Limitations of First-Order Surface Impedance Boundary Condition and Its Effect on 2D Simulations for PCB Transmission Lines

Yuandong Guo[1#], DongHyun Kim[2#], Jiayi He[3#], Shaohui Yong[4#], Yuanzhuo Liu[5#], Xiaoning Ye[6*], and Jun Fan[7#]
[#]Electromagnetic Compatibility Laboratory
Missouri University of Science and Technology
Rolla, Missouri, USA
[1]ydggdd, [2]kimdonghy, [3]hejiay, [4]sy2m5, [5]liuyuanz, [7]jfan@mst.edu
[*]Datacenter Group
Intel Corporation
Portland, Oregon, USA
[6]xiaoning.ye@intel.com

***Abstract*—Signal integrity (SI) issue is a critical concern as the data rate continues to increase and SI analysis is heavily dependent on simulations. Inaccurate simulation data may result in inadequate design decisions influencing high-speed digital design and optimization. The surface impedance boundary condition (SIBC) concept is generally utilized in commercial electromagnetic (EM) solvers, which is considered to be an efficient technique as the interior region of the conductor of interest does not need to be included in the numerical procedure. The first-order SIBC has been incorporated into many EM simulation tools widely used in industries. In this paper, the limitations of the first-order SIBC in 2D simulations for PCB transmission lines are analyzed and demonstarted for the first time. Different PCB transmission lines with various cross-sectional geometries are simulated in a commercial 2D EM solver with and without the implementation of the first-order SIBC to reveal the effect on the simulated transmission line behaviors. It is found that the accuracy of the simulations with the first-order SIBC decreases as the edge of the signal conductor in the cross-section becomes narrower. The possible solutions are proposed to overcome the issue.**

Keywords*—signal integrity, surface impedance boundary condition, transmission line, skin effect, per-unit-length resistance, internal inductance, conductor loss, surface roughness.

I. Introduction

As the data rate and clock frequencies increase, signal integrity (SI) issue has become a more critical concern in the design of high-speed interconnects. Often SI analysis relies on simulations. Inaccurate or unreliable simulation data may lead to inadequate design decisions which may result in delay in the market release of a product. The surface impedance boundary condition (SIBC) concept utilized in many commercial electromagnetic (EM) solvers is generally considered to be an efficient technique enabling accelerated simulations as the interior conducting region of the conductor of interest does not need to be included in the numerical procedure [1].

Frequency-dependent conductor loss resulted from skin effect is critical in SI analysis. The calculation of the field distribution inside the conductor can be simplified under skin effect condition [2]. SIBCs constructed upon skin effect theory include the material properties of the conductor and define the approximate relationships between the tangential components of the electric and magnetic fields or between the normal and tangential components of the magnetic field [3], [4]. In computational electromagnetics, SIBCs are categorized by the order of approximation. The first-order SIBC does not take into account the curvature of the interface between the conductor and dielectric, and the field variation along the conductor surface [5]. The inherent drawbacks limit the application area of the first-order SIBC. SIBCs of high order have been developed in [6]-[8] allowing the improvement in simulation accuracy and the expansion in the concrete application of surface impedance boundary concept. However, it was found that the first-order SIBC is widely used in the EM simulation tools with high market share in the industry [9]-[12].

Surface roughness is a principle consideration for high-speed digital design as it may introduce significant increase in the conductor loss of a PCB transmission line. The surface roughness correction factor is commonly treated as a modification to the SIBCs defined on the conductor/dielectric interface [13]. Therefore, suitable SIBC is essential for an accurate simulation.

In this paper, the limitations of the first-order SIBC in 2D simulations for PCB transmission lines are analyzed and the results are demonstrated for the first time. PCB transmission lines with various cross-sectional geometries are simulated in one commercial EM solver with and without the implementation of the first-order SIBC. The simulated transmission line behaviors are compared to reveal the effect of the first-order SIBC. It is found that the accuracy of the simulations with the first-order SIBC decreases as the edge of the signal conductor in the cross-section becomes narrower.

PCB striplines with smooth conductors are modeled and examined in Section III after the brief introduction to the first-order SIBC in Section II. It can be expected that similar conclusions are applicable for the microstrip-lines since SIBC is not a function of transmission line type. Through the comparisons of conductor loss and per-unit-length resistance, the impact of the first-order SIBC on the simulated conductor loss of smooth conductors is illustrated. Based on this finding,

This work was partly supported by the NSF No. IIP-1440110 and Intel Corporation, Oregon, USA.

Authorized licensed use limited to: Missouri University of Science and Technology. Downloaded on January 25,2021 at 01:36:54 UTC from IEEE Xplore. Restrictions apply.

the effect on the modeled conductor loss for rough conductors is further investigated in Section IV. The possible solutions are proposed in Section V to overcome the limitations of the first-order SIBC.

II. Physical Background of the First-Order SIBC

A. Derivations of the First-Order SIBC

The first-order SIBC, also known as Leontovich approximation [14], was introduced in the 1940's as a pioneering investigation on the problem of field distribution calculation inside the conductor accounting for skin effect. A simplified derivation procedure of the surface impedance expression for a flat smooth conductor is exhibited in [2]. The critical derivations and prominent assumptions are adopted in this paper in order to show the physical background of the first-order SIBC. It is assumed that the conductor has a conductivity of σ and the dielectric has a permeability of μ and permittivity of ε.

For a good conductor with high conductivity, the conduction current $\vec{J}$ is much larger than the displacement current $j\omega\vec{D}$. Consequently, Ampere's law can be written as

$$\nabla \times \vec{H} \approx \vec{J} = \sigma\vec{E} \tag{1}$$

Taking the curl of Faraday's law yields

$$\nabla \times \left(\nabla \times \vec{E}\right) = -j\omega\mu\nabla \times \vec{H} \tag{2}$$

In the derivation, the free charge is assumed to be negligible and the source of the electric field is assumed as the time-varying magnetic field, which allows the reduction to Gauss's law

$$\nabla \cdot \varepsilon\vec{E} = 0 \tag{3}$$

With the application of the vector identity, (2) is re-written as

$$\nabla^2\vec{E} = j\omega\mu\nabla \times \vec{H} \tag{4}$$

Substituting (1) into (4) and multiplying by σ at both sides gives the diffusion equation for the current density $\vec{J}$

$$\nabla^2\vec{J} = j\omega\mu\sigma\vec{J} \tag{5}$$

Assuming the current is in z-direction, the solution to the diffusion equation is found as

$$\vec{J}_z = \vec{J}_0 e^{-(1+j)x/\delta} \tag{6}$$

where δ is the skin depth and expressed as

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{7}$$

Integrating (6) gives the expression of the total current density $\vec{J}_T$

$$\vec{J}_T = \int_0^\infty \vec{J}_0\, e^{-(1+j)x/\delta} dx = -\frac{1}{2}\vec{J}_0\delta(j-1) \tag{8}$$

The surface impedance $\vec{Z}_s$ is defined as the ratio of the electric field at the conductor surface and the total current density, which yields

$$\vec{Z}_s = \frac{\vec{E}_0}{\vec{J}_T} = (1+j)\frac{1}{\sigma\delta} \tag{9}$$

The real part of the surface impedance is the per-unit-length frequency-dependent series resistance and the imaginary part is generally expressed as $j\omega L$, where L is the internal inductance. The real and imaginary parts should satisfy the K-K relationship [15], [16].

The first-order SIBC forces the tangential component of electric field equal to the term $n \times H_{tan}$ multiplying by the surface impedance Z_s.

B. Critical Assumptions for a Valid First-Order SIBC

The first-order SIBC is only valid under a set of critical assumptions. First, the incident wave upon a conductor is assumed to be perpendicular to the surface and there is no variation in the field distribution along the interface between conductor and dielectric. Second, the conductor has to be flat and infinite thick in half-space, which implies $\delta \ll T$, where T is the conductor thickness. Third, the source of the incident wave is at infinity and the dielectric material is linear, homogeneous and isotropic [4].

Noticeable violations of the aforementioned assumptions in the structure of typical PCB transmission lines are that the thickness of the conductors is not infinite large and the conductors are not flat. Therefore, the implementation of the first-order SIBC is considered to be problematic in the simulations, especially when there are sharp edges in the cross-sections of PCB transmission lines.

III. Simulation Based Analysis

Different PCB transmission lines with various cross-sectional geometries are simulated in 2D in one commercial EM tool with and without the application of the first-order SIBC. Smooth conductors with finite conductivity are employed in the simulations. A set of comparisons of the simulated transmission line behaviors are made to illustrate the impact of the first-order SIBC. The simulations without the implementation of first-order SIBC, namely, the simulations with the direct solve or solve inside option, are treated as the references.

Assuming 28 Gbps data rate for the simulated single-ended PCB channels, the conductor loss, per-unit-length resistance and internal inductance at 14 GHz and 28 GHz are singled out for the comparisons. As aforementioned, PCB striplines are used as examples for the illustration. It is assured that all simulated results converged.

A. The 2D Simulation Models

Representative cross-sections of the real PCB striplines are shown in Fig. 1 and Fig. 2 indicating the under etched and over etched scenarios, respectively. The first subscript in the geometrical parameters denote the cross-sectional shape of the signal trace, which is a trapezoid. The second subscript expresses under or over etched profile of the signal conductor. Table I and Table II summarize the values of the cross-sectional geometries in Fig. 1 and Fig. 2, respectively.

To reveal the limitations of the first-order SIBC and how the accuracy of the simulation changes with the cross-sectional shape of the signal trace when the first-order SIBC is applied, the striplines with the special cross-sections are simulated and the 2D models are demonstrated in Fig. 3. The subscripts in the dimensional parameters represent the shapes of the signal conductors in the cross-sections. Table III gives the values of the geometries.

Authorized licensed use limited to: Missouri University of Science and Technology. Downloaded on January 25,2021 at 01:36:54 UTC from IEEE Xplore. Restrictions apply.

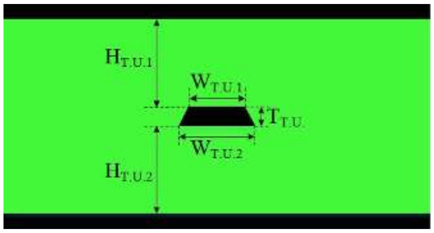


Fig. 1. A representative cross-section of real PCB striplines with under etched scenario.

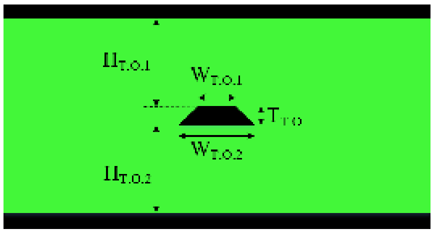


Fig. 2. A representative cross-section of real PCB striplines with over etched scenario.

TABLE I. VALUES OF THE GEOMETRIES IN FIG. 1

$H_{T.U.1}$	$H_{T.U.2}$	$W_{T.U.1}$	$W_{T.U.2}$	$T_{T.U.}$
3.2 mil	3.8 mil	4.8 mil	6.0 mil	1.2 mil

TABLE II. VALUES OF THE GEOMETRIES IN FIG. 2

$H_{T.O.1}$	$H_{T.O.2}$	$W_{T.O.1}$	$W_{T.O.2}$	$T_{T.O.}$
3.2 mil	3.8 mil	3.0 mil	6.0 mil	1.2 mil

The first-order SIBC is implemented on all conductor surfaces. The surfaces of the conductors are smooth and the conductivity is $5.8 \times 10^7\ S/m$. For all simulations, the relative permittivity and loss tangent of the dielectric at 1.0 GHz are 3.7 and 0.002, respectively, indicating the MEGTRON 6 material. The Djordjevic model described in [17] is applied to each 2D simulation.

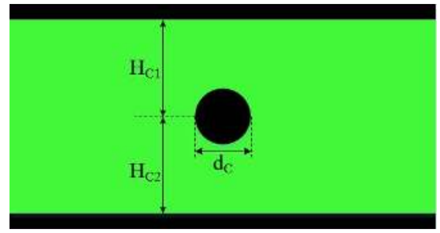
B. Simulation-based Analysis

The effect of the first-order SIBC on the per-unit-length resistance for various cross-sections is revealed in Fig. 4 to Fig. 8. The first-order SIBC starts to lose accuracy in the simulations as the edge of the signal conductor becomes narrower. The reason behind is the first-order SIBC is no longer valid at the edges due to the violations of the flat and thick conductor assumptions in the cross-sections.

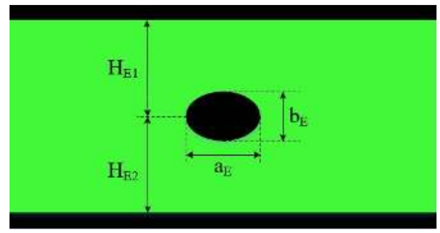
The over-etched trapezoidal shape has the narrowest edge. Thus, it is considered to be an extreme case for the comparison of the per-unit-length internal inductance. The results are exhibited in Fig. 9 with the zoom-in view at low frequencies where the simulation error reaches its maximum value of 58.02% at 10MHz. The error reduces to 14.99% and 14.46% at 14 GHz and 28 GHz, respectively.

With the per-unit-length resistance, capacitance and total inductance extracted from the 2D simulations, the per-unit-length conductor loss is calculated using [18]

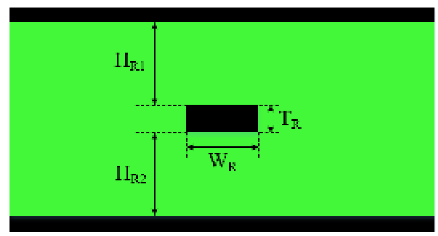
$$\alpha_c \approx 0.5R/\sqrt{Z_0} \tag{10}$$



(a)



(b)



(c)

Fig. 3. The 2D models of the striplines with special shaped signal conductors: (a) circle, (b) ellipse, (c) rectangle.

TABLE III. VALUES OF THE GEOMETRIES IN FIG. 3

Para.	Value	Para.	Value	Para.	Value
H_{C1}	4.2 mil	H_{E1}	4.2 mil	H_{R1}	3.2 mil
H_{C2}	3.8 mil	H_{E2}	3.8 mil	H_{R2}	3.8 mil
d_C	3.0 mil	a_E	4.0 mil	W_R	6.0 mil
/	/	b_E	2.0 mil	T_R	1.2 mil

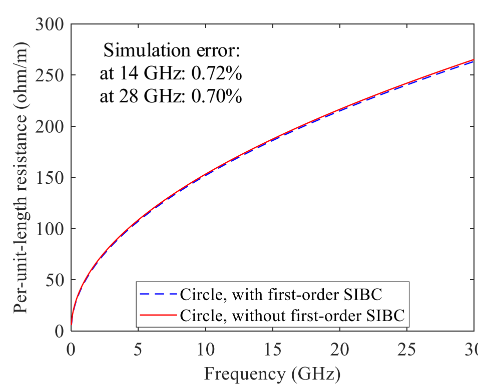


Fig. 4. Comparison of the per-unit-length resistance when the cross-section of the stripline signal conductor is a circle.

where the characteristic impedance $Z_0 \approx \sqrt{L/C}$.

Authorized licensed use limited to: Missouri University of Science and Technology. Downloaded on January 25,2021 at 01:36:54 UTC from IEEE Xplore. Restrictions apply.

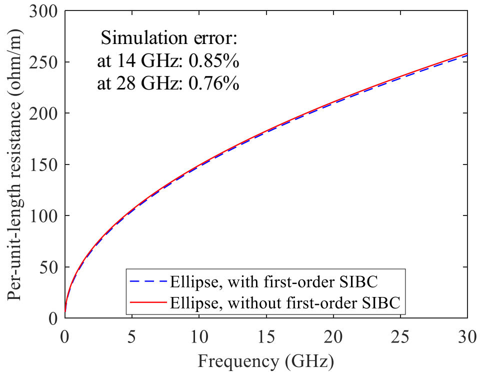


Fig. 5. Comparison of the per-unit-length resistance when the cross-section of the stripline signal conductor is an ellipse.

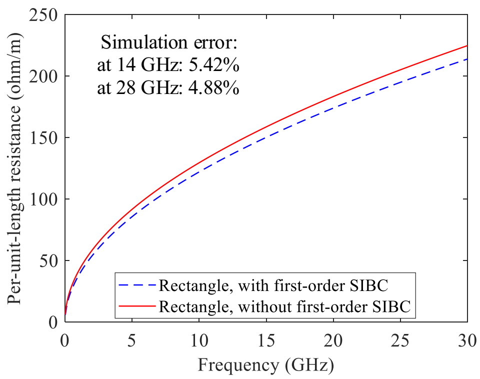


Fig. 6. Comparison of the per-unit-length resistance when the cross-section of the stripline signal conductor is a rectangle.

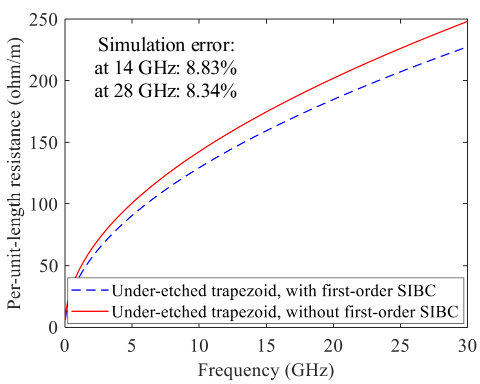


Fig. 7. Comparison of the per-unit-length resistance when the cross-section of the stripline signal conductor is an under-etched trapezoid.

Fig. 10 demonstrates the comparisons of conductor loss for the striplines with both under-etched and over-etched trapezoidal cross-sections when assuming 3 inches length. The conductor loss in the simulation with the first-order SIBC and over-etched trapezoidal signal trace is 6.16% and 5.33% smaller than the references at 14 GHz and 28 GHz, respectively.

The relatively smaller simulation errors in the conductor loss comparison against to that in the per-unit-length resistance and internal inductance comparisons is explainable based on (10). Since both per-unit-length *R* and *L* are under estimated with the first-order SIBC applied to the simulations, the error in conductor loss becomes relatively smaller.

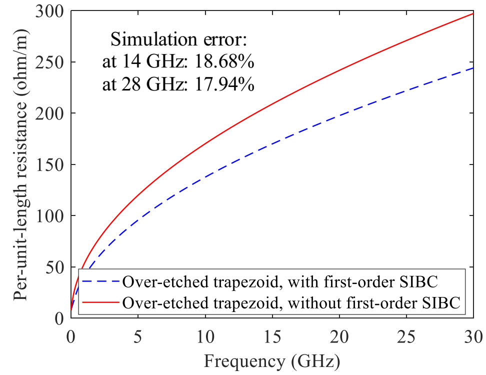
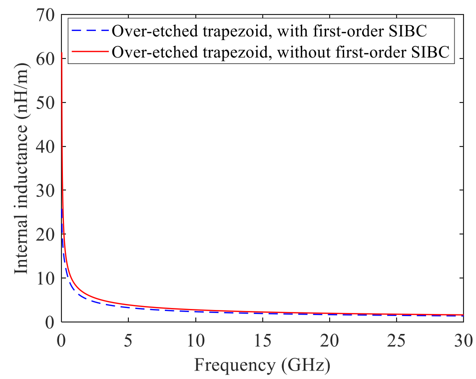
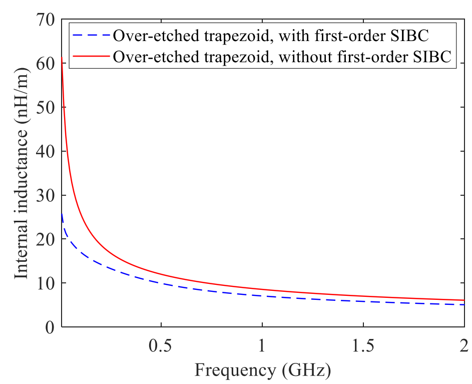


Fig. 8. Comparison of the per-unit-length resistance when the cross-section of the stripline signal conductor is an over-etched trapezoid.



(a)

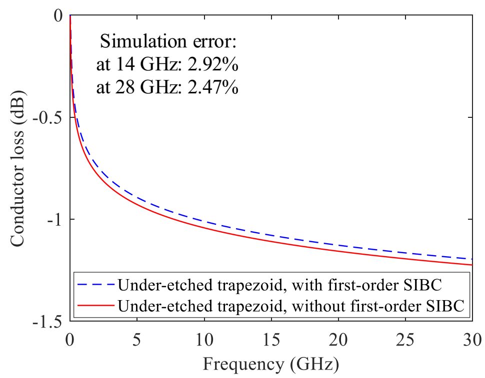


(b)

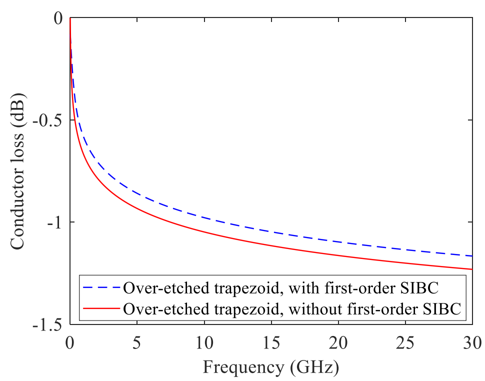
Fig. 9. Comparison of the per-unit-length internal inductance for the stripline with the over-etched trapezoidal signal conductor: (a) comparison of the per-unit-length internal inductance from 10 MHz to 30 GHz, (b) comparison of the per-unit-length internal inductance from 10 MHz to 2 GHz (the zoom-in view of Fig. 9 (a) at low frequencies).

Therefore, it is considered to be a coincidence for the simulations modeled in the 2D EM simulation tool under the study in this paper that the simulated conductor loss obtained with the first-order SIBC implemented could have a seemingly acceptable error when compared to the reference data. However, accuracy has been lost for the simulated per-unit-length resistance and internal inductance. The first-order SIBC should be avoid using for the PCB transmission lines with trapezoidal signal conductors in the cross-sections. Cautions should also be given for the applications where the surface impedance value is important for the analysis.

Authorized licensed use limited to: Missouri University of Science and Technology. Downloaded on January 25,2021 at 01:36:54 UTC from IEEE Xplore. Restrictions apply.



(a)



(b)

Fig. 10. Comparisons of the conductor loss when assuming 3 inches length: (a) comparison of the conductor loss for the stripline with the under-etched trapezoidal signal conductor, (b) comparison of the conductor loss for the stripline with the over-etched trapezoidal signal conductor.

IV. The Effect of the First-Order SIBC on the Conductor Loss for Rough Conductors

The complex-valued causal Huray roughness correction factor was introduced in [13] and the expression is

$$H_c(\omega) = 1 + \sum_{i=1}^{n} \frac{K_i}{1 + \left(\frac{2j\omega}{\omega_i}\right)^{-1/2}} \quad (11)$$

where

$$K_i = \frac{6\pi a_i^2 N_i}{A_{hex}} \quad (12)$$

$$\omega_i = \frac{2}{a_i^2 \mu\sigma} \quad (13)$$

where a_i, N_i and A_{hex} are the snowball radius, total snowball number and a section of smooth surface area defined in the classical Huray surface roughness model [19], respectively.

The surface impedance of a smooth conductor has the following expression

$$Z_{s\_smooth} = R_{smooth} + j\omega L_{int\_smooth} \quad (14)$$

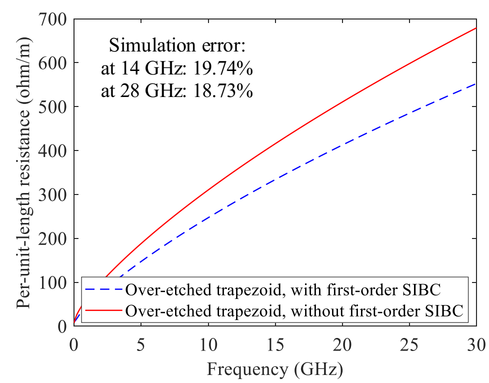
Multiplying (14) by (11) gives the surface impedance of the rough conductor. The real part of Z_{s_rough} is the constructed per-unit-length resistance of the rough conductor. The corresponding per-unit-length internal inductance can be calculated from the imaginary part using

$$L_{int\_rough} = \frac{Im(Z_{s\_rough})}{\omega} \quad (15)$$

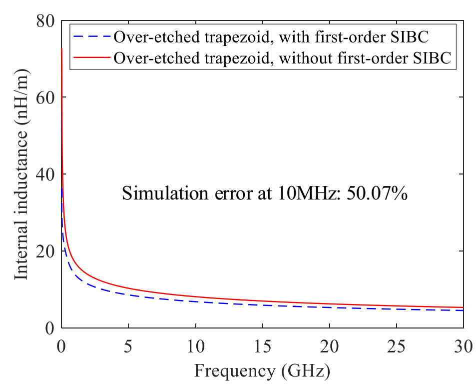
Assuming a_i, N_i and A_{hex} have the numbers listed in Table IV and taking the modeled striplines with the over-etched signal conductors as example, the comparisons of the constructed per-unit-length resistance and internal inductance are illustrated in Fig. 11, describing the impact of the first-order SIBC on the simulated results when the conductor surfaces are rough.

TABLE IV. Values of the Parameters in Huray Model

a_i	N_i	A_{hex}
0.5 μm	50	9.4×9.4 μm²



(a)



(b)

Fig. 11. Comparisons of the per-unit-length resistance and internal inductance for the stripline with the over-etched signal conductor and rough surfaces: (a) comparison of the per-unit-length resistance, (b) comparison of the per-unit-length internal inductance.

V. The Possible Solutions

The direct solve in the numerical procedure without the application of the first-order SIBC is considered to be the most accurate. However, the implementation of the surface roughness models requires the surface impedance concept. The possible solutions to this problem are proposed as the following:

Authorized licensed use limited to: Missouri University of Science and Technology. Downloaded on January 25,2021 at 01:36:54 UTC from IEEE Xplore. Restrictions apply.

1) Building the real rough surfaces in a 3D EM solver and using direct solve.

2) Incorporating SIBCs of higher order into the 2D EM tool.

3) For given a_i, N_i and A_{hex}, constructing the per-unit-length circuit parameters for a transmission line with rough surfaces based on the simulated results of the corresponding model with smooth conductors. Direct solve is needed in the 2D simulations.

VI. Conclusion

In this paper, the limitations of the first-order SIBC in the 2D simulations for PCB transmission lines with both smooth and rough conductor surfaces are analyzed for the first time. Various transmission lines are simulated in a commercial EM solver with and without the implementation of the first-order SIBC. It is found that the simulation accuracy decreases as the edge of the conductor in the cross-section becomes narrower. The possible solutions are discussed and proposed to overcome the limitations of the first-order SIBC.

References

[1] S. Yuferev, and L. Kettunen, "Implementation of high order surface impedance boundary conditions using vector approximating functions," IEEE Transcations on Magnetics, vol. 36, no. 9, pp. 1606-1609, Jul. 2000.

[2] S. H. Hall, and H. L. Heck, "Advanced signal integrity for high-speed digital designs," Hoboken, NJ, USA: John Wiley & Sons, 2009, pp. 204–222.

[3] S. Yuferev, and N. Ida, "Selection of the surface impedance boundary conditions for a given problem," IEEE Transcations on Magnetics, vol. 35, no. 3, pp. 1486-1489, May 1999.

[4] N. Ida, S. Yuferev, and L. D. Rienzo, "A systematic approach to the concept of surface impedance boundary conditions," Facta universitatis – series: Electronics and Energetics, vol. 22, no. 2, pp. 143-158, Aug. 2009.

[5] S. Yuferev, and L. D. Rienzo, "Surface impedance boundary conditions in terms of various formalisms," IEEE Transcations on Magnetics, vol. 46, no. 9, pp. 3617-3628, Sep. 2010.

[6] K. M. Mitzner, "An integral equation approach to scattering from a body of finite conductivity," Radio Sci., vol. 2, no. 12, pp. 1459-1470, Dec. 1967.

[7] S. Yuferev, and N. Ida, "Time domain surface impedance concept for low frequency electromagnetic problems—Part I: Derivation of high order surface impedance boundary conditions in the time domain," Inst. Elect. Eng. Proc.—Sci., Meas. Technol., vol. 152, no. 4, pp. 175-185, Jul. 2005.

[8] S. Barmada, L. D. Rienzo, N. Ida, and S. Yuferev, "The time domain surface impedance concept for low frequency electromagnetic problems—Part II: Application to transient skin and proximity effect problems in cylindrical conductors," Inst. Elect. Eng. Proc.—Sci., Meas. Technol., vol. 152, no. 5, pp. 207-216, Sep. 2005.

[9] User's Guide – High Frequency Structure Simulator (HFSS), Ansoft Corporation.

[10] G. Gold, "Modeling surface roughness in CST Microwave Studio," Institute of Microwaves and Photonics, Apr. 2016.

[11] Theory of operation for Momentum, online documentation of Keysight Technologies [Online]. Available: https://edadocs.software.keysight.com/display/ads2009/Theory+of+Operation+for+Momentum

[12] Electromagnetic – Advanced Design System, online documentation of Keysight Technologies [Online]. Available: http://edadownload.software.keysight.com/eedl/ads/2012_08/pdf/em.pdf

[13] J. E. Bracken, "A causal Huray model for surface roughness," in DesignCon 2012, Santa Clara, CA, USA, 2012.

[14] M. A. Leontovich, "Investigation of propagation of radiowaves," Moscow, U.S.S.R.: Acad. Sci., 1948, pt. II.

[15] X. Sun, Y. Guo, Y. Sun, K. Song, L. Ye, X. Ye, J. L. Drewniak, and J. Fan, "Causality analyzing for transmission line with surface roughness," 2019 IEEE International Symposium on Electromagnetic Compatibility, Signal & Power Integrity (EMC+SIPI), New Orleans, LA, USA, Jul. 22-26, 2019.

[16] Y. Guo, B. Chen, X. Sun, X. Ye, J. Hsu, and J. Fan, "Study of TDR impedance for better analysis to measurement correlation," 2019 Joint International Symposium on Electromagnetic Compability, Sapporo and Asia-Pacific International Symposium on Electromagnetic Compatibility (EMC Sapporo/APEMC), Sapporo, Japan, Jun. 3-7, 2019.

[17] A. R. Djordjevic, R. M. Biljie, V. D. Likar-Smiljanic, and T. K. Sarkar, "Wideband frequency-domain characterization of FR-4 and time-domain causality," IEEE Trans. Electromag. Compat., vol. 43, pp. 662-667, Nov. 2001.

[18] D. M. Pozar, "Microwave engineering," 4th ed., Hoboken, NJ, USA: John Wiley & Sons, 2012, pp. 78–80.

[19] P. G. Huray, O. Oluwafemi, J. Loyer, E. Bogatin and X. Ye, "Impact of copper surface texture on loss: a model that works," in DesignCon 2010, Santa Clara, CA, USA, 2010.

Authorized licensed use limited to: Missouri University of Science and Technology. Downloaded on January 25,2021 at 01:36:54 UTC from IEEE Xplore. Restrictions apply.