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# LIGHT ELEMENTS AND THE ISOTROPY OF THE UNIVERSE

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SUMMARY

Nucleosynthesis in homogeneous Bianchi-type universes admitting Robertson–Walker solutions is investigated. The requirement that helium and deuterium are synthesized cosmologically in the abundance range observed is shown to constrain the ratio of the shear to the expansion rate, $(\sigma/\theta)_0$, of the Universe today more severely than does the measured isotropy of the microwave background radiation. Rotation is shown to have a negligible effect upon nucleosynthesis and in particular if the Universe were Bianchi I then the abundances $X(\mathrm{He}^4) = 0{\cdot}29 \pm 0{\cdot}04$, $X(\mathrm{D}) \sim 2.10^{-5}$ indicate that $(\sigma/\theta)_0 \lesssim 4{\cdot}8.10^{-12}$. The local shear induced by large scale density perturbations is also considered together with some qualitative discussion of the physical effects of inhomogeneity.

## 1. INTRODUCTION

It is the aim of this paper to evaluate the constraints that can be placed upon the shear of the Universe today by the requirement that helium and deuterium are synthesized cosmologically in the abundance ranges observed. The permitted anisotropy is modelled by Bianchi types of spatially homogeneous universes which admit Robertson–Walker (R–W) models. Those Bianchi types which do not admit R–W solutions become unrealistically anisotropic at large times. The first quantitative estimates of the deviation of the Universe from isotropic, irrotational, shear-free behaviour were given by Kristian & Sachs (**1**) from counts of galaxies, fairly weak limits of $(\sigma/\theta)_0 \lesssim 0{\cdot}3$ and $(\omega/\theta)_0 \lesssim 1$ for the present ratios of the shear and rotation to the expansion rate being obtained. Hawking (**2**) and Hawking & Collins (**3**) considerably improved upon these limits by an analysis of the large scale microwave background (M–W) temperature isotropy, $\Delta T/T \lesssim 3.10^{-3}$ in isotropizing Bianchi-type universes. The most favourable cases gave limits of $(\sigma/\theta)_0 \lesssim 6.10^{-8}$ and $(\omega/\theta)_0 \lesssim 10^{-11}$ in Bianchi types I and IX respectively, which constitute the simplest generalizations of the flat and closed R–W models if the M–W radiation was last scattered at $z_s \sim 1000$. These limits indicate that, on large scales, the Universe has evolved remarkly like a R–W model at least since the last scattering of the radiation field.

The possibility of synthesizing light elements in the early hot stage of the Universe was first realized by Gamow and his co-workers (Alpher *et al.* (**4**)). Detailed calculations were performed by Peebles (**5**) and Wagoner *et al.* (**6**) in R–W models. The striking confirmation of their predictions in the case of helium, $X(\mathrm{He}^4) \sim 0{\cdot}3$ and deuterium, $X(\mathrm{D}) \sim 2.10^{-5}$ by mass in open R–W models both by solar system and interstellar measurements in the last 2 yr, Reeves (**7**), together with the lack of good sites for the local production of these light elements must be considered as very good evidence that the Universe has been close to a R–W model in its large scale evolution since $\sim 1$–$1000$ s after the initial singularity at least.

The effects of anisotropy upon the synthesis of helium in the early Universe were first examined semi-quantitatively by Hawking & Tayler (**8**) and later in some detail by Thorne (**9**), using the programs of Wagoner, in the simplest anisotropic cases. Peebles (**5**) and Wagoner (**10**) have also computed the effect of a constant scaling of the R–W expansion time scale on the light element abundances. It will be shown that this factor can be related to the dimensionless quantity $\sigma/\theta$ at the time of nucleosynthesis, $t_{es}$, and limits derived at $t_{es}$ from the present helium abundance range. By using the appropriate evolution equations this limit is extrapolated to the present day assuming that it is not augmented in any way by other processes; the dynamics of the Universe changing from radiation to matter domination at $t_{eq} = 10^{10}\,\Omega^{-2}$ s, where $\Omega$ is the usual density parameter. No calculation is made of the effects of possible and probable isotropizing agents like neutrino viscosity, particle creation or Mixmaster oscillations (Misner (**11**), Zeldovich (**12**)) since nucleosynthesis occurs at a time when these processes are no longer important. A limit is therefore obtained on the magnitude of shearing motions at $t_{es}$ following which only gravitational fields act as isotropizing agents, since at low densities the relaxation times for dissipative processes are very short and the entropy per particle remains virtually constant.

Section 2 gives a brief geometrical description of the Bianchi-type homogeneous models used; Section 3 considers the effects of shear, rotation and pressure gradients upon nucleosynthesis; Section 4 the evolution of shear in Bianchi models and Section 5 indicates the relation of these calculations to more inhomogeneous models of the Universe.

## 2. HOMOGENEOUS BIANCHI UNIVERSES

The Bianchi-type universes are a class of spatially homogeneous models with metric form $g_{00} = 1$, $g_{0i} = 0$ whose spatial sections are described by one of the three-dimensional spaces with groups of motions classified by Bianchi (**13**). Defining three invariant vectors fields $E_a{}^\alpha$ on the homogeneous surfaces dual to $E_\alpha{}^a$ such that $E_\alpha{}^a E_b{}^\alpha = \delta_b{}^a$ the metric is given in this basis by

$$ds^2 = dt^2 - \gamma_{ab}(t)\, E_\alpha{}^a E_\beta{}^b\, dx^\alpha\, dx^\beta \tag{2.1}$$

where $E_\alpha{}^a E_b{}^\beta (E_{\alpha,\beta}{}^c - E_{\beta,\alpha}{}^c) = C_{ab}{}^c = C_{[ab]}{}^c$ and $C_{[bc}{}^a C_{ef]}{}^d = 0$. The models are classified into various Bianchi types by the ten equivalence classes of structure constants $C_{ab}{}^c$ generated by the freedom of basis choice up to a general linear transformation $E_a{}'^\alpha = L_a{}^b E_b{}^\alpha$. These models have been described in detail by Ellis & MacCallum (**14**) and MacCallum (**31**). The following evolutionary characteristics have a bearing on what follows.

(i) In some types the effects of matter fields and spatial curvature cause shearing motions to decay and there is an approach to R–W behaviour at late times. As conjectured first by MacCallum and proved by Collins & Hawking (**15**) this can only occur in those Bianchi types which admit R–W models as a special case. These are: types I and VII$_0$ admitting $k = 0$, V and VII$_h$ admitting $k = -1$ and IX admitting $k = +1$; where $k$ is the curvature constant of the R–W models.

Types I and V necessarily have isotropic spatial curvature whereas, although the others admit cases with the curvature isotropic, in general it will be anisotropic and hinder the adiabatic decay of the shear.

(ii) Heckmann & Schucking (**16**) first examined the type I and V models, whilst recently Doroshkevich *et al.* (**17**) have investigated types $VII_0$, $VII_h$, IX in detail extending the perturbative analysis of Collins & Hawking (**15**). They consider the evolution to divide into three distinct eras; the vacuum stage where matter is dynamically negligible, types I, V, $VII_0$ and $VII_h$ have a Kasner-like behaviour but type IX displays an oscillatory behaviour caused by the large anisotropy in the spatial curvature. This is followed by the isotropization stage characterized by the decrease of $\sigma/\theta$ with time, nucleosynthesis occurs during this period which in types I, V and $VII_0$ extends to the present-day, $t_0$. Finally there can exist a time $t_m$ following which gravity ceases to be dynamically important, the matter having effectively escaped from its own gravitational field. This is called the Milne stage and occurs in type $VII_h$ at a time determined by the parameter $h$ and in type IX where it is of no interest to us since it begins at the time of maximum volume in these recollapsing models.*

## 3. NUCLEOSYNTHESIS

In hot models of the early Universe a state of thermal equilibrium is maintained when $T \sim 10^{11}$ K. The helium abundance is essentially determined by the value of the neutron–proton ratio, $n/p$ frozen in by the equality of the expansion time scale $\tau$ and the equilibrium mediating weak interaction time scale. Usually the final abundance of helium increases as $\tau$ decreases because the neutron is more abundant in the earlier stage and element synthesis is induced at an earlier time. The helium abundance becomes smaller again if $\tau$ decreases below the time scale of the helium-producing nuclear reactions. To consider the relative effects of the various kinematical quantities upon the crucial parameter, the expansion time scale of the Universe, we use the formalism of Ehlers (**18**) and Ellis (**19**).

Consider a perfect fluid with stress energy tensor $T_{ab} = U_a U_b + p h_{ab}$, where $\mu$ is the energy density, $p$ the pressure, $U_a$ the fluid velocity normalized such that $U_a U^a = -1$ and $h_{ab} = g_{ab} + U_a U_b$ the projection tensor into the 3-spaces orthogonal to $U^a$. By using the definition of the curvature tensor and the field equations we obtain a propagation equation for $\theta$ (Raychaudhuri (**20**))

$$\dot{\theta} + \frac{\theta^2}{3} = 2(\omega^2 - \sigma^2) - \tfrac{1}{2}(\mu + 3p) + \dot{u}_a{}^{;a} \qquad (3.1)$$

where $\theta \equiv u_a{}^{;a} = \dot{V}/V$ is the expansion rate scalar, $V$ the comoving volume, $\sigma$ the shear scalar, $\omega$ the rotation scalar and $\dot{u}_a = u_{a;b}u^b$ the acceleration. It is clear that there are three factors causing deviations from the R–W time scale obtained when $\sigma = \omega = \dot{u}_a = 0$. These are:

(i) A 'centrifugal energy density' $\omega^2$ causing expansion of the fluid flow lines, lengthening the time scale and acting as a negative energy density.

(ii) An 'anisotropy energy density' $\sigma^2$ causing contraction of the fluid flow lines, shortening the expansion time scale and acting as a positive energy density.

* No proof has actually been found to show that all type IX models recollapse, although the existing evidence seems to indicate that they all do (Matzner *et al.* (**36**)).

(iii) The divergence of the acceleration $\dot{u}_a{}^{;a}$ which can act as an energy density of either sign. This is induced by pressure gradients as can be explicitly seen by using the conservation equations $T_{ab}{}^{;b} = 0$ to obtain

$$\dot{u}_a{}^{;a} = \frac{1}{(\mu+p)^2}[\dot{p}+p^{,a}(p+\mu)_{,a}] - \frac{1}{(\mu+p)}[\Box^2 p + \ddot{p} - 2\dot{p}\theta]$$

where $\Box^2$ is the generalized D'Alembertian operator.*

The acceleration necessarily vanishes when $p = 0$ and is negligible compared with the other terms in (3.1) in the large class of velocity-dominated models of Eardley *et al.* (**21**). In the irrotational case there exist $\{t = \text{constant}\}$ 3-spaces orthogonal to $U_a$ with metric $h_{ab}$ and it can be shown (Ellis (**19**)) that for zero cosmological constant,

$$^3R_{ab} - \tfrac{1}{3}{}^3Rh_{ab} = h_a{}^f h_b{}^g[-V^{-1}(V\sigma_{fg}) + \dot{u}_{(f;g)}] + \dot{u}_a\dot{u}_b - \tfrac{1}{3}h_{ab}\dot{u}_{;c}{}^c \qquad (3.2)$$

is the trace-free part of the Ricci tensor of these 3-spaces. So in the case of isotropic space curvature and $\dot{u}_a = 0$ we have $\sigma \propto V^{-1}$. Such an expression permits the integration of (3.1) giving the generalized Friedmann energy equation

$$\theta^2 = 3\left(\sigma^2 + \mu - \frac{^3R}{2}\right). \qquad (3.3)$$

The influence of the curvature terms is negligible in the early period of evolution under consideration since the others terms on the right-hand side of (3.3) are $O(V^{-2})$ and $O(V^{-4/3})$, so the energy equation reduces to

$$\theta^2 = 3(\sigma^2 + \mu). \qquad (3.4)$$

We shall just consider the effect of shear on the dynamics at $t_{es}$ since one expects the influence of rotation to be negligible for the following reason. Any rotation will be governed by the conservation of angular momentum

$$\mu V^{5/3}\omega = \text{constant}$$

so the centrifugal energy density $\omega^2 \propto V^{-2/3}$ is negligible compared with the radiation density $\mu \propto V^{-4/3}$ and anisotropy energy density $\sigma^2 \propto V^{-2}$ at $t \sim t_{es}$. (The Bianchi I models are in any case necessarily irrotational.) This means that no limits can be derived for the rotation of the Universe from element synthesis analysis; they are best obtained from the M–W isotropy measurements, Collins & Hawking (**3**). However, in models that have no horizon in a particular direction the influence of rotation can become increasingly important, for example in the simple Kasner case, with orthogonal scale factors

$$R_i \propto t^{P_i}, \quad \sum_i P_i = \sum_i P_i^2 = 1,$$

which implies $p_1 \in [-\frac{1}{3}, 0]$, $p_2 \in [0, \frac{2}{3}]$, $p_3 \in [\frac{2}{3}, 1]$ the conservation of angular momentum must be expressed as $\mu R_i{}^5\omega = \text{const}$. The relative alignment of the principal axes of shear and rotation will be important, in particular along the 1-axis we have $\omega^2 \propto t^n$, $n \geqslant 0$, and so the model is unstable to the formation of rotational motions and their associated peculiar velocities along the axis on which no horizon is present. Also in the presence of shock waves caused by non-linear fluid motions and density irregularities rotation could be generated from irrotational motions and is not necessarily governed by the conservation of angular momentum.

* These considerations apply to inhomogeneous as well as homogeneous models.

Wagoner considers deviations from the R–W time scale by introducing the relative rate factor, $\xi$, defined by

$$\theta = \xi\sqrt{3\mu} \qquad (3.5)$$

where $\theta_0 = \sqrt{3\mu}$ is the R–W expansion rate.

Very large $\xi$ factors $\sim 10^4$, corresponding to anisotropy domination and 'Kasner-like' behaviour which can give helium abundances $\sim 30$ per cent are ruled by the very large ($\sim 10$ per cent) deuterium abundances they produce and Reeves, (**7**), gives $\xi = 1 \pm 0{\cdot}3$ as compatible with the $29 \pm 4$ per cent helium abundance range of Peimbert (**22**). Since the final abundances are fused over a very small temperature interval, $T \sim 5.10^9$–$10^9$ K, the rate is almost independent of $\xi$ as a function of density in this range as far as the helium synthesis is concerned. Using (3.4), (3.5) and $\xi \leqslant 1{\cdot}3$ we obtain

$$\left(\frac{\sigma}{\theta_0}\right)_{\text{es}} \leqslant 0{\cdot}48$$

at freeze-in of $n/p$ ratio at $t_{\text{es}} \sim 100$ s, say (from now on we write $\theta_0 \equiv \theta$ for the R–W expansion rate and the subscript will denote the value of a quantity at the present day).

## 4. ISOTROPIZATION

The subsequent evolution of the shear is now considered, taking $t_{\text{eq}} \simeq 10^{10}\ \Omega^{-2}$ as the time when the dynamics of the Universe ceased to be radiation dominated, an immediate transition to complete matter domination after $t_{\text{eq}}$ is assumed; and $t_0 \simeq 10^{17}$ s the present day. Representative densities of $\Omega = 1$ for types I, VII$_0$, IX and $\Omega = 10^{-2}$ for V, VII$_h$ are taken. The simplest cases with isotropic space curvature behave as follows

$$t_{\text{es}} \lesssim t < t_{\text{eq}};\ p = \frac{\mu}{3} \qquad t > t_{\text{eq}};\ p = 0$$

Type I

$$V \propto t^{3/2},\ \theta \propto t^{-1},\ \frac{\sigma}{\theta} \propto t^{-1/2} \qquad V \propto t^2,\ \theta \propto t^{-1},\ \frac{\sigma}{\theta} \propto t^{-1}$$

Type V

$$V \propto t^{3/2},\ \theta \propto t^{-1},\ \frac{\sigma}{\theta} \propto t^{-1/2} \qquad V \propto t^3,\ \theta \propto t^{-1},\ \frac{\sigma}{\theta} \propto t^{-2}.$$

The limit of $\sigma/\theta \lesssim 0{\cdot}48$ at $t_{\text{es}}$ gives

Type I

$$\left(\frac{\sigma}{\theta}\right)_{\text{eq}} \lesssim 4{\cdot}8.10^{-6}\, t_{\text{es}}^{1/2}; \qquad \left(\frac{\sigma}{\theta}\right)_0 \lesssim 4{\cdot}8.10^{-13}\, t_{\text{es}}^{1/2}$$

Type V

$$\left(\frac{\sigma}{\theta}\right)_{\text{eq}} \lesssim 4{\cdot}8.10^{-8}\, t_{\text{es}}^{1/2}; \qquad \left(\frac{\sigma}{\theta}\right)_0 \lesssim 4{\cdot}8.10^{-14}\, t_{\text{es}}^{1/2}.$$

These are considerably better limits than those obtained by analysis of the M–W isotropy measurements. The type V evolution considered is extreme for $t > t_{\text{eq}}$

and a more realistic transition giving an averaged evolution of $\sigma/\theta \propto t^{-3/2}$ gives $(\sigma/\theta)_0 \lesssim 1 \cdot 5 . 10^{-12}\, t_{es}^{1/2}$. So taking $t_{es} \sim 100$ s we have

Type I:

$$\left(\frac{\sigma}{\theta}\right)_0 \lesssim 4\cdot 8 . 10^{-12}$$

Type V:

$$\left(\frac{\sigma}{\theta}\right)_0 \lesssim 1\cdot 5 . 10^{-11}.$$

We now consider the more general types $\mathrm{VII}_0$, $\mathrm{VII}_h$, IX using the solutions of Doroshkevich *et al.* (**17**). The discussion of shear evolution in these models provokes a comment on the definitions of isotropization used by Collins & Hawking (**15**) in their perturbative analysis of Bianchi-type models who required (i) $V \to \infty$, (ii) $\sigma/\theta \to 0$, and (iii) $\beta_{ij} \to$ constant as $t \to \infty$ where $\beta_{ij}$ is the symmetric trace-free part of the metric(2.1) and is related to the shear tensor by $\sigma_{ij} = (e^{\beta})_{k(i}(e^{-\beta})_{j)k}$ and in general $\beta_{ij} \sim \int \sigma_{ij}\, dt$ to first approximation. (i) Eliminates from consideration type IX models which recollapse, (ii) eliminates all models in which $\sigma/\theta \to$ constant $\ll 1$ to be considered as isotropizing even though they can adequately describe the Universe today; (iii) is a statement about the cumulative distortion of the M–W background because its anisotropy measures the overall distortion since the last scattering surface. The conditions (i), (ii) and (iii) are neither necessary nor sufficient to guarantee that a model gives a good description of the Universe as we see it today.

Using metric form (2.1) the structure constants are given by

$$C_{23}{}^1 = 1; \qquad C_{21}{}^3 = C_{31}{}^2 = -\alpha; \qquad C_{13}{}^1 = C_{23}{}^2 = h$$

where $\alpha = 0$ in $\mathrm{VII}_0$ and $\mathrm{VII}_h$ and $h = 0$, $\alpha = 1$ in IX. In the isotropization era when the influence of the Milne curvature is negligible $\mathrm{VII}_0$, $\mathrm{VII}_h$ and IX models evolve similarly, the volume expansion being R–W like plus small perturbations. Element synthesis occurs when approximately

$$\mu = \frac{3}{4t^2}\left(1 - \frac{1}{\ln(At)}\right), \qquad A = \text{constant}$$

and subsequently

$$\frac{\sigma}{\theta} \propto \frac{1}{\ln(At)} \qquad \text{for} \qquad t < t_{eq},\ p = \frac{\mu}{3}$$

and

$$\frac{\sigma}{\theta} \propto \frac{1}{t^{2/3}} \qquad \text{for} \qquad t > t_{eq},\ p = 0.$$

The anisotropy in the spatial curvature also decays logarithmically, equation (3.2). As before we calculate

$$\left(\frac{\sigma}{\theta}\right)_{es} \lesssim 0\cdot 5 \qquad \text{for} \qquad \xi \leqslant 1\cdot 3.$$

The final Milne era, $t > t_m$ in type $\mathrm{VII}_h$ has

$$\frac{\sigma}{\theta} = \text{constant}, \qquad t > t_m.$$

The distortion is frozen in because of the weakness of the gravitational field at late times. Hence the value of $(\sigma/\theta)_0$ in VII$_h$ models will depend upon the arbitrary parameter $t_m$. We find for VII$_0$ and IX models ($\Omega = 1$) taking $t_{es} \sim 100$ s.

$$\left(\frac{\sigma}{\theta}\right)_{eq} \lesssim 3\cdot5 . 10^{-2} \quad \text{and} \quad \left(\frac{\sigma}{\theta}\right)_0 \lesssim 8\cdot0 . 10^{-7}$$

varying only by a factor $\sim 3$ for $t_{es} \sim 1\text{–}10^3$ s. And in VII$_h$ ($\Omega = 10^{-2}$) models

$$\left(\frac{\sigma}{\theta}\right)_{eq} \lesssim 3\cdot5 . 10^{-2} \quad \text{and} \quad \left(\frac{\sigma}{\theta}\right)_0 \lesssim 4\cdot5 . 10^{-5} \left(\frac{10^{17}}{t_m}\right)^{2/3}.$$

These limits can be compared with the results of Collins & Hawking (**3**) and also with an analysis of the M–W isotropy using the solutions of Doroshlevich *et al.* (**17**). The latter find for types VII and IX that,

$$\left(\frac{\Delta T}{T}\right)_{max} \sim \frac{8}{\ln (At_{eq})} \left(\frac{t_{eq}}{t_s}\right)^{2/3} \tag{4.1}$$

when $t_s$ is time of last scattering of the radiation. In the standard theory $z_s \sim 10^3$ although it is possible that $z_s \sim 8$ in types I, VII$_0$, IX (i.e. $\Omega \sim 1$) if there were an intergalactic gas ionized at this redshift. Given $\Delta T/T < 3 . 10^{-3}$, Partridge (**23**), on large angular scales, equation (4.1) gives for VII$_0$ and IX

$$\left(\frac{\sigma}{\theta}\right)_0 \lesssim 1\cdot9 . 10^{-6} \quad \text{if} \quad z_s = 10^3$$

$$\left(\frac{\sigma}{\theta}\right)_0 \lesssim 3\cdot0 . 10^{-4} \quad \text{if} \quad z_s = 8$$

inferior to the light element limit. Whereas in VII$_h$ models

$$\left(\frac{\sigma}{\theta}\right)_0 \lesssim 6\cdot5 . 10^{-6} \left(\frac{10^{17}}{t_m}\right)^{2/3}$$

the superiority over the light element value increasing with decreasing density.

These results are tabulated together with the Hawking & Collins (**3**) results for types I and V in Table I.

## 5. INHOMOGENEOUS UNIVERSES

In reality the Universe is not exactly spatially homogeneous. We observe systematic small scale inhomogeneity in the form of galaxies and clusters, but the overall structure should not deviate too greatly from homogeneity since the Cosmological Principal (Bondi (**24**)) and the M–W isotropy require the Universe to be almost isotropic about every point, and purely spatial radiation isotropy implies dynamic spatial isotropy (Ehlers *et al.* (**32**)) which implies spatial homogeneity (Walker (**25**)).

It would be reasonable to expect, however, that the evolution of shear in inhomogeneous cosmologies would differ from that in the Bianchi models, the presence of large scale density gradients generating shear. Two examples are considered.

TABLE I

*Maximum allowed anisotropy compatible with observed helium and deuterium abundances and microwave background compared*

| Type | | I | | V | $VII_0$ | | $VII_h$ | IX | |
|---|---|---|---|---|---|---|---|---|---|
| Admits R–W | | $k = 0$ | | $k + -1$ | $k = 0$ | | $k = -1$ | $k = +1$ | |
| Max. $(\sigma/\theta)_0$ | E–S | $4 \cdot 8 . 10^{-12}$ | | $1 \cdot 5 . 10^{-11}$ | $8 \cdot 0 . 10^{-7}$ | | $4 \cdot 5 . 10^{-5} (10^{17}/t_m)^{2/3}$ | $8 \cdot 0 . 10^{-7}$ | |
| $z_s$ | | 8 | 1000 | 1000 | 8 | 1000 | 1000 | 8 | 1000 |
| Max. $(\sigma/\theta)_0$ | M–W | $10^{-4}$* | $6 . 10^{-8}$* | $1 \cdot 5 . 10^{-4}$* | $3 . 10^{-4}$ | $1 \cdot 9 . 10^{-6}$ | $6 \cdot 5 . 10^{-6} (10^{17}/t_m)^{2/3}$ | $3 . 10^{-4}$ | $1 \cdot 9 . 10^{-6}$ |

* Hawking & Collins (3).

R–W = Robertson–Walker model; E–S = Limit based on light elements observed; M–W = Limit based on microwave radiation temperature isotropy.

The light element calculations are free of the parameter $z_s$ and give superior results in all cases except in $VII_h$ when $t_m < t_0$.

(i) *Self-similar solutions*

These have recently been examined geometrically by Eardley (**26**), but the simplest way of seeing the effects of this very simple form of inhomogeneity is to calculate the quantities $\sigma$, $\omega$ and $\theta$ in the two conformally related space-times with

$$\hat{g}_{\mathrm{ab}} = \phi^2(x^k)\, g_{\mathrm{ab}}, \quad \hat{h}_{\mathrm{ab}} = \phi^2(a^k)\, h_{\mathrm{ab}} \quad \text{and} \quad \hat{u}_{\mathrm{a}} = \phi(x^k)\, u_{\mathrm{a}}$$

these relations imply

$$\hat{\theta} = \frac{\theta}{\phi} + 3\,\frac{\dot{\phi}}{\phi^2}, \qquad \hat{\sigma} = \frac{\sigma}{\phi}, \qquad \hat{\omega} = \frac{\omega}{\phi}.$$

The relation of the Bianchi models we have considered to their self-similar inhomogeneous counterparts is clearly seen, in particular $\dot{\phi} = 0$ give $\hat{\sigma}/\hat{\theta} = \sigma/\theta$, and when $\dot{\phi} \neq 0$ then $\hat{\sigma}/\hat{\theta} = \sigma(\theta + 3\,\dot{\phi}/\phi)^{-1}$ and so if $\phi$ has a power law time dependence we have $\hat{\sigma}/\hat{\theta} \sim \sigma/\theta$ where $\sim$ indicates the quantities have the same time dependence and differ in magnitude only by a factor of order 1. This is what one would intuitively expect since these conformally related space times have the same free gravitational field which has a predominant influence upon the evolution of the shear.

(ii) *Small perturbations of homogeneous universes*

Liang (**27**) has explicitly demonstrated the generation of local shearing motions by large scale, $\lambda > ct$, irrotational inhomogeneity

$$\frac{\sigma}{\theta_0} \sim \frac{\delta\mu}{\mu_0} = A(\boldsymbol{x})\, t^{-1} + B(\boldsymbol{x})\, t^{2/3} \qquad \text{for} \qquad p = 0$$

$$\frac{\sigma}{\theta_0} \sim \frac{\delta\mu}{\mu_0} = A(\boldsymbol{x})\, t^{-1/2} + B(\boldsymbol{x})\, t \qquad \text{for} \quad p = \frac{\mu}{3}$$

where $\theta_0$, $\mu_0$ are the unperturbed R–W quantities and $\sim$ again means that both sides of the equation have the same time dependence and order of magnitude.* The $A$ mode reduces to the Bianchi I behaviour in the long wavelength limit. We might expect though that the growing $B$ mode effects would completely dominate at $t_{\mathrm{es}}$, but this is not the case for, as Liang points out, the $B$ mode is induced by primordial curvature fluctuations $\delta\mu \sim {}^3R$ and Gisler, Harrison & Rees (**28**) have shown that in general the effect of such fluctuations is very small in the irrotational case; any but the smallest curvature fluctuations collapsing to form compact objects. Large scale inhomogeneity in the form of small perturbations from R–W will in general induce local Bianchi I-type behaviour with small spatial 'wrinkles' at $t_{\mathrm{es}}$. Quantitatively some idea of the shear that can be induced by large scale inhomogeneity on some scales can be obtained by combining the relation $\sigma/\theta_0 \sim \delta\mu/\mu_0$ with the limits obtained by Sunyaev & Zeldovich (**29**) for $\delta\mu/\mu$ on certain mass

* We note, however, that for the 'Bianchi I-like' $p = \mu/3$ case, the linear relation of Liang may be invalid or only true for certain coordinate choices. This arises because while the density constant is not an invariantly gauge independent quantity in the sense of Stewart & Walker (**37**), the shear is, being zero in the unperturbed R–W background. Therefore the above linear relation may hold only in certain gauges. The referee points out that unpublished work of Liang shows that if the density perturbation is specifically defined relative to co-moving proper time, then $\delta\mu/\mu_0 \sim (\sigma/\theta_0)^2 \ln(\sigma/\theta_0)$. Consideration of the linear case is adequate for our purposes, however, since the induced shear is far smaller in the quasi-quadratic case.

scales in a particular redshift range by consideration of the allowed energy injection into the M–W background from the dissipation of irregularities by radiative viscosity after they enter the horizon. They give,

$$\frac{\delta\mu}{\mu_0} < 0{\cdot}2\ \Omega^{7/16} \quad \text{for} \quad 10^4\ \Omega^{-1/2} \lesssim z \lesssim 5{\cdot}4 . 10^4\ \Omega^{-6/5}$$

corresponding to mass scales

$$5 . 10^5\ \Omega^{4\cdot 9} \lesssim \frac{M}{M_\odot} \lesssim 10^9\ \Omega^{7/4} \quad \text{when} \quad \Omega \lesssim 0{\cdot}4.$$

So for example in our Bianchi V model with $\Omega = 10^{-2}$ the above relations give

$$\frac{\sigma}{\theta} \sim \frac{\delta\mu}{\mu} \lesssim 2{\cdot}6 . 10^{-2} \quad \text{for} \quad 10^5 \lesssim z \lesssim 1{\cdot}4 . 10^7 \quad \text{and} \quad 8{\cdot}0 . 10^{-5} \lesssim \frac{M}{M_\odot} \gtrsim 3{\cdot}2 . 10^5$$

and for $\Omega = 0{\cdot}1$

$$\frac{\sigma}{\theta} = \frac{\delta\mu}{\mu} \lesssim 7{\cdot}3 . 10^{-2} \quad \text{for} \quad 3{\cdot}2 . 10^4 \lesssim z \lesssim 8{\cdot}6 . 10^5 \quad \text{and} \quad 5 \lesssim \frac{M}{M_\odot} \lesssim 3{\cdot}2 . 10^5$$

whereas the evolution from $t_{es}$ gives for $z \sim 10^7$, $(\sigma/\theta) \lesssim 10^{-2}$.

So we see the limits are compatible, the shear induced by the large scale inhomogeneity, on these scales at least, being of the same order of magnitude as in the corresponding homogeneous Bianchi V ($\Omega = 10^{-2}$) situation. If the amplitude of density perturbations falls off on gradually increasing scales then the evolution of shear in perturbed homogeneous universes will be very similar to that in their exactly homogeneous counterparts.

Epstein & Petrosian (**30**) have investigated the effect of a gaussian density distribution on the synthesis of deuterium, helium-3 and lithium-7, although they associate no scales with the regions of differing densities. If scales were very large though, their analysis would break down because shearing motions would be induced, increasing the local expansion rate and altering their abundance calculations and in particular the helium-4 abundance which they assume is unaffected. Their calculations also neglect the fact that if nucleosynthesis were to occur in a very inhomogeneous density distribution the diffusion of particles and radiation between regions of different temperature and density would bring about a considerable deviation from the simple conditions they envisage. These diffusion processes would be particularly significant in models where horizons were absent in some or all directions at $t_{es}$.

## 6. CONCLUSION

We have seen that if the interstellar deuterium and helium we observe is of cosmological origin, then their abundances are potentially the most important probe we have of conditions in the early Universe. In particular the present abundance data of $X(\mathrm{He}^4) = 0{\cdot}29 \pm 0{\cdot}04$ and $X(\mathrm{D}) \sim 2 . 10^{-5}$ place finer constraints on the permitted distortion of the Universe today than the M–W temperature isotropy of $\Delta T/T \lesssim 3 . 10^{-3}$ in a class of homogeneous Bianchi type I, V, $\mathrm{VII}_0$ and IX models, but not in low density type $\mathrm{VII}_h$ models. The effects of strong magnetic fields and unknown particles have not been included in the picture of nucleosynthesis used here; these factors would also cause deviations from the standard R–W expansion rate at $t_{es}$ but their inclusion would strengthen the limits obtainable on $\sigma/\theta$ since other forms of positive energy density would be contributing to the expansion dynamics at $t_{es}$ and therefore the anisotropy energy density $\sigma^2$ would need to be smaller to avoid conflict with observation. The outstanding problem that remains is to discover why such small anisotropies are demanded rather than just permitted by physical processes in general models of the early Universe. The most promising candidates for plausible isotropizing agents are neutrino viscosity, $t_d \sim 1$ s (Misner (**11**)) and particle creation from the vacuum gravitational field $t_d \sim 10^{-43}$ s (Zeldovich (**12**)). However, Collins & Stewart (**33**), in an analysis of dissipative processes in irrotational Bianchi models have proved rigorously that for arbitrary initial conditions of non-zero measure in the space of all homogeneous initial data the shear anisotropy of the Universe could be arbitrarily large today. Their analysis is particularly powerful in that it is based on the theorems governing the general extension of a solution of a differential system over an open bounded region and is a consequence of the differential system satisfying the Lifschitz condition locally. Therefore similar restrictions on the efficiency of dissipation can be expected for any process whose action is governed by differential equations. It would appear that an adequate explanation of the isotropy and homogeneity of the Universe depends either on exotic quantum processes before the Planck time, $10^{-43}$ s or some form of anthropic or Machian selection effect (Carter (**34**); Raine (**35**)).

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