phase at each location on the aerofoil. The results obtained in this way are referred to as the "smoothed" results. In the second method, a set of pressure measurements were chosen as reliable from the evidence of the correlation work which produced the "smoothed" results. These particular measurements were then used to plot out the "actual" lift variation. With the transverse gust, the actual lift response was, in fact, closely sinusoidal. The actual lift response to the streamwise gust was, however, significantly different.

The results obtained have been compared with the available unsteady flow theory where it existed and where necessary with theory developed by the author.

THEORY

Sears (7) presented a solution for the fluctuating lift on an isolated aerofoil in a

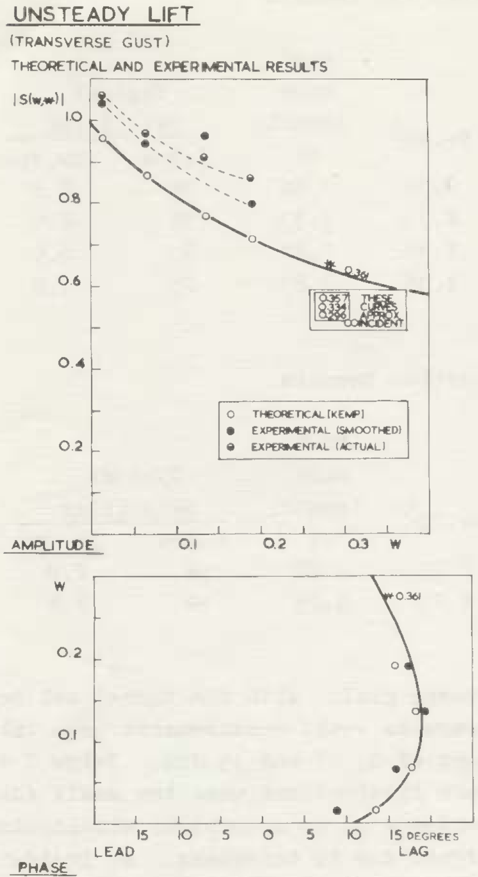

Fig. 7

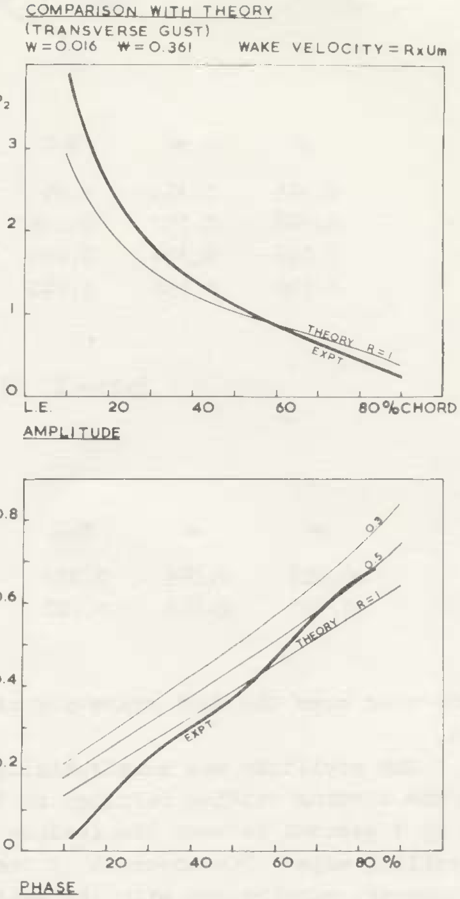

Fig. 8

transverse gust flow. The solution is limited to the case of a convecting gust. Kemp (1), in a later note, states the extended form of the solution that relates to the general non-convecting gust.

$$\text{LIFT} = \rho c U V_0 \pi e^{ift} S(\omega, \varpi) \qquad (1)$$

$$S(\omega,\varpi) = \left[J_0(\varpi) - i J_1(\varpi) \right] C(\omega) + i \left[\frac{\omega}{\varpi} \right] J_1(\varpi) \qquad (2)$$

where

$C(\omega)$ = Theodorsen function

$$= \frac{K_1(i\omega)}{K_0(i\omega) + K_1(i\omega)} \qquad (3)$$

This is the expression used by the author in the consideration of the results of the transverse gust experiment.

The case of the convecting streamwise gust was solved by Horlock (6). The extension of the theory to the non-convecting case has been made by the author (4).

$$\text{LIFT} = \rho c U U_0 \pi \alpha e^{ift} H(\omega,\varpi) \qquad (4)$$

$$H(\omega,\varpi) = S(\omega,\varpi) + J_0(\varpi) + i J_1(\varpi) \qquad (5)$$

Equation (5) compares directly with a relationship, established in the literature by Arnoldi (10), between the solutions of Horlock and of Sears

$$\text{HORLOCK Fn} = \text{SEARS Fn} + J_0(W) + iJ_1(W) \qquad (6)$$

The author has used his solution for the consideration of the results of the streamwise gust experiment.

Arnoldi draws attention to the significance of the relationship expressed in equation (6). His comments apply equally to equation (5). A streamwise gust (Up) at incidence (α) to a aerofoil may be regarded as equivalent to a small transverse gust (αUp) and a gust acting along the chordline (Up). The transverse gust, as considered by Kemp, causes a change in the circulation and the formation of a wake. The response to (αUp) is thus Kemp's function in equation (5). The chordwise gust (Up) causes no change in the

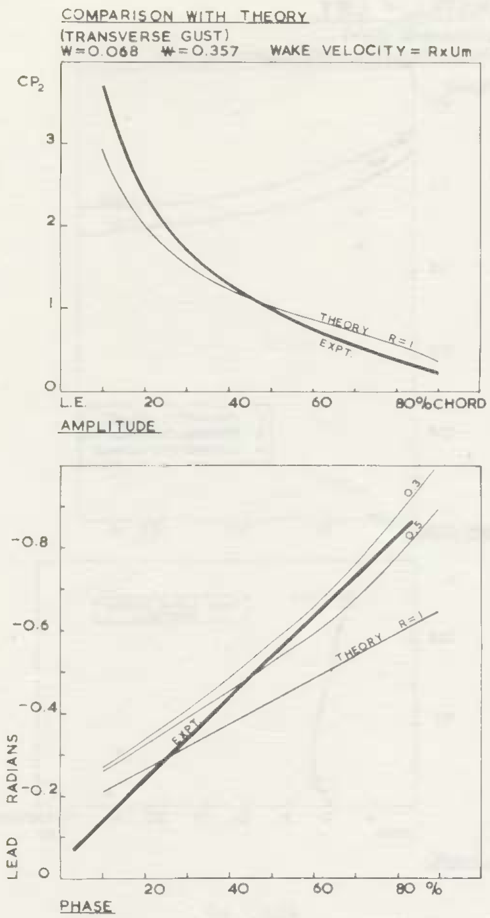

Fig. 9

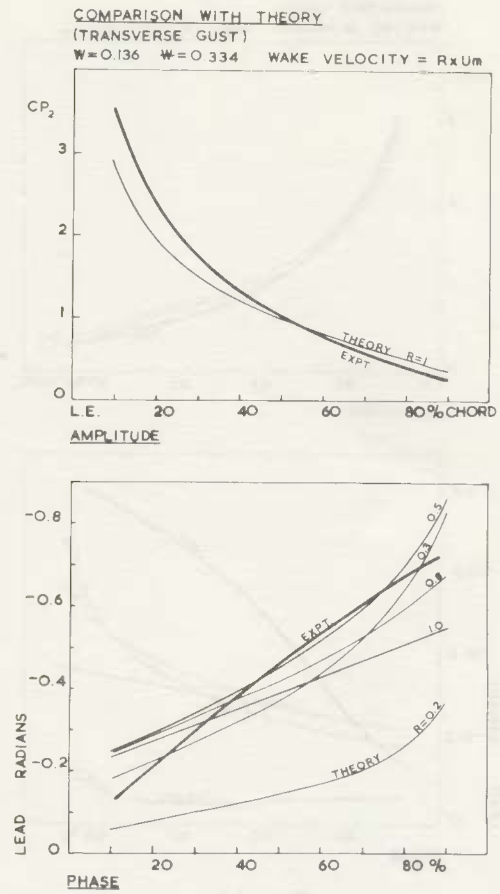

Fig. 10

circulation and, hence, no wake, but a lift fluctuation occurs because of the change in dynamic pressure. This response may be shown to be α times the remaining term on the right-hand side of equation (5).

The author has developed a theory for the solution of the pressure distribution. The theory does not yield an analytical solution directly, but provides the basis of a computer program which can quickly furnish solutions. The experimental results are compared with the results of this theory.

The experimental results are also compared with a theory developed by the author which assumes that the wake propagates at a velocity equal to R. U., where "R" is, in general, a constant of value less than one and "U" is the mean flow velocity.

Under this situation, it has been demonstrated that Kemp's solution may be generalized by redefining Theodorsen's function as a function of ω/R.

The generalized form of $H(\omega, \varpi)$ is given by the still valid equation (5). The associated pressure distribution has also been determined.

RESULTS

Transverse Gust Response

The experimental results are presented in Figs. 7 through 11. Fig. 7 shows the amplitude and phase of the lift fluctuations measured at each of the four flow conditions. The variation in the magnitude of the lift response from one flow condition to another supports the theory, although the absolute magnitude of the lift response is approximately 10 percent larger than predicted. The phase of the lift fluctuation agrees with the theory for all four flow conditions.

Figs. 8 through 11 compare the measured pressure fluctuations with the author's predictions. The magnitude of the pressure fluctuations varies smoothly across the chord. As the lift fluctuation falls with increasing frequency, the pressures across the chord reduce evenly. The pressure fluctuations are larger than predicted in the leading edge region and smaller in the

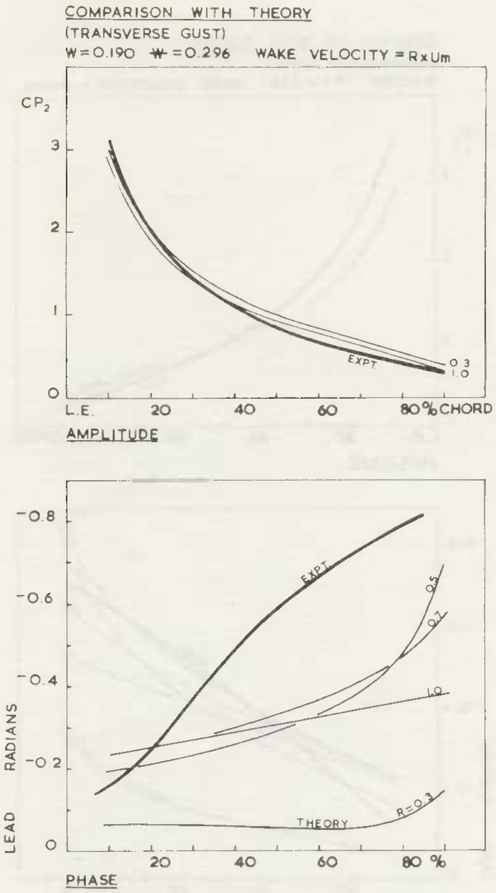

Fig. 11

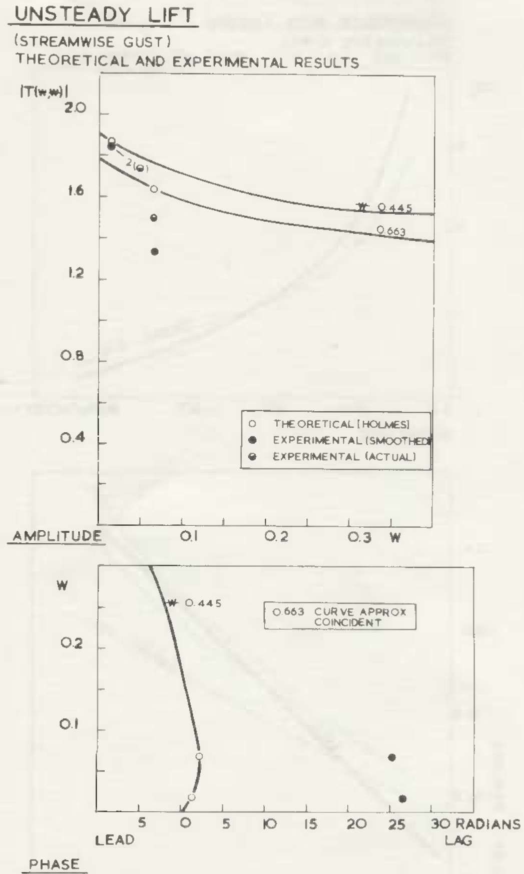

Fig. 12

trailing edge region. At the lower frequencies, the phase distribution corresponds, although not accurately, with the theory which was arrived at by assuming that the wake propagates at half the free stream velocity. The agreement at the highest frequency is bad whatever assumption is made.

Streamwise Gust Response

Fig. 12 shows that the measured lift responses are not as predicted. At both frequencies, the response lags the predicted phase by approximately 25 deg, and although the magnitude of the response agrees with the theory at the lower frequency, the agreement is not repeated at the higher frequency.

The pressure fluctuations due to incidence were obtained by subtracting, from the measurement at incidence, the pressure fluctuations measured with the aerofoil at zero incidence. The results are compared with the theoretical response at an incidence of 10 deg. The experimental results obtained at 7 and 9 deg have been scaled up to relate to an incidence of 10 deg. Figs. 13 and 14 summarize the results.

The amplitude distributions are significantly asymetrical, whereas the theory predicts a symetrical distribution. In the trailing edge region, on the upper surface, the pressure fluctuation is much smaller than predicted. On the lower surface, the fluctuation in the same region is much larger than predicted. The lower lift fluctuation at the higher frequency can be attributed to a significant reduction in the pressure loading over the front half of the chord.

A collapse is apparent in the upper surface phase distributions, while the lower surface distributions vary by a far greater amount than predicted. The phase distributions appear to be continuous around the trailing edge.

The results are not compared with the theory for a range of values of "R," since, for the streamwise gust, the theory is virtually independent of the wake propagation velocity.

DISCUSSION OF RESULTS

Transverse Gust Response

The results suggest that Kemp's theory accurately predicts the variation in the lift response from one flow condition to another, but

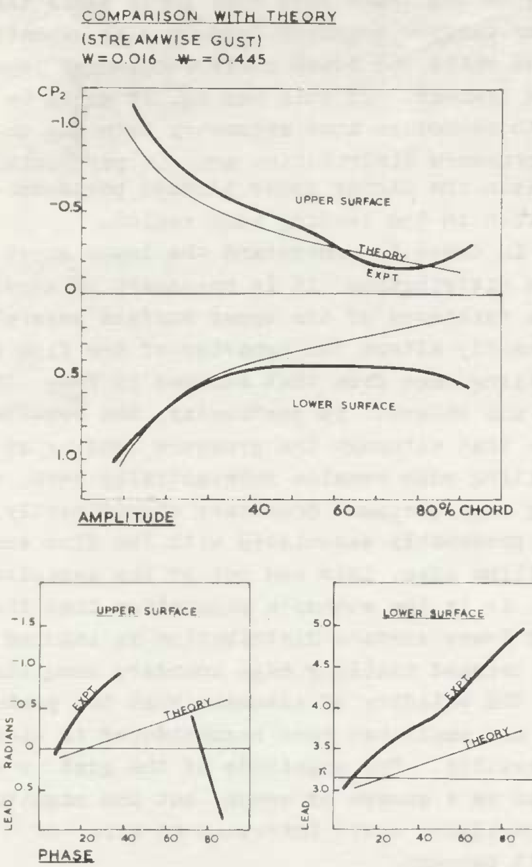

Fig. 13

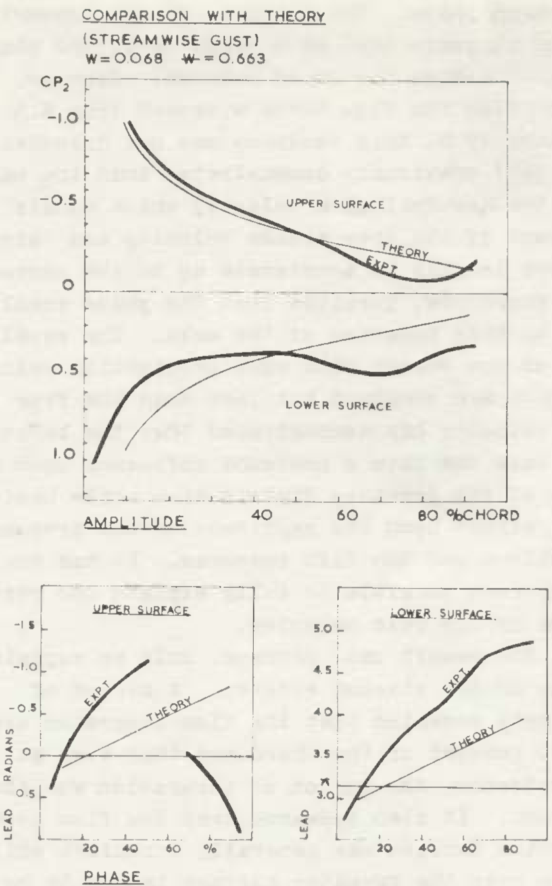

Fig. 14

not the absolute magnitude of the response. So that the significance of this result can be understood, it is worth examining Kemp's results in more detail.

Kemp's function, $S(\omega, \varpi)$, can be separated into three terms (see Appendix).

I_1 = The quasi-steady response: This is the solution derived for the aerofoil response if the steady Bernoulli equation is used and the wake is ignored.

I_2 = The term introduced by using the unsteady Bernoulli equation: $I_1 + I_2$ is the solution derived if the unsteady Bernoulli equation is used and the wake is ignored. Solutions of this type have been obtained by Parker (11).

I_3 = The term introduced by allowing for the influence of the wake: $I_1 + I_2 + I_3$ is the solution derived if the unsteady Bernoulli equation is used and the wake is allowed for. This corresponds to Kemp's solution.

An examination of the values of I_1, I_2, and I_3 corresponding to the four flow conditions studied reveals that:

1 The absolute value of the lift fluctuation was primarily a function of I_1.

2 The variation in the response from one flow condition to another was primarily a function of $I_2 + I_3$.

3 The phase of the response was essentially a function of I_3 alone.

The results obtained, therefore, suggest that Kemp's treatment of the unsteady potential flow and of the wake is substantially justified. It appears, however, that the quasi-steady term only predicted 90 percent of the quasi-steady response.

The author has not identified the different terms that contribute to the pressure distribution, but it is suggested that the observed discrepancies between the results and the theory might be associated with the quasi-steady part of the solution. The amplitude distribution certainly corresponds best with the theory when the conditions are quasi-steady.

Graham (8) demonstrated that for a flow where the gust travels at the free stream velocity ($U=\mho$), the pressure fluctuations are all tuned to a common phase. The author's theory supports this and suggests that as $\bar{U}$ tends to U, the phase difference across the chord reduces. However, although over the flow tests $\bar{U}$ varied from 4.5 to 64 percent of U, this tendency was not detectable. Takata (12) previously demonstrated that the wake leaves the aerofoil at a velocity which equals 25 percent of the free stream velocity and takes two chord lengths to accelerate up to 100 percent. It is, therefore, possible that the phase result is due to this behavior of the wake. The recalculation of the theory with wake propagation velocities which are constant but less than the free stream velocity has demonstrated that the behavior of the wake can have a profound influence upon the phasing of the pressure distribution while having a small effect upon the magnitude of the pressure fluctuations and the lift response. It has not, however, been possible to fully explain the results in terms of the wake behavior.

The result can, perhaps, only be explained in terms of the viscous effects. A series of smoke tests revealed that the flow separated over up to 40 percent of the chord and that even at zero incidence, the region of separation was about 15 percent. It also appeared that the flow over the suction surface was generally turbulent while the flow over the pressure surface tended to be laminar.

The existence of the separation is a likely reason for the low-pressure fluctuations in the trailing edge region. It is also possible that the separation is superimposing a phase on the pressure at the trailing edge and, hence, inducing the observed phase distribution. The photograph, Fig. 6, suggests the existence of a strong vortex in the separated region. It is worth noting that any periodic variation in the strength of this vortex must create a wake and, therefore, invalidate the simple model used by Kemp, the author, and others.

Streamwise Gust Response

As the streamwise gust tests were performed with the aerofoil at a steady incidence, it is likely that the flow was permanently separated over the trailing edge region on the upper surface and that the flow on the lower surface remained attached. In the absence of any variation of the incidence, the extent of the separated region could be expected not to vary. This model of the flow is supported by the pressure measurements. The sudden change in the phase distribution on the upper surface indicates a separation point. The low level of the pressure fluctuations downstream of that point is consistent with the flow being separated. These characteristics are not apparent on the lower surface. It is also probable on the evidence of the transverse gust smoke tests that the upper surface boundary layer was permanently turbulent while the lower surface boundary layer remained laminar. If this was so, it could be expected to introduce some asymmetry into the unsteady pressure distribution and, in particular, may explain the higher upper surface pressure fluctuation in the leading edge region.

In order to understand the lower surface pressure distribution, it is necessary to accept that the existence of the upper surface separation significantly alters the behavior of the flow at the trailing edge from that assumed by Kemp, the author, and others. In particular, the results indicate that although the pressure loading at the trailing edge remains substantially zero, the trailing edge pressure does vary significantly. This is presumably associated with the flow around the trailing edge, into and out of the separated region. It is the author's suggestion that the observed lower surface distribution is induced by this imposed trailing edge boundary condition.

The validity of assuming that the perturbations are small has been reconsidered in view of the results. The magnitude of the gust is unlikely to be a source of error, but the magnitude of the incidence could introduce an error of around 13 percent.

Tunnel interference, although insignificant in the transverse gust flow, could have a further effect of order 10 percent.

Secondary Results

It was observed that by setting the aerofoil at incidence to a transverse gust, many of the characteristics of the streamwise response could be reproduced. In particular, on the upper surface, the phase distribution collapsed and the pressure fluctuation in the trailing edge region became reduced.

CONCLUSIONS

Viscous effects, in particular separation, appear to have a significant influence upon the behavior of an aerofoil under unsteady flow conditions. The most noticeable effect is upon the phase of the pressure fluctuations in the trailing edge region. In the case of an aerofoil at zero incidence to a transverse gust, the region of separation is quite limited and the pressure fluctuations, whose phase are effected, are quite small. Kemp's theory, therefore, is largely applicable. In the case of an aerofoil at incidence to a streamwise gust, the region of separation is both larger and always present with the result that the unsteady thin aerofoil theory is not

Table 3

ω	Real Part I_1	Real Part I_2	Real Part I_3	Imaginary Part I_1	Imaginary Part I_2	Imaginary Part I_3
0.016	0.968	0.003	-0.042	-0.177	0.023	-0.073
0.068	0.968	0.012	-0.155	-0.176	0.099	-0.190
0.136	0.972	0.022	-0.259	-0.165	0.199	-0.276
0.190	0.978	0.028	-0.313	-0.146	0.280	-0.332

applicable.

The existence of a separation region, therefore, reduces the validity of applying the thin aerofoil theory unless the theory is modified to allow for the imposed trailing edge condition on the lower surface and the separation on the upper surface.

In problems where the reduced frequency is higher, it might also be necessary to allow for the separated region constituting a secondary source of shed vorticity.

Finally, it is worth noting that, although the agreement with the theory was only marginally improved by allowing for the wake propagating at less than the free stream velocity, the solution obtained is very dependent upon the behavior of the wake.

APPENDIX

$$I_1 = 2\rho U v_0 \pi \left[J_0(\varpi) - i J_1(\varpi) \right]$$

$$I_2 = 2\rho U v_0 \pi \frac{\omega}{\varpi} \left[\varpi J_1(\varpi) + i \left(J_1(\varpi) + \varpi J_0(\varpi) \right) \right]$$

$$I_3 = -2\rho U v_0 \pi \left[\left(\omega J_1(\varpi) + a J_0(\varpi) + b J_1(\varpi) \right) + i \left(\omega J_0(\varpi) + b J_0(\varpi) - a J_1(\varpi) \right) \right]$$

where

$$a + ib = 1 - C(\omega)$$

I_1, I_2, and I_3 are in general complex. The values of I_1, I_2, and I_3 which relate to the transverse gust flows examined are shown in Table 3.

A thorough analysis of the terms (I_1, I_2, and I_3) for $\varpi = \omega$; i. e., $I_1 + I_2 + I_3$ = Sears function reveals the following interesting points:

1 The frequency at which conditions may be considered to be quasi-steady is found to be very low. At a reduced frequency as low as 0.2, both I_2 and I_3 have magnitudes which are approximately 30 percent of the quasi-steady response.

2 The term due to the wake, I_3, is of the same order as the quasi-steady term, I_1, at a reduced frequency as low as 0.5.

3 At the higher frequencies, I_2 and minus I_3 tend independently to infinity, although the difference is always finite and of the same order as I_1.

It is, therefore, obvious that calculating the lift fluctuation without allowing for the effect of the wake will lead to a substantial error.

Furthermore, the accuracy of the solutions, which take the effect of the wake into account, are very dependent upon the validity of the model used to describe the wake.

As far as the current theories are concerned, it is important that the following assumptions hold:

1 The wake down not roll up but forms a rectilinear sheet.
2 The wake convects at the mean flow speed.
3 The circulation in the system is instantaneously constant.
4 Circulation can only exist in the flow in the form of a circulation around the aerofoil or as vorticity in the wake.

ACKNOWLEDGMENTS

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