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Light Propagation in Stratified Chiral Media. The 4×4 Matrix Method

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Abstract—Propagation of electromagnetic waves in stratified bianisotropic chiral structures is described by the 4×4 matrix method. At arbitrary layer parameters, the amplitude and the polarization characteristics (intensity, polarization azimuth, and ellipticity) of reflected and transmitted electromagnetic waves are studied as functions of the angle of the wave incidence onto the structure. © *2000 MAIK "Nauka/Interperiodica".*

INTRODUCTION

Recently, we have evidenced the fast development of the theory of electromagnetic waves (EMW) propagation in bianisotropic chiral media, such as composite materials, liquid crystals, and other optically active substances [1]. Chiral media exhibit two basic properties: the optical activity (difference of phase velocities for the left- and right-handed circular polarizations) and chiral dichroism. Within the microwave range, such media are formed by the inclusion of metallic or ceramic helicoids into a dielectric matrix [2, 3]. Within the optical range, the role of such helicoids can be played by molecules possessing no mirror symmetry [4]. These properties are inherent in cholesteric and smectic liquid crystals, which, despite the appreciable differences in the properties and structures, have one common characteristic—they are all formed by molecules possessing the left- or right-hand symmetry (chiral molecules) and spatially periodic with the period usually lying within the optical range [5, 6]. Similar phenomena can also be obtained by using helicoid swastika or Ω like inclusions [7, 8]. In the general case of arbitrary orientations of the anisotropy axes, the symmetry axis of the stratified structure and the propagation directions, the analysis of the characteristics of EMW propagation in bianisotropic chiral media is an important but rather complicated problem. Today, a number of particular problems of electrodynamics of bianisotropic and chiral media has been solved. The studies in this field are progressing rapidly [9–12].

Below, the propagation of electromagnetic waves in chiral structures is described by the method of 4×4 matrices, which is efficient for any kind of anisotropy and number of layers. Vanous modifications of this method are used to describe the EMW propagation in media with the anisotropy of different nature, in particular, in dielectric, magnetic and liquid–crystal structures [13, 14]. In the majority of publications, the authors restricted themselves to the consideration of particular cases of chirality and the simplest stratified structures. Below, such cases are considered as particular cases of the general method. In these cases the parameters of the reflected and the transmitted EMWs (intensity, polarization azimuth, and ellipticity) are determined.

4×4 PERMITTIVITY MATRIX

Consider a medium composed of anisotropic chiral layers parallel to the *XY*-plane, with the *Z*-axis coinciding with the symmetry axis of the structure. Let a plane monochromatic EMW with the wave vector **k**, parallel to *XZ*-plane propagate in this medium. Then, the electric and the magnetic fields of the wave, **E**, **D**, **H**, and **B**, are proportional to $\exp[i(\omega t - k_x x)]$, and the Maxwell equations have the form

$$\nabla \mathbf{E} = -ik_0\mathbf{B}, \quad \nabla \mathbf{H} - ik_0\mathbf{D}, \tag{1}$$

where $\nabla = \left(-ik_x, 0, \frac{\partial}{\partial z}\right)$, $k_0 = \omega/c$, ω is frequency, and c is the velocity of light in vacuum. To describe the bianisotropic chiral medium in the general form, we write the material equations as [15]:

$$\mathbf{D} = \hat{\varepsilon}\mathbf{E} - \hat{\alpha}\mathbf{H}, \quad \mathbf{B} = \hat{\mu}\mathbf{H} + \hat{\beta}\mathbf{E}. \tag{2}$$

The above equations include four tensors—those of dielectric $\hat{\varepsilon}$, magnetic $\hat{\mu}$, and magnetooptical $\hat{\alpha}$ and $\hat{\beta}$ permittivities, which relate the strengths of the electric and magnetic fields with the electric and magnetic inductions. Substituting (2) into (1), we arrive at the following system of equations:

$$E'_y = ik_0(\hat{\mu}\mathbf{H} + \hat{\beta}\mathbf{E})_x, \quad H'_y = -ik_0(\hat{\varepsilon}\mathbf{E} + \hat{\alpha}\mathbf{H})_x,$$
$$E'_x = -ik_0[(\hat{\mu}\mathbf{H} + \hat{\beta}\mathbf{E})_y + n_x E_z], \tag{3}$$
$$H'_x = ik_0[(\hat{\varepsilon}\mathbf{E} + \hat{\alpha}\mathbf{H})_y - n_x H_z],$$

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$$n_x E_y = (\hat{\mu}\mathbf{H} + \hat{\beta}\mathbf{E})_z, \quad n_x H_y = -(\hat{\varepsilon}\mathbf{E} + \hat{\alpha}\mathbf{H})_z,$$

where prime denotes differentiation with respect to z and $n_x = k_x/k_0$. Excluding the field components E_z and H_z parallel to the structure axis and introducing the vector $\mathbf{g} = (E_x, -E_y, H_x, H_y)$, having four tangential field components, we can represent the system (3) of the wave equations for planar layered medium in terms of the following differential matrix equation:

$$\mathbf{g}' = -ik_0\hat{G}\mathbf{g}, \tag{4}$$

where, the matrix $\hat{G}$ of the dimension 4×4 is determined by the local properties of the medium, i.e., has the same form in both homogeneous and inhomogeneous media and contains no differential operators. It is constructed using four permittivity tensors and allows the most general consideration of bianisotropic and chiral properties of the medium. In order to write the $\hat{G}$ matrix in the most concise form, introduce the following notation:

$$\mathcal{E}_{ij} = \begin{vmatrix} \varepsilon_{ij} & \varepsilon_{iz} & \alpha_{iz} \\ \varepsilon_{zj} & \varepsilon_{zz} & \alpha_{zz} \\ \beta_{zj} & \beta_{zz} & \mu_{zz} \end{vmatrix}, \quad \mathcal{M}_{ij} = \begin{vmatrix} \mu_{ij} & \beta_{iz} & \mu_{iz} \\ \alpha_{zj} & \varepsilon_{zz} & \alpha_{zz} \\ \mu_{zj} & \beta_{zz} & \mu_{zz} \end{vmatrix},$$

$$\mathcal{A}_{ij} = \begin{vmatrix} \alpha_{ij} & \varepsilon_{iz} & \alpha_{iz} \\ \alpha_{zj} & \varepsilon_{zz} & \alpha_{zz} \\ \mu_{zj} & \beta_{zz} & \mu_{zz} \end{vmatrix}, \quad \mathcal{B}_{ij} = \begin{vmatrix} \beta_{ij} & \beta_{iz} & \mu_{iz} \\ \varepsilon_{zj} & \varepsilon_{zz} & \alpha_{zz} \\ \beta_{zj} & \beta_{zz} & \mu_{zz} \end{vmatrix},$$

$$e_{ij} = \begin{vmatrix} \varepsilon_{ij} & \alpha_{iz} \\ \beta_{zj} & \mu_{zz} \end{vmatrix}, \quad m_{ij} = \begin{vmatrix} \mu_{ij} & \beta_{iz} \\ \alpha_{zj} & \varepsilon_{zz} \end{vmatrix}, \quad d = \begin{vmatrix} \varepsilon_{zz} & \alpha_{zz} \\ \beta_{zz} & \mu_{zz} \end{vmatrix},$$

$$a_{iz} = \begin{vmatrix} \alpha_{iz} & \varepsilon_{iz} \\ \alpha_{zz} & \varepsilon_{zz} \end{vmatrix}, \quad a_{zj} = \begin{vmatrix} \alpha_{zj} & \alpha_{zz} \\ \mu_{zj} & \mu_{zz} \end{vmatrix},$$

$$b_{iz} = \begin{vmatrix} \beta_{iz} & \mu_{iz} \\ \beta_{zz} & \mu_{zz} \end{vmatrix}, \quad b_{zj} = \begin{vmatrix} \beta_{zj} & \beta_{zz} \\ \varepsilon_{zj} & \varepsilon_{zz} \end{vmatrix},$$

where $i, j = x, y$, and $|A|$ is the determinant of the matrix A. Now, the matrix $\hat{G}$ can be represented as a sum of three terms proportional to different powers of n_x:

$$\hat{G} = \frac{1}{d}\begin{pmatrix} \mathcal{B}_{yx} & -\mathcal{B}_{yy} & \mathcal{M}_{yx} & \mathcal{M}_{yy} \\ \mathcal{B}_{xx} & -\mathcal{B}_{xy} & \mathcal{M}_{xx} & \mathcal{M}_{xy} \\ -\mathcal{E}_{yx} & \mathcal{E}_{yy} & -\mathcal{A}_{yx} & -\mathcal{A}_{yy} \\ \mathcal{E}_{xx} & -\mathcal{E}_{xy} & \mathcal{A}_{xx} & \mathcal{A}_{xy} \end{pmatrix}$$

$$+\frac{n_x^2}{d}\begin{pmatrix} 0 & \alpha_{zz} & 0 & -\mu_{zz} \\ 0 & 0 & 0 & 0 \\ 0 & -\varepsilon_{zz} & 0 & \beta_{zz} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{5}$$

$$-\frac{n_x}{d}\begin{pmatrix} e_{zx} & m_{yz} - e_{zy} & a_{zx} & a_{zy} + b_{yz} \\ 0 & m_{xz} & 0 & b_{xz} \\ b_{zx} & -a_{yz} - b_{zy} & m_{zx} & m_{zy} - e_{yz} \\ 0 & a_{xz} & 0 & e_{xz} \end{pmatrix}.$$

For a homogeneous medium, $\hat{G}$ is independent of the z-coordinate, and the solution of matrix equation (4) is the superposition of the eigenwaves

$$\mathbf{g} = \sum a_j \mathbf{g}_j \exp(-ik_{zj}z), \quad j = 1, \ldots, 4, \tag{6}$$

where a_j are waves amplitudes corresponding to the eigenvectors $\mathbf{g}_j$ of the $\hat{G}$ matrix. The eigenvalues $n_{zj} = k_{zj}/k_0$ of this matrix are the roots of the dispersion equation

$$\det(\hat{G} - n_z\hat{I}) = 0, \tag{7}$$

where $\hat{I}$ is the unit matrix. In general case, it follows from (6) and (7) that there are four eigenvalues with different polarizations, propagation directions, and refractive indices $n_j = \sqrt{n_x^2 + n_{zj}^2}$ quartic in n_{zj} and defined by equation (7).

If the medium is homogeneous along the z-axis, the study of EMW propagation is reduced to the solution of the boundary–value problem: the medium is divided into thin layers, whose boundaries lie in the XY plane and the material parameters are constant within each layer.

PLANAR LAYERED STRUCTURE

Consider EMW propagation in a planar layered medium. The tangential components of the electric- and magnetic-fields strengths or, which is equivalent, the four-component vector $\mathbf{g}$, should be continuous across the boundaries of the adjacent layers. Let the superposition of the eigenwaves with the amplitudes $a_j^{(n)}$ be incident onto the boundary between the nth and $(n+1)$th layers. For the wave entering the $(n+1)$th layer, the amplitudes obtained from the continuity conditions for the $\mathbf{g}$ vector components at the boundary are determined by the matrix equation:

$$a_i^{(n+1)} = M_{ij}^{(n)} a_j^{(n)}, \tag{8}$$

where the elements of propagation matrix $\hat{M}^{(n)}$ at the nth boundary have the form

$$M_{ij}^{(n)} = \tilde{\mathbf{g}}_i^{(n+1)}\mathbf{g}_j^{(n)}. \quad (9)$$

Here, $\tilde{\mathbf{g}}_i^{(n)}$ are the vectors complementary to $\mathbf{g}_j^{(n)}$, i.e., the vectors satisfying the condition $\tilde{\mathbf{g}}_i^{(n)}\mathbf{g}_j^{(n)} = \delta_{ij}$. The EMW propagation through the homogeneous nth layer, with no account for the boundary, is described by the diagonal $\hat{T}^{(n)}$ matrix with the elements

$$T_{ij}^{(n)} = \delta_{ij}\exp(-ik_{zj}^{(n)}l_n), \quad (10)$$

where l_n is the thickness of the nth layer. For the system consisting of p layers, the resultant propagation matrix is the product of the propagation matrices for particular boundaries and layers

$$\hat{M} = (\hat{M}^{(p)}\hat{T}^{(p)})(\hat{M}^{(p-1)}\hat{T}^{(p-1)})\ldots(\hat{M}^{(1)}\hat{T}^{(1)})\hat{M}^{(0)}. \quad (11)$$

The amplitude of the transmitted wave is given by

$$\mathbf{a}^{(p)} = \hat{M}\mathbf{a}^{(0)}. \quad (12)$$

Let us mark the eigenwaves propagating in the forward direction with subscripts 1 and 2 ($n_z > 0$), and those propagating in the backward direction, with 3 and 4 ($n_z < 0$). The waves with subscripts 1, 3 and those with subscripts 2, 4 have the same polarization. Now, introduce the matrix $\hat{N} = \hat{M}^{-1}$ inverse with respect to $\hat{M}$ and write down the corresponding elements of reflection and transmission matrices of the layered structure:

$$\begin{aligned}
r_{11} &= \left.\frac{a_3^{(0)}}{a_1^{(0)}}\right|_{a_2^{(0)}=0} = \frac{L_{31}^{22}}{L_{11}^{22}}, \quad r_{12} = \left.\frac{a_4^{(0)}}{a_1^{(0)}}\right|_{a_2^{(0)}=0} = \frac{L_{41}^{22}}{L_{11}^{22}},\\
r_{21} &= \left.\frac{a_3^{(0)}}{a_2^{(0)}}\right|_{a_1^{(0)}=0} = \frac{L_{32}^{11}}{L_{11}^{22}}, \quad r_{22} = \left.\frac{a_4^{(0)}}{a_2^{(0)}}\right|_{a_1^{(0)}=0} = \frac{L_{42}^{11}}{L_{11}^{22}},\\
t_{11} &= \left.\frac{a_3^{(p)}}{a_1^{(0)}}\right|_{a_2^{(0)}=0} = \frac{N_{22}}{L_{11}^{22}}, \quad t_{12} = \left.\frac{a_4^{(p)}}{a_1^{(0)}}\right|_{a_2^{(0)}=0} = \frac{N_{21}}{L_{11}^{22}},\\
t_{21} &= \left.\frac{a_3^{(p)}}{a_2^{(0)}}\right|_{a_1^{(0)}=0} = -\frac{N_{12}}{L_{11}^{22}}, \quad t_{22} = \left.\frac{a_4^{(p)}}{a_2^{(0)}}\right|_{a_1^{(0)}=0} = \frac{N_{11}}{L_{11}^{22}}.
\end{aligned} \quad (13)$$

Above, we used the notation $L_{ij}^{kl} = N_{ij}N_{kl} - N_{il}N_{kj}$.

In semi-infinite media, labelled with subscripts "0" and "p," separated by the layered structure, the vectors $\mathbf{g}_j$ are normalized in such a way that the energy fluxes corresponding to each wave are equal (e.g., $|\mathbf{S}_j| = |\mathbf{E}_j \times \mathbf{H}_j^* + \mathbf{E}_j^* \times \mathbf{H}_j| = 1$). Then the quantities $|r|^2 = |r_{j1}|^2 + |r_{j2}|^2$ and $|t|^2 = |t_{j1}|^2 + |t_{j2}|^2$ determine the ratios of energy fluxes of the reflected and the transmitted waves to that of the incident wave. Other types of normalization are also possible, e.g., such that r and t would be the ratios of the amplitudes of the corresponding fields. In intermediate layers, normalization is not necessary, because no determination of the eigenwave amplitudes is required.

The method under consideration is a unified approach to the problem of EMW propagation in planar layered structures. It allows the consideration of various problems of electro- and magnetooptics, including the optics of bianisotropic and chiral media.

APPLICATION OF THE METHOD TO SIMPLEST CHIRAL STRUCTURES

1. Bianisotropic medium. For an isotropic medium, $\hat{\varepsilon}$, $\hat{\mu}$, $\hat{\alpha}$, and $\hat{\beta}$ are the diagonal tensors of the type $\varepsilon_{ij} = \varepsilon\delta_{ij}$, so that the $\hat{G}$ matrix acquires the simple form

$$\hat{G} = \begin{pmatrix} 0 & s\alpha-\beta & 0 & \mu(1-s) \\ \beta & 0 & \mu & 0 \\ 0 & \varepsilon(1-s) & 0 & s\beta-\alpha \\ \varepsilon & 0 & \alpha & 0 \end{pmatrix}, \quad s = \frac{n_x^2}{\varepsilon\mu-\alpha\beta}. \quad (14)$$

Then, the solution of dispersion equation (7) yields the following eigenvalues of the $\hat{G}$ matrix:

$$n_z^{\pm} = \left[\varepsilon\mu(1-s) + \frac{1}{2}[\alpha(s\beta-\alpha) + \beta(s\alpha-\beta)] \pm i(\alpha-\beta)\sqrt{\varepsilon\mu-(\alpha+\beta)^2/4}\right]^{1/2}. \quad (15)$$

Using the expressions for $n_z^{\pm}$, we can determine the refractive indices of the eigenwaves as:

$$n_{\pm} = \sqrt{\varepsilon\mu-(\alpha+\beta)^2/4} \pm i(\alpha-\beta)/2. \quad (16)$$

The eigenvalues of the $\hat{G}$ matrix, determined from the equation $(\hat{G} - n_z\hat{I})\mathbf{g}^{(c)} = 0$, have the following components:

$$\mathbf{g}^{(c)} = (n_z^{\pm}(n_{\pm} \mp i\alpha), \pm i\varepsilon\mu + \beta(n_{\pm} \mp i\alpha), \pm i\varepsilon n_z^{\pm}, \varepsilon n_{\pm}). \quad (17)$$

The above vectors specify the eigenwaves of the biisotropic medium, which are the left-hand (upper signs) or right-hand (lower signs) polarized waves propagating in the forward ($n_z^{\pm} > 0$) or the backward ($n_z^{\pm} < 0$) directions.

Introducing the parameters of nonreciprocity $\chi = (\alpha+\beta)/2$ and chirality $\kappa = i(\alpha-\beta)/2$ instead of magnetoelectric permittivities, we obtain the refractive indices for the eigenwaves in the medium in the form: $n_{\pm} = \sqrt{\varepsilon\mu-\chi^2} \pm \kappa$. With due regard of complexity of the introduced parameters ($\chi = \chi' + i\chi''$, $\kappa = \kappa' + i\kappa''$), the above relations lead to two general types of biisotropic non-absorbing media. For such media, the imaginary part of the chirality parameter is zero, while either imaginary or real part of the nonreciprocity parameter has nonzero value. The dependence of the refractive indices of eigenwaves on nonreciprocity for two types of media is quite different. For the first type ($\chi'' = 0$) the refracting index monotonically decreases; for the second type ($\chi' = 0$), the value of $n_\pm$ monotonically increases.

2. Reflection from the dielectric–chiral medium interface. Let a wave from dielectric with material parameters ε_0, μ_0 be incident onto the plane interface with a semi-infinite chiral medium. For an isotropic dielectric, all the waves irrespectively of their polarization are eigenwaves; therefore, we may resolve the field into the p- and s-polarized waves, for which the vector $\mathbf{g}$ has the components

$$\mathbf{g}_p^{(0)} = (\pm\sigma_0, 0, 0, \eta_0), \quad \mathbf{g}_s^{(0)} = (0, 1, \pm\eta_0\sigma_0, 0),$$

where $\sigma_0 = \sqrt{1 - n_x^2/\varepsilon_0\mu_0}$, $\eta_0 = \sqrt{\varepsilon_0/\mu_0}$, and the signs "±" correspond to two opposite directions of wave propagation. According to (9), the propagation matrix $M_{ij}^{(0)}$ at the boundary between the media can be written as $M_n^{(0)} = \tilde{\mathbf{g}}_i^{(c)}\mathbf{g}_j^{(0)}$, then the $N_{ij} = \tilde{\mathbf{g}}_i^{(0)}\mathbf{g}_j^{(c)}$ matrix has the form

$$\hat{N} = \begin{pmatrix} \eta_0 & 0 & 0 & \sigma_0 \\ 0 & \eta_0\sigma_0 & 1 & 0 \\ -\eta_0 & 0 & 0 & \sigma_0 \\ 0 & \eta_0\sigma_0 & -1 & 0 \end{pmatrix}$$
$$\times \begin{pmatrix} n_z^+(n_+ - i\alpha) & n_z^-(n_- + i\alpha) & -n_z^+(n_+ - i\alpha) & -n_z^-(n_- + i\alpha) \\ i\varepsilon\mu + \beta(n_+ - i\alpha) & -i\varepsilon\mu + \beta(n_- + i\alpha) & i\varepsilon\mu + \beta(n_+ - i\alpha) & -i\varepsilon\mu + \beta(n_- + i\alpha) \\ i\varepsilon n_z^+ & -i\varepsilon n_z^- & -i\varepsilon n_z^+ & i\varepsilon n_z^- \\ \varepsilon n_+ & \varepsilon n_- & \varepsilon n_+ & \varepsilon n_- \end{pmatrix}. \tag{18}$$

Using of (13) and (18), one may find the coefficients of the EMW reflection from the interface between the dielectric and chiral media. For normal incidence, these coefficients for the s- and p-polarized waves are

$$r_{pp} = -r_{ss} = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2 + 2\eta\eta_0\gamma},$$
$$r_{ps} = r_{sp} = \frac{2\chi\eta\eta_0/\sqrt{\varepsilon\mu}}{\eta^2 + \eta_0^2 + 2\eta\eta_0\gamma}, \tag{19}$$

where $\gamma = \sqrt{1 - \chi^2/\varepsilon\mu}$ an $\eta = \sqrt{\varepsilon/\mu}$. Thus, the characteristics of reflected wave are independent of the medium chirality specified by the parameter κ but are essentially dependent on the nonreciprocity parameter χ. The polarization characteristics of the reflected wave, i.e., polarization azimuth θ_r and ellipticity angle E_r, are obtained from the relationship

$$\tan(\theta_r - iE_r) = \frac{r_{ps}}{r_{pp}} = \frac{2\chi\eta\eta_0/\sqrt{\varepsilon\mu}}{\eta^2 - \eta_0^2}. \tag{20}$$

At low values of the nonreciprocity parameter ($|\chi| \ll 1$), they have the form

$$\theta_r = \frac{2\chi'\sqrt{\varepsilon_0\mu_0}}{\varepsilon\mu_0 - \varepsilon_0\mu}, \quad E_r = \frac{2\chi''\sqrt{\varepsilon_0\mu_0}}{\varepsilon\mu_0 - \varepsilon_0\mu}. \tag{21}$$

For a non-absorbing chiral medium, either the rotation of the polarization plane (the medium of the first type) or the ellipticity (the medium of the second type) of the reflected radiation can take place.

For an oblique EMW incidence onto the interface between two media, we obtain in the first approximation in small parameters κ and χ:

$$r_{pp} = \frac{\eta\sigma_0 - \eta_0\sigma}{\eta\sigma_0 + \eta_0\sigma},$$
$$r_{ps} = \frac{2\eta\eta_0\sigma_0(\chi\sigma^2 + i\kappa(\sigma^2 - 1))}{\sigma\sqrt{\varepsilon\mu}(\eta_0\sigma_0 + \eta\sigma)(\eta\sigma_0 + \eta_0\sigma)},$$
$$r_{sp} = \frac{2\eta\eta_0\sigma_0(\chi\sigma^2 - i\kappa(\sigma^2 - 1))}{\sigma\sqrt{\varepsilon\mu}(\eta_0\sigma_0 + \eta\sigma)(\eta\sigma_0 + \eta_0\sigma)}, \tag{22}$$
$$r_{ss} = \frac{\eta_0\sigma_0 - \eta\sigma}{\eta_0\sigma_0 + \eta\sigma},$$

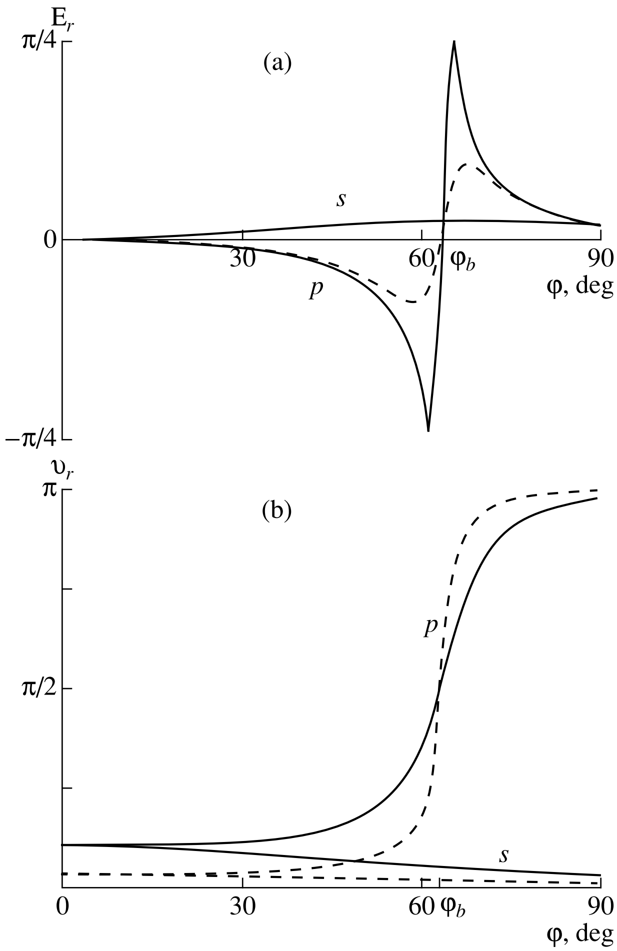

Fig. 1. (a) Ellipticity angle E_r and (b) polarization azimuth θ_r as functions of the incidence angle φ of *p*- and *s*-polarized reflected wave: (a) κ = 0.1, χ = 0 (solid curve), χ = 0.04 (dashed curve); (b) κ = 0, χ = 0.06 (solid curve), χ = 0.02 (dashed curve).

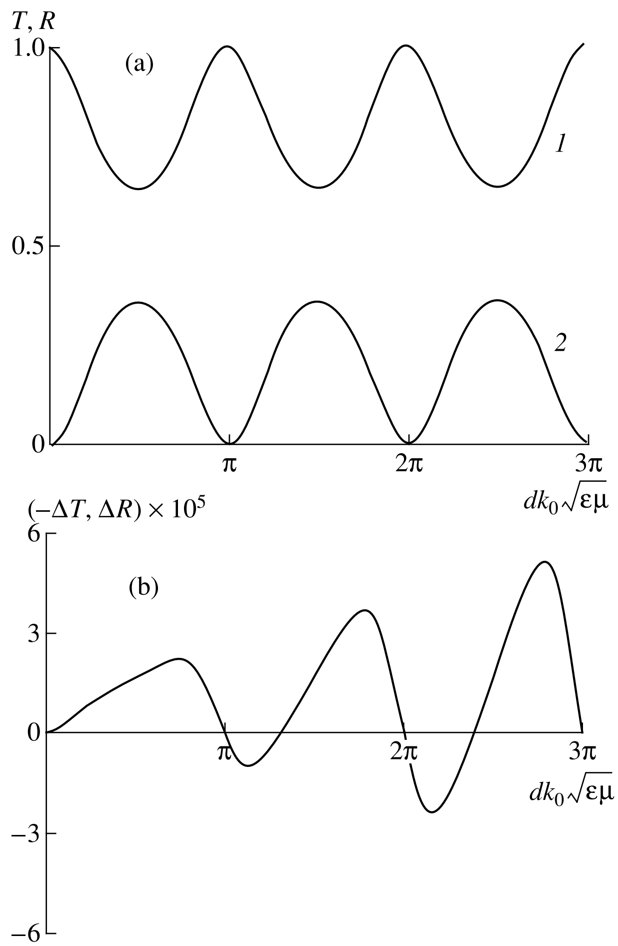

Fig. 2. Power transmittance *T* (a, curve *1*) and reflectance *R* (a, curve *2*) versus chiral layer thickness at χ = 0.01 and (b) their variations, Δ*T* and Δ*R*, with respect to the dielectric layer with the same permittivities and χ = 0.

where $\sigma = \sqrt{1-s}$. In this approximation, r_{pp} and r_{ss} are of the standard form, i.e. coincide with the well-known expressions for reflection of an electromagnetic wave from the interface between two dielectrics. The polarization characteristics of the reflected wave depend on both nonreciprocity and chirality of biisotropic medium. Figure 1a shows the ellipticity and Fig. 1b the polarization azimuth of the reflected wave as a function of the angle of incidence φ at the interface between the dielectric and chiral media with ε = 4 and μ = 1. The curves are obtained at various values of the nonreciprocity and chirality parameters. The dependence $E_r(\varphi)$ is plotted for the medium with chirality κ = 0.1 and nonreciprocity χ = 0 (solid curve) and χ = 0.04 (dashed curve). The dependence $E_r(\varphi)$ is plotted for the medium with chirality κ = 0 and χ = 0.06 (solid curve) and ξ = 0.02 (dashed curve). If the incident wave is *s*-polarized, the polarization characteristics of the reflected wave are almost independent of the incidence angle. For a *p*-polarized incident wave, the changes in θ_r and E_r are most pronounced near the angle φ close to the Brewster angle φ_b. At $\varphi = \varphi_b$, the reflected *p*-wave is linearly polarized with the polarization plane being rotated by angle of π/2 with respect to that of the incident wave. On departure of φ from φ_b-value the ellipticity angle first rapidly increases (at χ = 0 it reaches the value π/4, i.e., the wave becomes circularly polarized) and then gradually decreases and becomes almost zero, at the normal and the grazing incidence. When the incidence angle φ attains the value of the Brewster angle, the polarization plane of the reflected *p*-wave is rotated by an angle close to 180°. This rotation occurs the slower, the higher the nonreciprocity of the medium.

Typical values of the chirality parameter κ normalized to the refractive index $\sqrt{\varepsilon\mu}$ for natural and synthesized biisotropic media range from 0.05 to 0.3. The nonreciprocity effect observed in Cr_2O_3 natural crystals is much weaker, the corresponding parameter for these crystals is also lower, $\chi \approx 10^{-5}$ [16]. For a clearer representation of the nonreciprocity effects, we use higher values of this parameter.

3. Chiral layer in dielectric. To find the reflectance and transmittance of an EMW in a layer of thickness d in a dielectric with the material parameters ε_0 and μ_0, we represent the resultant propagation matrix (11) as the product of transmission matrices for the first interface, layer, and second interface:

$$\hat{M} = \hat{M}_0^{-1}\hat{T}\hat{M}_0, \quad T_{ij} = \delta_{ij}\exp(-ik_0 n_{zj} d). \tag{23}$$

Using this relationship and formula (13), we can obtain the expressions for amplitude coefficients of reflection and transmission for an EMW normally incident onto the layer:

$$\begin{aligned} t_{pp} &= t_{ss} = (2/D)\eta\eta_0\gamma\cos(\kappa k_0 d), \\ t_{ps} &= -t_{sp} = (2/D)\eta\eta_0\gamma\sin(\kappa k_0 d), \end{aligned}$$

$$\begin{aligned} r_{pp} &= -r_{ss} = (1/D)i(\eta^2 - \eta_0^2)\sin(dk_0\gamma\sqrt{\varepsilon\mu}), \\ r_{ps} &= r_{sp} = (1/D)2i\eta\eta_0\frac{\chi}{\sqrt{\varepsilon\mu}}\sin(dk_0\gamma\sqrt{\varepsilon\mu}), \end{aligned} \tag{24}$$

$$\begin{aligned} D &= 2\eta\eta_0\gamma\cos(dk_0\gamma\sqrt{\varepsilon\mu}) \\ &+ i(\eta^2 + \eta_0^2)\sin(dk_0\gamma\sqrt{\varepsilon\mu}). \end{aligned}$$

In this case, the ratio r_{ps}/r_{pp}, determining the polarization characteristics of the reflected wave, coincides with the analogous expression (20) for the interface between two semi-insinite media. Polarization characteristics of the transmitted wave linearly depend on the layer thickness and are determined by the chirality parameter, namely, $\theta_t - iE_t = \kappa k_0 d$. For non-absorbing medium ($\kappa'' = 0$), the transmitted wave shows only the rotation of the polarization plane by the angle $\theta_t = \kappa' k_0 d$. Figure 2 presents dependences of the transmittance $T = |t_{pp}|^2 + |t_{ps}|^2$ and reflectance $R = |r_{pp}|^2 + |r_{ps}|^2$ on the layer thickness (Fig. 1a), and also their variations $\Delta T = T(\chi) - T(0)$ and $\Delta R = R(\chi) - R(0)$ (Fig. 1b) due to medium nonreciprocity. For a non-absorbing medium, the total energy of the reflected and transmitted waves is conserved, with ΔT being equal to $-\Delta R$.

CONCLUSIONS

The above solutions and their analysis demonstrate the efficiency and versatility of the method based on the reduction of the Maxwell equations for plane EMW propagating in a layered bianisotropic medium, to the matrix first-order differential equation for a four-component vector, with the tangential field components. The method proposed can be used for determining the intensity and polarization characteristics of transmitted and reflected EMWs for continuous inhomogeneous chiral structures and structures with an arbitrary number of uniform layers.

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