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LIGHT-CONE PHYSICS AND DUALITY

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I. INTRODUCTION

In recent years important contributions to our understanding of the hadron physics have been obtained following two rather different approaches.

In the framework of a pure theory of strong interactions large attention has been dedicated to the study of the properties of the Dual Resonance Models (DRM). The interest of these models is based on the fact that they incorporate some general requirements of S matrix theory, such as analyticity of first and second kind and crossing, and they exhibit very attractive physical features such as a large degeneracy of the spectrum and linear Regge trajectories with an universal slope [1].

On the other hand, the study of the electromagnetic and weak interactions of hadrons was instrumental in obtaining new important physical insights. At this conference the properties of the products of currents for light-like distances, especially of current commutators, and their consequences for the asymptotic behaviour (Bjorken limit) of scattering processes have been extensively discussed [2].

At first sight these two lines of investigation appear quite unrelated to each other. We can mention, for example, the difficulties met by the program of building dual amplitudes for currents. Nevertheless, a feeling of the existence of a possible relationship between these two different types of approaches is now developing. Let us consider, for example, the extremely large degeneracy of the spectrum, exponentially increasing with the energy, that is perhaps the most striking feature of the DRM. This degeneracy, when seen from the point of view of the statistical models of the hadrons, suggests an underlying picture of the hadrons as made up of an infinite number of constituents moving almost freely. This picture then is not remote from the one advocated by the parton model, in which the e.m. current interacts directly with the almost free constituents of the nucleon.

Furthermore, recent developments in the analysis of the properties of the DRM are leading to an extremely simple and attractive physical picture for a specific critical dimension D of the space-time. In fact, it has been shown that for $D \leq 26$, for the ordinary $\alpha(0) = 1$ model ($D \leq 10$ for the Neveu-Schwarz model), the model is free from negative norm states (ghosts)[3]. In addition, at the critical dimension, the physical picture given by the model is extremely similar to the infinite momentum frame picture. In fact, any physical state can be obtained by applying to the "vacuum" (corresponding to the lower energy level, the tachyon with squared mass equal to -1) an arbitrary number of purely transverse "photon" operators with collinear momenta[4].

The purpose of this talk is to further investigate these analogies, starting from the light-cone point of view and recovering some interesting properties of the DRM. This talk follows essentially the line of a previous paper[5] by the same authors, but more emphasis is given here to the discussion of the deep inelastic structure functions that can be explicitly obtained as an output of our approach.

II. BASIC ASSUMPTIONS

We start with the assumption that the elementary particles are, in some way, composite systems made of quarks which, at least in the present deep inelastic experiments, appear as point-like objects.

The experimental validity of the Bjorken scaling suggests that for what concerns the light-cone properties of the hadrons the "true" quark field can be approximated by an "asymptotic" field which obeys the free massless equation

$$\Box \phi(x) = 0 \tag{II.1}$$

For simplicity we will start with spinless quarks. We recall that the above equation is not only invariant under the Poincaré group, but also under the larger conformal group.

Another fundamental property of Eq. (II.1) is that its general integral can be written in the form

$$\phi(\vec{x},t) = \int f(t-\vec{x}\cdot\vec{u};\vec{u})\, d\Omega_{\vec{u}} =$$

$$= \int d\rho \int d\Omega_{\vec{u}}\, f(\rho,\vec{u})\, \delta(t-\vec{x}\cdot\vec{u}-\rho) \tag{II.2}$$

where the integral in $d\Omega_{\vec{u}}$ is evaluated over all the directions of a unit vector $\vec{u}$:

$$\vec{u}^{\,2} = 1 \tag{II.3}$$

In order to avoid the appearance of unpleasant divergent quantities which are present anytime we consider matrix elements of a product of operators $f(\rho,\vec{u})$ belonging to the same direction $\vec{u}$, we define the averaged operator:

$$P_i(\rho) = \frac{1}{\sqrt{\Delta\Omega_i}} \int_{\Delta\Omega_i} d\Omega\, f(\rho,\vec{u}) \tag{II.4}$$

so that Eq. (II.2) can be expressed as

$$\phi(\vec{x},t) = \sqrt{\Delta\Omega} \sum_i P_i(t-\vec{x}\cdot\vec{u}_i) \tag{II.5}$$

The commutation relations and the vacuum expectation value of the field $P_i(\rho)$ are given by

$$[P_i(\rho_1), P_j(\rho_2)] = \delta_{ij} \frac{i}{8\pi^2} \delta'(\rho_1-\rho_2) \tag{II.6}$$

$$\langle 0| P_i(\rho_1) P_j(\rho_2) |0\rangle = -\frac{1}{16\pi^3} \delta_{ij} \frac{1}{(\rho_1-\rho_2-i\varepsilon)^2} \tag{II.7}$$

Equation (II.6) shows that each direction gives rise to orthogonal subspaces.

Up to now we have only discussed, very briefly, the kinematical properties of the parton field. We now introduce our main dynamical assumptions. We assume that not only the quark field but also the compound field obeys, near the light-cone, the d'Alembert equation

$$\Box A(x) = 0 \tag{II.8}$$

In analogy with what we did in the case of the quark field, we can make the following decomposition

$$A(\vec{x},t) = \Delta\Omega \sum_i \chi_i(\rho) \tag{II.9}$$

The field $\chi_i(\rho)$ is then constructed as a function of the quark field $P_i(\rho)$:

$$\chi_i(\rho) = F\left[P_i(\rho)\right] \tag{II.10}$$

As a consequence of the assumptions (II.8) and (II.10) we see that among all possible multiquark states the single particle states are those obtained by the product of an arbitrary number of collinear quark fields. This shows the analogy of our model both with the parton model in the infinite momentum frame and with the DRM where for a critical value of the space-time dimension the physical states are obtained by a product of an arbitrary number of "photon" operators all aligned along a certain common light-like direction [3]. We must notice, however, that our model is unable to give a correct description of the "wee" partons, i.e., partons of very low momentum that it is impossible to assign to any precise direction. Therefore, we expect that our model will fail in correctly describing features, such as the behaviour of the structure functions for $\omega \to 0$, strictly related, in the parton model to the distribution of the "wee" partons.

On the other hand, we notice that the DRM gives rise to masses of the physical particles which are non-vanishing. This is a consequence of the fact that even if the "photon" operators carry a light-like four-momentum, as in the case of Eqs. (II.1) and (II.8), they are applied to a vacuum state (which, in the DRM, is associated with the ground state with a definite momentum) whose momentum combined with the momentum of the "photons" gives rise to a non-vanishing mass for the excited particles.

It is clear that the "collinearity assumption" in our present form is too crude and it needs probably to be modified for a future development.

III. COMPOUND STATES AND THEIR DEEP INELASTIC STRUCTURE FUNCTIONS

We can now discuss the transformation properties of our parton field under the conformal group. As we have discussed previously, we will be interested only in the case of partons all moving along the same direction (say the z direction). Then we will be interested only in the subgroup of the full conformal group leaving the z direction invariant.

The generators of such a subgroup are, in the usual notations,

$$J_3 = M_{12} \qquad \Lambda = \frac{D - M_{03}}{2} \tag{III.1}$$

$$\begin{aligned} L_+ &= \frac{i}{2}(P_0 + P_3) \\ L_0 &= \frac{i}{2}(M_{03} + D) \\ L_- &= \frac{i}{2}(K_0 - K_3) \end{aligned} \tag{III.2}$$

The algebraic properties of the five operators defined in Eqs. (III.1) and (III.2) are most interesting. First of all, J_3 and Λ commute with all other operators.

The three fundamental operators L_i obey the $O(2,1)$ algebra given by:

$$[L_0, L_\pm] = \pm L_\pm$$

$$[L_+, L_-] = -2L_0 \qquad \text{(III.3)}$$

Finally the commutation relations of L_i with $P(\rho)$ are

$$[L_+, P(\rho)] = \frac{dP}{d\rho}$$

$$[L_0, P(\rho)] = \left(\rho \frac{d}{d\rho} + 1\right) P(\rho)$$

$$[L_-, P(\rho)] = \left(\rho^2 \frac{d}{d\rho} + 2\rho\right) P(\rho) \qquad \text{(III.4)}$$

The operators Λ and J_3 commute with $P(\rho)$. The fact that Λ commutes with $P(\rho)$ is quite general. On the other hand, the zero commutator of J_3 with $P(\rho)$ is a consequence of the fact that $\phi(x)$ is a spinless field. In the case of quarks endowed with spin, J_3 will have non-vanishing commutators with the relevant operators.

We notice at this point that, since we are working in the case of massless quarks oriented along the same direction, the value of the total angular momentum will coincide with that of its component along the direction of motion. This means that the spin of the composite particles will have no orbital component and will only originate from the spin of the quarks.

So in the present unrealistic case of spinless quarks we shall thus deal with spinless composite particles.

We wish to define now a new quantum number which will allow us to classify the different states that can be obtained in terms of collinear quarks.

The operator W associated with such quantum number should be diagonalizable at the same time as the momentum and should have the same eigenvalue independently of the Lorentz frame we are using.

In other terms

$$[W, L_0] = 0$$

$$[W, L_+] = 0 \tag{III.5}$$

Such an operator is provided by the Casimir operator of the $O(2,1)$ group:

$$W = L_0^2 - \frac{1}{2}(L_+L_- + L_-L_+) \tag{III.6}$$

Then we can classify the single particle compound states by means of the eigenvalues of the Casimir operator.

In other words, we require that the compound field belong to an irreducible representation of $O(2,1)$:

$$[L_+, \chi(\rho)] = \frac{d\chi}{d\rho}$$

$$[L_0, \chi(\rho)] = \left(\rho\frac{d}{d\rho} + \lambda\right)\chi(\rho)$$

$$[L_-, \chi(\rho)] = \left(\rho^2\frac{d}{d\rho} + 2\lambda\rho\right)\chi(\rho) \tag{III.7}$$

We can now obtain the general expression for the operator $\chi(\rho)$ associated with a compound state.

The most general operator constructed out of the quark field whose dimensionality is equal to N, is given by

$$[P(\rho)]^{n_1}\left[\frac{dP(\rho)}{d\rho}\right]^{n_2}\left[\frac{d^2P(\rho)}{d\rho^2}\right]^{n_3}\cdots\cdots \tag{III.8}$$

with the condition

$$n_1 + 2n_2 + 3n_3 + \cdots \quad = N \tag{III.9}$$

The number of these operators is given by the partition function of N objects; we denote it by T(N). We have to take an arbitrary linear combination of them and impose the correct commutation relations (III.7) The number of operators of the type (III.8) which are covariant under O(2,1) transformations with $\lambda = N$ is then given by

$$P(N) = T(N) - T(N-1) \tag{III.10}$$

The action of the operator W on such states which we denote by $|\psi, \lambda\rangle$ is given by

$$W\, |\psi, \lambda\rangle = \lambda(\lambda - 1)\, |\psi, \lambda\rangle \tag{III.11}$$

The degeneracy of physical states for large values of λ is given by the asymptotic expression of the partition function:

$$d(\lambda) \sim e^{\frac{2\pi}{\sqrt{6}}\sqrt{\lambda}} \tag{III.12}$$

If we now make the formal substitution

$$\lambda \longrightarrow b m^2 \tag{III.13}$$

we recover the exponential degeneracy of the spectrum given by the D.R.M. The "correspondence principle" (III.12) leads also to a quantization for integer values of the physical particles as we would expect from a model based on linear Regge trajectories.

We see that in our extreme "collinear" model we lose track of the mass operator which is not important for consideration valid at short distances; however, from our approach we get a new quantum number λ which seems to enjoy properties very similar to the mass of the physical states in the DRM.

To further investigate this analogy, we have to go behind this extremely unrealistic model in which both the quarks and the compound states are spinless. To this purpose in next section we will discuss the case of a spin $\frac{1}{2}$ quark field.

In order to gain some insight on the physical consequences of our model we conclude this section discussing the properties of the electromagnetic structure function in our model.

Let us consider the matrix element of the bilocal operator evaluated on the light cone, between states of one compound particle of momentum p, directed along the direction $\vec{u}_z$:

$$< P \,|\, :\left(\phi(x)\,\phi(y)\right)_{L.C.}:\, | P > \qquad \text{(III.14)}$$

The Fourier transform of this matrix element gives the momentum distribution of the parton inside the compound particle and, when generalized to a more realistic case, is related to the structure functions measured in deep inelastic electron scattering. The state $<p|$ is given in terms of the field of the compound particle $\chi_\lambda(\rho)$ by:

$$< P, \lambda | = C(P) \int_{-\infty}^{\infty} < 0 | \chi_\lambda(\rho)\, e^{ip\rho} d\rho \qquad \text{(III.15)}$$

where the constant $C(p)$ has to be fixed through the normalization requirements. For the first few levels the field $\chi_\lambda(\rho)$ is uniquely determined by our conformal requirement, so that the matrix element (III.14) can be evaluated. As an example, we can consider the most general states which are coupled to two partons. The structure of these fields is the following:

$$\Psi_n = \, : \sum_{k=0}^{n-2} c_k \frac{d^k P}{d\rho^k} \, \frac{d^{(n-2-k)} P(\rho)}{d\rho^{n-2-k}} \, : \qquad \text{(III.16)}$$

where $n = 0, 2, 4, \ldots$ and

$$c_k = (-1)^k \frac{(n-1)(n-k-1)}{k!\,(k+1)!} \left[\frac{\Gamma(n-1)}{\Gamma(n-k)} \right]^2 c_0 \tag{III.17}$$

Following the same line of Appendix C of Ref. 5) we find that the structure function associated with the field ψ_n is given by

$$f_n(\omega) \sim (1-\omega)\,\omega^2\, F(2-n, n+1; 2; 1-\omega) \tag{III.18}$$

Recalling that

$$F(2-n, n+1; 2; 0) = F(2-n, n+1; 2; 1) = 1 \tag{III.19}$$

we see that the behaviour near the points $\omega \sim 0$ and $\omega \sim 1$ is universal for such states.

We can also consider the states corresponding to the compound fields $:P^n(\rho):$. Their structure functions are the following:

$$F_n(\omega) \sim (1-\omega)^{2n-3}\,\omega^2 \tag{III.20}$$

The above examples give an idea of the validity of our model. The structure functions that we obtain are quite reasonable for $\omega \sim 1$, but they fall off too fast when $\omega \to 0$. This problem is common to all parton models with a finite number of partons and clearly suggests that our collinearity assumption is too strong for the "wee" partons that contribute at $\omega \sim 0$.
We notice that the behaviour near $\omega \sim 0$ of the structure functions (III.19) and (III.21) is the same; for a general compound state $|\psi>$ the structure function behaves near $\omega \sim 0$ as

$$\omega^{2+2j}$$

where j is the smallest derivative of the quark field present in $|\psi>$.

For $\omega \sim 1$ the structure function of a general compound state $|\psi>$ behaves:

$$(1-\omega)^{2n-3+2i}$$

where n is the number of quarks of which $|\Psi>$ is made and i is the highest derivative of the quark field present in $|\Psi>$.

IV. SPIN $\frac{1}{2}$ QUARKS

In this Section we discuss the main results of our model, concerning the particle spectrum and the structure functions, in the more realistic case of spin $\frac{1}{2}$ quarks.

We start with the free massless Dirac equation for the quark field

$$\gamma_\mu \frac{\partial}{\partial x_\mu} \psi(x) = 0 \tag{IV.1}$$

We will write for $\Psi(x)$ an unspecified Poisson bracket, without committing ourselves to a Fermi or a Bose statistics

$$\left[\Psi_\alpha^+(x), \Psi_\beta(x') \right]_{t=t'} = \delta^{(3)}(\vec{x}-\vec{x}')\, \delta_{\alpha\beta} \tag{IV.2}$$

As in Section II we perform the "velocity" decomposition (II.2)

$$\Psi_\pm(x) = \int d\Omega \int_{-\infty}^{\infty} d\rho\, \delta'(t-\vec{x}\cdot\vec{u}-\rho)\, g_\pm(\rho,\vec{u})\, \xi^\pm(\vec{u}) \tag{IV.3}$$

where

$$\Psi_\pm(x) = \frac{1}{2}(1 \pm i\gamma_5)\Psi(x) \tag{IV.4}$$

and $\xi^\pm(\vec{u})$ are normalized spinors obeying the equation

$$\xi^\pm(\vec{u}) = \pm(\vec{\sigma}\cdot\vec{u})\,\xi^\pm(\vec{u}) \tag{IV.5}$$

The reasons for the appearance of $\delta'(t-\vec{x}\cdot\vec{u}-\rho)$ in Eq. (IV.3), rather than a $\delta(t-\vec{x}\cdot\vec{u}-\rho)$ as in Eq. (II.2) are discussed thoroughly in Appendix A of Ref. 5).

As in the scalar case, it is useful to average the operator $g_\pm(\rho,\bar{u})$ around a given direction $\vec{u}_i$ and we define:

$$S_\pm^{(i)}(\rho) = \frac{1}{\Delta\Omega}\int_{\Delta\Omega} g_\pm(\rho,\vec{u})\,d\Omega \tag{IV.6}$$

Then Eq. (IV.3) reads

$$\Psi_\pm(x) = \sqrt{\Delta\Omega}\,\sum_i \frac{dS_i(\rho)}{d\rho} \tag{IV.7}$$

Using Eqs. (IV.2) and (IV.7) we obtain the following Poisson brackets for the operators $S_\pm^i(\rho)$:

$$\left[S_\pm^{(i)}(\rho), S_\mp^{(j)}(\rho')\right] = 0$$

$$\left[S_\pm^{(i)}(\rho), S_\mp^{+(j)}(\rho')\right] = 0$$

$$\left[S_\pm^{(i)}(\rho), S_\pm^{(j)}(\rho')\right] = 0$$

$$\left[S_\pm^{(i)}(\rho), S_\pm^{+(j)}(\rho')\right] = \frac{1}{4\pi^2}\,\delta(\rho-\rho')\,\delta_{ij} \tag{IV.8}$$

The non-vanishing Green functions are:

$$<0|S_\pm^{(i)}(\rho)S_\pm^{+(j)}(\rho')|0> = -\frac{i}{(2\pi)^3}\frac{1}{\rho-\rho'-i\varepsilon}\,\delta_{ij} \tag{IV.9}$$

The commutators of $S_\pm(\rho)$ with the fundamental operators J_z, L_o, $L_\pm$ are:

$$\left[J_z, S_\pm(\rho)\right] = \pm S_\pm(\rho)$$

$$\left[L_+, S_\pm(\rho)\right] = \frac{d}{d\rho} S_\pm(\rho)$$

$$\left[L_o, S_\pm(\rho)\right] = \left(\rho\frac{d}{d\rho} + \frac{1}{2}\right) S_\pm(\rho)$$

$$\left[L_-, S_\pm(\rho)\right] = \left(\rho^2\frac{d}{d\rho} + \rho\right) S_\pm(\rho) \tag{IV.10}$$

We see that $S_{\pm}(\rho)$ belongs to a representation $\lambda = \frac{1}{2}$ of $O(2,1)$. In order to obtain the structure of the compound states we can now proceed as in the scalar case.

The most general compound field of conformal quantum number λ is a superposition with appropriate coefficients of terms of the type:

$$S_+^{m_1}\left[\frac{dS_+}{d\rho}\right]^{n_2}\cdots\cdots\; S_-^{m_1'}\left[\frac{dS_-}{d\rho}\right]^{n_2'}\cdots\cdots$$
$$\cdots\; S_+^{+m_1}\left[\frac{dS_+^+}{d\rho}\right]^{m_2}\cdots\cdots\; S_-^{+m_1'}\left[\frac{dS_-^+}{d\rho}\right]^{m_2'} \tag{IV.11}$$

where

$$\lambda = m_1 + 2n_2 + 3n_3 + \cdots + n_1' + 2n_2' + \cdots$$
$$+ m_1 + 2m_2 + 3m_3 + \cdots + m_1' + 2m_2' + 3m_3' + \cdots \tag{IV.12}$$

The coefficients of the superposition are fixed by the requirement that the compound field be covariant under the group $O(2,1)$.

Then, we again obtain, for large values of λ, the Hagedorn exponential increase of levels. Since we are dealing with quarks having a spin different from zero, we can now derive some interesting properties concerning the spin of excited levels. It is easy to see, in fact, that the spin of the compound states will never exceed λ, and that the most probable value of the spin is proportional to $\sqrt{\lambda}$ for large values of λ.

In order to obtain more detailed information on the compound particle states in terms of the quark field, we still lack two main ingredients:

1) a precise commitment to the quark statistics;
2) the introduction of internal quantum numbers.

Then, in order to calculate the electron-proton structure function, we introduce internal quantum numbers for the field $S_{\pm}(\varsigma)$ and define

$$S_{\pm}(\varrho) \equiv \begin{pmatrix} U_{\pm}(\varrho) \\ D_{\pm}(\varrho) \end{pmatrix} \tag{IV.13}$$

whose $U_{\pm}(\varrho)$ and $D_{\pm}(\varrho)$ are respectively fields corresponding to the value $\pm\frac{1}{2}$ of the third component of the isotopic spin.

We also assume the validity of an SU(6) scheme and the fact that the quarks satisfy the Fermi statistics.

Then the nucleon and the $N^{*}_{\frac{3}{2}\frac{3}{2}}$ resonance have the same space structure. The lowest possible value of λ allowed is then $\lambda = \frac{9}{2}$ corresponding to the $N^{*}_{\frac{3}{2}\frac{3}{2}}$ field

$$:\, U_{+}(\varrho)\, \dot{U}_{+}(\varrho)\, \ddot{U}_{+}(\varrho)\, :$$

We can now build two expressions having the correct commutation relations with L_{-}, both corresponding to the values $J = \frac{1}{2}$, $T_3 = \frac{1}{2}$, namely:

$$K(\varrho) = :\, U_{+}(\varrho)\dot{U}_{+}(\varrho)\ddot{D}_{-}(\varrho) + \ddot{U}_{+}(\varrho)U_{+}(\varrho)\dot{D}_{-}(\varrho) + \dot{U}_{+}(\varrho)\ddot{U}_{+}(\varrho)D_{-}(\varrho)\, : \tag{IV.14}$$

and

$$\begin{aligned} H(\varrho) = :\, & U_{+}\dot{U}_{-}\ddot{D}_{+} + \ddot{U}_{+}U_{-}\dot{D}_{+} + \dot{U}_{+}\ddot{U}_{-}D_{+} + \\ & + U_{-}\dot{U}_{+}\ddot{D}_{+} + \ddot{U}_{-}U_{+}\dot{D}_{+} + \dot{U}_{-}\ddot{U}_{+}D_{+}\, : \end{aligned} \tag{IV.15}$$

The linear combination

$$2\,K(\varrho) - H(\varrho) \tag{IV.16}$$

is the desired pure isotopic spin $\frac{1}{2}$ which is identified with the proton field.

Using techniques similar to the ones discussed in Appendix C of Ref. 5) we can now explicitly calculate the structure function $F_2^{ep}(\omega)$. The behaviour of such function for $\omega \sim 0$ and $\omega \sim 1$ is the following:

$$F_2^{ep}(\omega) \sim \omega^2 \qquad \omega \to 0$$

$$F_2^{ep}(\omega) \sim (1-\omega)^3 \qquad \omega \to 1 \tag{IV.17}$$

Had we started with a Bose quark field we would have obtained the following proton field:

$$: U_+^2 D_- - U_+ U_- D_+ : \tag{IV.18}$$

and the following behaviour for the structure function:

$$F_2^{ep} \sim \omega^2 \qquad \omega \to 0$$

$$F_2^{ep} \sim 1-\omega \qquad \omega \to 1 \tag{IV.19}$$

V. CONCLUSIONS

To conclude we summarize the results we have obtained so far. We have introduced a conformal quantum number λ to classify our states. In terms of λ we have the following properties:

1) the conformal quantum number λ takes only integer values;
2) the states corresponding to small values of λ are non-degenerate;
3) for large values of λ the degeneracy increases exponentially;

$$N(\lambda) \approx e^{\frac{2\pi}{\sqrt{6}}\sqrt{\lambda}} \tag{V.1}$$

4) the spin of the compound particle cannot be larger than

$$J \leq \lambda \tag{V.2}$$

5) the average value of the spin for large values of λ is proportional to $\sqrt{\lambda}$. All those results are translated into well-known properties of the dual models if one introduces the "correspondence principle" ;

$$\lambda \sim b m^2 \tag{V.3}$$

The constant b present in Eq. (V.3) introduces a fundamental length which in the dual models is related to the universal slope of the Regge trajectories.

We see then, that in our "asymptotic" model, the mass of the compound particles can only be obtained "a posteriori" on the basis of the "correspondence principle" [Eq. (V.3)]. This is clearly due to the fact that the quark fields have been studied essentially from the point of view of their "light-cone" properties, so that they have been taken to be massless and collinear

This hypothesis of extreme collinearity seems to be a very weak point in our model. As we have already discussed, this hypothesis prevents the introduction of "wee partons" in our model. As a consequence, we obtain structure functions - for example in the "realistic" spin $\frac{1}{2}$ case - that although quite good at $\omega \sim 1$, are in disagreement with any reasonable Regge model in the asymptotic limit $\omega \to 0$. This disease is common to all parton models with only optical quarks.

We can conclude by saying that both the program to recover the main features of the DRM starting from the light-cone approach and the one of obtaining information on the "parton distribution" inside the nucleon have been only partially successful. Any further progress seems clearly to be bound to a reformulation or a weakening of the collinearity assumption.

REFERENCES

1) See for instance:
G. Veneziano, Lectures given at the Erice Summer School (1970);
S. Fubini, Lectures given at Les Houches Summer School (1971).

2) See for instance Ref. 5) in our previous paper.

3) R. Brower, MIT preprint CTP 277 (1972);
P. Goddard and C. Thorn, CERN preprint TH.1493 (1972).

4) E. Del Giudice, P. Di Vecchia and S. Fubini, Ann.Phys. 70, 378 (1972).

5) E. Del Giudice, P. Di Vecchia, S. Fubini and R. Musto, MIT preprint CTP 271 (1972). To be published in Nuovo Cimento.