Limits to Quantum Gravity Effects on Energy Dependence of the Speed of Light from Observations of TeV Flares in Active Galaxies

S. D. Biller,[1] A. C. Breslin,[2] J. Buckley,[3] M. Catanese,[4] M. Carson,[2] D. A. Carter-Lewis,[4] M. F. Cawley,[5] D. J. Fegan,[2] J. P. Finley,[6] J. A. Gaidos,[6] A. M. Hillas,[7] F. Krennrich,[4] R. C. Lamb,[8] R. Lessard,[6] C. Masterson,[2] J. E. McEnery,[9] B. McKernan,[2] P. Moriarty,[10] J. Quinn,[11] H. J. Rose,[7] F. Samuelson,[4] G. Sembroski,[6] P. Skelton,[7] and T. C. Weekes[11]
[1]*Oxford University, Oxford, United Kingdom*
[2]*University College, Dublin, Ireland*
[3]*Washington University, St. Louis, Missouri 63130*
[4]*Iowa State University, Ames, Iowa 50011*
[5]*Maynooth University, Maynooth, Ireland*
[6]*Purdue University, West Lafayette, Indiana 47907*
[7]*University of Leeds, Leeds, United Kingdom*
[8]*California Institute of Technology, California 91125*
[9]*University of Utah, Salt Lake City, Utah 84112*
[10]*Galway-Mayo Institute of Technology, Galway, Ireland*
[11]*Whipple Observatory, Amado, Arizona 85645*
(Received 13 October 1998)

We have used data from a TeV γ-ray flare associated with the active galaxy Markarian 421 to place bounds on the possible energy dependence of the speed of light in the context of an effective quantum gravitational energy scale. Recent theoretical work suggests that such an energy scale could be less than the Planck mass and perhaps as low as 10^{16} GeV. The limits derived here indicate this energy scale to be in excess of 6×10^{16} GeV for at least one approach to quantum gravity in the context of D-brane string theory. To the best of our knowledge, this constitutes the first convincing limit on such phenomena in this energy regime.

PACS numbers: 98.70.Rz, 03.30.+p, 04.60.–m

It has recently been pointed out that many quantum gravity scenarios may result in an observable time dispersion for high energy radiation originating at large distances from the Earth [1–3]. This would result from an effective energy dependence to the velocity of light in vaccum owing to propagation through a gravitational medium containing quantum fluctuations on distance scales near the Planck length, $L_P \simeq 10^{-33}$ cm, with time scales on the order of $1/E_P$, where E_P is the Planck mass ($\simeq 10^{19}$ GeV). In particular, it has been indicated [1] that different approaches to quantum gravity lead to a similar description of the first-order effects of such a time dispersion:

$$\Delta t \simeq \xi \frac{E}{E_{\mathrm{QG}}} \frac{L}{c}, \qquad (1)$$

where Δt is the time delay relative to the standard, energy-independent speed of light, c; ξ is a model-dependent factor of order 1; E is the energy of the observed radiation; E_{QG} is the assumed energy scale for quantum gravitational effects which can couple to electromagnetic radiation; and L is the distance over which the radiation has propagated. While E_{QG} is generally assumed to be on the order of E_P, recent work within the context of string theory suggests that the onset of noticeable quantum gravitational effects may correspond to a characteristic energy scale smaller than the Planck mass and perhaps as low as 10^{16} GeV [4]. Thus, any experimental probe of such scales or higher would be of great interest.

In a recent paper [1], it was suggested that γ-ray bursts (GRBs) could provide a natural way to test such predictions owing to the short duration, high energies, and the apparent cosmological origin of at least some of these bursts. Based on current data, these authors indicate that if (1) time structure on the scale of 0.01 sec or smaller can be established for energies $\sim$200 keV and (2) an association of such a burst can be made with an object possessing a redshift of order 1, energy scales of $E_{\mathrm{QG}} \sim 10^{16}$ GeV could be probed. Unfortunately, establishing the distance of any particular GRB from earth has proven to be nontrivial, with only a handful positively associated with optical counterparts. Also, some of the highest energies seen from GRBs are associated with an "afterglow" which seems to occur over much longer time scales than the initial burst. However, more stringent and robust limits to E_{QG} can already be set based instead on the rapidly rising TeV flares seen to occur in active galaxies.

The Whipple Observatory γ-ray telescope, located in Arizona, detects the Čerenkov light generated by electromagnetic cascades resulting from the interaction of high-energy γ rays in the atmosphere. Images taken of such cascades are used to discriminate backgrounds and derive energies of the primary γ rays in the regime above $\sim$250 GeV. To date, three extragalactic sources, all active galaxies of the blazar class, have been identified as emitters of TeV radiation [5–7]. Two of these, Markarian 421 and Markarian 501, produce particularly strong emission with energy spectra approximated by an $\sim E^{-2.5}$ power law (although Markarian 501 shows evidence for additional curvature) between energies of 350 GeV and 10 TeV [8]. These same sources have also exhibited dramatic changes in flux level on time scales ranging from minutes to days. On several occasions, such variations have been simultaneously studied and correlated with x-ray, UV, and optical measurements [9,10].

The most rapid flare observed thus far was seen from Markarian 421 on 15 May 1996 [11]. These data are shown in Fig. 1, where the excess rate of γ-ray selected events above a threshold of 350 GeV is binned in intervals of 280 sec duration, as it appeared in the original publication of this observation. To avoid confusion (and potential bias), we will retain this same binning throughout the current analysis. The doubling time of the flare is less than 15 min, although variability is apparent on the scale of the binning at the 99% confidence level. Because of the rapidly falling energy spectrum, the γ-ray data are dominated by events near the triggering threshold. Thus, the peak of the flare is almost entirely defined by events with γ-ray energies less than 1 TeV, as shown at the top of Fig. 2 where the average background level is ~ 12 events per bin. The lower plot in Fig. 2 shows the same distribution for events with γ-ray energies in excess of 2 TeV, where ~ 1 of the seven events is expected to be background.

It is worth noting that the bin containing the largest number of higher energy events out of the 36 intervals shown in Fig. 1, is the same 280 s interval which contains the largest number of lower energy events. If a time lag on the order of the binning scale was present due to quantum gravity effects, one would expect this to show up as a "smearing" of the signal at the trailing and/or leading edges of the peak bin. The absence of events in either of the immediately adjacent bins therefore suggests that no such lag is present on scales greater than that of the binning. To explicitly quantify this, we first note that the excess of low energy events above the average background level can be used to define a probability density function (PDF), binned in time. This may then be used to compute the relative likelihood for the observed distribution of higher energy events (bottom half of Fig. 2) to be drawn from an identical distribution which is shifted in time with respect to the lower energy events. The PDF must be suitably normalized over those bins which allow such a mapping owing to the "edges" of the 28 min uninterrupted data run. For example, when considering a possible time lag equivalent to one time interval for the higher energy data, only the first five bins of the PDF derived from lower energies can be used since the sixth bin would map to a shifted time interval outside of the range of data. Furthermore, only high energy data in the latter five bins can be used since there is no corresponding lower energy bin at earlier times from which to map the probability. Comparisons with the null hypothesis of no shift must also account for this truncation of the higher energy data by appropriately renormalizing the PDF.

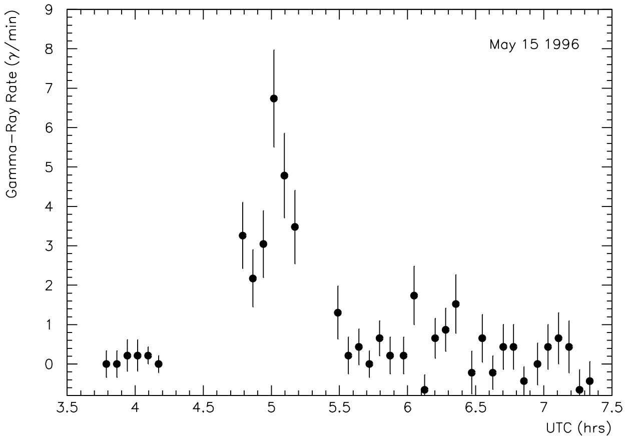

FIG. 1. TeV γ-ray flare from Markarian 421 observed on 15 May 1996 by the Whipple γ-ray observatory. The rate of excess γ-ray selected events is binned in intervals of 280 sec (taken from Ref. [11]).

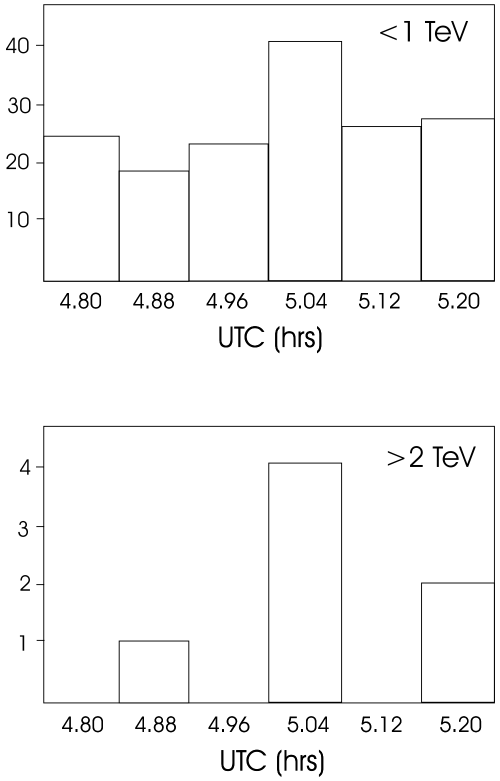

FIG. 2. Total number of γ-ray selected events occurring in each 280 sec interval near the peak of the 15 May 1996 flare from Markarian 421. The top plot consists of events with γ-ray energies less than 1 TeV, whereas the bottom plot is for energies greater than 2 TeV.

The quantity $-2\log(L_r)$, where L_r is the relevant likelihood ratio, should be approximately distributed as a χ^2 distribution with one degree of freedom [12]. However, in order to insure accuracy in the regime of small numbers, the confidence levels were determined by Monte Carlo sampling of the specific PDFs used. As expected, the likelihood ratio peaks for the hypothesis of zero time lag and the results for various other time lag/lead scenarios are shown in Table I.

Hence, at the greater than 95% confidence level, emission above 2 TeV appears to keep in step with emission below 1 TeV for variability time scales less than 280 sec. A caveat to this analysis is that it is possible to conceive of a scenario in which high and low energy emission are emitted at slightly different times from the source in just such a way as to compensate for time delays in the propagation of the radiation due to quantum gravity effects. However, we regard this scenario as being overly conspiratorial in nature and note that future studies of sources at different redshifts will resolve this issue beyond doubt.

The redshift of Markarian 421 is 0.031, which translates to 1.1×10^{16} light seconds for an assumed Hubble constant of 85 km/s/Mpc. From Eq. (1), our results then lead to a lower bound on E_{QG}/ξ of 4×10^{16} GeV. Recent calculations in the context of D-brane theory [13] indicate a value of $\xi \sim 3/2$, leading to a bound of $E_{\mathrm{QG}} > 6 \times 10^{16}$ GeV for this model. On the other hand, calculations in the context of loop gravity [3] lead to a value of ξ as large as 4, suggesting an energy scale in excess of 1.6×10^{17} GeV.

Given that the theory of quantum gravity is still in its infancy, it is possible that a predicted time dispersion with an energy dependence other than that of Eq. (1) may yet arise from other approaches. Accordingly, we would like to consider a dispersion of the form:

$$\Delta t \simeq \frac{L}{c}\left(\frac{aE}{E_{QG}}\right)^{b}, \qquad (2)$$

where a and b are arbitrary constants. Figure 3 shows lower bounds to E_{QG}/a derived from the observations presented here as a function of the assumed power law index b.

In more recent work it has also been suggested that a "stochastic" broadening of the time spectrum of higher energy radiation may also result from quantum gravity [15]. The suggested dependence of this effect on energy is similar to that of Eq. (1). We therefore adopt the generic form

$$\sigma_t \simeq \frac{L}{c}\left(\frac{\alpha E}{\Lambda_{QG}}\right)^{\beta}, \qquad (3)$$

where σ_t is the rms time spread, Λ_{QG} is the quantum gravitational energy scale of relevance to this effect (not necessarily equal to E_{QG}), with α and β as arbitrary constants. Based on the bottom half of Fig. 2, we take σ_t to be less than 3 time intervals, or 840 s. The dashed line in Fig. 3 shows the resulting lower bounds to $\Lambda_{\mathrm{QG}}/\alpha$ as a function of β.

TABLE 1. Confidence levels for the exclusion of several hypotheses for the lag or lead of radiation above 2 TeV in time with respect to radiation below 1 TeV.

Hypothesis	$2\log(L_r)$	C.L. for exclusion
Zero lag ($H0$)	0	0
Lag by 280 s	−4.74	99.3%
Lead by 280 s	−5.76	99.3%
Lag by 560 s	−3.70	96.6%
Lead by 560 s	−7.70	99.8%

We note that an earlier limit on the energy dependence of the speed of light, which would be more restrictive than that given here, had been derived from the possible ultrahigh-energy detection of anomalous pulsed emission from Hercules X-1 in 1986 [14]. However, more recent analyses and the lack of further such detections suggests that the interpretation of that observation as a statistical fluctuation is not an unreasonable one [16]. We therefore believe that the limits presented in this paper represent the most credible and stringent bounds thus far obtained.

The next generation of proposed ground-based instruments, such as VERITAS and HESS, will feature multitelescope systems with much improved sensitivity, energy coverage, and resolution, along with the ability to track candidate sources of flares more continuously using dedicated telescopes. This will allow for both a more detailed study of the time structure of currently known TeV sources and the prospect of discovering and studying more distant objects. It is therefore reasonable to expect to probe E_{QG} to even higher energies in the near future from further studies of TeV flares. As has already been pointed out [1], the distinctive dependence of the shortest observable variability time scale on both energy and source distance

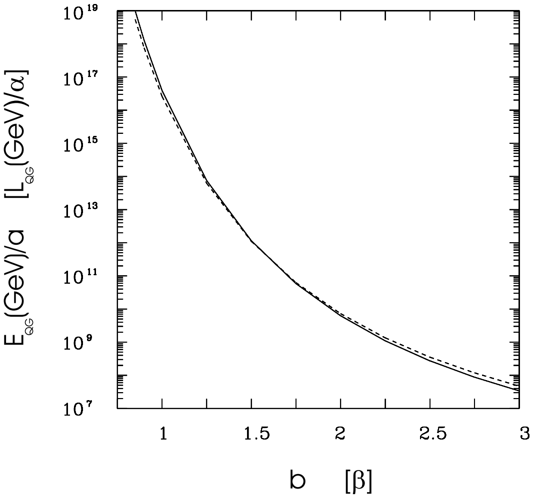

FIG. 3. Derived lower bounds to E_{QG}/a as a function of index b (solid line) and to $\Lambda_{\mathrm{QG}}/\alpha$ as a function of index β (dashed line) for the more generic forms of quantum gravity dispersion and stochastic broadening, respectively, as defined by Eqs. (2) and (3).

for quantum gravitational dispersion should allow source-specific effects to be distinguished. Thus, future TeV studies could conceivably provide convincing evidence for quantum gravity, particularly if the resulting time-dispersion effects are associated with characteristic energy scales less than the Planck mass. We hope that this prospect, in addition to the bounds derived here, will encourage more detailed predictions of such phenomena to be calculated in the context of specific quantum gravity frameworks.

We would like to thank Subir Sarkar and Nick Mavromatos at Oxford for several useful discussions on this topic. This work has been supported in part by PPARC, Forbairt, the U.S. Department of Energy, NASA, and the Smithsonian Institution.

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