Limited Automata and Context-Free Languages

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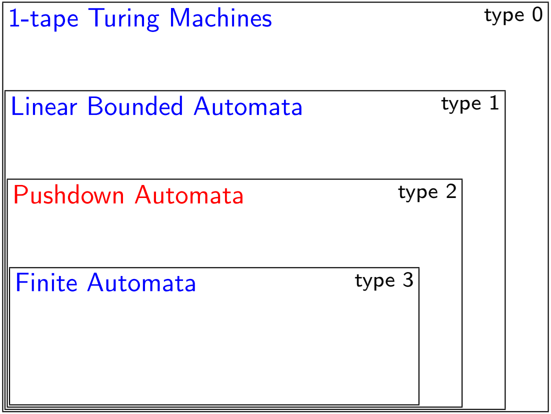
DI MILANO

The Chomsky Hierarchy



Limited Automata [Hibbard'67]

One-tape Turing machines with restricted rewritings

Definition

Fixed an integer $d \geq 1$, a *d-limited automaton* is

- a one-tape Turing machine
- which is allowed to rewrite the content of each tape cell *only in the first d visits*

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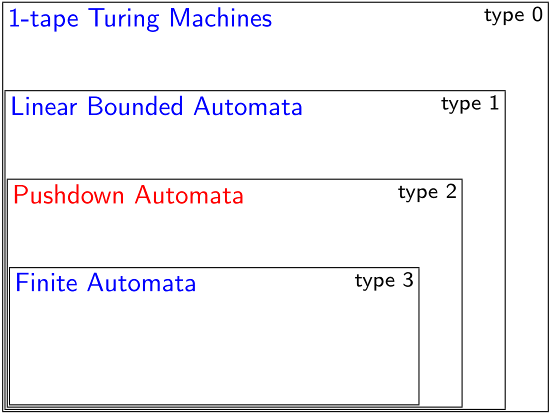
Computational power

- For each $d \geq 2$, d-limited automata characterize context-free languages [Hibbard'67]
- 1-limited automata characterize regular languages [Wagner&Wechsung'86]

The Chomsky Hierarchy



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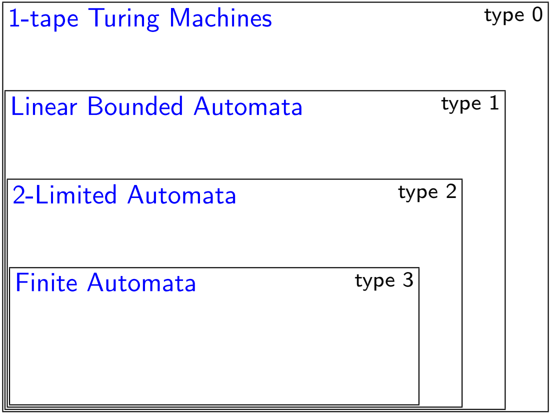

Our Contributions

- 2-Limited Automata ≡ Pushdown Automata:
  descriptional complexity point of view

- Determinism vs Nondeterminism

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2-LAs → PDAs

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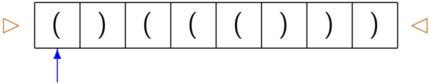
Deterministic Context-Free Languages ≡ Deterministic 2-LAs

Example: Balanced Parentheses



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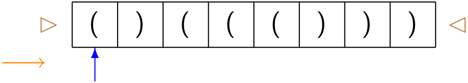

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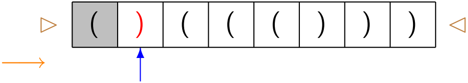

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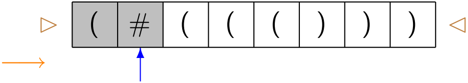

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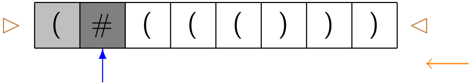

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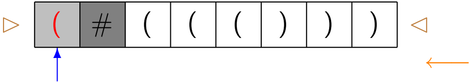

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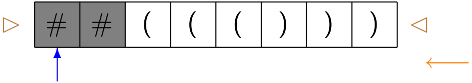

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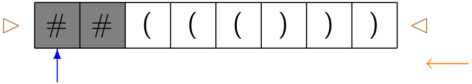

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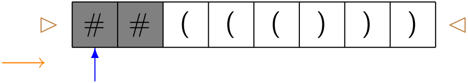

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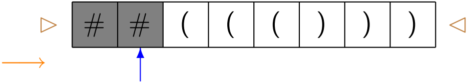

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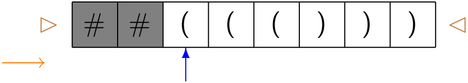

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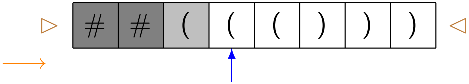

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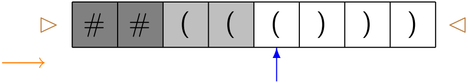

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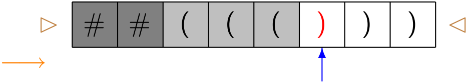

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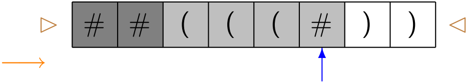

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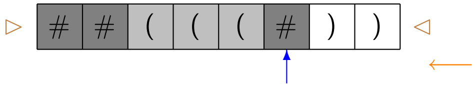

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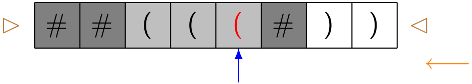

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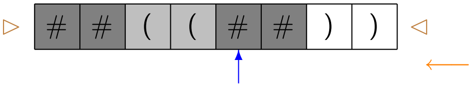

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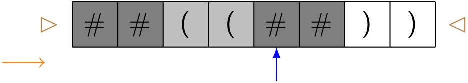

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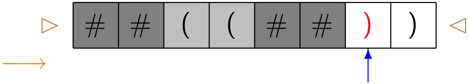

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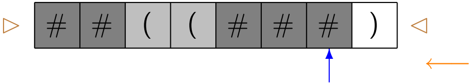

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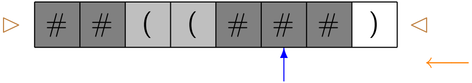

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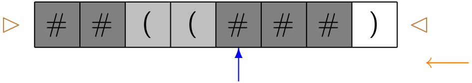

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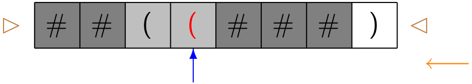

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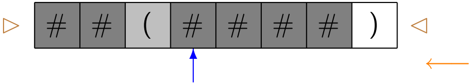

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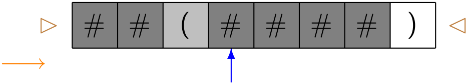

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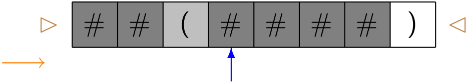

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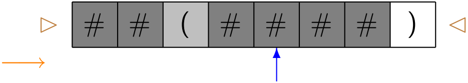

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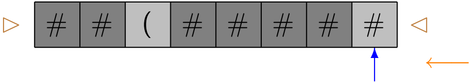

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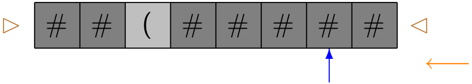

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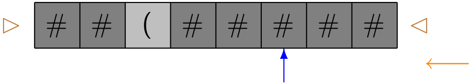

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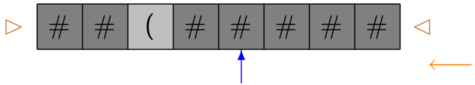

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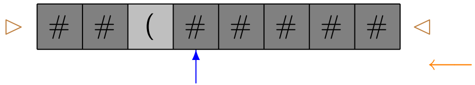

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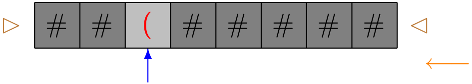

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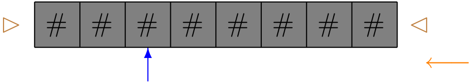

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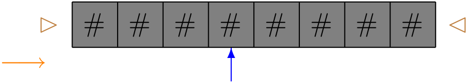

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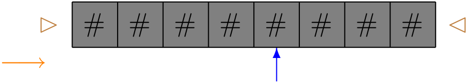

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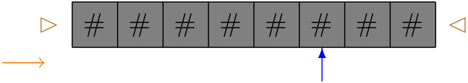

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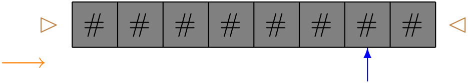

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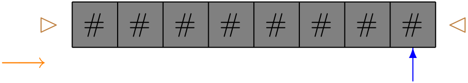

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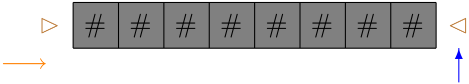

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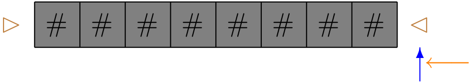

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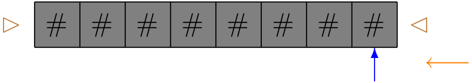

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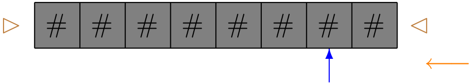

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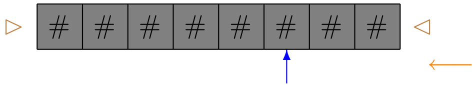

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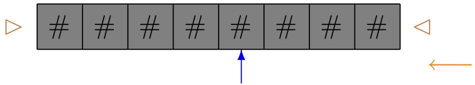

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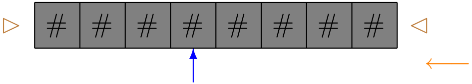

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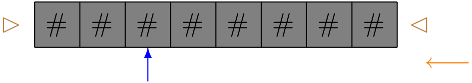

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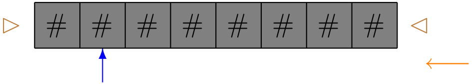

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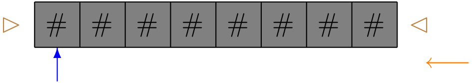

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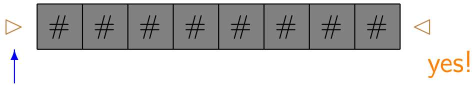

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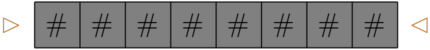

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Each cell is rewritten only in the first 2 visits!

Simulation of 2-Limited Automata by Pushdown Automata

Problem

How much it costs, in the description size, the simulation of 2-LAs *by* PDAs*?*

This work

Exponential cost!

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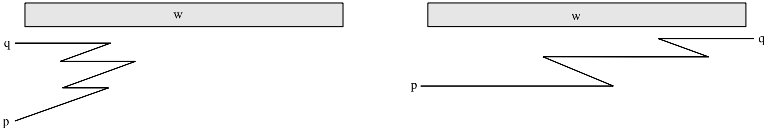
Transition Tables of 2-LAs

- Fixed a 2-limited automaton
- *Transition table* τ_w $\qquad$ w is a "frozen" string

$$\tau_w \subseteq Q \times \{-1, +1\} \times Q \times \{-1, +1\}$$



$(q, d', p, d'') \in \tau_w$ iff M on a tape segment containing w has a computation path:

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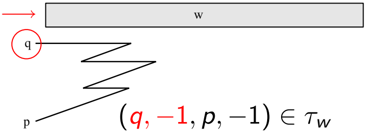

$$(q, -1, p, -1) \in \tau_w$$

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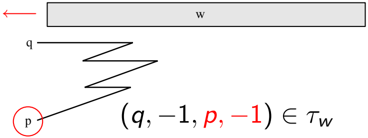

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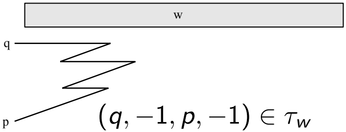
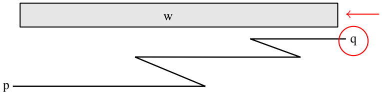


$(q,-1,p,-1) \in \tau_w$



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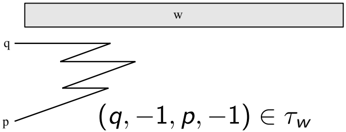

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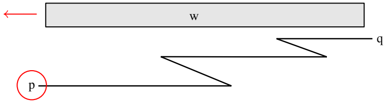

$(q, +1, p, -1) \in \tau_w$

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Simulation of 2-LAs by PDAs

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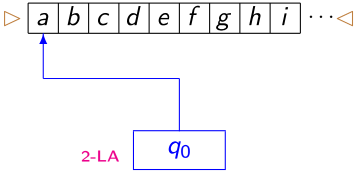

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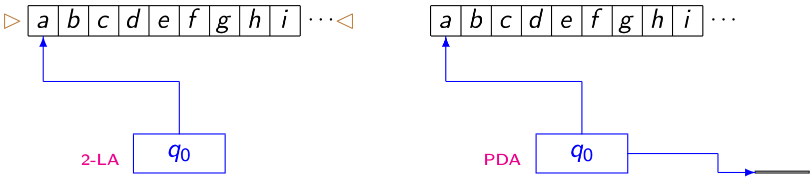

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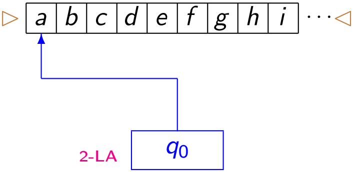

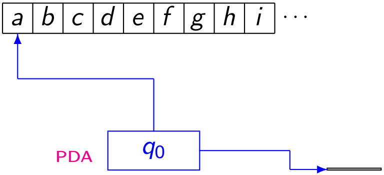

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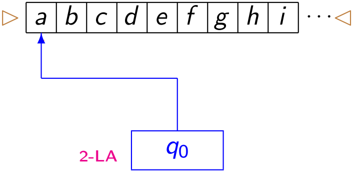

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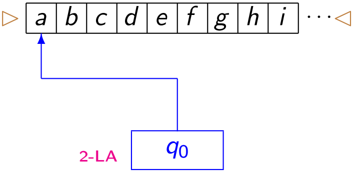

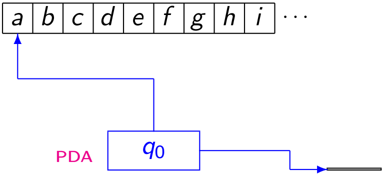

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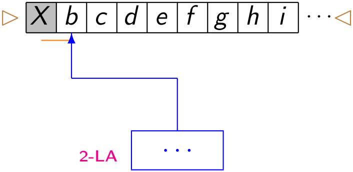

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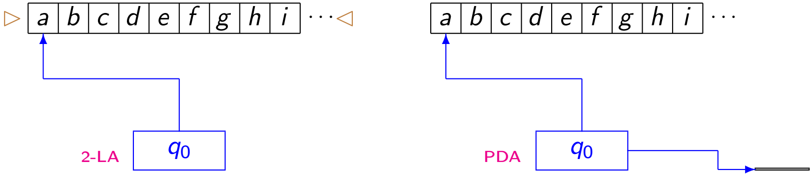

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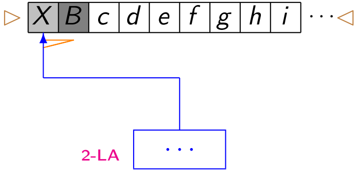

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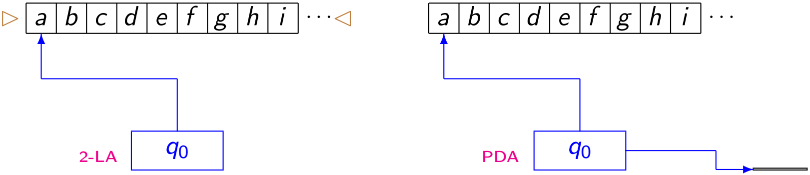

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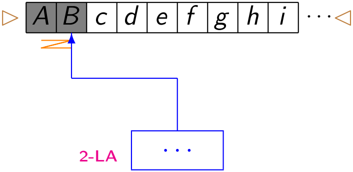

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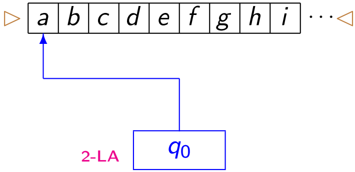

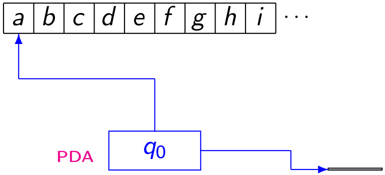

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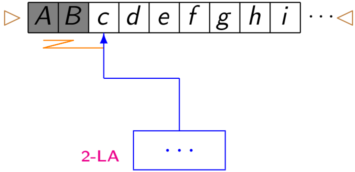

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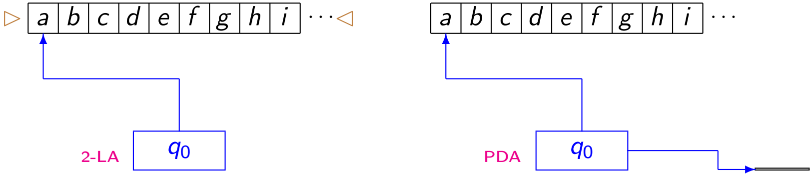

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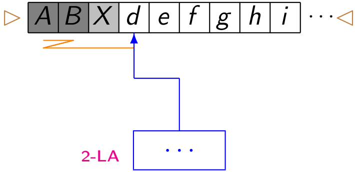

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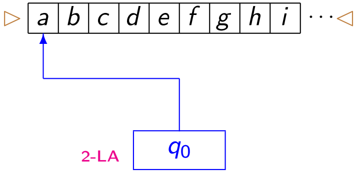

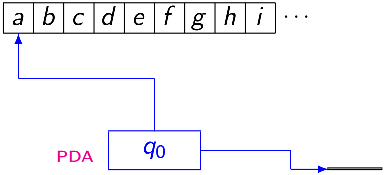

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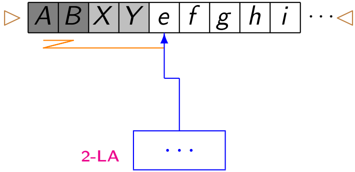

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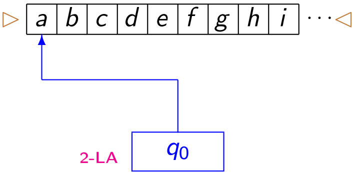

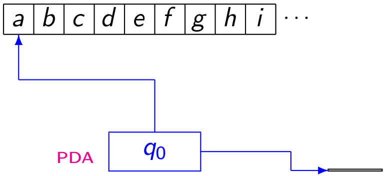

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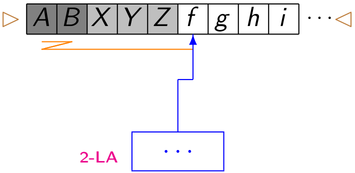

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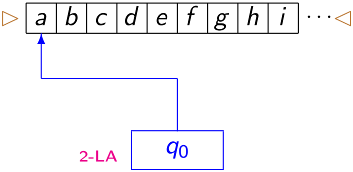

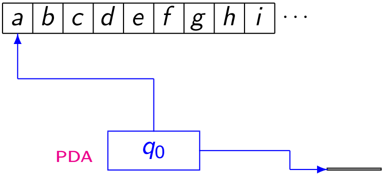

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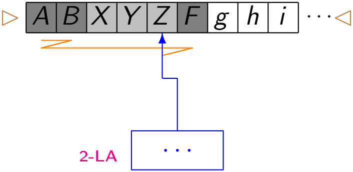

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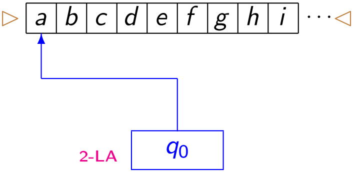

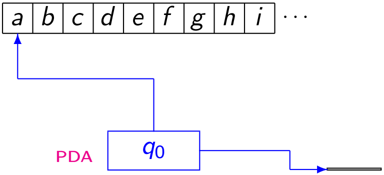

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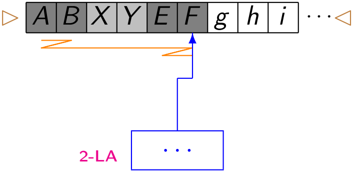

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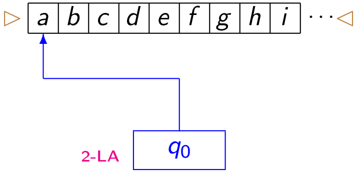
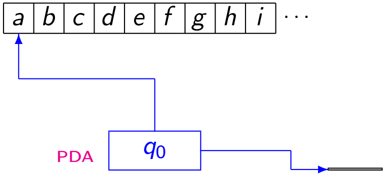
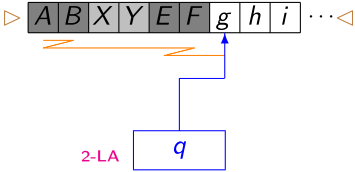
Initial configuration





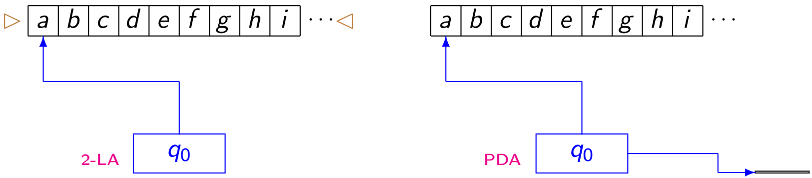
Some computation steps...



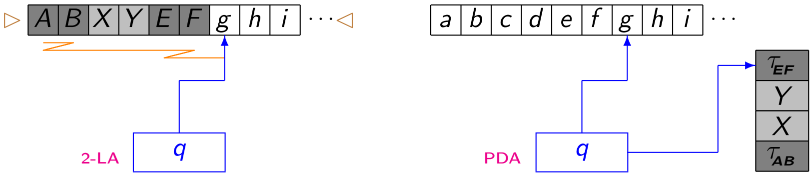
...

Simulation of 2-LAs by PDAs

Initial configuration

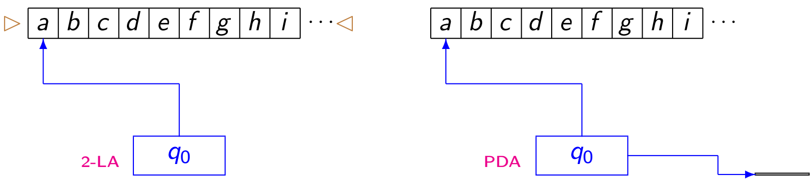


After some steps...

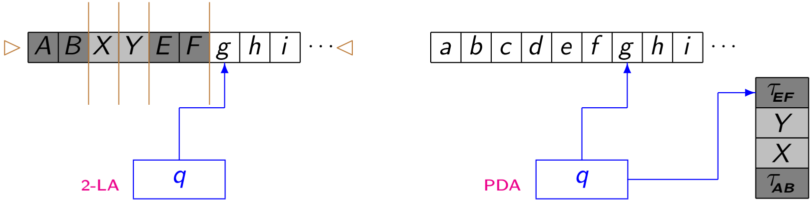


Simulation of 2-LAs by PDAs

Initial configuration

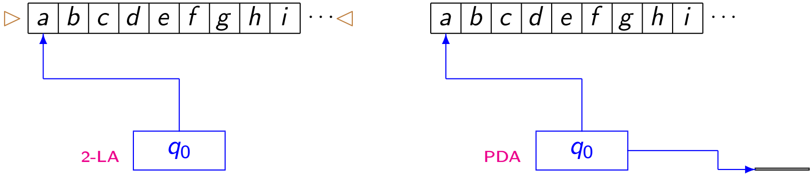


After some steps...

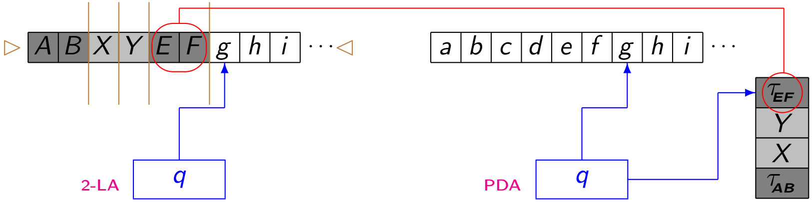


Simulation of 2-LAs by PDAs

Initial configuration

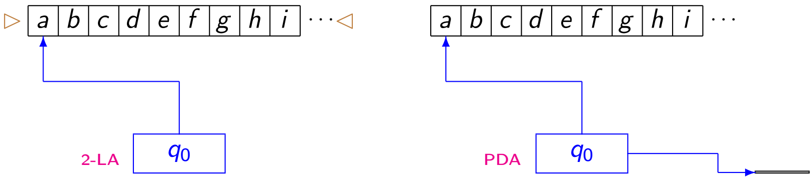


After some steps...

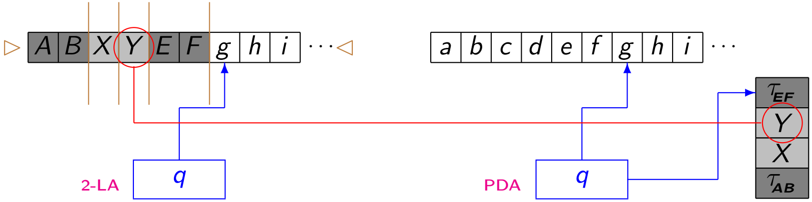


Simulation of 2-LAs by PDAs

Initial configuration

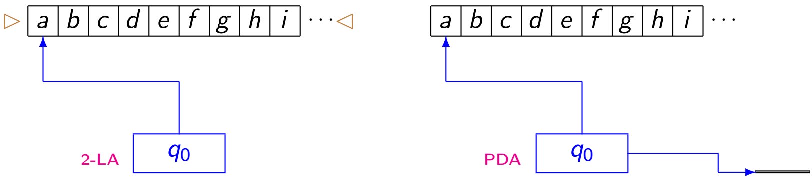


After some steps...

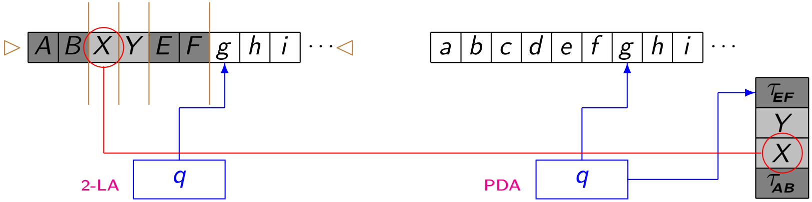


Simulation of 2-LAs by PDAs

Initial configuration

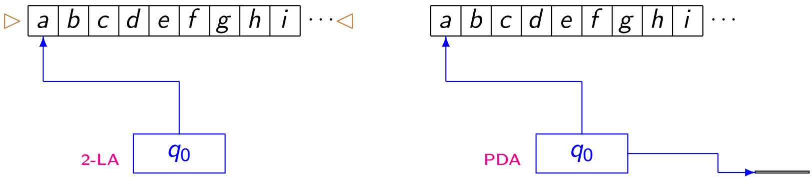


After some steps...

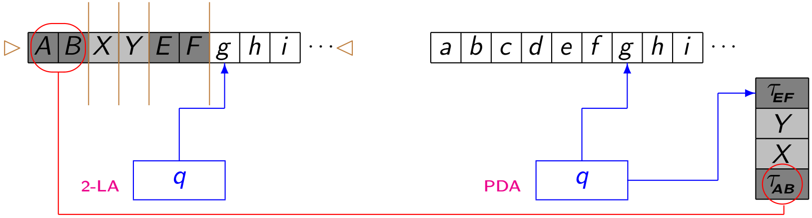


Simulation of 2-LAs by PDAs

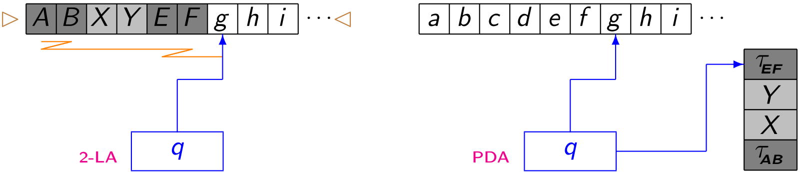
Initial configuration



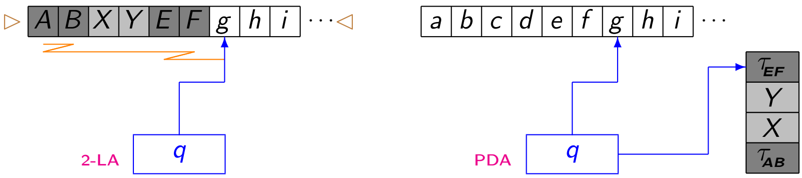
After some steps...



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs

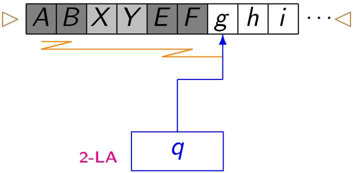


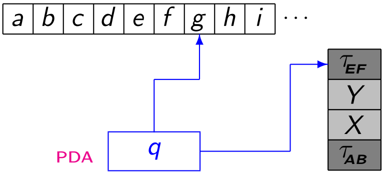
$\delta(q, g) \ni (p, Z, +1)$

move to the right

$\Downarrow$

Simulation of 2-LAs by PDAs

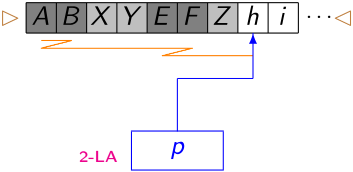




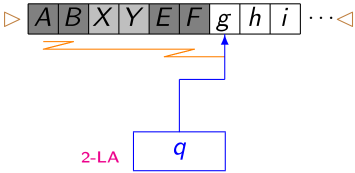
$\delta(q, g) \ni (p, Z, +1)$

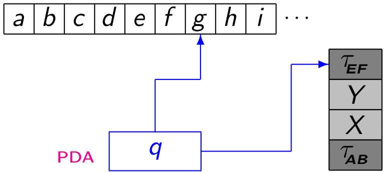
move to the right

$\Downarrow$



Simulation of 2-LAs by PDAs





$\delta(q, g) \ni (p, Z, +1)$

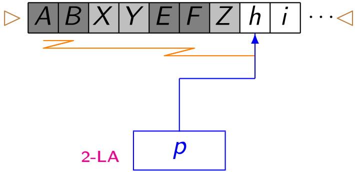
move to the right

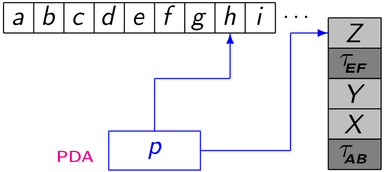
$\Downarrow$

normal mode

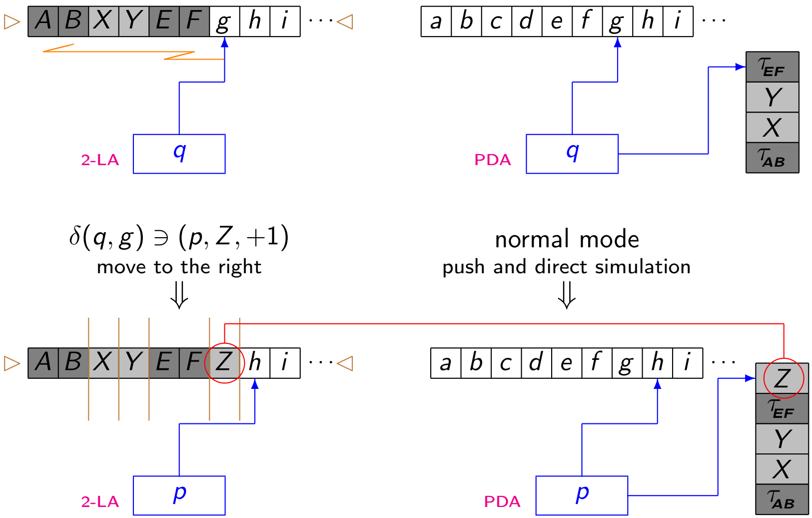
push and direct simulation

$\Downarrow$

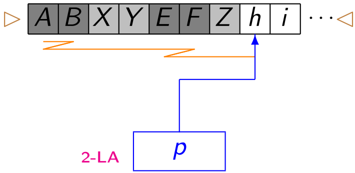


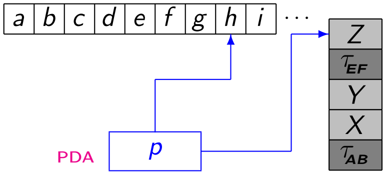


Simulation of 2-LAs by PDAs

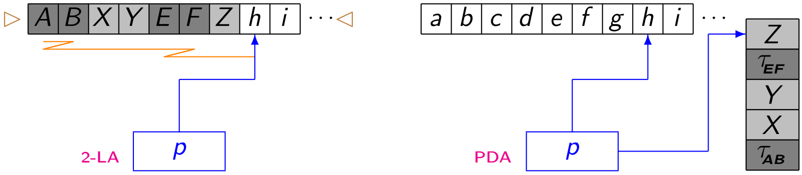


Simulation of 2-LAs by PDAs





Simulation of 2-LAs by PDAs

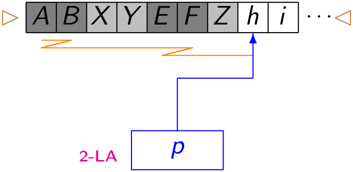


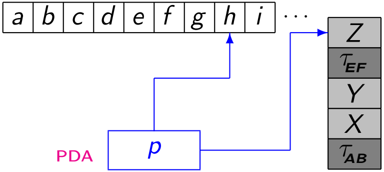
$\delta(p, h) \ni (r, H, -1)$

move to the left

$\Downarrow$

Simulation of 2-LAs by PDAs

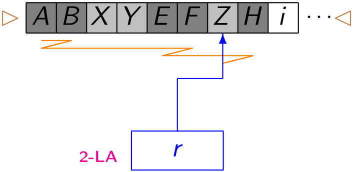




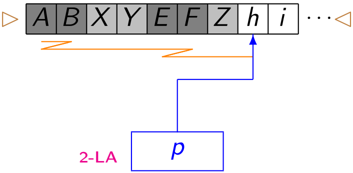
$\delta(p, h) \ni (r, H, -1)$

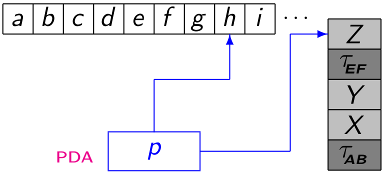
move to the left

$\Downarrow$



Simulation of 2-LAs by PDAs





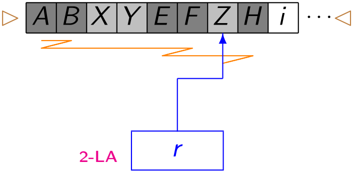
$\delta(p, h) \ni (r, H, -1)$

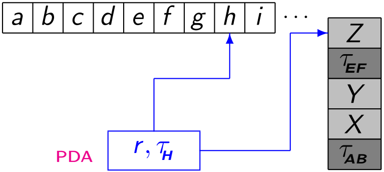
move to the left

$\Downarrow$

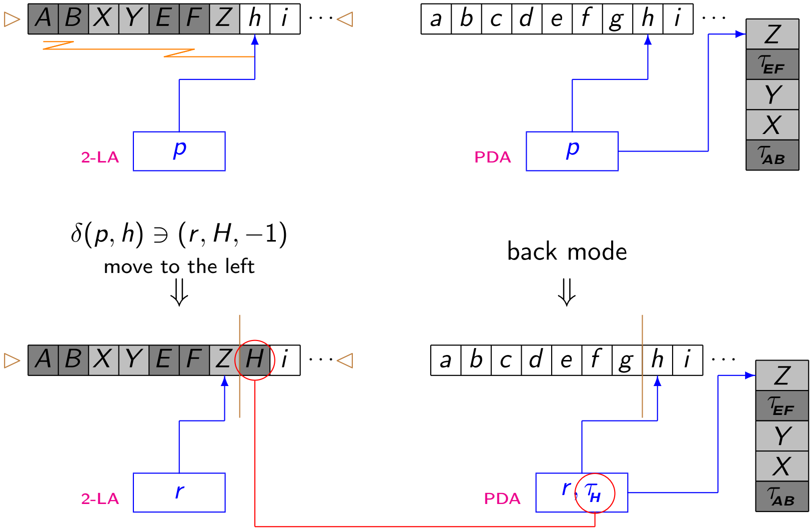
back mode

$\Downarrow$

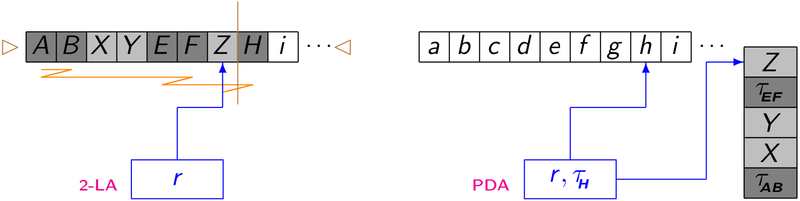




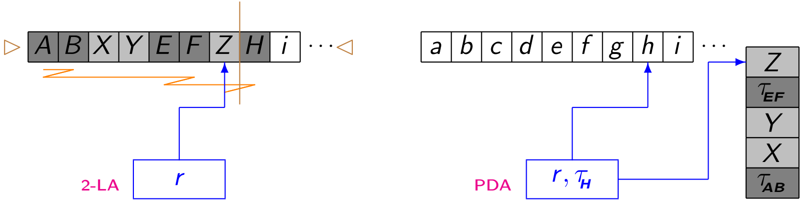
Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs

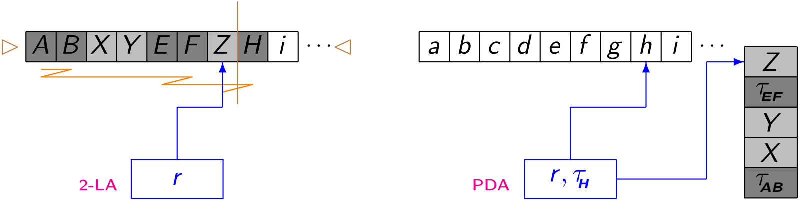


$\delta(r, Z) \ni (q, G, -1)$

move to the left

$\Downarrow$

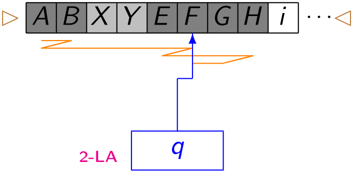
Simulation of 2-LAs by PDAs



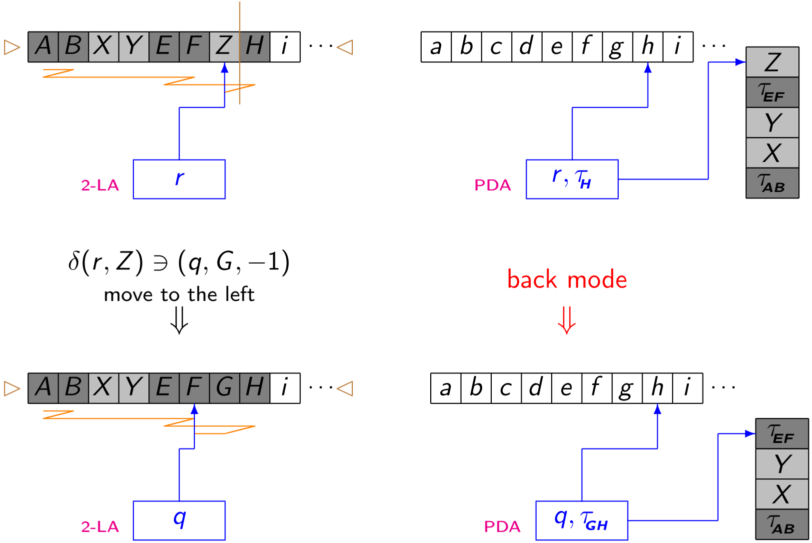
$\delta(r, Z) \ni (q, G, -1)$

move to the left

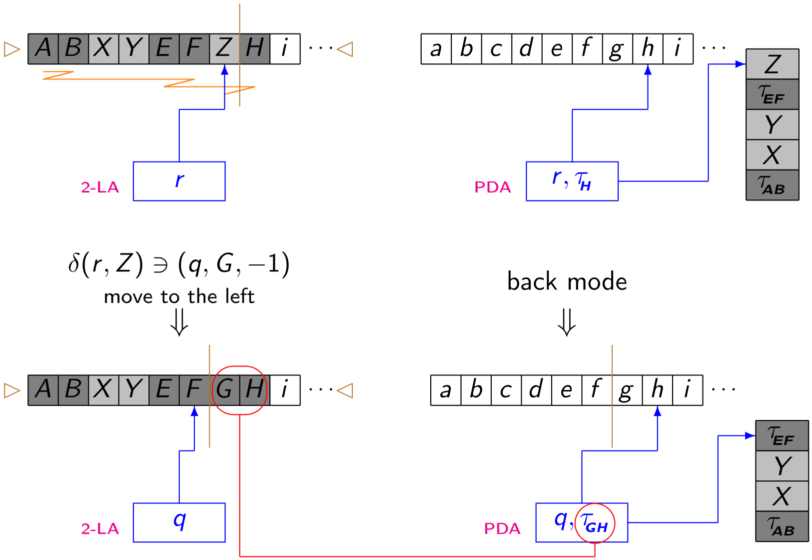
$\Downarrow$



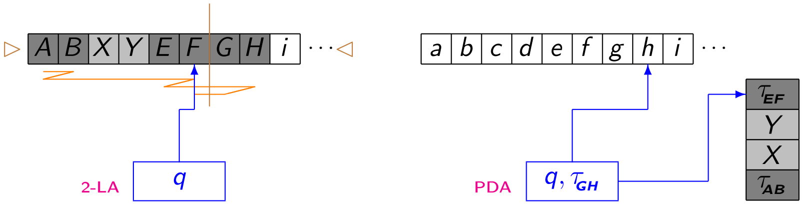
Simulation of 2-LAs by PDAs



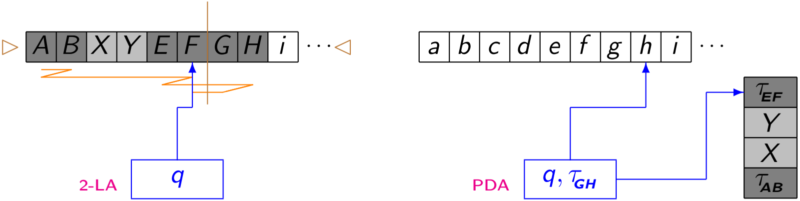
Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs

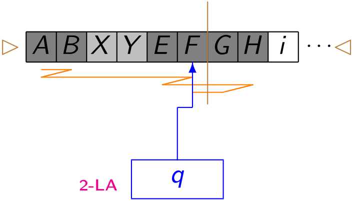


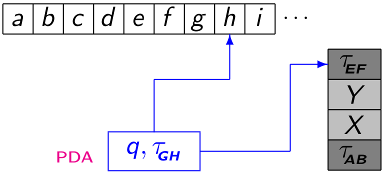
$(q, +1, s, -1) \in \mathcal{T}_{EF}$

exit to the left

$\Downarrow$

Simulation of 2-LAs by PDAs

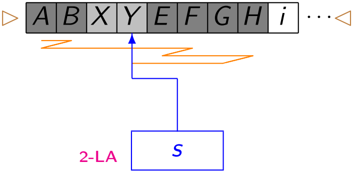




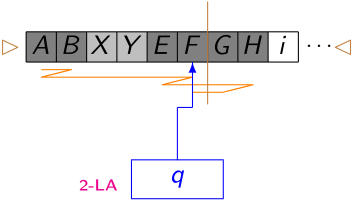
$(q, +1, s, -1) \in \mathcal{T}_{EF}$

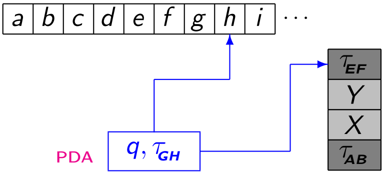
exit to the left

$\Downarrow$



Simulation of 2-LAs by PDAs





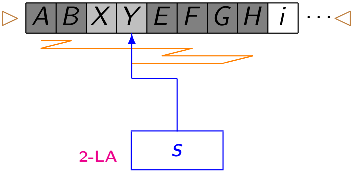
$(q, +1, s, -1) \in \mathcal{T}_{EF}$

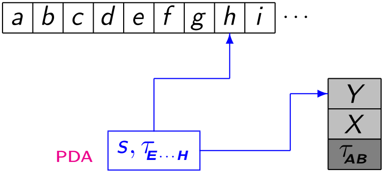
exit to the left

$\Downarrow$

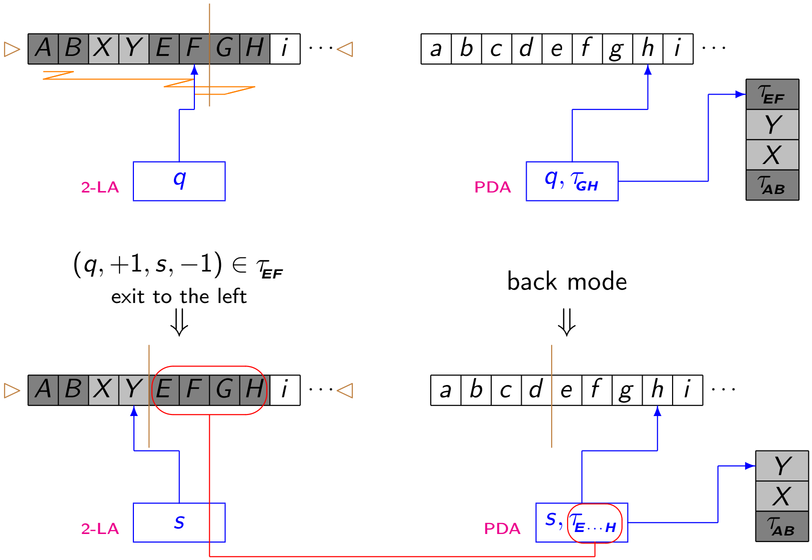
back mode

$\Downarrow$

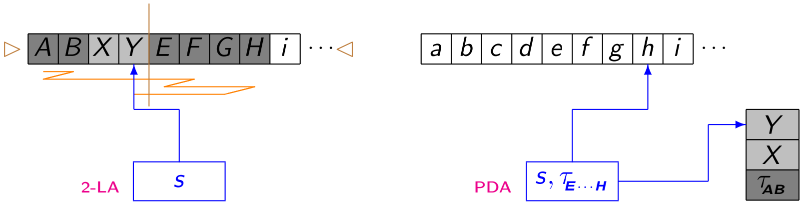




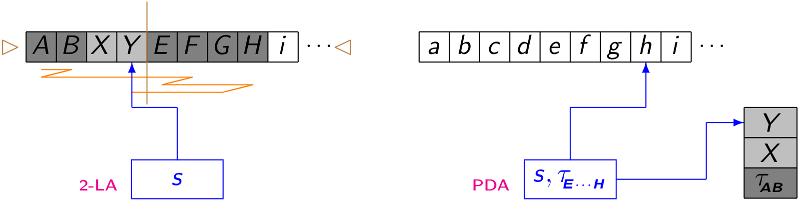
Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs

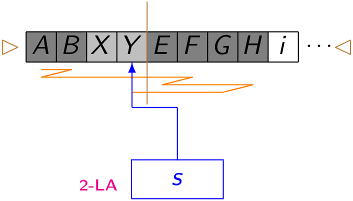


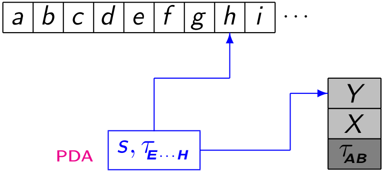
$$\delta(s, Y) \ni (p, D, +1)$$

move to the right

$\Downarrow$

Simulation of 2-LAs by PDAs

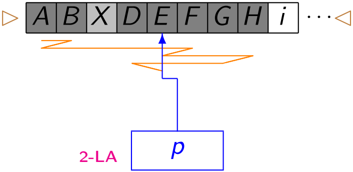




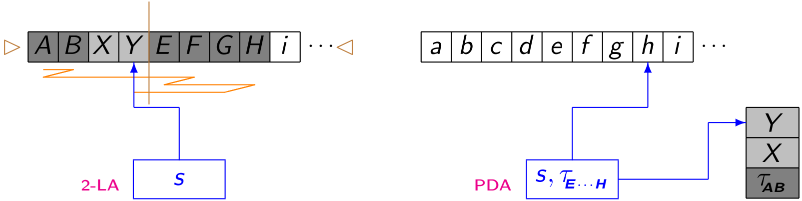
$\delta(s, Y) \ni (p, D, +1)$

move to the right

$\Downarrow$



Simulation of 2-LAs by PDAs



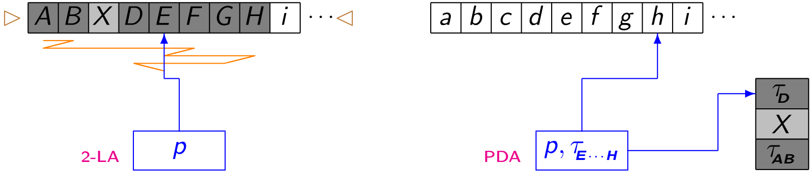
$\delta(s, Y) \ni (p, D, +1)$

move to the right

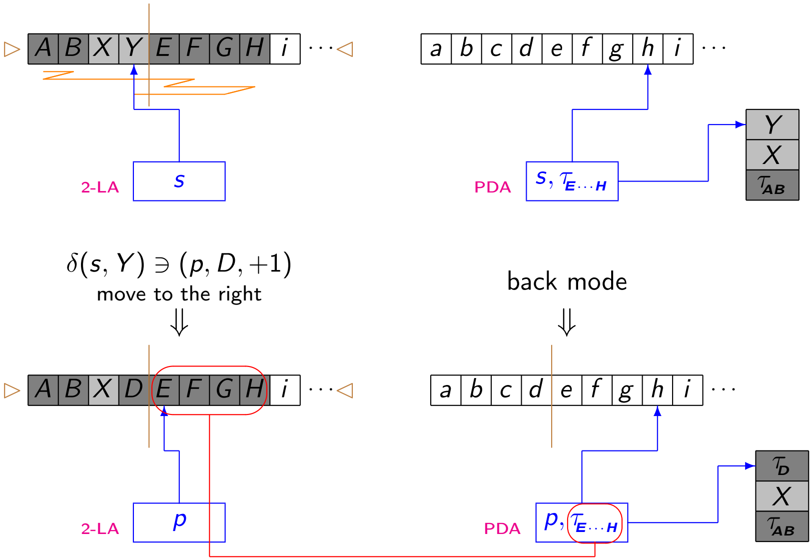
$\Downarrow$

back mode

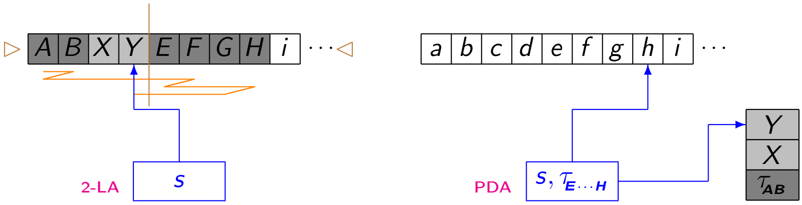
$\Downarrow$



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs

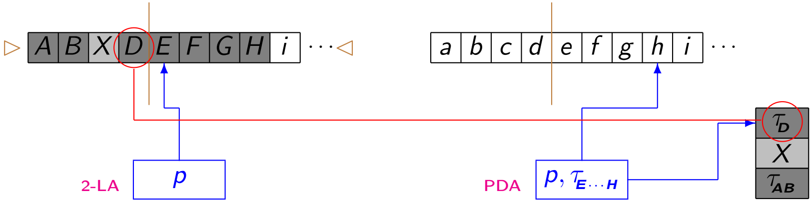


$\delta(s, Y) \ni (p, D, +1)$

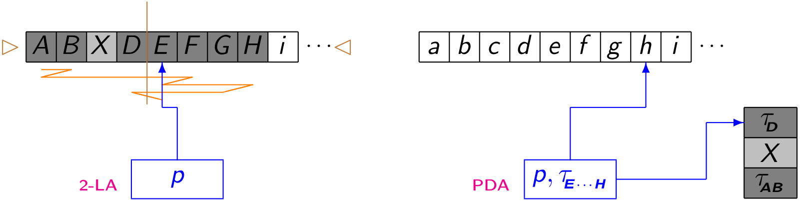
move to the right

back mode

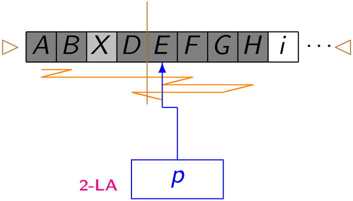
$\Downarrow$

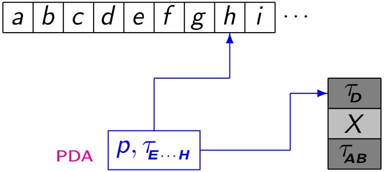


Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



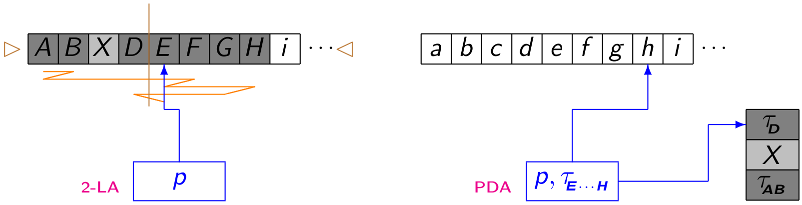


$(p, -1, r, +1) \in \tau_{E\cdots H}$

exit to the right

$\Downarrow$

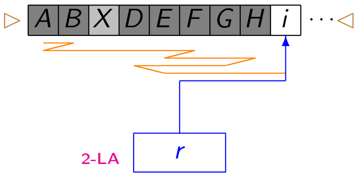
Simulation of 2-LAs by PDAs



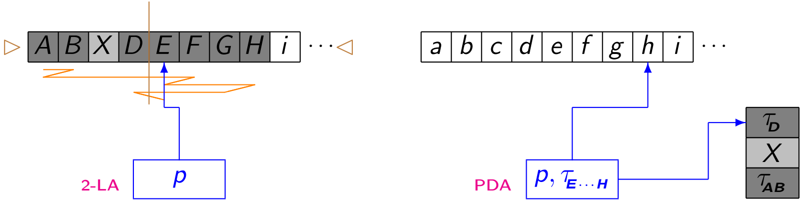
$(p, -1, r, +1) \in \tau_{E\cdots H}$

exit to the right

$\Downarrow$



Simulation of 2-LAs by PDAs



$(p, -1, r, +1) \in \mathcal{T}_{E\cdots H}$

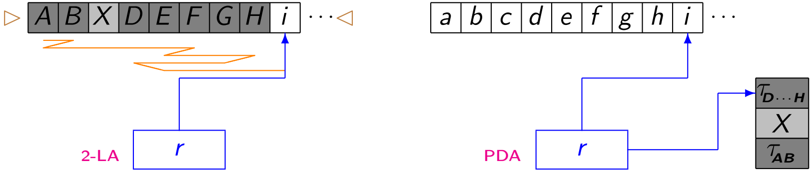
exit to the right

$\Downarrow$

resume normal mode

move to the right

$\Downarrow$



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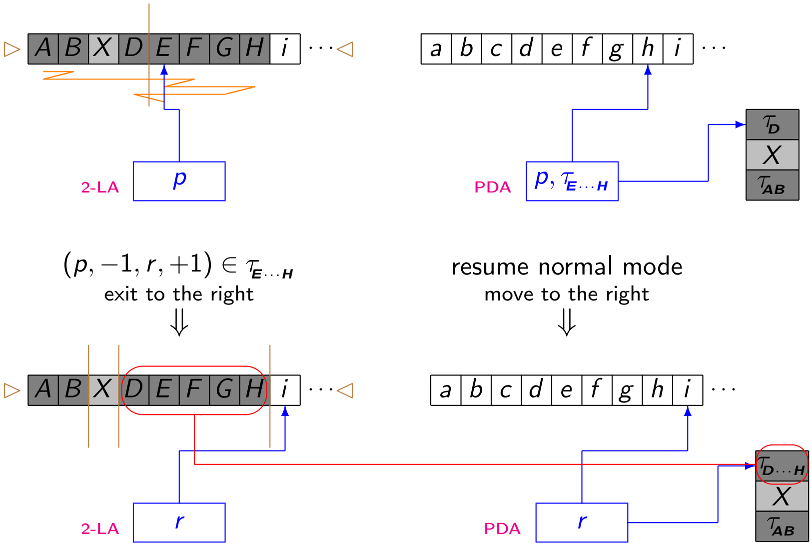

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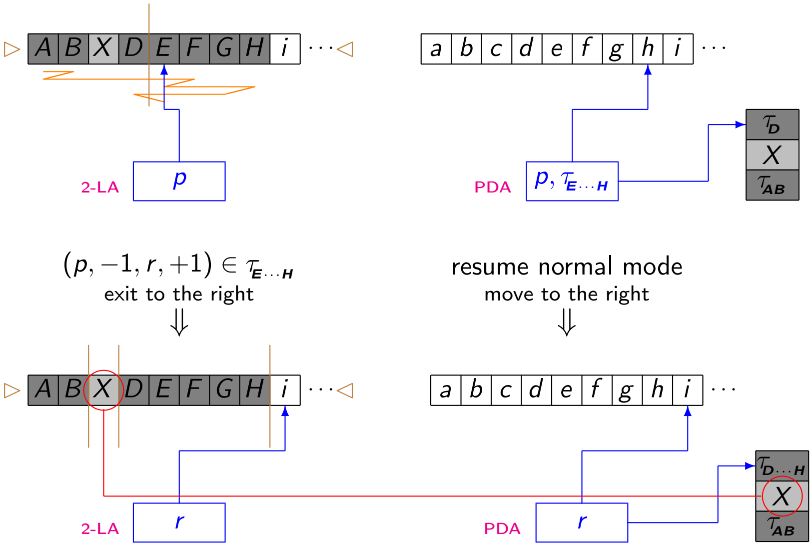

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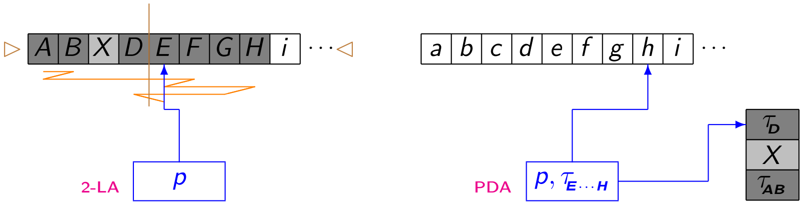

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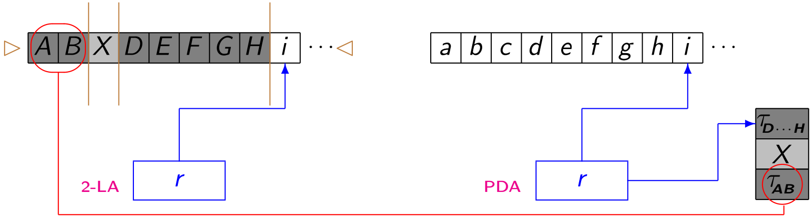

Simulation of 2-LAs by PDAs

Summing up...

Cost of the simulation

- In the resulting PDA transition tables are used for
  - states
  - pushdown alphabet
- *Exponential upper bound* for the size of the resulting PDA
- Optimal

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- Determinism is preserved by the simulation
  provided that the input of the PDA *is right end-marked*
- *Double exponential* size for the simulation of D2-LAs by DPDAs
- Conjecture: this cost cannot be reduced

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Simulation of Pushdown Automata by 2-Limited Automata

PDAs $\longrightarrow$ 2-LAs

Polynomial cost!

(in the description size)

Simulation of Pushdown Automata by 2-Limited Automata

PDAs $\rightarrow$ 2-LAs

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DPDAs $\rightarrow$ D2-LAs

Polynomial cost!

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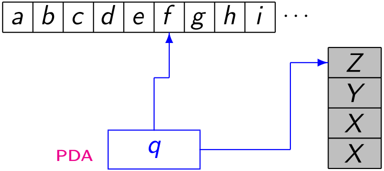
Simulation of PDAs by 2-LAs



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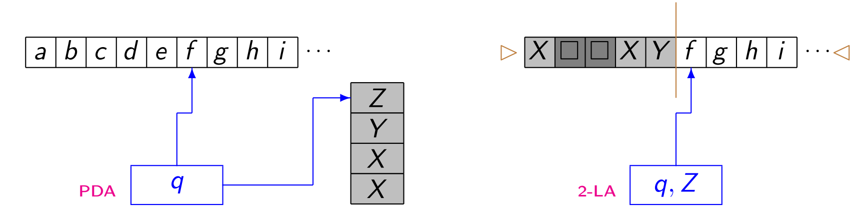

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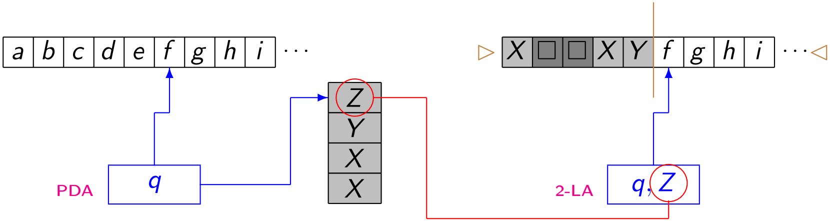

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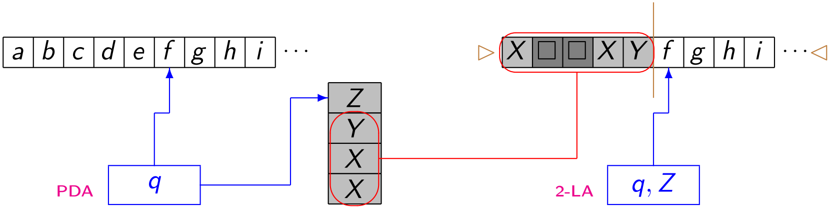

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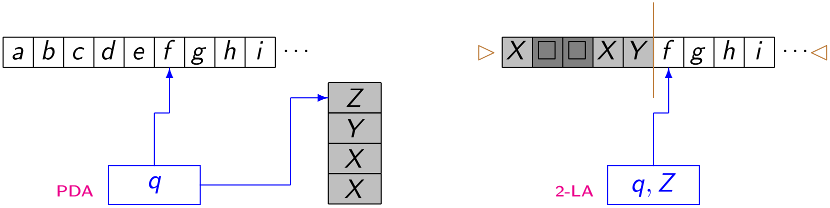

Normal form for (D)PDAs:

- at each step, the stack height increases at most by 1
- ϵ-moves cannot push on the stack

Each (D)PDA can be simulated by an equivalent (D)2-LA of polynomial size

Determinism vs Nondeterminism in Limited Automata

Corollary of the simulations

Deterministic 2-LAs $\equiv$ Deterministic Context-Free Languages

On the other hand, the language

$$L = \{a^n b^n c \mid n \geq 0\} \cup \{a^n b^{2n} d \mid n \geq 0\}$$

is accepted by a *deterministic* 3-LA, but it is not a DCFL

Infinite hierarchy [Hibbard'67]

> *For each $d \geq 2$ there is a language which is accepted by a deterministic d-limited automaton and that cannot be accepted by any deterministic $(d-1)$-limited automaton*

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Futher Investigations

- Descriptional complexity aspects for $d > 2$

  We conjecture that for $d > 2$ the size gap from d-limited automata to PDAs remains exponential

- Descriptional complexity aspects in the unary case
  - Unary context-free language are regular [Ginbsurg&Rice'62]
  - Ex: $L_n = (a^{2^n})^*$

	size
2-LA	$O(n)$
DPDA	$O(n)$
minimal DFA	2^n
minimal 2NFA	2^n

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Thank you for your attention!