

# Lightning Electromagnetic Fields and Induced Voltages: Influence of Channel Tortuosity

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## Abstract

Models for calculation of lightning induced overvoltages usually assume a straight and vertical lightning channel. However, it is well known that the lightning path is tortuous on scales ranging from 1 m to 1 km. In this paper the tortuosity effect is analyzed for both lightning-generated electromagnetic fields and induced voltages. For a schematic representation of tortuous lightning channel, it is shown that at close and intermediate ranges the predominant effect is due to the inclination of the lowest channel segment; only for fields at relatively far ranges the overall tortuosity effect becomes appreciable.

## 1. Introduction

Power quality issues are nowadays increasing in their importance. In particular, MV distribution lines are very sensitive to effects of nearby lightning. An accurate evaluation of the lightning induced overvoltages is therefore essential to address those issues. Most models for the calculation of lightning induced overvoltages assume a straight vertical lightning path, although it is well known that the return stroke channel is tortuous on scales ranging between 1m (and even less) up to 1 km or so. The inclination of each segment of the lightning channel can cause attenuation or amplification of the electric and magnetic field, due to a change in the direction of propagation of the current wave, and can lead to appreciable variations of the field waveforms and hence influence the induced voltages.

While the effect of tortuosity on electromagnetic field has been studied in a number of papers e.g. [1-3], this effect on induced voltages has been rarely discussed [4,5]. In [4], the inducing potentials at an arbitrary space-time point are calculated by using a step lightning current, and the induced surge voltages are obtained by solving the transmission line equations [6] using the finite difference time domain (FDTD) method. In that paper only peak values of the induced voltages are derived. In [5], the authors, by using a double exponential channel-base current waveform and the MTLE model, calculate the far electromagnetic field according to the theory developed in [1]. The coupling model for induced voltage calculations is based on the theory developed in [7]. The far-field assumption leads to inaccurate results for close distances (e.g. 100 m).

The aim of the present paper is to further investigate the effect of channel tortuosity both on the electromagnetic fields and on the induced voltages. In this paper, we have chosen, as a preliminary step, schematic channel geometries by using an approach similar to that adopted in [4].

## 2. Electromagnetic Field Evaluation and Coupling Model

In the calculations of fields, the lightning channel is decomposed into  $N$  arbitrarily oriented segments which make their individual contributions to the total field or induced voltage. Each segment is treated as a single line radiator  $\mathbf{C}$  of length  $h$ , with arbitrary slope and location above ground [2]. By adopting a proper cylindrical reference system  $\mathcal{R}$ , with the  $z$ -axis coincident with the axis of the channel segment and the origin coincident with the starting point  $\mathbf{C}$ , the mathematical expression of the fields can be simplified since only the  $\phi$  component of the magnetic field  $\mathbf{H}$  and the  $r$  and the  $z$  components of the electric field  $\mathbf{E}$  are present. By assuming that the channel is traversed by a unit step function current:

$$i(z', t) = \left\{ u \left( t - \frac{z'}{v} \right) [u(z') - u(z' - h)] \right\} \quad (1)$$

where  $v$  is the return stroke front speed and  $u$  is the Heaviside step function, the generated EM field calculated in cylindrical reference system  $\mathcal{R}$  at the observation point  $P(r, \phi, z)$  can be formulated as given in [2].

Since a perfectly conducting plane is considered, the method of images can be applied and the contribution of the image sources can be obtained in the same manner by adopting a cylindrical coordinate system with the  $z$ -axis coincident with the axis of the image channel segment. Once the electric and magnetic fields due to a unit step current have been calculated, by adopting a suitable convolution summation, the field associated with an arbitrary current source  $i(t)$  can be obtained using Duhamel's integral:

$$y(t) = \int_0^t \frac{d i(\tau)}{d \tau} s(t - \tau) d\tau \quad (2)$$

where  $s(t)$  is the field component generated by a unit step current  $u(t - z'/v)$  travelling along  $C$  and  $y(t)$  is the field component generated at the same point by the arbitrary current  $i(t)$ .

The model for the calculation of the induced voltage on a lossless single-wire overhead line is the one found in [8]. The induced voltage will be calculated at the line point closest to the base of the lightning channel. In this case, solution of the partial differential equation of the coupling model is [9]:

$$v_i(0, t) = - \int_0^h e_z(0, d, z, t) dz - \frac{1}{2} \int_{-\infty}^{+\infty} e_L \left( x, d, t - \frac{|x|}{c} \right) \text{sgn}(x) dx \quad (3)$$

### 3. Channel Geometry

We will consider 4 different geometries which are shown in Fig. 1. Starting from the vertical channel (Fig. 1a), then a straight inclined channel will be considered (Fig. 1b) with a channel inclination angle with respect to vertical  $\alpha=15^\circ$ . Successively slanted channel segments with the same inclination, will be considered for a total of 2, and 4 segments of the same length (Figs. 1c,d). All channels segments lie in the  $x$ - $z$  plane.

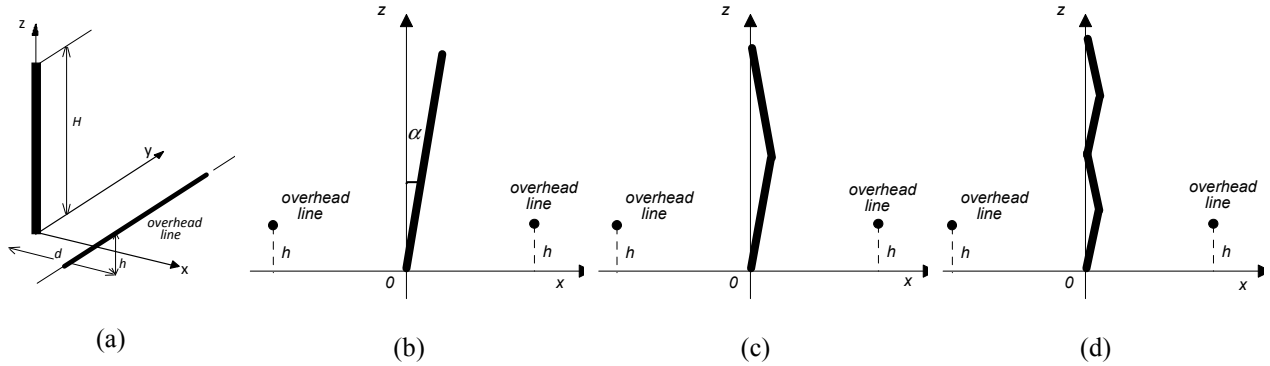


Fig. 1 Different channel geometries. (a) is not to scale with (b), (c) and (d).

### 4. Results

The specified channel-base current is [10]

$$i(0, t) = \frac{I_0}{\eta} \frac{(t/\tau_1)^n}{1 + (t/\tau_1)^n} e^{-\frac{t}{\tau_2}} + I_{02} (e^{-\frac{t}{\tau_3}} - e^{-\frac{t}{\tau_4}}) \quad (4)$$

with all the parameters set as in [10], which propagates at a velocity  $v=1.2 \cdot 10^8$  m/s. As far as the return stroke propagation model, we will use the MTLL [11], which considers a linear current decay along the channel, from a maximum at the channel base to zero at the channel top (at height  $H=7.5$  km above ground level). Vertical electric fields have been calculated at ground level at distances of 50 m, 500 m, 1000 m, and 5000 m from the channel base, at symmetrical points (with respect to  $x=0$ ) on the  $x$  axis (Fig. 2 and Fig. 3). One can see that for short and intermediate ranges, the effect of the inclination of the lowest segment is more important than overall tortuosity. Observation points toward which the lowest segment is inclined experience an electrical field which is significantly higher than the points on the opposite side of the ground termination point. At relatively far ranges, the inclination effect is weaker and the overall tortuosity plays a more important role.

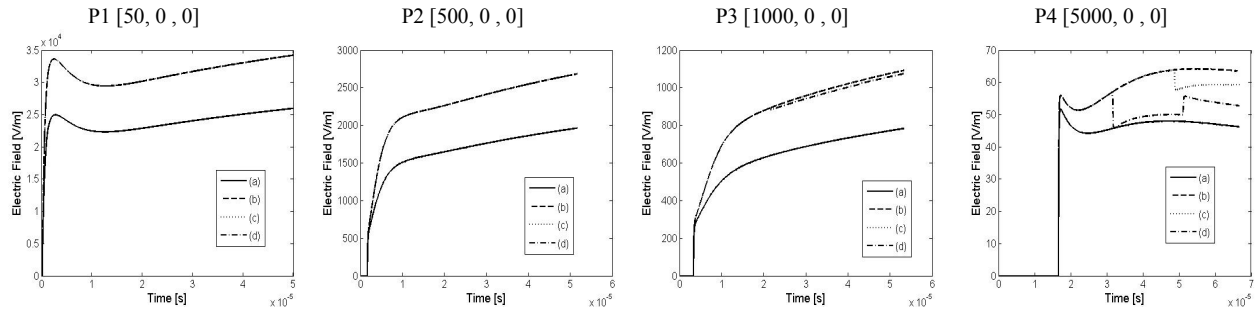


Fig. 2. Vertical electric field at different distances from the channel base (at  $x=0$ ) for channel geometries shown in Figs.1 (a), (b),(c) and (d) (observation points the  $x$  axis for  $x>0$ )

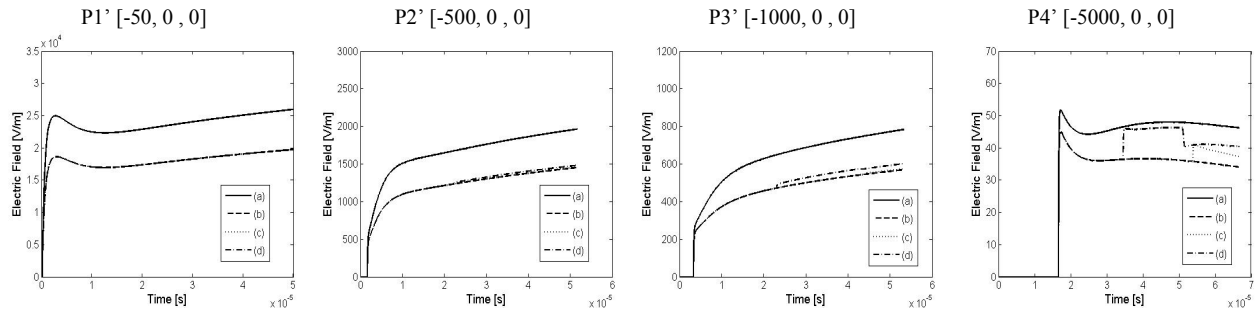


Fig. 3. Vertical electric field at different distances from the channel base (at  $x=0$ ) for channel geometries shown in Figs. 1 (a), (b),(c) and (d) (observation points on the  $x$  axis for  $x<0$ )

Induced voltages have been evaluated for an overhead line composed of a single conductor, at height  $h=10$  m above ground, perpendicular to the  $x$ - $z$  plane (see Fig. 1), at distances  $d$  of 50 m and 1000 m. It is clearly seen in Fig. 4 that for short and intermediate ranges, similar to electromagnetic fields, the effect of the inclination of the lowest segment is more important than overall tortuosity. Compared to the vertical channel, the amplitude of the induced voltage at a distance of 50 m is about 25% higher for points toward which the lowest segment is inclined and about 20% lower for points on the opposite side. Similar behavior is observed at 1000 m, although the induced voltage waveform exhibits a larger width. No overall tortuosity effect is noticed.

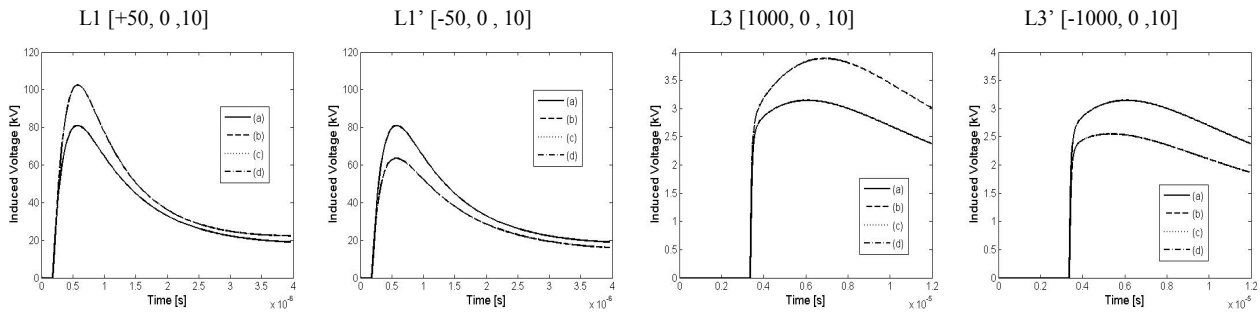


Fig. 4. Induced voltages at different distances from the channel base for channel geometries shown in Figs.1 (a), (b),(c) and (d)

## 5. Conclusion

In this paper, we analyzed the influence of tortuosity of schematic lightning channel on electromagnetic fields and induced voltages on an overhead line. It has been found that at close and intermediate ranges the predominant effect is due to the inclination of the lowest channel segment both for the fields and for the induced voltages; only for fields at relatively far ranges the overall tortuosity effect becomes appreciable.

## 6. References

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