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LIMIT PRICING, UNCERTAIN ENTRY, AND THE ENTRY LAG †

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1. Introduction.

Recently progress has been made in the formal characterization of the pricing policy, maximizing the long-run profits of a supplier in a market with potential entry. Kamien and Schwartz (hereafter K-S) [9], assume the seller views the appearance of rivals as uncertain and dependent on its current price. They conclude that the optimal pre-entry policy is, (a) constant through time, (b) typically below the short-run monopoly price but above the limit price, and (c) tends to fall as certain non-price barriers to entry drop. Results similar to (b) and (c) were also obtained by Gaskins [5] and Baron [2] supposing that the supplier's current price affects the rate of entry in the industry.

In this paper we explore some consequences for the seller's price policy of an entry lag between a rival's decision to enter and its appearance as an entrant. Former focus on entry induced by current policies, assumed rivals to enter immediately upon deciding to do so. A priori arguments as to the significance in this context of such a lag were made by Bain [1] and Hicks [7], among others.

We proceed in the K-S uncertain entry framework. First appropriate adaptions are indicated, whereafter through time constant and freely variable pre-entry policies are discussed. Besides obtaining additional insights, we show that for a positive lag, prediction (c) may be reversed and that conclusion (a) generally does not hold, in case there are no adjustment costs associated with price changes.

2. The K-S Model Adapted.

Following K-S, a cartel's pricing problem is studied in a two period setting. We retain their assumptions 1 and 2, referring respectively to current pre-entry profits $e^{gt}\pi_1(p(t))$ and (expected) current post-entry gains $e^{gt}\pi_2$, where p(t) denotes product price at time t and g a constant rate of growth or decline. To allow for an entry lag, the stochastic framework governing the transition between both periods is adapted.

We suppose that once new rivals decide to enter, it takes them a fixed time interval τ, $0\leq\tau\leq\infty$, before they are actually entrants. This entry lag τ is exogenous to the cartel and may be thought of as a characteristic of the industry, (Bain p.11). Where new plants and/or distribution channels have to be planned, built, and put into operation, τ may be of considerable magnitude. In other situations, rivals may be able to convert their existing operations and enter almost instantaneously upon deciding to do so, (Hines [8]).

In addition, uncertainty prevails as to when rivals decide to enter, and therefore as to when entry will occur. At time zero, the probability of entry by a time t, F(t), is assessed by the cartel in the following way :

Assumption 3 : $F(0)=0$, $F'(t)/(1-F(t))=k$ for all $0\leq t<\tau$, $0\leq k<\infty$; $F(\tau)=1-e^{-k\tau}$, $F'(t)/(1-F(t))=h(p(t-\tau);g)\geq 0$ for all $t\geq\tau$, $h(0;g)=0$, $\delta h/\delta p\geq 0$, $\delta^2 h/\delta p^2\geq 0$, $p\geq 0$, $\delta h/\delta g\geq 0$.

At the time of the pricing decision no entry has occurred. During a "closed period" (Hicks), $0\leq t<\tau$, rivals can only enter if they had already decided to do so before time zero, since the entry lag is τ. For this interval the likelihood of entry is unaffected by the policy to be decided on, but we assume it to be directly related to the degree of initial rivalry as represented by the parameter k. For k=0, a monopoly position is kept at least until time τ.

In the "open period", $t \geq \tau$, if rivals enter at a given instant t, they must have decided to do so at time $t-\tau$. The indicated specifications are therefore in accordance with K-S's modelling of uncertain entry. (We supress g in much of the following) Finally, the assumption on $F(\tau)$, together with the sign restrictions on k and h, assure that F is continuous and non-decreasing.

The cartel's pricing problem is to choose a pre-entry policy so as to maximize long-run profits V, under the assumptions and any other restrictions as to the variability of this policy due to adjustment costs, where,

$$(1) \quad V = \int_0^\infty e^{-\rho t}[\pi_1(p(t))(1-F(t)) + \pi_2 F(t)]dt$$

$\rho = r-g>0$, and r is a constant rate of time discount.

We consider consecutively the polar cases of infinite and zero adjustment costs; which case is more appropriate would depend on the nature of the industry. The K-S original model obtains for the latter assumption and $\tau=0$.

3. Constant Pre-Entry Policies.

With $p(t)=p$ for all t in the pre-entry period, noting Assumption 3 and (1), the problem becomes :

$$(2) \quad \max_p V(p) = l(p)\pi_1(p) + m(p)\pi_2$$

with,

$$(3) \qquad l(p) = (1-e^{-(\rho+k)\tau})/(\rho+k) + e^{-(\rho+k)\tau}/(\rho+h(p))$$

$$(4) \qquad m(p) = 1/\rho - l(p)$$

The expected time until entry is $[(1-e^{-k\tau})/e^{-k\tau} + e^{-k\tau}/h(p)]$. In the absence of initial rivalry, the entry lag τ is expected to be prolonged with a lag in inducing entry decisions $1/h(p)$.

Let p° denote an optimal interior solution to (2), then :

(5) $V'(p^\circ) = l(p^\circ)\pi_1'(p^\circ) + l'(p^\circ)(\pi_1(p^\circ)-\pi_2) = 0$

(6) $V''(p^\circ) = \left[l(p^\circ)\pi_1''(p^\circ) - e^{-(\rho+k)\tau}(\pi_1(p^\circ)-\pi_2)h''(p^\circ)/(\rho+h(p^\circ))^2\right.$

$$\left. + 2(1-e^{-(\rho+k)\tau})\pi_1'(p^\circ)h'(p^\circ)/(\rho+k)(\rho+h(p^\circ))\right] \leq 0$$

understanding that $h'=\delta h/\delta p$ and $h''=\delta^2 h/\delta p^2$.

Equation (5) can readily be interpreted, as done by K-S. To evaluate (6) we inquire into the relative magnitude of p°.

In three trivial pricing situations one verifies $p^\circ=p^m$, with $\pi_1'(p^m)=0$. Specifically, for :

- $h'(p^m)=h(p^m)=0$ (K-S), sufficiently high non-price barriers blockade entry in the open period;
- $\tau\to\infty$, even if such barriers were sufficiently low to induce entry decisions when $p^\circ=p^m$, this does not cause actual entry due to a prohibitive large entry lag (natural or legal monopoly);
- $h\to\infty$ (for $p>0$) and $\tau>0$, all non-price barriers have been removed; entry will occur by time τ and monopoly profits are reaped while it is still possible.

In each of these circumstances entry before time τ due to initial rivalry may still be recognized. However the short-run monopoly price is adopted because this does not affect the eventual appearance of entrants.

Proceeding for non-trivial situations ($h'(p^m)>0$, $\tau<\infty$ and $h<\infty$) we have, with $\underline{p}$ the smaller solution to $\pi_1(p)=\pi_2$,

(7) $\underline{p} < p^\circ < p^m$

Other feasible values of p can be excluded recalling the assumptions. In particular let $\bar{p}$ and $\bar{\bar{p}}$ denote the larger solutions to $\pi_1(p)=\pi_2$ and $\pi_1(p)=0$ respectively. From (5) we obtain for $0 \leq p \leq \underline{p}$, $V'(p)>0$ and for $p^m \leq p < \bar{p}$, $V'(p) \leq 0$. Policies $\bar{p} \leq p \leq \bar{\bar{p}}$ can be improved upon, say by charging p^a, $\underline{p}<p^a<p^m$. Indeed, noting (2), (3) and (4) one has, $l(p^a)>l(p)$, $(\pi_1(p^a)-\pi_2)>(\pi_1(p)-\pi_2)$, and thus $V(p^a)>V(p)$. Characterization (7), derived by K-S along different routes, remains valid for any positive finite entry lag and constant pre-entry policies. However a complication arises, as compliance with the second-order condition (6) does not follow for $0<\tau<\infty$, while it does for $\tau=0$.

This difficulty will be resolved by making the additional assumption that $p°$ is differentiable with respect to τ for $0 \leq \tau < \infty$. (Alternatively, one can derive sufficient restrictions on the π_1 and/or h functions). Unless otherwise stated, it is understood that all functions are evaluated at $p°$. We claim for $0 \leq \tau < \infty$,

(8) $(dp°/d\tau) = [\pi_1'/(-V'')] > 0$

Compute from (5), $(dp°/d\tau)(-V'')=\pi_1'$. This expression is now defined by assumption for finite values of the lag. As for those values, $\pi_1'>0$ by (7), we have $(dp°/d\tau)\neq 0$ and $(-V'')\neq 0$. Hence the expression in (8) is defined. Now suppose the claimed inequality did not hold. This would imply there exists at least one finite τ where $(dp°/d\tau)=0$, because $(dp°/d\tau)|_{\tau=0}>0$. But this was already excluded and (8) follows. Consequently, in view of (7) and (8), we may continue with $V''<0$.
Thus, the optimal constant pre-entry price increases towards p^m as the lag increases towards infinity. Intuitively, the larger τ, the less any given sacrifice of current gains is worthwile as the effect on forestalling entry in the open period is postponed to a more distant future.

Turning to further implications, we note that $p°$ does not fall below the limit price (highest price for which h=0). It is assumed that h is twice differentiable for all prices, so that h' is continuous, while also by non-triviality $h'(p^\circ)>0$. Hence $h(p^\circ)>0$. For $\tau=0$, p° may coincidentally be arbitrarily close to the limit price, compare with K-S. If this were the case a still larger price would apply for a positive lag.

Next we compute, with $V^\circ = V(p^\circ)$,

$$(9)\quad (dV^\circ/d\tau) = [e^{-(\rho+k)\tau}(h-k)(\pi_1-\pi_2)/(\rho+h)] \gtreqless 0 \quad \text{as} \quad h \gtreqless k$$

observing (5) and (7).

For a high degree of initial rivalry, say $k \geq h(p^m)$, a longer lag reduces profits. The closed period with a high (exogenous) threat of entry is prolonged, while the beginning of the open period with a comparatively low (endogenous) threat is postponed. Only in the absence of, or with moderate initial rivalry, say $0 \leq k \leq h(\underline{p})$, is a longer lag beneficial to the cartel.

Given a change in the intensity of initial rivalry, we obtain for $0<\tau<\infty$,

$$(10)\quad (dp^\circ/dk) = [(\tau(\rho+k)-1+e^{-(\rho+k)\tau})/(\rho+k)^2](\pi_1'/(-V'')) > 0$$

$$(11)\quad (dV^\circ/dk) = [(\tau e^{-(\rho+k)\tau}(h-k)/(\rho+k)(\rho+h) - (1-e^{-(\rho+k)\tau})/(\rho+k)^2](\pi_1-\pi_2)$$

noting with regard to (10) that the expression between square brackets is zero for $\tau=0$ and monotonically increases with τ.

The cartel does not try to compensate for additional likelihood of entry outside its influence by further discouraging entry it may be able to forestall. In fact just the opposite is done. Because entry during the open period becomes more likely anyway, it tries to reap more pre-entry profits while it still can. However expected profits will decline provided already intense rivalry prevailed, so that $k \geq h$. With a low intensity such that $k<h$, it is possible that as a result total profits will increase.

To study a shift in the h function, let $h=\gamma M(p)$, $\gamma=1$ originally. Then,

$$(12)\quad \text{sign } (dp^\circ/d\gamma) = \text{sign } \left[(h^2-\rho^2) - e^{-(\rho+k)\tau}(h^2+\rho k)\right]$$

For $h>\rho$, denote by $z(\tau)$ the expression on the right hand side of (12). We have $z(0)<0$ and $z(\infty)=(h(p^m)-\rho)(h(p^m)+\rho)>0$, as by non-triviality $h(p^m)>h$. Also, noting (8), $(dz/d\tau)=[2(1-e^{-(\rho+k)\tau})(hh')(dp^\circ/d\tau)+e^{-(\rho+k)\tau}(\rho+k)(h^2+\rho k)]>0$. Since z is an increasing function in τ over the interval $0\leq\tau<\infty$, negative at one endpoint and positive at the other, z=0 for exactly one value of τ which we indicate by τ^b. We conclude :

$$\text{for } h > \rho,\ (dp^\circ/d\gamma) \lesseqgtr 0 \quad \text{as} \quad \tau \lesseqgtr \tau^b, \text{ and}$$

(13)

$$\text{for } h \leq \rho \text{ or } \tau=0,\ (dp^\circ/d\gamma) < 0$$

Interpreting, with K-S, a positive shift in h as a decrease in certain non-pri- barriers to entry, it follows that such a fall may lead to a higher pre-entry price. This reversal of the more conventional prediction will not occur as long as the lag is small ($0\leq\tau<\tau^b$) or entry in the open period is relatively unlikely, say $h(p^m)\leq\rho$. However it will apply when the traditional barriers are low, say $h(\underline{p})>\rho$, and in addition the lag is large enough ($\tau>\tau^b$). Intuitively, when an already high (endogenous) threat of entry intensifies and the effect of foregoing current gains on tempering this threat is postponed to a too distant future, it may be worthwile to sacrifice less and instead reap profits while the opportunity persists.

Another insight emerges, given that the cartel views the decisions to enter of say n potential rivals as statistically independent. A larger γ then corresponds to a larger n, ceteris paribus. Hence an increase in the number of potential entrants may yield a higher or lower constant price, in situations which can be readily interpreted given the comments above. Comparable claims, though on the grounds of stochastic dependence between entrants' decisions, were made by Sherman and Willett [10] and Goldberg and Moirao [6].

One may also verify that easier entry in the open period reduces expected profits regardless of the (finite) magnitude of τ.

Finally, with respect to the remaining parameters π_2, r and g, no additions to the K-S predictions follow (although some of those could only be obtained under slightly more restrictive conditions).

4. Freely Variable Pre-Entry Policies.

Provided there are no adjustment costs associated with pre-entry price changes, the cartel's problem is to maximize V (expression (1)), subject to the differential and differential-difference equations of assumption 3.

In trivial instances the corresponding comments of the previous section are still valid. For non-trivial situations and $\tau>0$ the problem constitutes an optimal control problem with delay τ. (For $\tau=0$, see K-S). Following the procedure outlined by Budelis and Bryson [4] it can be shown that if a price policy $\hat{p}$ is optimal, it satisfies :

$$(14)\quad e^{-(\rho+k)t}\pi_1'(\hat{p}(t)) + \lambda(t+\tau)h'(\hat{p}(t))(1-F(t+\tau)) = 0 \qquad 0 \leq t < \tau$$

$$(15)\quad e^{-\rho t}\pi_1'(\hat{p}(t))(1-F(t)) + \lambda(t+\tau)h'(\hat{p}(t))(1-F(t+\tau)) = 0 \qquad t \geq \tau$$

where F obeys the differential-difference equation and boundary condition of Assumption 3, and the multiplier function λ obeys,

$$(16)\quad \lambda'(t) = e^{-\rho t}[\pi_1(\hat{p}(t))-\pi_2] + \lambda(t)h(\hat{p}(t-\tau)) \qquad t \geq \tau$$

with transversality condition $\lim_{t\to\infty} \lambda(t)=0$

Due to the difficulties in analyzing this system of mixed differential-difference equations, we did not succeed in completely characterizing $\hat{p}$. Still, inquiring into the possibility of a constant policy, we obtain :

(17) $\hat{p}(t)=p$, all t in the pre-entry period, $\Rightarrow$ (α) for $0<\tau<\infty$, $k=h(p)$, and

(β) for $\tau\geq 0$, $p=\tilde{p}$

where $\tilde{p}$ is defined by,

(18) $R(\tilde{p}) = \pi_1'(\tilde{p})(\rho+h(\tilde{p})) - e^{-(\rho+h(\tilde{p}))\tau}h'(\tilde{p})(\pi_1(\tilde{p})-\pi_2) = 0$

To verify (α), note that with $\hat{p}(t)=p$ we have for $t\geq\tau$, $\lambda(t)=-e^{-\rho t}(\pi_1(p)-\pi_2)/(\rho+h(p))$ and $F(t)=1-e^{-k\tau-h(p)(t-\tau)}$, from respectively (16) and assumption 3. Substituting these functions and $\hat{p}(t)=p$ in (14), changing variables where appropriate, one obtains for $0\leq t<\tau$, $[\pi_1'(p)e^{-(k-h(p))t}-e^{-(\rho+k)\tau}h'(p)(\pi_1(p)-\pi_2)/(\rho+h(p))]=0$. Differentiating this expression with respect to t, we see that $k=h(p)$ and/or $p=p^m$. But the latter possibility is excluded, since by non-triviality $h(p^m)>0$.
With $k=h(p)$, (β) and admissibility of $\tilde{p}$, can be checked from (15), (16) and the indicated functions λ and F.

By arguments analog to those following (7), we infere that $\underline{p}<\tilde{p}<p^m$. Therefore, in non-trivial instances with no adjustment costs and $\tau>0$, a constant policy is not optimal if either $0\leq k\leq h(\underline{p})$ or $k\geq h(p^m)$. Such a policy may be adopted only when $h(\underline{p})<k<h(p^m)$ in the razor's edge case of $k=h(\tilde{p})$. It is not clear whether in the latter situation $\tilde{p}$ will in fact be optimal, that is, whether the conditions (14) through (16) are also sufficient. For $\tau=0$, $\tilde{p}$ is optimal (K-S).

Finally, by implicit differentiation of (18) and assuming differentiability of $\tilde{p}$, one obtains additional properties of $\tilde{p}$, similar to those of section three.

However, some remarks are in order. First, one can only study changes in parameters different from k which is supposed to adjust so as to preserve k=h. Second, this symmetry slightly changes some of the inferences. Notably, the effect of a shift in the h function will now only depend on whether the lag is small or large (compare (13)). Also, expected profits now decline with an increase in τ.

5. Conclusions.

Earlier formal work did not encompass the possibility, sensible on a priori and empirical grounds (Bain, p.208 and Blackstone [3, p.60]), that an established seller able to discourage entry, may act so as to make hay while the sun shines. Our analysis suggests that such behavior may be consistent with long-run profit maximization in industries where rivals need a time interval to make their entries effective. Specifically, for constant pre-entry prices, the supplier is more likely to reap higher short-run profits, the longer the entry lag, the more intense eventual exogenous initial rivalry, and the easier entry in a distant future. For freely variable policies these insights could only be obtained in more limited circumstances. It remains to be seen whether a complete characterization of such price policies would yield different conclusions.

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