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LIGHTLIKE REAL HYPERSURFACES WITH TOTALLY UMBILICAL SCREEN DISTRIBUTIONS

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Abstract. In this paper, we study the geometry of lightlike real hypersurfaces of an indefinite Kaehler manifold. The main result is a characterization theorem for lightlike real hypersurfaces M of an indefinite complex space form $\bar{M}(c)$ such that the screen distribution is totally umbilic.

1. Introduction

It is well known that the normal bundle $TM^{\perp}$ of the lightlike hypersurface M of a semi-Riemannian manifold $\bar{M}$ is a subbundle of the tangent bundle TM, of rank 1. Thus there exists a non-degenerate complementary vector bundle $S(TM)$ of $TM^{\perp}$ in TM, called a *screen distribution* on M, such that

$$TM = TM^{\perp} \oplus_{\text{orth}} S(TM), \tag{1.1}$$

where $\oplus_{\text{orth}}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $(M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M. We use the same notation for any other vector bundle. We known [2] that, for any null section ξ of $TM^{\perp}$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique vector bundle $\text{tr}(TM)$ in $S(TM)^{\perp}$ satisfying

$$\bar{g}(\xi, N) = 1, \ \bar{g}(N, N) = \bar{g}(N, X) = 0, \ \forall\, X \in \Gamma(S(TM)). \tag{1.2}$$

Then the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$T\bar{M} = TM \oplus \text{tr}(TM) = \{TM^{\perp} \oplus \text{tr}(TM)\} \oplus_{\text{orth}} S(TM). \tag{1.3}$$

We call $\text{tr}(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to $S(TM)$ respectively.

We recall that Tashiro-Tachibana [6] and Bejancu-Duggal [1] proved the non-existence of totally umbilical non-degenerate and lightlike real hypersurfaces of an indefinite complex space form $\bar{M}(c)$ $(c \neq 0)$ of constant holomorphic

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sectional curvature c, in case $\bar{g}$ is positive definite and indefinite respectively. In 1996, Duggal-Bejancu have proved the following theorem for lightlike real hypersurfaces of $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in their book [2]:

Theorem A ([2]). *Let $(M, g, S(TM))$ be a lightlike real hypersurface of $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then $S(TM)$ is totally geodesic.*

The purpose of this paper is to prove a new characterization theorem for lightlike real hypersurfaces M of $\bar{M}(c)$ such that $S(TM)$ is totally umbilic:

Theorem 1.1. *Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then we have both $c = 0$ and $C = 0$, on any $\mathcal{U} \subset M$. Moreover we show that*
(1) $c = 0$ *implies that the ambient space $\bar{M}(c)$ is a semi-Euclidean space,*
(2) $C = 0$*, on any $\mathcal{U} \subset M$, implies that $S(TM)$ is totally geodesic in M.*

Comparing our Theorem 1.1 with above Theorem A, we observe that Theorem 1.1 has the following new features of geometric significance. We prove that the holomorphic sectional curvature c satisfies $c = 0$ if $S(TM)$ is totally umbilic in M. This is a very significant result. Contrary to this, there is no discussion on such a relationship in Theorem A's above result. Thus we also prove the non-existence of lightlike real hypersurfaces of $\bar{M}(c)(c \neq 0)$ such that $S(TM)$ is totally umbilic. For the rest of this paper, using Theorem 1.1, we prove several additional theorems for lightlike real hypersurfaces M of $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Recall the following structure equations:

Let $\bar{\nabla}$ be the Levi-Civita connection of $\bar{M}$ and P the projection morphism of $\Gamma(TM)$ on $\Gamma(S(TM))$ with respect to the decomposition (1.1). Then the local Gauss and Weingartan formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N, \tag{1.4}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N, \tag{1.5}$$

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi, \tag{1.6}$$

$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi \tag{1.7}$$

for any $X, Y \in \Gamma(TM)$, where ∇ and ∇^* are the liner connections on TM and $S(TM)$ respectively, B and C are the local second fundamental forms on TM and $S(TM)$ respectively, A_N and A^*_ξ are the shape operators on TM and $S(TM)$ respectively and τ is a 1-form on TM.

Since $\bar{\nabla}$ is torsion-free, the induced connection ∇ is also torsion-free and B is symmetric. From the fact that $B(X, Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we show that B is independent of the choice of a screen distribution and satisfies

$$B(X, \xi) = 0, \quad \forall\, X \in \Gamma(TM). \tag{1.8}$$

The induced connection ∇ of M is not metric and satisfies

$$(\nabla_X g)(Y, Z) = B(X, Y)\,\eta(Z) + B(X, Z)\,\eta(Y) \tag{1.9}$$

for any X, Y, $Z \in \Gamma(TM)$, where η is a 1-form on TM such that

$$\eta(X) = \bar{g}(X, N), \quad \forall\, X \in \Gamma(TM). \tag{1.10}$$

But the connection ∇^* on $S(TM)$ is metric. The above two local second fundamental forms B and C are related to their shape operators by

$$B(X, Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{1.11}$$

$$C(X, PY) = g(A_N X, PY), \quad \bar{g}(A_N X, N) = 0. \tag{1.12}$$

From (1.11), A^*_ξ is $S(TM)$-valued and self-adjoint on TM such that

$$A^*_\xi \xi = 0. \tag{1.13}$$

We denote by $\bar{R}$, R and R^* the curvature tensors of the Levi-Civita connection $\bar{\nabla}$ of $\bar{M}$, the induced connection ∇ of M and the connection ∇^* on $S(TM)$, respectively. Using the Gauss-Weingarten equations for M and $S(TM)$, we obtain the Gauss-Codazzi equations for M and $S(TM)$ such that, for any vector fields X, Y, Z, $W \in \Gamma(TM)$,

$$\bar{g}(\bar{R}(X, Y)Z, PW) = g(R(X, Y)Z, PW) + B(X, Z)C(Y, PW) - B(Y, Z)C(X, PW), \tag{1.14}$$

$$\begin{aligned} \bar{g}(\bar{R}(X, Y)Z, \xi) = g(R(X, Y)Z, \xi) &= (\nabla_X B)(Y, Z) - (\nabla_Y B)(X, Z) \\ &\quad + B(Y, Z)\tau(X) - B(X, Z)\tau(Y), \end{aligned} \tag{1.15}$$

$$\bar{g}(\bar{R}(X, Y)Z, N) = g(R(X, Y)Z, N), \tag{1.16}$$

$$g(R(X, Y)PZ, PW) = g(R^*(X, Y)PZ, PW) + C(X, PZ)B(Y, PW) - C(Y, PZ)B(X, PW), \tag{1.17}$$

$$\begin{aligned} g(R(X, Y)PZ, N) &= (\nabla_X C)(Y, PZ) - (\nabla_Y C)(X, PZ) \\ &\quad + C(X, PZ)\tau(Y) - C(Y, PZ)\tau(X). \end{aligned} \tag{1.18}$$

2. Lightlike real hypersurfaces

Let $\bar{M} = (\bar{M}, J, \bar{g})$ be a real $2m$-dimensional indefinite Kaehler manifold, where $\bar{g}$ is a semi-Riemannian metric of index $q = 2v$, $0 < v < m$, and J is an almost complex structure on $\bar{M}$ satisfying, for all X, $Y \in \Gamma(T\bar{M})$,

$$J^2 = -I, \quad \bar{g}(JX, JY) = \bar{g}(X, Y), \quad (\bar{\nabla}_X J)Y = 0. \tag{2.1}$$

An indefinite complex space form, denoted by $\bar{M}(c)$, is a connected indefinite Kaehler manifold of constant holomorphic sectional curvature c such that

$$\begin{aligned} \bar{R}(X, Y)Z \;=\; & \frac{c}{4}\{\bar{g}(Y, Z)X - \bar{g}(X, Z)Y + \bar{g}(JY, Z)JX \\ & -\bar{g}(JX, Z)JY + 2\bar{g}(X, JY)JZ\} \end{aligned} \tag{2.2}$$

for all X, Y, $Z \in \Gamma(TM)$. Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite Kaehler manifold $\bar{M}$, where g is the degenerate induced metric of M. Then the screen distribution $S(TM)$ splits as follow [2]:

Let $\{\xi, N\}$ be a pair of local sections of $TM^{\perp} \oplus \mathrm{tr}(TM)$. Then we have

$$\bar{g}(J\xi, \xi) = \bar{g}(J\xi, N) = \bar{g}(JN, \xi) = \bar{g}(JN, N) = 0, \ \bar{g}(J\xi, JN) = 1. \tag{2.3}$$

This show that $J\xi$ and JN are vector fields tangent to M. Thus $J(TM^{\perp})$ and $J(\mathrm{tr}(TM))$ are distributions on M of rank 1 such that $TM^{\perp} \cap J(TM^{\perp}) = \{0\}$ and $TM^{\perp} \cap J(\mathrm{tr}(TM)) = \{0\}$. Thus $J(TM^{\perp}) \oplus J(\mathrm{tr}(TM))$ is a vector subbundle of $S(TM)$ of rank 2. There exists a non-degenerate almost complex distribution D_o on M with respect to J, i.e., $J(D_o) = D_o$, such that

$$S(TM) = \{J(TM^{\perp}) \oplus J(\mathrm{tr}(TM))\} \oplus_{\mathrm{orth}} D_o. \tag{2.4}$$

Consider the 2-lightlike almost complex distribution D such that

$$D = \{TM^{\perp} \oplus_{\mathrm{orth}} J(TM^{\perp})\} \oplus_{\mathrm{orth}} D_o, \quad TM = D \oplus J(\mathrm{tr}(TM)) \tag{2.5}$$

and the local lightlike vector fields U and V such that

$$U = -JN, \quad V = -J\xi. \tag{2.6}$$

Denote by S the projection morphism of TM on D. Then, by the second equation of (2.5)[(2.5)-2], any vector field on M is expressed as follows

$$X = SX + u(X)U, \quad JX = FX + u(X)N, \tag{2.7}$$

where u and v are 1-forms locally defined on M by

$$u(X) = g(X, V), \quad v(X) = g(X, U) \tag{2.8}$$

and F is a tensor field of type $(1, 1)$ globally defined on M by

$$FX = JSX, \ \forall\, X \in \Gamma(TM).$$

Differentiate (2.6)-1 with X and use (1.5), (1.7), (2.1)-3 and (2.7)-2, we have

$$B(X, U) = v(A^*_{\xi}X) = u(A_N X) = C(X, V), \ \forall\, X \in \Gamma(TM), \tag{2.9}$$

$$\nabla_X U = F(A_N X) + \tau(X)U, \quad \nabla_X V = F(A^*_{\xi}X) - \tau(X)V. \tag{2.10}$$

3. Totally umbilical screen distributions

Definition 1. We say that (each integral leaf of) $S(TM)$ is totally umbilic[2] in M if, on any coordinate neighborhood $\mathcal{U} \subset M$, there is a smooth function γ such that $A_N X = \gamma PX$ for any $X \in \Gamma(TM)$, or equivalently,

$$C(X, PY) = \gamma g(X, Y) \tag{3.1}$$

for all $X, Y \in \Gamma(TM)$. In case $\gamma = 0$ (or $\gamma \neq 0$) on $\mathcal{U}$, we say that $S(TM)$ is totally geodesic (or proper totally umbilic) in M.

In general, $S(TM)$ is not necessarily integrable. The following result gives equivalent conditions for the integrability of $S(TM)$:

Theorem 3.1 ([2, 3])**.** *Let $(M, g, S(TM))$ be a lightlike hypersurface of a semi-Riemannian manifold $(\bar{M}, \bar{g})$. Then the following are equivalent*:

(1) *$S(TM)$ is integrable.*
(2) *C is symmetric on $\Gamma(S(TM))$.*
(3) *A_N is self-adjoint on $\Gamma(S(TM))$ with respect to g.*

Note 1. If $S(TM)$ is totally umbilic in M, then C is symmetric on $\Gamma(S(TM))$. Thus, by Theorem 3.1, $S(TM)$ is integrable and M is locally a product manifold $L_\xi \times M^*$, where L_ξ is a null curve and M^* is a leaf of $S(TM)$ [2, 3].

Proof of Theorem 1.1. Using the equations (2.9) and (3.1), we have

$$B(X, U) = \gamma g(X, V), \quad \forall\, X \in \Gamma(TM). \tag{3.2}$$

Replace X by U and V by turns in (3.2), we obtain

$$B(U, U) = \gamma, \qquad B(U, V) = 0. \tag{3.3}$$

From (1.9), (1.16), (1.18) and (3.1), for any X, Y, $Z \in \Gamma(TM)$, we obtain

$$\begin{aligned} &\gamma\, B(Y, PZ)\eta(X) - \{X[\gamma] - \gamma\tau(X) - \frac{c}{4}\eta(X)\}g(Y, PZ) \\ = \;&\gamma\, B(X, PZ)\eta(Y) - \{Y[\gamma] - \gamma\tau(Y) - \frac{c}{4}\eta(Y)\}g(X, PZ) \\ &+ \frac{c}{4}\{\bar{g}(JX, PZ)v(Y) - \bar{g}(JY, PZ)v(X) - 2\bar{g}(X, JY)v(PZ)\}. \end{aligned}$$

Replacing X by ξ in this equation and using (1.8), (2.6) and (2.8), we have

$$\begin{aligned} \gamma\, B(Y, PZ) = &\{\xi[\gamma] - \gamma\tau(\xi) - \frac{c}{4}\}g(Y, PZ) \\ &- \frac{c}{4}\{u(PZ)v(Y) + 2u(Y)v(PZ)\}, \ \forall\, Y,\, Z \in \Gamma(TM). \end{aligned} \tag{3.4}$$

Taking $Y = U$, $PZ = V$; $Y = V$, $PZ = U$ and $Y = PZ = U$ by turns in (3.4), and then, use (2.8) and (3.3), we have

$$\xi[\gamma] - \gamma\tau(\xi) - \frac{3c}{4} = 0, \quad \xi[\gamma] - \gamma\tau(\xi) - \frac{c}{2} = 0, \quad \gamma^2 = 0,$$

respectively. This shows that $c = 0$ and $\gamma = 0$. Thus we have Theorem 1.1. □

Corollary 1. *There exist no lightlike real hypersurfaces of an indefinite complex space form $\bar{M}(c)(c \neq 0)$ such that $S(TM)$ is totally umbilic in M.*

Corollary 2. *There exist no lightlike real hypersurfaces of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is proper totally umbilic.*

Proposition 3.2. *Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$. If $S(TM)$ is totally umbilic in M, then the vector field U is conjugate to any vector field on M. In particular, U is an asymptotic vector field. Moreover, B is degenerate on $\Gamma(S(TM))$.*

Proof. Since $\gamma = 0$ on any $\mathcal{U} \subset M$, from (3.2), we have $B(X, U) = 0$ for any $X \in \Gamma(TM)$. Thus we have our assertion. □

Theorem 3.3. *Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then $H = D_o \oplus_{\text{orth}} J(\text{tr}(TM)) \oplus_{\text{orth}} TM^\perp$ is a parallel distribution with respect to the induced connection ∇ and M is locally a product manifold $L_v \times M^\natural$, where L_v is a null curve tangent to $J(TM^\perp)$ and $M^\natural$ is a leaf of H.*

Proof. In general, by using (1.4), (2.1) and (2.10), we derive

$$g(\nabla_X \xi, U) = -g(\xi, \bar{\nabla}_X U) = -B(X, U), \quad g(\nabla_X U, U) = 0,$$
$$g(\nabla_X Y, U) = -g(Y, \bar{\nabla}_X U) = -g(Y, \nabla_X U) = 0$$

for all $X \in \Gamma(TM)$ and $Y \in \Gamma(D_o)$. Since $S(TM)$ is totally umbilic in M, we have $B(X, U) = 0$ by (3.2) with $\gamma = 0$. Thus H is parallel with respect to ∇ and both H and $J(TM^\perp)$ are integrable distributions. Thus we obtain our theorem. □

Definition 2. We say that M is totally umbilic[2] in $\bar{M}$ if, on any coordinate neighborhood $\mathcal{U} \subset M$, there is a smooth function β such that

$$B(X, Y) = \beta g(X, Y) \tag{3.5}$$

for all $X, Y \in \Gamma(TM)$. In case $\beta = 0$ on $\mathcal{U}$, we say that M is totally geodesic.

Theorem 3.4. *Let $(M, g, S(TM))$ be a totally umbilical lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then M is totally geodesic in $\bar{M}$.*

Proof. From the equations (3.2) with $\gamma = 0$ and (3.5), we show that

$$\beta g(X, U) = B(X, U) = 0 \tag{3.6}$$

for all $X \in \Gamma(TM)$. Replacing X by V in (3.6), we have $\beta = 0$, i.e., $B = 0$. Thus M is totally geodesic in $\bar{M}$. Therefore, we have our theorem. □

Theorem 3.5. *Let $(M, g, S(TM))$ be a totally umbilical lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then D is a parallel distribution with respect to the induced connection ∇ and M is locally a product manifold $L_u \times M^\sharp$, where L_u is a null curve tangent to $J(\text{tr}(TM))$ and $M^\sharp$ is a leaf of D.*

Proof. In general, by using (1.4), (2.1) and (2.10), we derive

$$g(\nabla_X \xi, V) = -g(\xi, \bar{\nabla}_X V) = -B(X, V), \quad g(\nabla_X V, V) = 0,$$
$$g(\nabla_X Y, V) = -g(Y, \nabla_X V) = -g(Y, F(A^*_\xi X)) = B(X, FY)$$

for all $X \in \Gamma(TM)$ and $Y \in \Gamma(D_o)$. Since M is totally umbilic, we have $B = 0$ by Theorem 3.4. Thus D is parallel with respect to ∇ and both D and $J(\text{tr}(TM))$ are integrable distributions. Thus we obtain our theorem. □

Theorem 3.6. *Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then M and each leaf M^* of $S(TM)$ are spaces of constant curvature 0.*

Proof. Consider the induced quasi-orthonormal frame field $\{\xi; W_a\}$ on M such that $\mathrm{Rad}(TM) = \mathrm{Span}\{\xi\}$ and $S(TM) = \mathrm{Span}\{W_a\}$. Using this quasi-orthonormal frame field, we obtain

$$R(X,Y)Z = \sum_{a=1}^{2m-2} \epsilon_a g(R(X,Y)Z, W_a)W_a + g(R(X,Y)Z, N)\xi$$

for any $X, Y \in \Gamma(TM)$ and $\epsilon_a = g(W_a, W_a)$. Using (1.14), (1.16) and the last equation, we have $R(X,Y)Z = 0$ for any $X, Y, Z \in \Gamma(TM)$, due to the facts that $c = 0$ and $C = 0$ by Theorem 1.1. Thus M is a space of constant curvature 0. Also, from (1.14) and (1.17), we also have $R^*(X,Y)Z = 0$ for any $X, Y, Z \in \Gamma(S(TM))$. Thus M^* is also a space of constant curvature 0. □

Combining Note 1 and Theorem 1.1 and 3.6, we have:

Theorem 3.7. *Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then M is a lightlike space form of constant curvature* 0 *and locally a product manifold $L_\xi \times M^*$, where L_ξ is a null curve and M^* is a semi-Euclidean space.*

Nomizu and Pinkall [4] defined an affine immersion as follows: Let $f : M \to \bar{M}$ be an immersion of a manifold M as a hypersurface of a manifold $\bar{M}$ and ∇ and $\bar{\nabla}$ be torsion-free linear connections on M and $\bar{M}$ respectively. Then f is an *affine immersion* if there exists locally a transversal vector field N along f such that

$$\bar{\nabla}_{f_*X} f_*Y = f_*(\nabla_X Y) + B(X,Y)N, \quad \forall\, X, Y \in \Gamma(TM),$$

where f_* is the differential map of f. Then, as usual, we put

$$\bar{\nabla}_{f_*X} N = -A_N(f_*X) + \tau(f_*X)N.$$

Clearly, by (1.4), any lightlike isometric immersion is an affine immersion. Suppose $\dim M = 2m-1$ and ∇ is a flat connection on M. Let $\psi : M \to \mathbb{R}^{2m-1}$ such that every point $x \in M$ has a neighborhood $\mathcal{U}$ on which ψ is an affine connection preserving differmorphism with an open neighborhood $\mathcal{W}$ of $\psi(x)$ in $\mathbb{R}^{2m-1}$. Consider $\mathbb{R}^{2m-1}$ as a hyperplane of $\mathbb{R}^{2m}$ and let N be a parallel vector field transversal to $\mathbb{R}^{2m-1}$. Define, for any differentiable function $F : M \to \mathbb{R}$,

$$f : M \to \mathbb{R}^{2m}\ ;\ f(x) = \psi(x) + F(x)N, \quad \forall\, x \in M.$$

Thus, f is an affine immersion with $A_N = 0$, called the *graph immersion* with respect to F. Now, we recall the following result.

Theorem 3.8 ([2])**.** *Let M be a lightlike hypersurface of $\mathbb{R}^{2m}_q$ with a parallel screen distribution $S(TM)$. Then the immersion of M is affinely equivalent to the graph immersion of a certain function $F : M \to \mathbb{R}$.*

Theorem 3.9. *Let $(M, g, S(TM))$ be a lightlike real hypersurface of an indefinite complex space form $\bar{M}(c)$ such that $S(TM)$ is totally umbilic in M. Then* *the immersion of M is affinely equivalent to the graph immersion of a certain function $F : M \to \mathbb{R}$.*

Proof. By Theorem 1.1, we have $C = 0$, on any $\mathcal{U} \subset M$, and $c = 0$, i.e., the screen distribution $S(TM)$ is totally geodesic in M and the constant holomorphic sectional curvature c of the ambient space $\bar{M}(c)$ satisfies $c = 0$. By (1.6), $C = 0$, on any $\mathcal{U} \subset M$, implies $S(TM)$ is parallel with respect to the induced connection ∇. Also $c = 0$ implies that the ambient space $\bar{M}(c)$ is $\mathbb{R}^{2m}_q$. Therefore, by Theorem 3.8, we have our theorem. □

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