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LIMITED MATCHING ON CONCURRENT-SCHEDULE REINFORCEMENT OF ACADEMIC BEHAVIOR

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Three adolescent students with special educational needs were given a choice between completing one of two available sets of math problems. Reinforcers (nickels) across these alternatives were arranged systematically in separate experimental phases according to three different concurrent variable-interval schedules (reinforcement ratios of 2:1, 6:1, and 12:1). Time allocated to the two stacks of math problems stood in linear relationship to the reinforcement rate obtained from each stack, although substantial undermatching and bias were observed for all subjects. However, changes in the schedules were not followed by changes in allocation patterns until adjunct procedures (e.g., changeover delays, limited holds, timers, and demonstrations) were introduced. The necessity of adjunct procedures in establishing matching in applied situations is discussed as a limitation to quantitative applications of the matching law in applied behavior analysis.

DESCRIPTORS: matching law, matching theory, concurrent schedules, limited matching, academic behavior, schedule discrimination

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The matching law is a quantitative description of the functional relation between relative reinforcer frequency and relative rate of responding across multiple response alternatives (Herrnstein, 1961, 1970). Given that most human behavior can be characterized as a choice among concurrently available response alternatives, a matching-law account of human choice may have considerable significance for applied behavior analysis (Mace, 1994; McDowell, 1981, 1988, 1989; Myerson & Hale, 1984; Pierce & Epling, 1983; Rachlin, 1989).

Initial attempts to establish the generality of the matching law to human choice were conducted in the human operant laboratory. Procedures used commonly in nonhuman matching studies were translated into laboratory preparations suitable for humans. For example, subjects were typically presented with a two-choice situation in which responses on two separate manipulanda, such as levers or buttons, were reinforced according to independent concurrent variable-interval variable-interval (conc VI VI) schedules of reinforcement. Different pairs of conc VI VI schedules were presented in separate experimental phases to permit the parametric study of relative response frequency as a function of relative rate of reinforcement using simple linear regression methods (McDowell, 1989; Pierce & Epling, 1983).

Reviews of the basic human literature have reported mixed results across and within studies (Lowe & Horne, 1985; Pierce & Epling, 1983). Using Baum's (1974) log linear transformation of the generalized matching equation, $\log(B1/B2) = a \log(r1/r2) + \log(k)$, some investigators have found relative rates of responding ($B1/B2$) and rein-

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forcement ($r1/r2$) to be approximately equal (i.e., where the slope of the regression line, a, is at or near 1.0), with no constant bias for one response alternative (i.e., where the intercept, log(k), is at or near zero) (e.g., Bradshaw, Szabadi, & Bevan, 1976; Bradshaw, Szabadi, Bevan, & Ruddle, 1979). On the other hand, there have been several reports of significant departures from matching, with undermatching (i.e., $a < 1.0$) the most commonly reported deviation (e.g., Oscar-Berman, Heyman, Bonner, & Ryder, 1980; Schmitt, 1974; Schroeder & Holland, 1969), and biased responding observed in excess of 0.15 intercept (e.g., Ruddle, Bradshaw, Szabadi, & Bevan, 1979).

Laboratory studies with nonhumans and humans have also used various procedures adjunctive to conc VI VI schedules to increase the sensitivity of choice behavior to scheduled rates of reinforcement and to improve the prospects of obtaining matching. With few exceptions (e.g., Bradshaw et al., 1979), experiments with humans have imposed a changeover delay (COD) interval for schedule switching that ranged from 1 s (e.g., Schroeder & Holland, 1969) to 30 s (Schroeder, 1975), although some studies employed varying COD durations across subjects without expressly treating the COD as an independent variable (e.g., Baum, 1975). Less common in human matching experiments is the imposition of a limited hold on the availability of scheduled reinforcers in order to encourage schedule switching and increase contact with scheduled contingencies. Limited-hold durations have varied from 0.3 s to 30 s (Schroeder, 1975).

Other adjunct procedures have been devised specifically for use with humans. The most commonly used procedure is to provide subjects with instructions that describe important aspects of the choice situation. These include indications that (a) the tasks are independent (e.g., Schmitt, 1974), (b) switching between alternatives is profitable (e.g., Ruddle et al., 1979; Schmitt, 1974), (c) exteroceptive stimuli are correlated with different outcomes (e.g., Cliffe & Parry, 1980; Horne & Lowe, 1993), and (d) the object of the task is to optimize the number of reinforcers received (Schroeder & Holland, 1969). Another tactic has been to expose subjects to the specific schedule conditions singly before arranging the schedules concurrently (e.g., Bradshaw et al., 1976; Ruddle et al., 1979). Schedule conditions have also been correlated uniquely with different exteroceptive stimuli across all phases rather than only within phases, as is common in nonhuman matching studies (e.g., Bradshaw et al., 1976, 1979; Hanna, Blackman, & Todorov, 1992; Oscar-Berman et al., 1980).

Takahashi and Iwamoto (1986) examined the extent to which such adjunct procedures are necessary for human concurrent performance to conform to the matching law. In conjunction with three separate ratios of conc VI VI reinforcement, eight groups of subjects received different combinations of (a) prior exposure to single schedule conditions, (b) unique schedule-correlated stimuli, and (c) instructions about schedule independence. Subjects who received only one or two of these adjunct procedures failed to show orderly variations in choice ratios with reinforcement ratios. Only the group receiving all three adjunct procedures showed a linear relationship between choice and reinforcement ratios, although substantial undermatching was evident for all subjects. In a similar vein, Horne and Lowe (1993) examined human performance on concurrent schedules alone and with various adjunct procedures. Only 13 of 30 subjects had allocation patterns that could be described by the matching law. Matching was observed most frequently when subjects were provided with either visual cues and/or verbal instructions indicating the ordinal relations between schedule-correlated stimuli and their corresponding reinforcement frequencies. A COD presented alone or in conjunction with these adjunct procedures did not contribute to matching. Together, these two studies suggest that, even in the operant laboratory, matching on concurrent schedules cannot be assumed for humans. Moreover, one or more adjunct procedures are frequently needed to promote matching, although no consistent finding indicates which procedure or combination of procedures best encourages matching allocation of behavior to obtained reinforcement.

Our objectives for the present investigation were twofold. First, we sought to extend the generality of the matching law to human concurrent performance when behaviors of social significance are targeted. The vast majority of human studies that parametrically varied conc VI VI schedules have targeted arbitrary responses such as key pressing or lever pulling. We are aware of only three studies that have examined human performance of socially relevant behavior on different ratios of conc VI VI schedules. Conger and Killeen (1974) and Pierce, Epling, and Greer (1981) found that rates of agreement statements by listeners predicted the proportion of time speakers oriented to each listener. Similarly, Hamblin, Clairmont, and Chadwick (1975) reported that gamblers bet on game options in proportion to the payoffs derived from the betting alternatives. The present experiment extends this work to academic behavior by reinforcing adolescents' completion of math problems according to three pairs of conc VI VI schedules. Our second objective was to assess informally how students' choices between alternatives responded to changes in reinforcement ratios of conc VI VI schedules and whether the use of various adjunct procedures may be necessary to increase responsiveness to the scheduled contingencies. If the matching law is to have direct implications for applied work, socially relevant human choice should be sensitive to relative rate of reinforcement when reinforcement ratios are both constant over time and when the ratios change from one value to another.

METHOD

Participants

Three teen-age students with behavior disorders and learning difficulties participated in the study. All students were enrolled in a special education program and were referred by their classroom teacher for participation in this study because of performance deficits in arithmetic skills. Informed consent to participate was obtained from the students and their guardians before commencement of the study.

Matt was a 17-year-old male with a full-scale Wechsler Intelligence Scale for Children—Revised (WISC–R) IQ of 83. He was diagnosed with attention deficit and hyperactivity disorder (ADHD) and with severe deficits in visual-motor performance and perceptual organization. Standardized educational tests placed him below the 3.0 grade level in mathematics and at the 10.0 grade level overall. Jean was a 19-year-old female who tested in the borderline range of intelligence. Her achievement scores placed her at the 7.2 grade level in mathematics and 7.3 overall. Terry was a 15-year-old girl with a WISC–R IQ of 76 and diagnosis of disruptive behavior disorder. Achievement tests placed Terry at the 6.6 grade level in mathematics. Jean and Terry were participants in a previous matching study that had exposed them to performance of math problems on concurrent schedules of reinforcement.

The experimenter was a research assistant who was unaware of the specific hypotheses of the study. Sessions were conducted in a small office at the school, with the experimenter and student seated facing each other at a table.

Procedure

Each student participated in two 10-min sessions per day, 3 days per week. Arithmetic problems printed on index cards (8.9 cm by 12.7 cm) were placed in two separate stacks approximately 30 cm apart on the table in front of the participant. The stacks contained duplicate sequences of arithmetic problems printed on different-colored index cards (e.g., green vs. yellow). Different pairs of colors were used as discriminative stimuli for each pair of conc VI VI schedules. Left/right placement of the colors alternated each session. When a student selected one problem to work on, the experimenter removed the unselected duplicate card from the other stack so that the choice was always between identical problems. Participants were free to select a new problem immediately after completing each problem until the session ended. The classroom teacher identified arithmetic problems that the students performed with approximately 80% accuracy. Matt's problems were three-digit by three-digit subtraction, Jean's were two-digit by four-digit long division, and Terry's were two-digit by one-digit multiplication. The experimenter provided the participant with a pencil and began the session with the statement, "You can earn nickels doing these math problems. You may work on either stack you choose. You may start when I say 'begin.' " Sessions ended with the experimenter saying, "Time is up. You can take a break. (I'll see you next time.)"

During the session, the experimenter provided nickels to the student contingent on correct answers to the arithmetic problems according to one of three different conc VI VI schedules. The experimenter used a key to correct each problem immediately after it was completed and marked incorrect problems with an "X." Nickels were deposited in one of two transparent plastic cups (located directly behind each stack) immediately after the first correctly completed problem following elapse of the reinforcement interval. Reinforcement intervals were recorded on an audiotape and were signaled to the experimenter through an earphone. Interval values were generated from a software program (Hantula, 1991) using iterative equations by Fleshler and Hoffman (1962).

The participants showed an insensitivity to the scheduled rates of reinforcement with each change in the schedule conditions, having a tendency to persist in their patterns of time allocation in the immediately preceding schedule condition. One or more of the following four procedures were used to improve each student's sensitivity to the scheduled rates of reinforcement: (a) Changeover delay intervals of 4 s, 8 s, and/or 10 s were imposed for schedule switches; (b) scheduled reinforcers were available only for a 15-s limited-hold interval; (c) a digital kitchen timer, signaling the time remaining in each reinforcement interval, was placed behind each stack of problems facing the subject; and (d) an experimenter, guided by the timers, provided a demonstration of alternating between the two response options to produce the maximum number of reinforcers available. Figures 1, 2 and 3 show the use of these procedures during each schedule condition. With two exceptions, these additional procedures were discontinued after time-allocation patterns changed and became stable under the schedule condition. A COD remained in effect for Matt and Jean during some schedule conditions (see Figures 1 and 2). Otherwise, regression analyses were conducted on sessions in which only the conc VI VI schedule operated. Adjunct procedures were introduced unsystematically based on the experimenter's estimation of which procedure(s) a given subject may be responsive to.

Experimental Design

The study employed a concurrent schedules design to establish the differential control of each VI schedule in the concurrent schedule pair (Kazdin, 1982). Experimental control is established with this design when more time is spent responding on the richer of the two concurrent VI VI schedules.

Each student's arithmetic performance was subject to three different conc VI VI schedules in separate experimental phases with reinforcement ratios of 2:1, 6:1, and 12:1. Sessions were conducted under each schedule condition until responding showed evidence of changing in response to the altered contingencies and was stable (by visual inspection of the data) in a minimum of five consecutive sessions. The order of schedule conditions was determined randomly.

Data Collection and Interobserver Agreement

An experimenter recorded the time students allocated to performing problems in each stack during continuous 1-min intervals using a stopwatch. Time allocated to a problem was defined as the student being visually oriented to an index card. Independent measures of time allocation were taken on 31% of the sessions by a second observer who was unaware of the schedule conditions and experimental hypotheses. An agreement was defined as both observers recording the same duration ±2 s for a given 1-min interval. Interobserver agreement was calculated on a interval-by-interval basis by dividing the number of agreements by the total number of intervals per session and multiplying by 100%. Mean interobserver agreement was at least 95.9% for all subjects.

Observers also recorded the experimenter's delivery of nickels during continuous 1-min intervals.

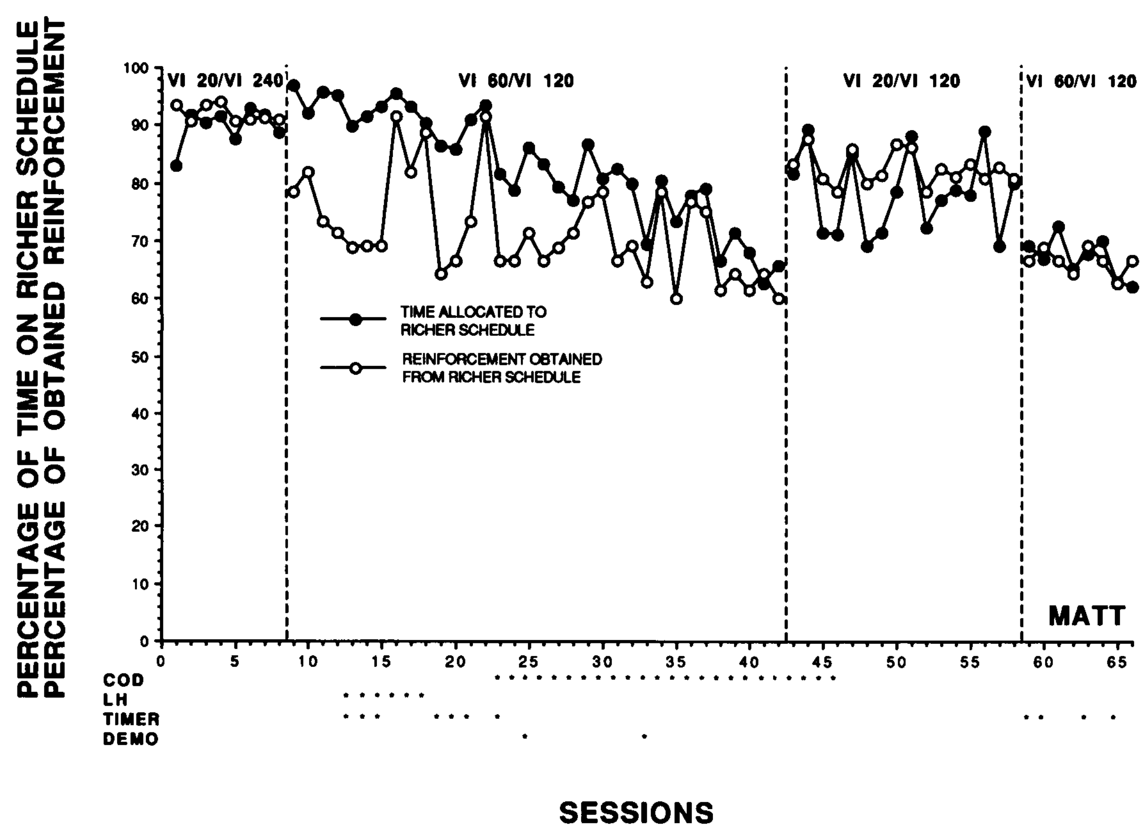

Figure 1. Percentage of time Matt allocated to performing arithmetic problems subject to the richer of two conc VI VI schedules (closed circles) and the percentage of reinforcement obtained from the richer schedule. Data are presented by session across different schedule conditions. Sessions in which adjunct procedures (changeover delay, COD; limited hold, LH; timer; demonstration, DEMO) were operative are indicated by asterisks at the bottom.

Interobserver agreement, calculated on a point-by-point basis using the above formula, was 99% or higher for all subjects.

RESULTS

Figures 1, 2, and 3 present the percentage of time participants allocated to the richer of the two concurrent VI VI schedules ($T1/T2$) and the percentage of reinforcers obtained from that schedule ($r1/r2$) under each schedule condition. All participants showed stable response patterns after six to eight sessions under the first conc VI VI schedule condition they experienced. However, their allocation patterns were slow to adjust to changes in the schedule conditions, requiring between 11 and 36 sessions and the use of additional training procedures to reach stability in the second and third schedule conditions. Following each change in schedule conditions, participants showed a distinct tendency to continue the allocation patterns established in the preceding condition. One exception was that Matt's choice patterns (Figure 1) reached stability after eight sessions with no additional training procedures during his fourth experimental condition that replicated the conc VI 60-s VI 120-s schedule.

Students' patterns of time allocation and obtained reinforcement may also be analyzed during the last five sessions in each experimental condition when schedule control and stability became evident. A comparison of the mean $T1/T2$ ratio with the mean $r1/r2$ ratio for the last five sessions in each schedule condition reflects the extent to which rel-

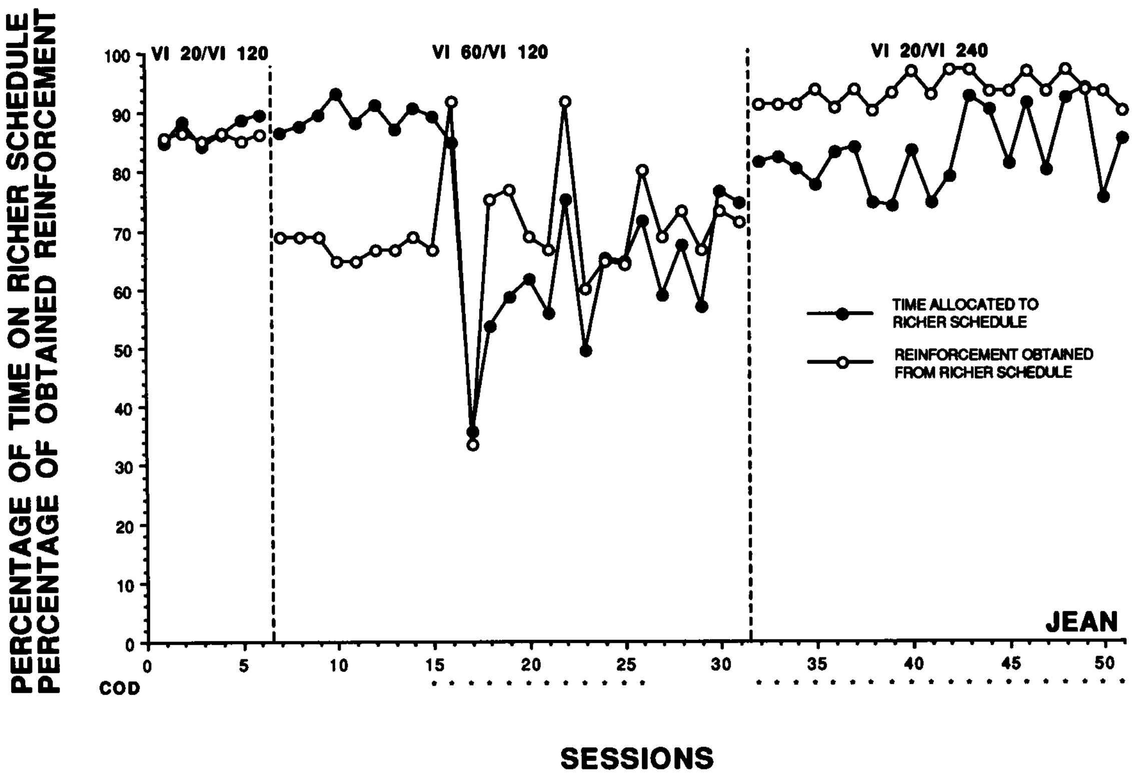

Figure 2. Percentage of time Jean allocated to performing arithmetic problems subject to the richer of two conc VI VI schedules (closed circles) and the percentage of reinforcement obtained from the richer schedule. Data are presented by session across different schedule conditions. Sessions in which a changeover delay (COD) adjunct procedure was operative are indicated by asterisks at the bottom.

ative time allocation approximated relative obtained reinforcement. Under the conc VI 20-s VI 240-s condition, mean $T1/T2$ and $r1/r2$ ratios were, respectively, 9.6:1 and 10.9:1 for Matt, 5.9:1 and 14.6:1 for Jean, and 4.9:1 and 9.5:1 for Terry. These values are compared with the 12:1 ratio of scheduled rates of conc VI VI reinforcement. During the conc VI 20-s VI 120-s condition, $T1/T2$ and $r1/r2$ ratios, respectively, averaged 3.1:1 and 4.5:1 for Matt, 7.1:1 and 6.2:1 for Jean, and 4.3:1 and 5.5:1 for Terry. By contrast, the programmed reinforcement ratio was 6:1 for this condition. Finally, mean $T1/T2$ and $r1/r2$ ratios in the conc VI 60-s VI 120-s condition were 2.0:1 and 1.8:1 for Matt, 2.0:1 and 2.4:1 for Jean, and 2.5:1 and 2.5:1 for Terry. These values closely approximated the 2:1 ratio of scheduled rates of reinforcement.

Least squares regression lines were fitted to a bivariate distribution of log ratios of time allocation ($T1/T2$) and log ratios of obtained reinforcement ($r1/r2$) pooled across the last five sessions in each schedule condition (Bradshaw et al., 1976).[1] Figure

[1] The practice of conducting simple linear regression analyses on subjects' log behavior ratios ($B1/B2$ or $T1/T2$) and their corresponding log obtained reinforcement ratios ($r1/r2$) is common in basic concurrent schedules research. The analysis quantitatively describes the degree to which choosing one alternative over another varies in relation to the frequency of reinforcement derived from each alternative (i.e., the proportion of variance in relative behavior frequency accounted for by relative reinforcement frequency). It also quantifies two distinct parameters (the slope and intercept) that represent systematic departures from a one-to-one match between relative behavior frequency and relative reinforcement frequency. Pierce and Epling (1983) and McDowell (1989) provide excellent primer discussions of regression analyses in matching research.

Applied researchers may note the limitations of conducting regression analyses on the small number of data points corresponding to the number of experimental phases of different pairs of conc VI VI schedules. The need to achieve stability

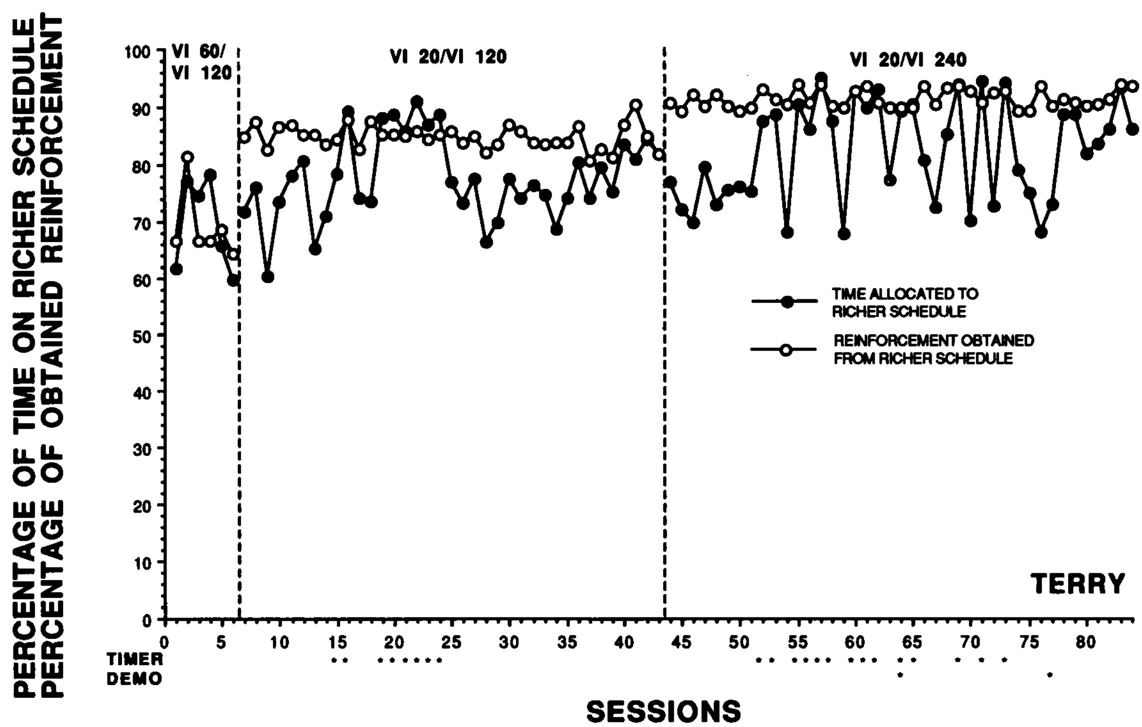

Figure 3. Percentage of time Terry allocated to performing arithmetic problems subject to the richer of two conc VI VI schedules (closed circles) and the percentage of reinforcement obtained from the richer schedule. Data are presented by session across different schedule conditions. Sessions in which adjunct procedures (timer; demonstration, DEMO) were operative are indicated by asterisks at the bottom.

4 shows these regression analyses for individual students and a pooled average for all participants. All 3 students exhibited substantial undermatching and bias. The fitted regression lines were $y = 0.15 + 0.84(x)$ for Matt, $y = 0.16 + 0.64(x)$ for Jean, and $y = 0.22 + 0.52(x)$ for Terry. Regression analysis for the pooled average across participants resulted in a similar pattern of undermatching and bias [$y = 0.19 + 0.65(x)$]. Although undermatching and bias were prominent, correlations between relative proportion of time allocation and relative rates of obtained reinforcement were moderate to high. Pearson product moment values were $r = 0.75$ for Matt, $r = 0.85$ for Jean, and $r = 0.99$ for Terry.

within experimental phases and maintain subjects' cooperation with the experiment hinders efforts to obtain a larger sample of paired observations. Although regression analyses are often performed with as few as three to five data points with both nonhuman and human subjects (e.g., Baum, 1974; Horne & Lowe, 1993), the reliability of the parameter estimates will be improved with larger samples of observations.

DISCUSSION

Adolescents presented with three different ratios of conc VI VI reinforcement allocated time to completion of two stacks of math problems in a linear relation to the rates of nickels received from each stack following exposure to various adjunct procedures. Three findings from this experiment extend the literature on human choice and may have implications for quantitative applications of the matching law in educational and clinical settings.

First, this study joins a handful of other parametric investigations demonstrating that socially relevant human behavior subject to conc VI VI reinforcement conforms to allocation patterns predicted by the matching law (Conger & Killeen, 1974; Hamblin et al., 1975; Pierce et al., 1981). In the present study, after schedule control and steady-state responding were observed, correlations between relative time allocated to performing math problems and relative rates of obtained reinforcement were moderate to strong, ranging from 0.75

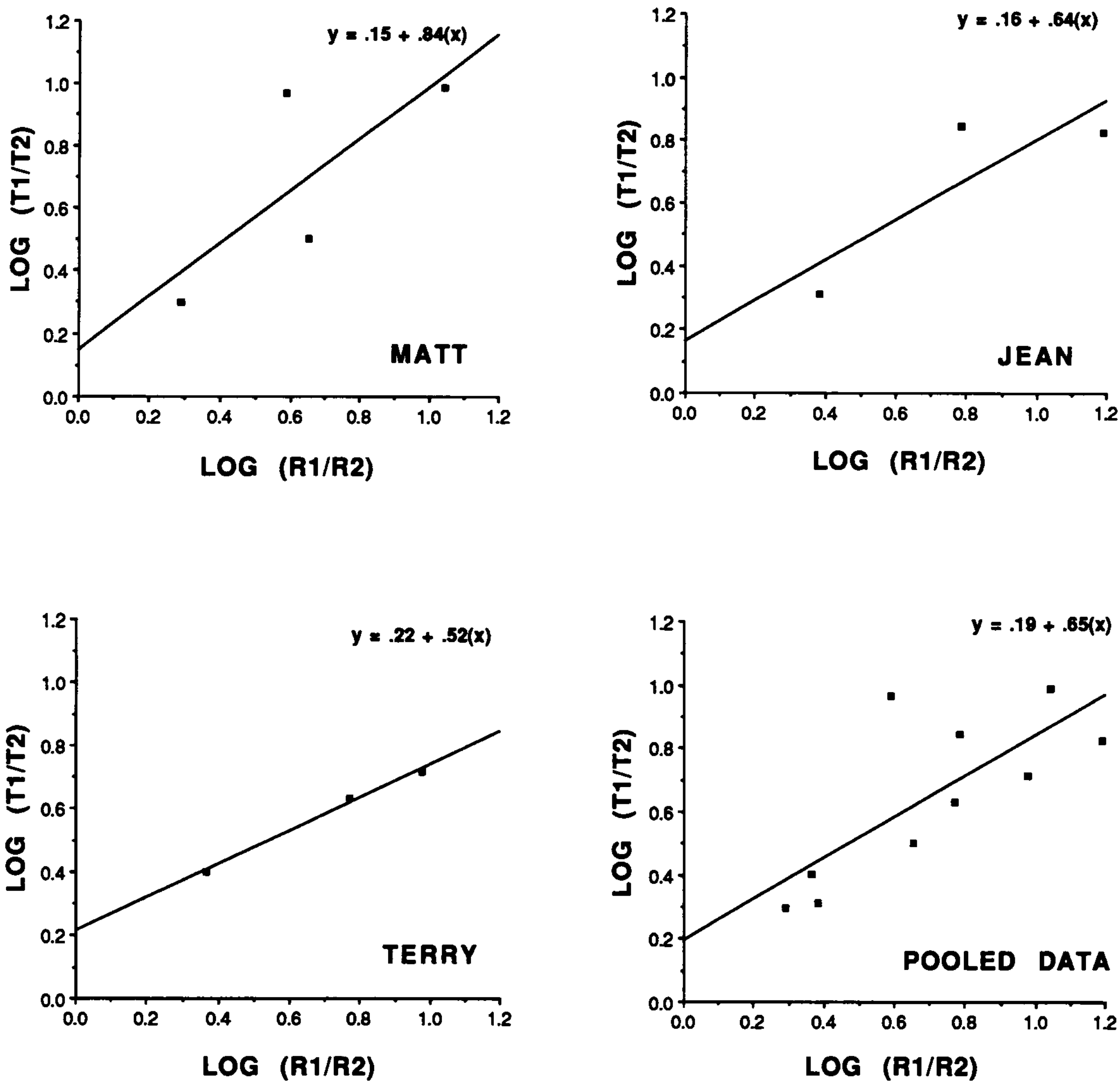

Figure 4. Bivariate distributions of $\log(T1/T2)$ (y axis) and $\log(r1/r2)$ (x axis) for Matt, Jean, Terry, and data pooled for all 3 participants. Each data point represents the mean for the last five data points in each schedule condition. The least squares regression equation for each relation is presented in the upper right portion of each distribution.

to 0.99 across subjects. These successful replications of the matching law to socially relevant human behavior suggest that the matching relation is not limited to arbitrary response topographies. In addition, the methods used to quantify the functional relation between reinforcement ratios and response ratios in nonhuman studies may also be used to quantitatively describe human choice behavior in natural contexts. These findings complement a growing body of nonparametric studies on concurrent performance and those that have examined single-alternative situations and have found that the matching law adequately describes a variety of socially relevant human behaviors (Mace, McCurdy, & Quigley, 1990; Martens, Halperin, Pummel, & Kilpatrick, 1990; Martens & Houk, 1989; Martens, Lochner, & Kelly, 1992; McDowell, 1981; Neef, Mace, & Shade, 1993; Neef, Mace, Shea, & Shade, 1992; Neef, Shade, & Miller, 1994).

A second significant finding is that, although our subjects' time allocation was a positive function of relative rates of reinforcement, the linear relation was characterized by substantial undermatching and bias. That is, the proportion of time subjects spent on the richer schedule was somewhat less than the proportion of reinforcers derived from that schedule. In addition, subjects showed a consistent preference for one of the two response alternatives, independent of its correlated rate of reinforcement. However, the source of this bias was not apparent in the present study. With typical sources of bias held constant (reinforcer quality and delay, response effort) or counterbalanced (left/right position), the factors influencing preference remain unclear.

Some degree of undermatching has been reported in most matching studies with humans (Lowe & Horne, 1985; McDowell, 1989; Pierce & Epling, 1983), such that undermatching may actually be the norm rather than the exception. On one hand, pervasive reports of undermatching seem to threaten the generality of at least the strict form of the matching law, in which relative response and reinforcement ratios are equal. However, slope values (short of complete indifference) may be less important than the strength of the linear relation, especially for those concerned with application of the matching law. For example, although Terry showed marked undermatching (a = .52), 98% of the variation in her time allocation ratios was explained by her relative rates of obtained reinforcement. This information could be used by an applied behavior analyst to select rates of reinforcement that will result in acceptable levels of engagement in various academic alternatives. Without this information, selection of reinforcement schedules would be left to guesswork.

A third finding from this study suggests a potentially important limitation of matching-law accounts of human choice in natural environments. Without the use of adjunct procedures, time allocation for all 3 subjects in this experiment came quickly under control of the first pair of conc VI VI schedules presented, and stability was reached within six to eight sessions. However, subsequent changes in the conc VI VI schedules failed to generate patterns of time allocation that matched the relative rates of reinforcement. Instead, subjects generally tended to persist in their allocation patterns of the previous schedule condition. In response to this situation, we unsystematically introduced various adjunct procedures (such as a COD, limited hold, timers, and demonstrations of allocation patterns yielding the maximum available reinforcers). After repeated exposure to one or more of these procedures, allocation became responsive to the scheduled contingencies and remained stable when most or all adjunct procedures were discontinued.

This finding suggests the need for explicit study of transitional performance that leads to steady-state responding under concurrent schedules. For example, Hanna et al. (1992) found that pigeons showed discriminated responding sooner following changes in conc VI VI schedules when each VI schedule was uniquely correlated with a discriminative stimulus versus when stimuli were correlated with different VI values. Although our study contained unique schedule-correlated stimuli, this condition alone was not sufficient to produce differential responding when conc VI VI schedules changed, at least not given the current amount of exposure to the schedule conditions. As in the studies by Takahashi and Iwamoto (1986) and Lowe and Horne (1993), additional adjunct procedures were necessary in our study for allocation ratios to vary in relation to relative reinforcement. The adjunct procedures used in this study and others are aimed to a large extent at enhancing the discriminability of the properties of concurrent schedules of reinforcement. Demonstrations and timers may have functioned in a manner similar to the cues and instructions used by Lowe and Horne, indicating the ordinal quantitative difference between schedules. However, reports that human matching is dependent on the use of these procedures (e.g., Baum, 1975) raises questions about the adequacy of the matching law alone to account for natural human choice behavior (Fuqua, 1984).

Our results seem to suggest that when humans are exposed to novel situations, their concurrent performance is susceptible to control by contingencies. All 3 students responded quickly to the scheduled contingencies during the first phase of conc

VI VI reinforcement. However, familiarity with concurrent schedules in a particular context appeared to weaken the students' sensitivity to changing consequences. Such insensivity to control by contingencies is considered by many investigators to be a defining characteristic of rule-governed behavior (Catania, Shimoff, & Matthews, 1989; Hayes, Brownstein, Haas, & Greenway, 1986). This finding is consistent with other studies that have shown insensitivity to concurrent schedules of reinforcement and the speculation that human choice may be contingency shaped at times and rule governed at others (Horne & Lowe, 1993; Takahashi & Iwamoto, 1986). For applied behavior analysts, this suggests that assessment methodologies are needed to distinguish rule-governed and contingency-maintained performance. Further work is also needed to develop intervention strategies for altering rule-governed choice in applied settings.

Our results showed lengthy transition phases following changes in reinforcement ratios and the apparent need for adjunct procedures to achieve allocation patterns predicted by the matching law. These findings suggest the need for caution in extending the matching law to applied work. First, this experiment, like most matching studies (Davison & McCarthy, 1988; Pierce & Epling, 1983), held the relative rates of reinforcement available for response alternatives constant across several sessions. However, this degree of consistency is uncharacteristic of many applied situations. It is more likely that a given situation will be correlated with a variety of different concurrent rates of reinforcement. That participants in this experiment were slow to discriminate changes in reinforcement ratios suggests that the matching law alone may not be a good predictor of precise allocation patterns in changeable human environments. Second, the adjunct procedures (COD, limited hold, timers, and demonstrations) used to aid discrimination of the schedule parameters are less prevalent in applied situations. The present findings suggest that, without adjunctive aids, relative response rates may not match relative reinforcement rates. Finally, the present findings were obtained using symmetrical response alternatives. That is, both response alternatives involved identical response effort, and both resulted in identical reinforcers with no delay to reinforcement; the alternatives differed only by their correlated rates of reinforcement. However, response alternatives are typically asymmetrical in most applied situations (Fuqua, 1984). Recent applied research has found that the effects of rate of reinforcement on allocation patterns can be substantially altered when alternatives differ by quality of reinforcement (Neef et al., 1992) and delay to reinforcement (Neef et al., 1993). Because these variables can interact differently and idiosyncratically as situations vary (Neef et al., 1994), mathematical predictions of choice in applied settings may be especially difficult to realize.

Although the present findings raise some questions about quantitative applications of the matching law in applied behavior analysis, we see greater applied potential in the general theoretical framework of matching theory. According to matching theory, all behavior is choice behavior, including the distribution of responses across concurrently available alternatives and choice of one alternative over another in a discrete-trial situation. For applied problems that are characterized as excessive levels of problem behavior and insufficient levels of desirable behavior, matching theory suggests that interventions alter the variables known to influence choice. Thus, the general intervention strategy is to manipulate, to the extent possible, response effort and rate, quality, and delay to reinforcement to favor the occurrence of desirable alternative behavior, while concurrently discouraging choice of problem behavior. Such a theoretical framework places emphasis on manipulating the variables that affect choice until socially significant changes in behavior occur. Although this viewpoint deemphasizes quantitative prediction of allocation patterns, the strategy of altering the independent variables that are functionally related to choice is derived directly from basic matching research and illustrates the value of integrating basic and applied research efforts (Mace, 1994).

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