

*Limit Distributions for
Sums of Independent
Random Vectors*

Heavy Tails in Theory and Practice

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