Spring 5-31-1967

Linear and adaptive delta modulation

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LINEAR AND ADAPTIVE DELTA MODULATION

BY

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A DISSERTATION

PRESENTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE

OF

DOCTOR OF ENGINEERING SCIENCE

AT

NEWARK COLLEGE OF ENGINEERING

Newark, New Jersey
1967

TO MY WIFE, MARY

ABSTRACT

New results are presented offering insight into the performance and optimization of linear and adaptive delta modulation, together with a comparison with pulse code modulation. The results are applied to three cases of practical importance: television, speech, and broadband signals.

The results presented can be grouped into the following three categories. First, a performance characterization of linear delta modulation (DM) is given. With the aid of certain empirical observations made from computer simulations, closed form expressions are found for granular noise, overload noise, and minimum quantization noise powers. These results permit the prediction of the optimum performance obtainable from DM at various bandwidth expansion factor values for many classes of signals. A defined quantity called the slope loading factor is usefully employed in the characterization of DM performance. It is shown that the slope loading factor is a normalizing variable when used to describe S/N_Q performance. The optimum performance of DM with signals such as television and speech having an integrated spectrum exceeds that with a broadband signal having a uniform spectrum. It was also found

that DM performance obtained with a Gaussian message signal amplitude probability density is essentially the same as that obtained with an exponential density.

Second, the advantages to be gained when adaptive control is introduced into the DM system are investigated. If the message signal ensemble is nonstationary, a companding function is required. It is shown that this may be provided in a DM system by forcing the step size to respond adaptively to changes in the derivative of the input signal. Adaptive DM may take either a discrete or continuous form. It is shown that discrete adaptive DM does not sacrifice optimum linear DM performance to achieve companding, and further that large values of companding improvement are possible. Because of the nonstationary nature of television and speech signals, it is concluded that adaptive DM appears better suited than linear DM to such signals. Finally, linear DM is shown to be a special case of discrete adaptive DM.

Third, the noise performance of PCM with Gaussian and exponential signal densities is presented together with a comparison between DM and PCM for television, speech, and broadband message signals. It is shown that the characteristic form of the performances of PCM and DM are similar when the independent variables are the amplitude loading factor and slope loading factor respectively. The effects of logarithmic companding and signal amplitude limiting on PCM performance are investigated. It has been found that adaptive DM appears capable of realizing a larger companding improvement than PCM, and that amplitude limiting in PCM is the counterpart of slope limiting in DM. For a television signal, it is concluded that DM provides a greater maximum S/N_Q performance than PCM for values of the bandwidth expansion factor less than eight. For a speech signal, it is concluded that the performance of discrete adaptive DM with a bandwidth expansion factor value of four and a final gain factor value of only eight is approximately the same as that of companded PCM with a compression parameter value of one hundred. For a broadband signal, it is concluded that the performance of PCM is superior to that of DM. Finally, because of the complex nature of television and speech communication, it is concluded that subjective tests are needed before further conclusions regarding the performance advantages of discrete adaptive DM can be reached.

For an abridgment of the material in this dissertation, the reader is referred to a paper of the same title, written by the author, appearing in the Proceedings of the IEEE, March, 1967.

APPROVAL OF DISSERTATION

LINEAR AND ADAPTIVE DELTA MODULATION

BY

JOHN EDWARD ABATE

FOR

DEPARTMENT OF ELECTRICAL ENGINEERING

NEWARK COLLEGE OF ENGINEERING

BY

FACULTY COMMITTEE

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NEWARK, NEW JERSEY

JUNE, 1967

ACKNOWLEDGMENTS

The author is indebted to Dr. J. J. Padalino, Professor in Electrical Engineering for his advice and constructive commentaries during the course of this investigation. He also wishes to give acknowledgment to the faculty of the Department of Electrical Engineering, Newark College of Engineering for many helpful suggestions, and to thank his many associates at Bell Telephone Laboratories, Incorporated, who during the past four years have provided many stimulating discussions. Particular thanks are given to J. B. O'Neal and W. H. Jules.

The financial support of Bell Telephone Laboratories, Incorporated, is gratefully acknowledged.

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1. INTRODUCTION

In recent years, systems designed for transmitting continuous messages but containing discrete signals have become widespread. Pulse Code Modulation (PCM) and Delta Modulation (DM) belong to this class of communication systems into which is included a discrete communication channel. Shannon proposed that such systems be called mixed. In the general case, a mixed system consists of: (1) an encoder which transforms the continuous message into a discrete one; (2) a discrete channel or digital transmission network which conveys the transformed message to a receiver; and (3) a decoder or receiver which transforms the discrete message back into its continuous state. These transformations, however, are not achieved without incurring some penalty upon the quality of the received continuous message. This penalty generally takes the form of a type of distortion termed quantization noise, which is attributed in the encoding process to the dividing of a continuous signal into a finite number of representative levels. The quantization noise can be made arbitrarily small at the expense of channel bandwidth. Obviously, the challenge to be taken here is the optimization of system performance; that is, the minimization of both quantization noise and channel bandwidth. It is necessary, in order to accomplish such an optimization, to understand how the quantization noise is affected by the characteristics of the signal and the parameters of the encoding system.

One of the purposes of this dissertation is to provide insight into the noise behavior and optimization of linear DM by characterizing its performance by relatively simple closed form approximate solutions. The fidelity criterion used to define optimum performance is that of minimum mean square error or noise power. Linear DM is a simple type of predictive quantizing system and is essentially a one digit differential pulse code modulation system.[29,31,33] Such systems are based primarily on an invention by Cutler[7] and de Jager,[11] who used one or more integrators to perform the prediction function. Their invention is based on transmitting the quantized difference between successive sample values rather than the samples themselves. When the quantizer contains only two levels, the system is reduced to its simplest form and is referred to as delta modulation, or simply DM. Both the encoder and decoder make an estimate or prediction of the signal's value based on the previously transmitted signal. In linear DM, the value of the signal at each sample time is predicted to be a particular linear function of the past values of the quantized signal.

O'Neal[32] has given a good description of linear DM and was the first to compare the results of digital computer simulation with those of analysis. Van De Weg[41] has provided an expression for granular noise power, and Protonotarios[35] has described slope overload noise in detail. In addition to the above, the literature abounds with discussion, modification and application of linear DM (e.g., see References 1, 2, 3, 10, 11, 13, 16, 17, 18, 19, 20, 23, 24, 25, 30, 34, 36, 37, and 44).

For problems concerning the performance and optimization of DM, it is convenient to have a model, involving only a few essential parameters, which will satisfactorily characterize the noise performance of the DM system. Present formulations of DM are complex and unwieldy. In Section Three the description of linear DM performance is simplified by employing useful approximations and observations of computer simulation results. Using simple closed form expressions to describe DM noise performance, we can gain insight into the operation of linear DM, especially with an eye toward characterizing adaptive systems. These simple formulations do suggest adaptive systems as well as their characterization.

Unfortunately, the performance of linear DM is sensitive to changes in the mean power of the message signal. As a result, optimum performance from the linear DM system is limited to a very narrow range of message signal mean power variation. This is indeed a severe restriction for many signals of practical importance. It will be shown that by incorporating an adaptive technique into the DM system, the restriction is abated.

The second purpose of this dissertation is to introduce and investigate an adaptive DM concept which appears to provide a promising means for the binary encoding of television and speech signals. In adaptive DM, the value of the signal at each sample time is predicted to be a nonlinear function of the past values of the quantized signal. Introducing nonlinear prediction into DM by forcing the system to respond adaptively to changes in the slope of the input signal provides a useful means of extending the range over which the delta system yields its optimum performance. This would not be necessary if the message signal ensemble were stationary. However, ensembles of many communication signals are nonstationary. These include speech, television, facsimile signals and the like. It is, therefore, useful to consider a means of incorporating adaptive techniques into the delta process, enabling the system to encode nonstationary ensembles in an optimal way.

In Section Four, an adaptive DM system which seems promising for the encoding of television and speech signals is presented. From the simple closed form approximations of Section Three, the expected performance of the adaptive system is found, and presented in Section Five. Computer simulations are used to verify the predictions of performance and aid in system optimization. The amount of companding improvement achieved by the adaptive system is found and presented along with expressions relating to the optimum selection of linear and adaptive DM parameters.

The third and final purpose of this dissertation is to quantitatively compare the performance of linear and adaptive DM with that of PCM. Since encoding a continuous message by DM may be much simpler and lower cost than by pulse code modulation (PCM), there is considerable interest in determining how the performance of DM relates to that of PCM. In comparison with PCM, DM has a number of important differences and several advantages. Since DM overloads on slope, its optimum performance is a function of the message signal spectrum. Since PCM overloads on amplitude, its optimum performance is a function of the message signal amplitude probability density function. When companding is used for nonstationary ensembles, the optimum performance range of PCM is extended, as it is in the adaptive DM system. The fundamental differences in the overload characteristics of DM and PCM require that the optimum performance range of each be well defined for the classes of message signals to be considered.

In Section Six, a performance comparison is made between PCM and linear and adaptive DM. First, a characterization of PCM granular and overload noise powers is given for the following cases.

(1) Gaussian and exponential message signal amplitude probability densities

(2) With and without logarithmic companding

(3) With and without message signal amplitude limiting

Then the optimum performance of PCM with a television signal is compared with that of adaptive DM. Next, a comparison of the performances of adaptive DM and companded PCM is made when the message signal is speech. Finally, linear DM performance is compared to that of PCM having uniform quantization for the case of a broadband signal.

The computer simulations cited herein and described in Appendix D were obtained using a FORTRAN program reported by O'Neal,[32] who used random numbers to represent sample values of the message signal. His program, written for linear DM, was modified to incorporate the parameters necessary for the adaptive case.

The results of this work are applied mainly to three cases of practical importance: television, speech, and broadband message signals. The first two will be approximated by a signal having an integrated power spectrum and an exponential probability density function. The integrated spectrum is defined as one having an asymptote of negative six decibels per octave of increasing frequency starting at ω_3 and bandlimited to some maximum frequency ω_m. The suitability of the integrated spectrum and exponential density for describing television and speech signals can be established by examining the results of Kretzmer,[22] O'Neal,[33] Davenport,[9] and Fletcher.[14] The broadband signal (e.g., frequency division multiplexed signals) will be approximated by one having a uniform or white spectrum bandlimited to ω_m, and a Gaussian amplitude probability density function. The results also can be applied directly to other communication or stochastic signals which have the spectrum and density characteristics described above. The assumptions and restrictions used in this work are that (1) error free transmission exists in the digital channel, (2) the encoder sampling rate and digital transmission channel bit rate are constant, and (3) both the DM encoder and decoder employ a single ideal integrator.

2. LINEAR DM, A QUALITATIVE DISCUSSION

2.1 System Description and Performance

The basic linear DM system consists merely of a two level quantizer and a feedback path containing a single integrator, as illustrated in Figure 2-1. A sampler is included either in the quantizer or prior to the subtractor. The quantizer produces at each sampling instant a pulse of uniform duration and amplitude k, the latter commonly referred to as the step size. The pulse or step is of positive polarity if the error signal or quantizer input is positive, and of negative polarity if the error signal is negative. The sequence of binary pulses produced by the quantizer is transmitted via the digital channel to the decoder where a replica of the original input signal is reconstructed. The decoder consists of an integrator identical to that in the encoder, and a low pass filter having the same bandwidth as the input signal.

In the delta system, quantization noise manifests itself in two forms. The first of these is granular noise which results from the fact that the continuous signal is forced to assume discrete values which are multiples of the quantizer step size. Granular noise can be viewed as being similar to PCM quantizing noise, and as in PCM, is a monotonic function of step size

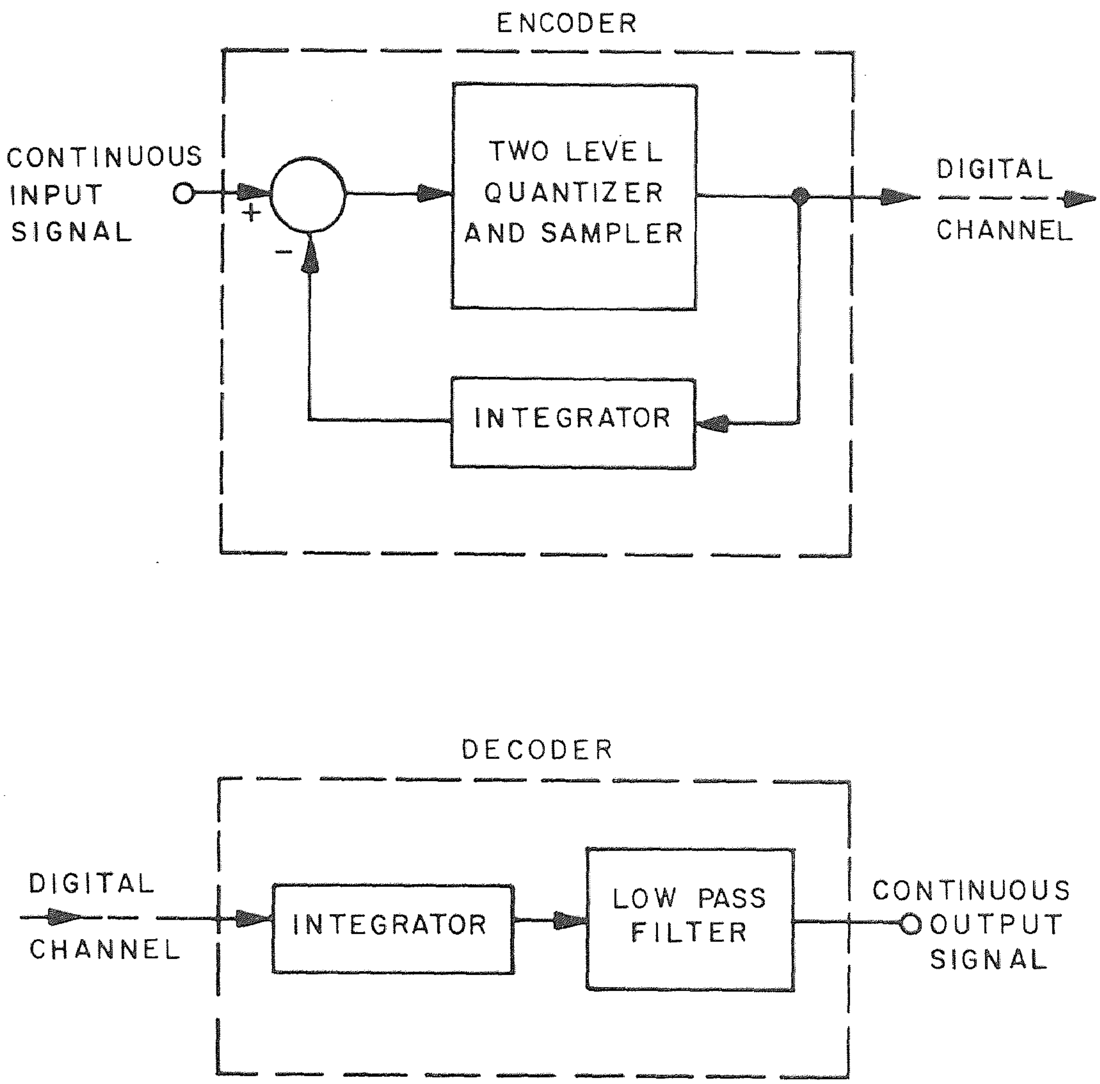


FIG. 2-1 DELTA MODULATION (DM) SYSTEM WITH SINGLE INTEGRATION.

(i.e., as the step size increases, granular noise increases). The second form of DM quantization noise is overload noise which is also a monotonic function of step size, but instead decreases with increasing step size. Typical waveforms of the DM system with single integration are illustrated in Figure 2-1. The quantization noise is illustrated at the bottom of Figure 2-2. If the step size is not too large relative to the standard deviation of the signal, the autocorrelation of the granular portion of the quantization noise becomes zero for time intervals which are large compared to the sampling period.[11] For relatively large step sizes, periodic patterns and tendencies appear in granular noise waveforms. Figure 2-3 illustrates the characteristic periodic behavior with large step sizes.

For small step sizes, overload noise predominates. As the step size approaches zero, the difference between the output and input approaches the input itself. Therefore, the overload noise power approaches the signal power, while the granular noise power approaches zero. This behavior is illustrated in Figure 2-4, which portrays granular noise power N_G, overload noise power N_O, and their sum or total quantization noise power N_Q as a function of the DM step size k, assuming

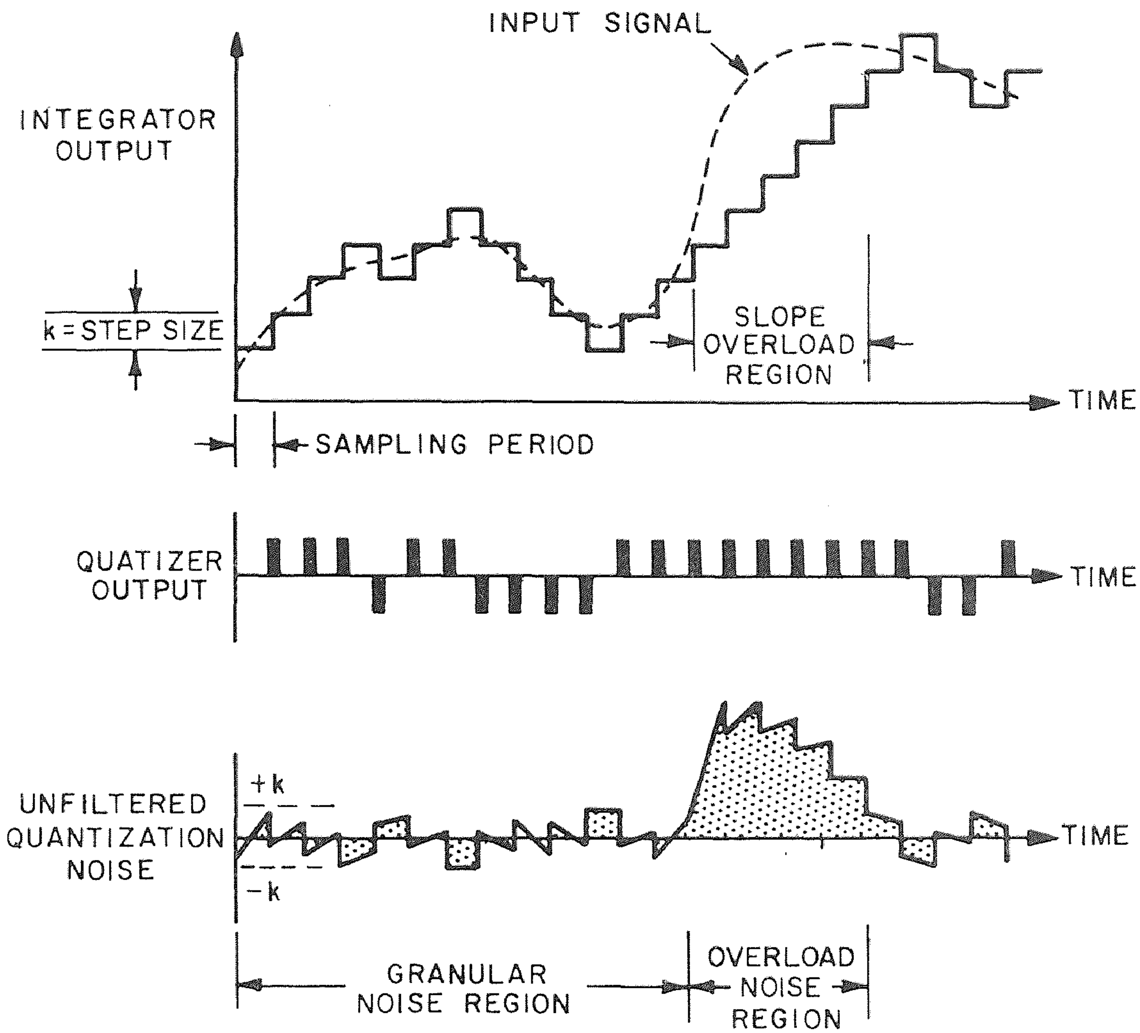


FIG. 2-2 WAVEFORMS OF DM SYSTEM WITH SINGLE INTEGRATION.

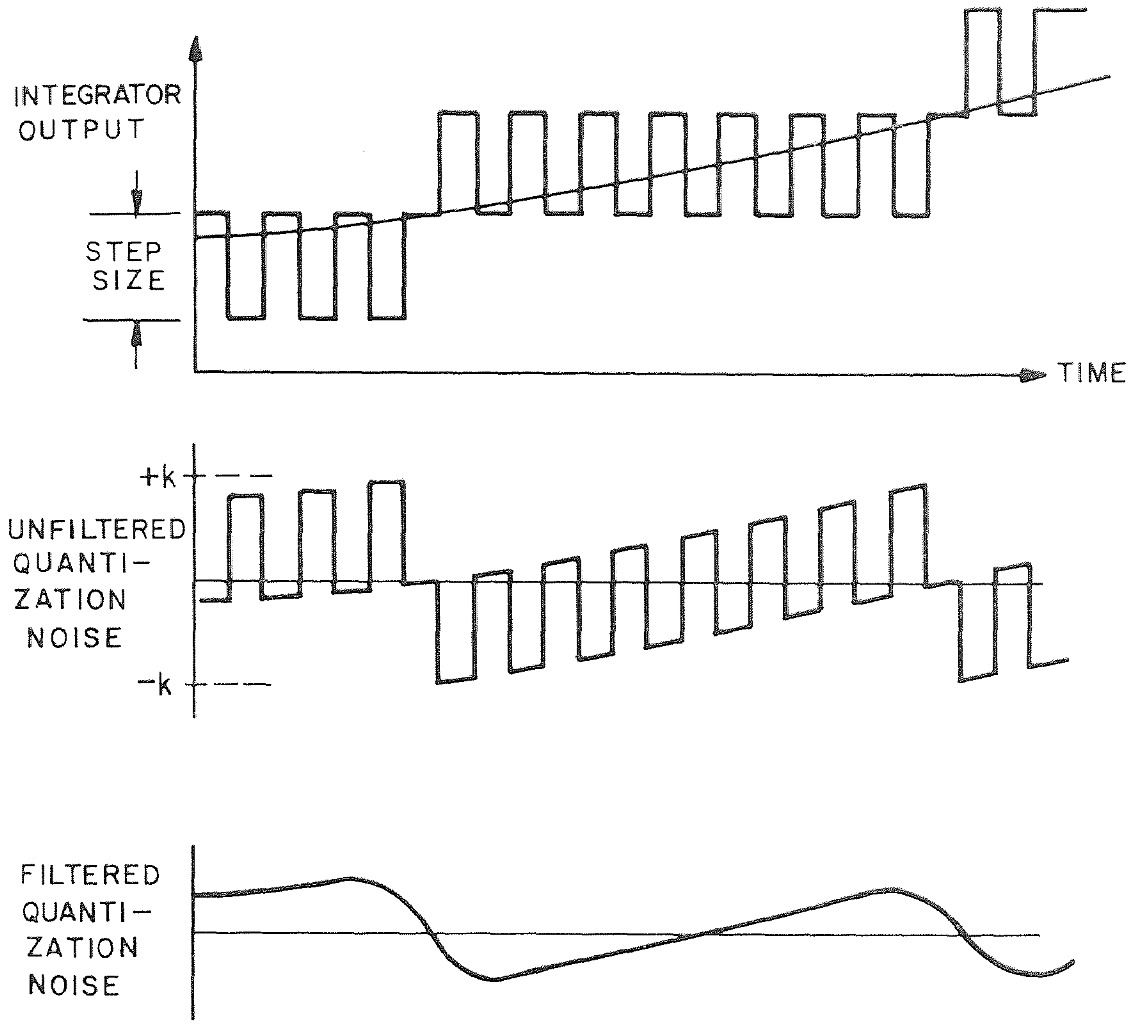


FIG. 2-3 DM WAVEFORMS WITH LARGE STEP SIZE.

a signal whose mean power, S, does not vary with time. Figure 2-4 illustrates that optimum performance (i.e., minimum N_Q) occurs for only a small range of variation of k. Alternatively, it could be stated, as will be shown quantitatively in Section Three, that optimum performance occurs for only one value of the signal standard deviation, and that for other RMS values of the signal the performance is degraded. Unfortunately, this represents a serious limitation of linear DM, but one which can be removed by recourse to adaptive techniques, as will be discussed in Sections Four and Five.

Because the DM quantizer in the encoder contains only two levels, the digital transmission channel pulse rate P is equal to the DM sampling rate f_s. The minimum bandwidth f_D required of the transmission channel is then equal to one half the sampling rate. The ratio of transmission channel bandwidth to message signal bandwidth f_m which shall be termed the bandwidth expansion factor and denoted by B in this work, is then simply one half of the ratio of sampling rate f_s to signal bandwidth f_m, or since,

$$P = f_s = 2f_D \tag{2-1}$$

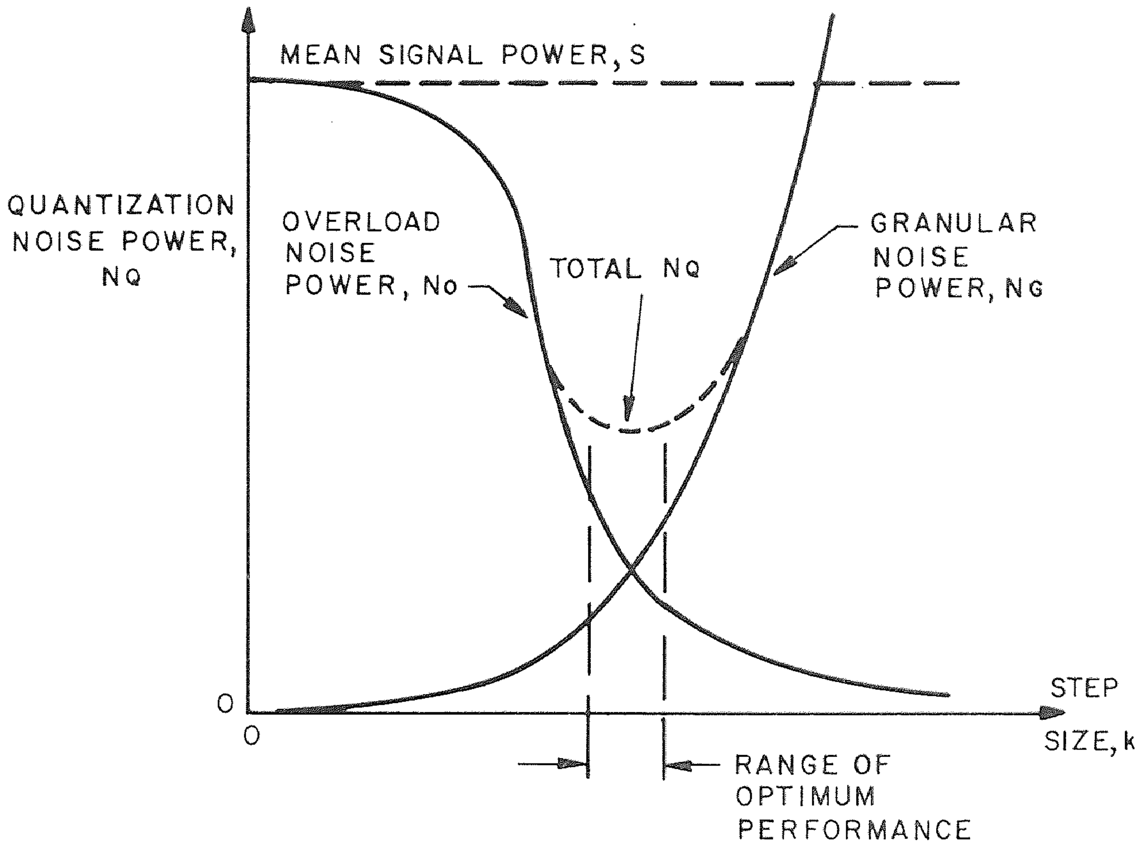


FIG. 2-4 LINEAR DM QUANTIZATION NOISE POWER

and,

$$B = \frac{f_D}{f_m} \qquad (2\text{-}2)$$

then,

$$B = \frac{f_s}{2f_m} \qquad (2\text{-}3)$$

2.2 Comparisons With PCM

As in DM, the quantization noise in PCM manifests itself into two forms. The first is the noise resulting from the discrete quantization process. We shall refer to this as granular noise so as to draw an analogy with its DM counterpart. In the literature, however, this is commonly referred to as quantizing noise, since the second form of noise is usually ignored. This second form of PCM quantization noise is caused by the limiting of the message signal to the maximum and minimum levels of the quantizer. We shall refer to this noise as overload noise. As opposed to DM overload noise which is produced when the message signal slope exceeds the slope capability of the DM quantizer, PCM overload noise is produced when the message signal amplitude exceeds the maximum level of the PCM quantizer. Exact analytical expressions for both PCM granular and overload noise powers are given in Section Six as a function of the bandwidth expansion factor and a defined quantity called the "amplitude loading factor." It will be shown later that the relationship between quantization noise and amplitude loading factor produces results similar in form to those illustrated in Figure 2-4.

DM and PCM are functionally different in a number of ways. First, in a PCM system the signal is generally sampled at a rate commonly known as the Nyquist rate which is twice that of the highest frequency present in the signal. In a DM system, by comparison, the sampling rate is generally many times that of the Nyquist rate. In a PCM system, the pulse rate is the sampling rate multiplied by the number of digits of encoding. The bandwidth expansion factor for PCM is then simply equal to the number of digits of encoding.

The number of quantizing levels in a PCM system is generally many times greater than two (e.g., in the order of 128 levels, or seven digits, for voice signals), whereas in DM it is only two levels. It should be noted here that a feedback quantizing system with a quantizer having more than two levels is generally referred to as differential PCM, or DPCM. Although the DPCM system has many of the characteristics of DM, it requires much more terminal equipment.

In PCM the signal is converted into pulse amplitude samples, which are then encoded into pulse words or groups. As a result, information concerning the pulse groupings referred to as "framing" must be inserted into the binary pulse sequence. In DM, since the quantizer consists of only two levels, the encoding into binary form is done in a single operation. As a result, no framing is required in DM. The consequence resulting from the lack of required framing as well as only two levels of quantizing is the outstanding simplicity and economy of the DM system.

The PCM system encodes the signal itself whereas the DM system, because of its feedback loop integrator, encodes the derivative of the signal.[11] As a result, if the signal amplitude is greater than the largest representative level of the quantizer, the PCM system is overloaded. With deterministic signals, this condition can be prevented through simple design. With stochastic signals, however, there will always be a finite probability that overload will exist. The optimum design in this case, then, is one that minimizes the quantization noise power as a function of the mean power of the signal.

In the DM system, overload will not be a function of the signal amplitude as in PCM, but instead will occur when the slope or derivative of the signal exceeds the slope capability of the DM system. Again, overload cannot be prevented if the signal is stochastic, it can only be minimized with respect to the mean power of the signal. If, however, the stochastic signal ensemble is nonstationary, then there can be no optimum linear DM system, and it will be shown that only an adaptive system will suffice.

In the PCM system, performance optimization is dependent on the amplitude probability density function of the input signal, but is independent of the signal's power spectrum. As a result, a PCM quantizer can be optimum only with respect to one input signal probability distribution, which of course requires that the statistics of the ensemble be stationary. Thus, even if the signal power remains constant, if the probability density of the signal changes, the PCM system may be no longer optimum. By contrast, DM performance will be shown to be dependent on the signal power spectrum and, for the densities considered in Sections Three and Five, independent of the signal amplitude probability density function. A summary of some comparisons between PCM and DM is given in Table 2-1.

TABLE 2-1

Some Comparisons Between PCM and DM

Characteristic	PCM	Linear DM	Adaptive DM
1. Prediction	None	Linear	Nonlinear
2. Number of Quantization Levels	Usually Many More Than Two	Two	Two, But of Variable Size
3. Sampling Rate	$2 f_m$	f_s	f_s
4. Signal Function Encoded	Amplitude	Derivative	Derivative
5. Overloading Function	Amplitude	Slope	Slope
6. Optimization is a Function of:	Signal Amplitude Density	Power Spectrum	Power Spectrum
7. Range of Optimum Performance With Nonstationary Signals	Large With, But Small Without, Companding	Small	Very Large
8. Bandwidth Expansion Factor, B, Equals	Number of Digits	$\frac{1}{2}\left(\frac{f_s}{f_m}\right)$	$\frac{1}{2}\left(\frac{f_s}{f_m}\right)$
9. Framing Required	Yes	No	No

3. LINEAR DM, A PERFORMANCE CHARACTERIZATION AND OPTIMIZATION

3.1 Slope Loading Factor Defined

In order to avoid slope overload, the slope capability of the DM system must be greater than the slope of the input signal. Since the former is given by the product of step size k and sampling rate f_s, then in order that the system not be overloaded, the following condition must be satisfied:

$$kf_s > |f'(t)| \tag{3-1}$$

where $|f'(t)|$ represents the magnitude of the input signal derivative with respect to time. If we denote the mean power of the derivative of the stationary stochastic signal by D, then we shall define a term, denoted by Δ and called the slope loading factor, as follows:

$$\Delta \equiv \frac{kf_s}{\sqrt{D}} \tag{3-2}$$

The slope loading factor given by Equation (3-2) represents the ratio of the slope capability of the system to the effective value of the slope of the stationary signal. It is, therefore, a dimensionless quantity and a measure of the degree by which the input is loading the capability of the DM system. In terms of the one sided power spectrum $F(\omega)$ of the signal, the mean power of the signal derivative is given by

$$D = \int_0^{\omega_m} \omega^2 F(\omega) d\omega \tag{3-3}$$

where $\omega_m = 2\pi f_m$ is the maximum angular frequency to which the signal is bandlimited prior to encoding.

In Table 3-1, the values of $F(\omega)$ and Δ are given for the types of signals to be considered in this work. For television and speech, the integrated power spectrum as given in Table 1 will be used with values of ω_3/ω_m of 0.011 and 0.23 respectively. These values, which will be used consistently herein are obtained from the results of O'Neal[32] and Fletcher.[14] The slope loading factor is expressed in Table 3-1 in terms of the bandwidth expansion factor, B, which for DM is given by Equation (2-3).

3.2 Quantization Noise Power

It is shown in Appendix B that granular noise power N_G as a function of Δ can be given with reasonable accuracy by two asymptotes. The first of these has a

TABLE 3-1

Power Spectrum and Slope Loading Factor

For Uniform and Integrated Signal Spectra

	Uniform Spectrum	Integrated Spectrum
$F(\omega)$	$\frac{1}{\omega_m}$	$\frac{1}{\omega_3 \tan \frac{\omega_m}{\omega_3}} \left[\frac{1}{1 + \left(\frac{\omega}{\omega_3}\right)^2} \right]$
s	$\frac{\sqrt{3}}{\pi} kB$	$\frac{Bk}{\pi \sqrt{\frac{\omega_3/\omega_m}{\tan^{-1}\omega_m/\omega_3} - \left(\frac{\omega_3}{\omega_m}\right)^2}}$

slope of six decibels per octave, that is granular noise power increases by six decibels per octave increase of Δ, and exists in the region $\Delta < 8$. The second asymptote has a slope of nine decibels per octave, and exists in the region $\Delta > 8$. The asymptotes are

$$N_G = \frac{\pi^2}{6}\left(\frac{D}{\omega_m^2}\right)\frac{\Delta^2}{B^3} \quad \text{for} \quad \Delta < 8, \tag{3-4}$$

and

$$N_G = \frac{\pi^2}{48}\left(\frac{D}{\omega_m^2}\right)\frac{\Delta^3}{B^3} \quad \text{for} \quad \Delta > 8. \tag{3-5}$$

For uniform and integrated spectra, these expressions are given in Table 3-2, where for convenience the mean signal power, S, and all impedances are assumed to be unity. When S is not unity, it is of course simply necessary to include it in the numerators of both $F(\omega)$ and N_G, and to include $\sqrt{S}$ in the denominator of Δ (i.e., divide k by $\sqrt{S}$, the standard deviation of the signal). Noise power is of course expressed in watts.

In DM systems, granular noise predominates for large values of Δ, and overload noise predominates for small values of Δ. From the computer simulation results

TABLE 3-2

Linear DM Results With Uniform And

Integrated Signal Spectra

	From Equation	Uniform Spectrum	Integrated Spectrum
N_G, $s < 8$	(3-4)	$\frac{\pi^2}{18} \frac{s^2}{B^3}$	$\frac{\pi^2 s^2}{6B^3}\left[\frac{\frac{\omega_3}{\omega_m}}{\tan^{-1}\frac{\omega_m}{\omega_3}} - \left(\frac{\omega_3}{\omega_m}\right)^2\right]$
N_G, $s > 8$	(3-5)	$\frac{\pi^2}{144} \frac{s^3}{B^3}$	$\frac{\pi^2 s^3}{48B^3}\left[\frac{\frac{\omega_3}{\omega_m}}{\tan^{-1}\frac{\omega_m}{\omega_3}} - \left(\frac{\omega_3}{\omega_m}\right)^2\right]$
N_O	(3-7)	$\frac{8\pi^2}{81} e^{-3s}(3s+1)$	$\frac{8\pi^2}{27}\left[\frac{\frac{\omega_3}{\omega_m}}{\tan^{-1}\frac{\omega_m}{\omega_3}} - \left(\frac{\omega_3}{\omega_m}\right)^2\right] e^{-3s}(3s+1)$

TABLE 3-2 (Cont)

Linear DM Results With Uniform And

Integrated Signal Spectra

	From Equation	Uniform Spectrum	Integrated Spectrum
Minimum N_Q	(3-8)	$\frac{\pi^2}{18}\left[\frac{(\ln B)^2 + 2.06 \ln B + 1.17}{B^3}\right]$	$\frac{\pi^2}{6}\left[\frac{\frac{\omega_3}{\omega_m}}{\tan^{-1}\frac{\omega_m}{\omega_3}} - \left(\frac{\omega_3}{\omega_m}\right)^2\right] \cdot \left[\frac{(\ln B)^2 + 2.06 \ln B + 1.17}{B^3}\right]$

given in Appendix D, it has been observed that minimum quantization noise power occurs at a value of the slope loading factor given approximately by

$$\Delta = \ln 2B. \qquad (3\text{-}6)$$

This relationship is illustrated in Figure 3-1 along with points obtained by computer simulation for the cases of uniform, television, and speech spectra. In the computer simulation, both Gaussian and exponential signal amplitude distributions were used with each of the three spectra cited. It was found that the results were substantially the same, that is neither the value of minimum quantization noise power nor the points illustrated in Figure 3-1 changed significantly when the amplitude distribution of the signal was changed. More will be said about this in Section Five.

Using Equation (3-6) and the fact that at its minimum the derivative of quantization noise with respect to slope loading factor must vanish, closed form empirical expressions for overload noise power N_O and minimum quantization noise power N_Q can be obtained. The results from Appendix B are as follows:

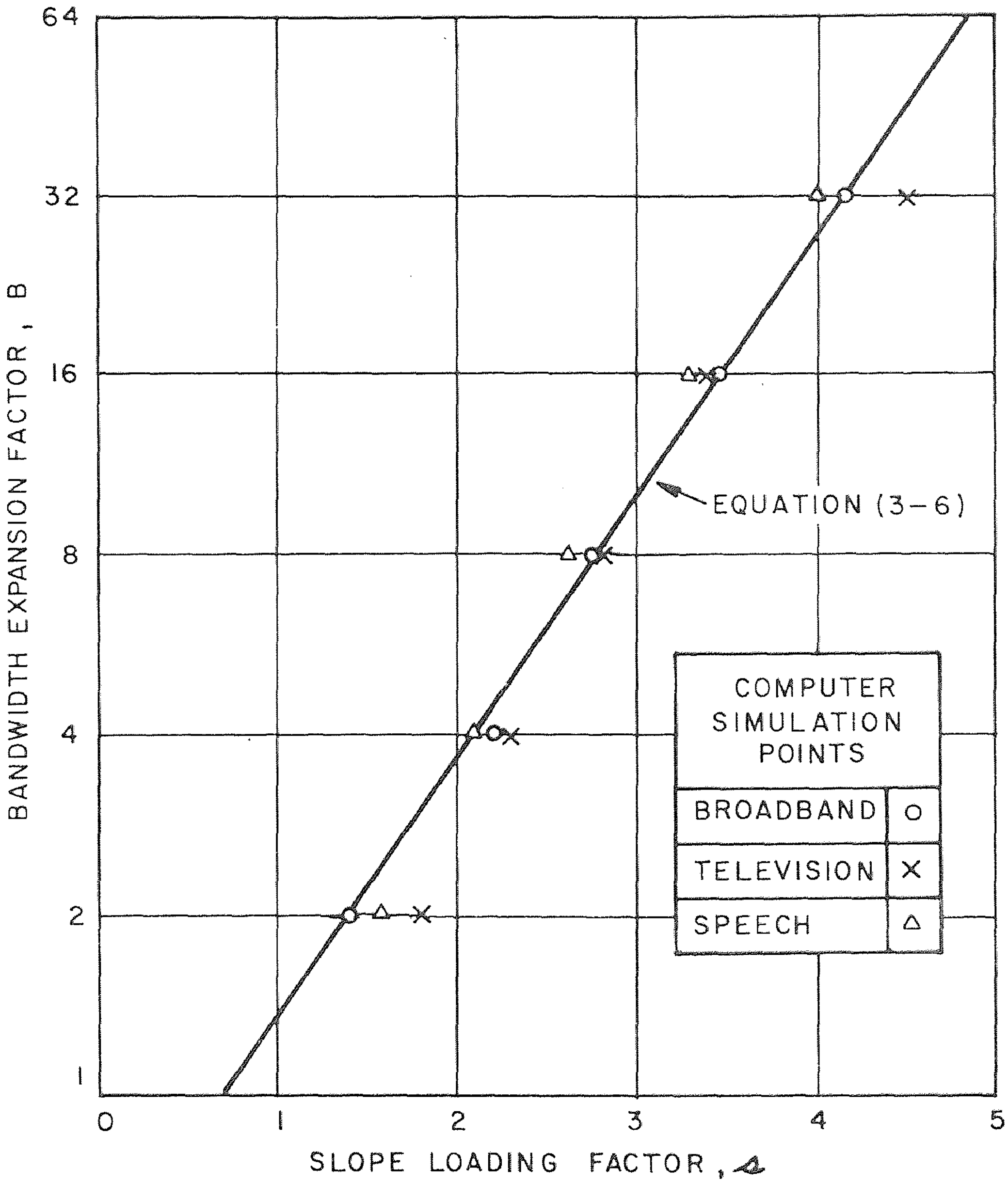


FIG. 3-1 RELATIONSHIP BETWEEN B AND $\mathcal{s}$ IN LINEAR DM AT MINIMUM QUANTIZATION NOISE.

$$N_O = \frac{8\pi^2}{27}\left(\frac{D}{\omega_m^2}\right)e^{-3\Delta}(3\Delta+1) \tag{3-7}$$

$$\text{minimum } N_Q = \frac{\pi^2}{6}\left(\frac{D}{\omega_m^2}\right)\left[\frac{(\ln B)^2 + 2.06 \ln B + 1.17}{B^3}\right]. \tag{3-8}$$

For uniform and integrated spectra, Equations (3-7) and (3-8) are given in Table 3-2. The optimum performance (i.e., maximum S/N_Q) expressed in decibels is the ratio of mean signal power to minimize N_Q, or simply

$$\text{maximum } S/N_Q = -\ 10 \log_{10}(\text{minimum } N_Q) \tag{3-9}$$

and where S has been assumed unity for convenience, as stated earlier. Throughout this work, signal-to-noise power ratio computations will be accomplished using the method shown by Equation (3-9).

Equations (3-2) through (3-9) provide a complete noise performance characterization of the linear DM system. Equation (3-8) indicates that the optimum delta system is capable of trading noise improvement with bandwidth expansion at a rate somewhat less than nine decibels per octave increase of B. A factor to note from Equation (3-8) is the strong dependence of maximum

S/N_Q on signal power spectrum. In the examples to follow, it will be shown that this characteristic of its performance gives the DM system an advantage over PCM for the class of signals having an integrated spectrum.

3.3 Application to Television, Speech, and Broadband Signals

The optimum performance (i.e., maximum S/N_Q)for uniform (e.g., broadband signal), television, and speech spectra are given in Table 3-3 and illustrated in Figure 3-2 as a function of the bandwidth expansion factor, along with points obtained by computer simulation.

The S/N_Q performance as a function of the slope loading factor is illustrated in Figure 3-3 for the uniform signal spectrum and Gaussian density (i.e., broadband signal) case at several values of B. For the integrated spectrum case, the performance curves are identical to those of Figure 3-3, the only change required being a shifting of the ordinate scale. It is clear that this is so from Equations (3-4), (3-5), and (3-7), since noise power at some specified value of Δ is proportional only to derivative power D. Similarly, for a specified value of B, the minimum quantization

TABLE 3-3

Linear DM Performance With

Television, Speech, and Broadband Signals

Parameter	From Equation	Television	Speech	Broadband
ω_3/ω_m	References 6 and 34	0.011	0.23	(Uniform Spectrum)
Δ	(3-2)	3.8 Bk	0.93 Bk	0.55 Bk
Maximum S/N_Q (in Decibels)	(3-9)	$\left[19.4 + 30 \log_{10}B - 10 \log_{10}([\ln B]^2 + 2.06 \ln B + 1.17)\right]$	$\left[7.1 + 30 \log_{10}B - 10 \log_{10}([\ln B]^2 + 2.06 \ln B + 1.17)\right]$	$\left[2.6 + 30 \log_{10}B - 10 \log_{10}([\ln B]^2 + 2.06 \ln B + 1.17)\right]$
Maximum S/N_Q Improvement Relative to Uniform Spectrum (in Decibels)	(3-10)	16.8	4.5	-

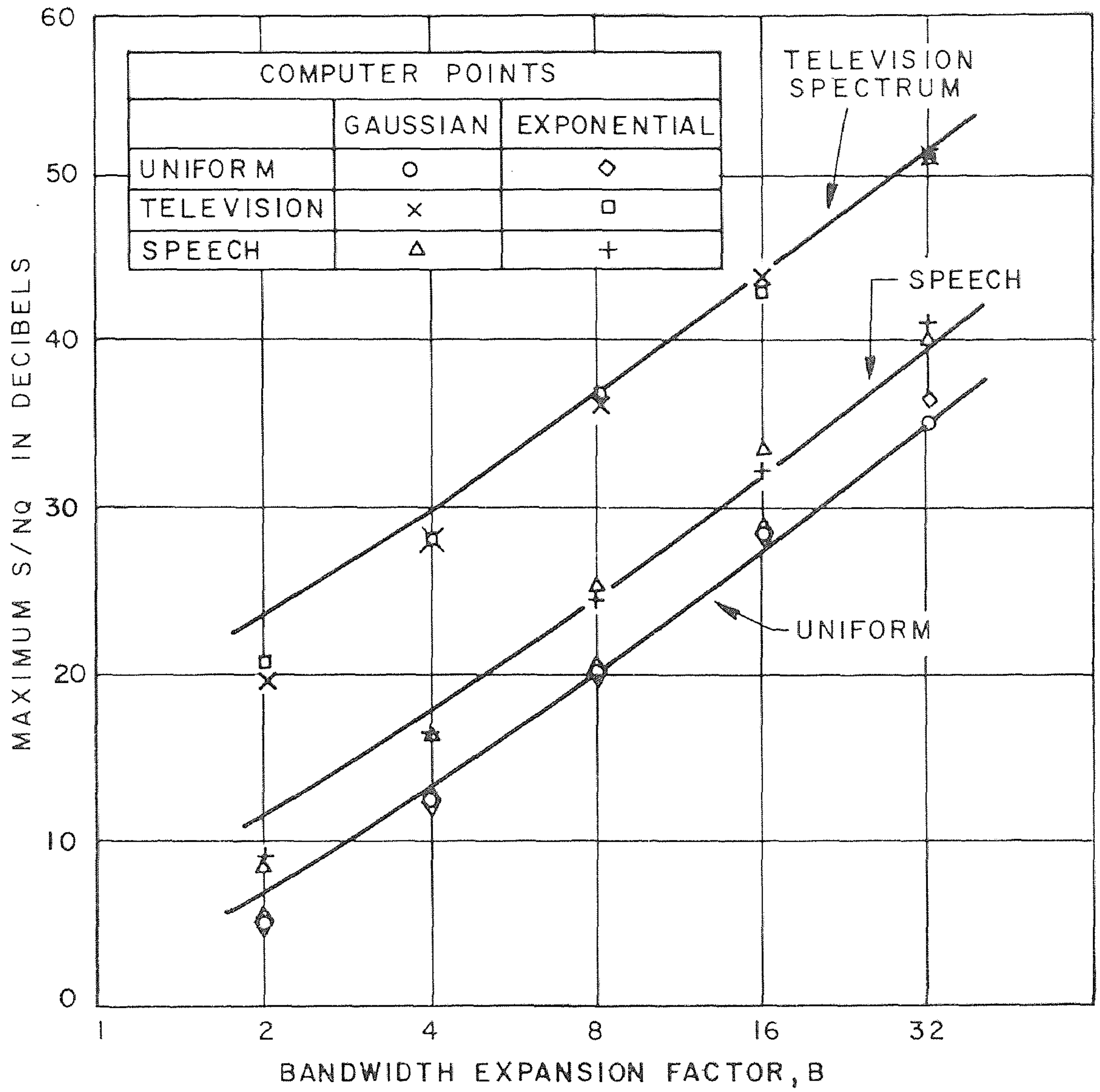


FIG. 3-2 OPTIMUM PERFORMANCE OF LINEAR DM, CURVES OBTAINED FROM TABLE 3-3, POINTS FROM COMPUTER SIMULATION.

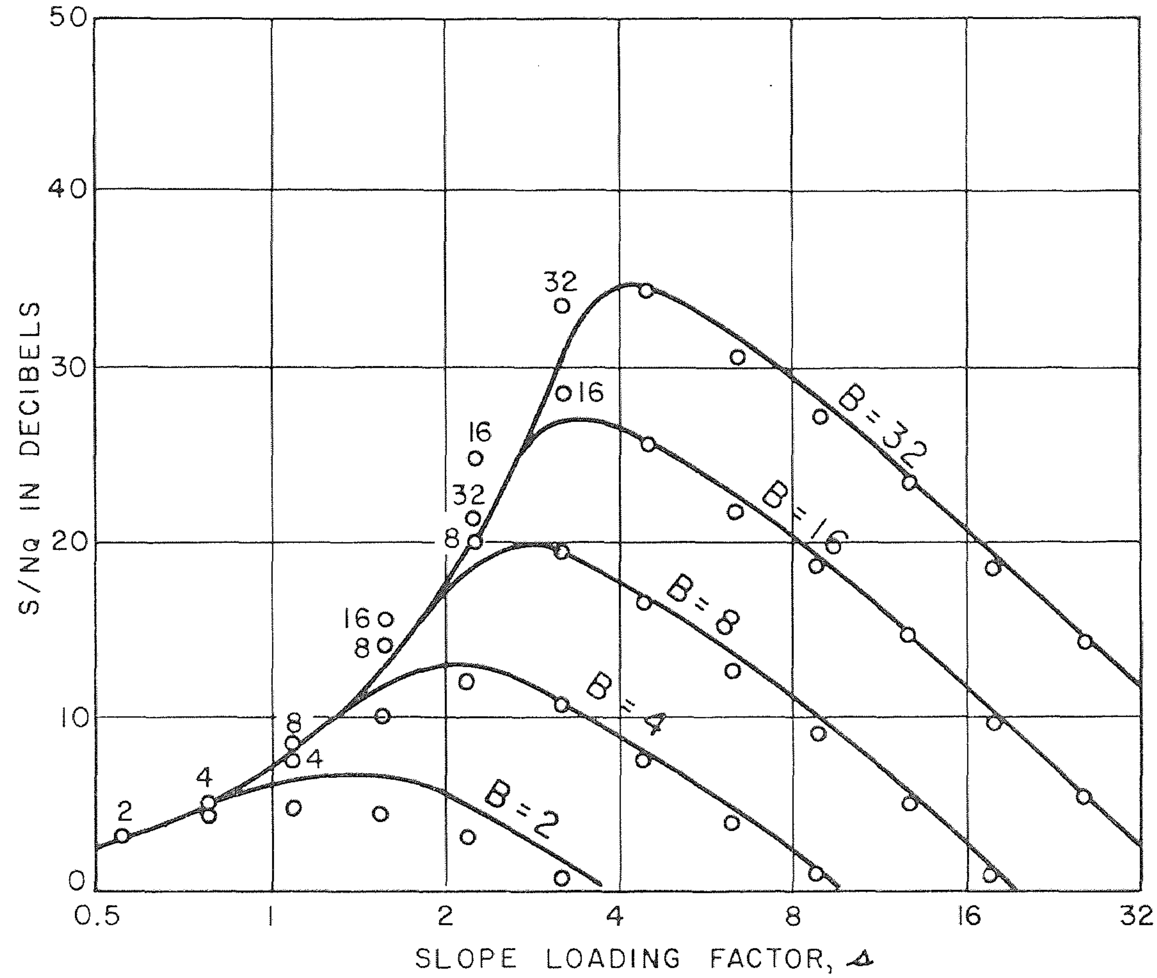


FIG. 3-3 S/NQ PERFORMANCE OF LINEAR DM WITH UNIFORM SIGNAL SPECTRUM; CURVES OBTAINED FROM TABLE 3-2, POINTS FROM COMPUTER SIMULATION, GAUSSIAN SIGNAL DENSITY.

noise power given by Equation (3-8) is proportional to the derivative power. For example, to obtain the S/N_Q performance of television or speech, it is simply necessary to add 16.9 dB or 4.5 dB respectively to the S/N_Q values that appear on the ordinate scale in Figure 3-3. The slope loading factor is shown, therefore, to be a normalizing variable for describing the S/N_Q performance of linear DM. The computer points shown in Figure 3-3 were first reported by O'Neal;[32] his normalized step size can be shown to be related to the slope loading factor.

From Equations (3-8) and (3-9), the improvement in maximum S/N_Q of the integrated spectrum (e.g., television and speech signals) relative to the uniform spectrum (e.g., broadband signal), expressed in decibels, is given by

$$\left\{\begin{array}{l}\text{Maximum } S/N_Q \text{ Improvement} \\ \text{of Integrated Spectrum} \\ \text{Relative to Uniform} \\ \text{Spectrum (in decibels)}\end{array}\right\} = \left[10 \log_{10}\left(\frac{\omega_m}{\omega_3}\right) - 4.8 - 10 \log_{10}\left(\frac{1}{\tan^{-1}\frac{\omega_m}{\omega_3}} - \frac{\omega_3}{\omega_m}\right)\right] \tag{3-10}$$

Applied to the cases of television and speech, Equation (3-10) is given in Table 3-3.

For a large class of signals, the ratio (ω_3/ω_m) is much less than unity. Television and facsimile signals, for example, are members of this class. As a result, Equation (3-10) can be reduced to

$$\left\{\begin{array}{l}\text{Maximum } S/N_Q \text{ Improvement}\\ \text{of Integrated Spectrum}\\ \text{Relative to Uniform}\\ \text{Spectrum (in decibels)}\end{array}\right\} = \left[10 \log_{10} \frac{\omega_m}{\omega_3} - 2.8\right] \qquad (3\text{-}11)$$

Equations (3-10) and (3-11) are illustrated in Figure 3-4 along with points obtained by computer simulation.

Before leaving the subject of linear DM, it may be interesting to consider one digression, namely, exploring the possibility that integrating the input signal could perhaps improve DM performance. That this is in fact not the case will be seen from the following example. Given an input signal having a uniform spectrum, it is desired to determine what performance can be expected from DM if the signal is integrated prior to encoding and differentiated after decoding. The rationale for

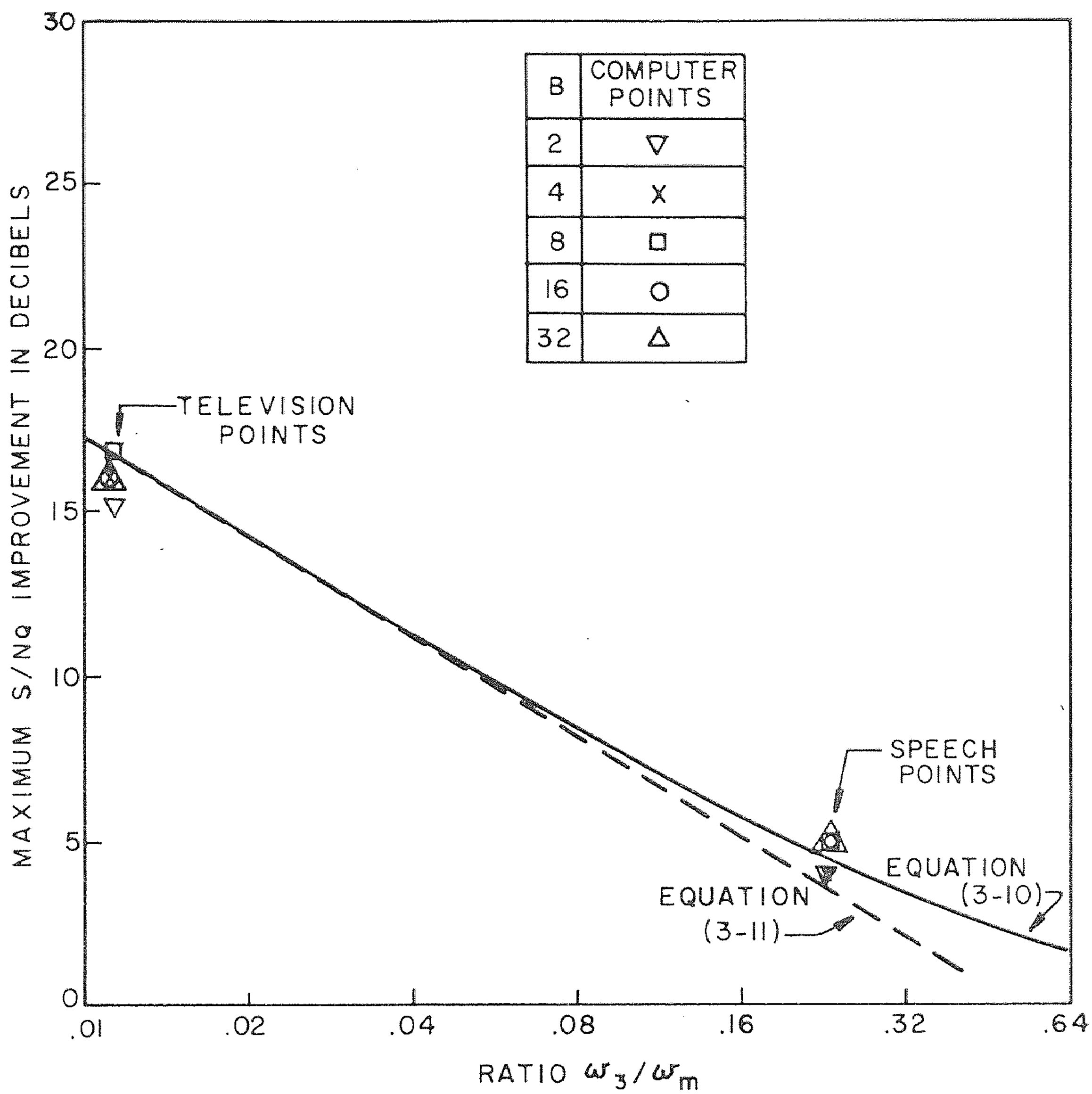


FIG. 3-4 MAXIMUM S/NQ IMPROVEMENT OF INTEGRATED SPECTRUM RELATIVE TO UNIFORM SPECTRUM

such filtering might be that in slope limiting the input signal, the DM system yields a lower value for minimum noise than if the original uniform spectrum were encoded. The falacy with such logic is that the additional noise produced by the differentiation process at the decoder output compensates for noise reduction through signal integration. The proof of this statement is arrived at directly through the use of the relationships for minimum quantization noise power in the cases of uniform and integrated signal spectra. If the original uniform spectrum signal is integrated with a network having a transfer response such that the power spectrum density at the output of the network becomes that of the integrated spectrum; and if the DM system step size is adjusted such that the quantization noise power is minimized, and given by Equation (3-8), then the minimum quantization noise power is less than that which would have resulted had the original uniform spectrum signal been encoded. The noise reduction can be expressed by the ratio of the minimum quantization noise obtained with an integrated spectrum to that obtained with a uniform spectrum, or

$$\frac{\text{Minimum } N_Q \text{ (Integrated Spectrum)}}{\text{Minimum } N_Q \text{ (Uniform Spectrum)}} = 3\left[\frac{\frac{\omega_3}{\omega_m}}{\tan^{-1}\frac{\omega_m}{\omega_3}} - \left(\frac{\omega_3}{\omega_m}\right)^2\right] \tag{3-12}$$

At the output of the DM decoder, a differentiator network (i.e., the inverse of that which integrated the original uniform spectrum signal) processes both the decoded signal and quantization noise. As a result, the mean power of both is increased. The ratio of the S/N at the differentiator output to the S/N at its input is given by

$$\frac{S/N\begin{pmatrix}\text{Differentiator}\\ \text{output}\end{pmatrix}}{S/N\begin{pmatrix}\text{Differentiator}\\ \text{input}\end{pmatrix}} = \frac{1}{\frac{\omega_3}{\omega_m}\left(\tan^{-1}\frac{\omega_m}{\omega_3}\right)\left[1+\frac{1}{3}\left(\frac{\omega_m}{\omega_3}\right)^2\right]} \tag{3-13}$$

Then, by combining Equations (3-12) and (3-13), the ratio of the differentiator output maximum S/N_Q to the maximum S/N_Q realizable with a uniform signal spectrum becomes

$$\frac{\text{Maximum } S/N_Q\begin{pmatrix}\text{Differentiator}\\ \text{output}\end{pmatrix}}{\text{Maximum } S/N_Q\begin{pmatrix}\text{Uniform}\\ \text{Spectrum}\end{pmatrix}}$$

$$= \frac{1}{1-\left(\frac{\omega_3}{\omega_m}\right)\tan^{-1}\frac{\omega_m}{\omega_3}\left[1+\left(\frac{\omega_3}{\omega_m}\right)^2\right]+3\left(\frac{\omega_3}{\omega_m}\right)^2} \tag{3-14}$$

Equation (3-14) shows that at the differentiator output, the DM performance approaches that of the case of the uniform spectrum. Thus, no significant performance improvement is gained through the use of an integration performed on the input uniform spectrum signal. This is not to say, however, that such networks are useless. Their effect in the DM system is clearly one of changing the spectrum characteristics of the quantization noise. In the example above, the differentiator at the decoder output has the effect of increasing the power spectrum of the noise at high frequencies. For some applications, such as television, this can be advantageous since the sensitivity of the human eye to random noise decreases with increasing frequency. In general, it can be stated that although signal spectrum shaping prior to delta encoding and complimentary reshaping after decoding can accomplish a net effect of shaping the noise power spectrum, it cannot produce for a uniform signal spectrum a significant performance improvement.

3.4 Discussion of Results

In this section, it has been shown that the granular, overload, and minimum quantization noise powers of linear DM can be described by simple closed form solutions. As a result, it is possible to predict with a simple expression the optimum performance obtainable by DM at various values of the bandwidth expansion factor. A defined quantity called the slope loading factor has been shown to be a useful parameter in characterizing DM performance. It has been shown that minimum quantization noise power is proportional to the mean power of the signal derivative. As a result, S/N_Q performance with an integrated spectrum such as television or speech exceeds that of a broadband (i.e., uniform spectrum) signal. Furthermore, it has been found that S/N_Q performance with a signal having a Gaussian density is approximately the same as that obtained with a signal having an exponential density.

It has been shown that the slope loading factor is a normalizing variable when used to describe S/N_Q performance. That is, the S/N_Q performance characteristic curves for broadband, television, and speech signals are identical in form, the only difference between them being one of the magnitude of the ordinate scale.

Unfortunately, in the linear DM system the quantization noise is sensitive to small changes in the mean power of the signal. As a result, the range of Δ over which S/N_Q is near maximum is small. From Equation (3-2) it is clear that a change in signal power produces a change in slope loading factor Δ. If Δ is substantially different in value from that given by Equation (3-6), then the value of N_Q will be greater than the minimum value and the DM system is suboptimum. As an example, for the case of B = 8 in Figure (3-3) if the quantization noise power is to be held to less than twice its minimum value (i.e., $S/N_Q \geq 17$ db), the slope loading factor must be constrained such that $2 < \Delta < 4$. This in turn requires that the effective value of the signal must be constrained to a variation of less than approximately ±40 percent. This is indeed a severe restriction for signals of practical importance such as television and speech. Forcing the DM system to respond adaptively to changes in the input signal by changing the slope loading factor with time, overcomes the restriction of a narrow optimum performance range. This adaptation of linear DM will be the subject of the next section.

4. ADAPTIVE DM, A QUALITATIVE DISCUSSION

It has been shown in Section Three that DM system performance is a function of the slope loading and bandwidth expansion factors. For any specified sampling rate, the total quantization noise reaches a minimum at a particular value of the slope loading factor. For any sampling rate then, there exists some value of step size k such that for a given signal spectrum, the ratio of signal power to quantization noise power is a maximum. Implicit in the above statements, is the constraint that the signal mean power and spectrum density are stationary with time. Unfortunately large and important classes of stochastic communication signals processed today are either nonstationary or at best only short term stationary. Two examples of such signals are television and speech.

In order to give the DM system the capability of encoding nonstationary signals in an optimal way, the restraint that exists in linear delta (i.e., that slope loading factor is fixed) must be removed. That is, the system should be permitted to become self-regulating or adaptive so that optimum performance (i.e., maximum S/N_Q) is achieved over a broad range of input signal variation. If the signal is stationary, then the

DM system is optimally loaded when the slope loading factor is made to satisfy Equation (3-6). If it is nonstationary, the DM system will be optimally loaded if and only if the slope loading factor is changed in accordance with the changing signal parameter. The objective of the adaptive DM system discussed herein is to maintain optimal loading and performance (i.e., maximum S/N_Q) by controlling the value of the slope loading factor. Since the sampling rate is assumed constant for a given system, it is clear from Equation (3-2) that by controlling the step size, the slope loading factor may be assigned any specified value.

The problem is to decide how to measure the nonstationary of the signal, and hence, the changing slope loading factor. That is, what measurement should be made and how should it be accomplished so that signal variations can bring about a reassignment of the value of k. Undoubtedly there are many approaches to this problem. In this work, a solution that appears promising is presented. It involves monitoring the instantaneous derivative of the encoded signal, determining if the condition specified by Equation (3-1) is satisfied, and changing the step size if necessary in a discrete manner to prevent slope overload.

Essentially, there can be both a discrete and a continuous method of adapting the system to changes in the signal derivative. The former observes the binary pulse sequence at the quantizer output and changes the step size in finite increments. The latter observes the continuous input signal and changes the step in a continuous manner. The former method will be called "discrete adaptive DM" and is illustrated in Figure 4-1. The latter method will be called "continuous adaptive DM" and is illustrated in Figure 4-2. In this work, only the discrete adaptive system is quantitatively discussed. Brown and Brolin[6] have discussed a system similar to the continuous adaptive DM system for speech application.

In the discrete adaptive system, the switch control chooses, in effect, a gain K_i by which to increase the quantum step size. The choice made by the control is dictated by a logical decision process based on observations of the sequence of pulses leaving the quantizer. For example, when slope overload occurs, causing suboptimum performance, the quantizer output is a series of pulses of the same polarity (i.e., a series of plus one's or minus one's). In response to this series of consecutive pulses, the switch control

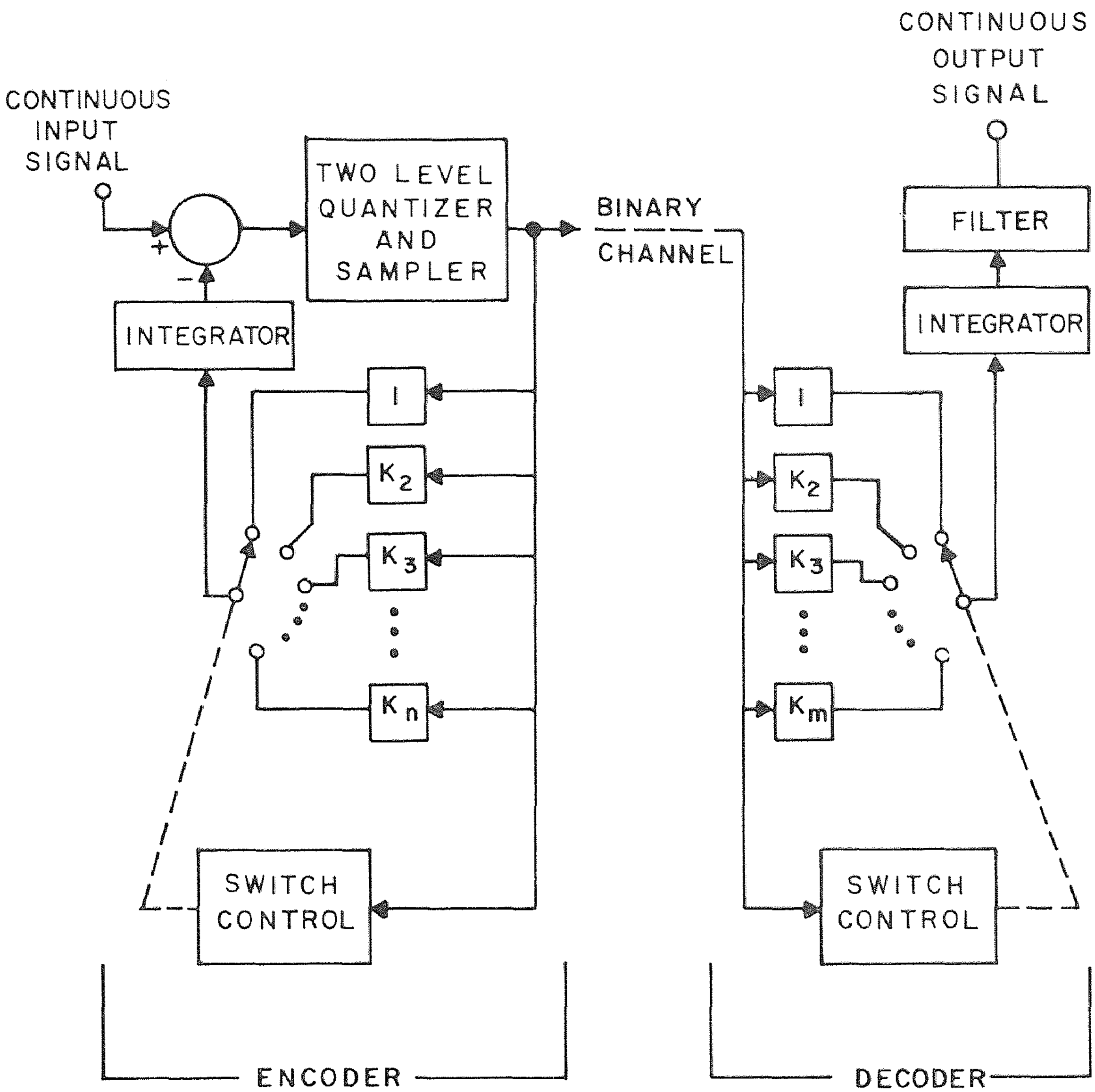


FIG. 4-1 DISCRETE ADAPTIVE DM SYSTEM.

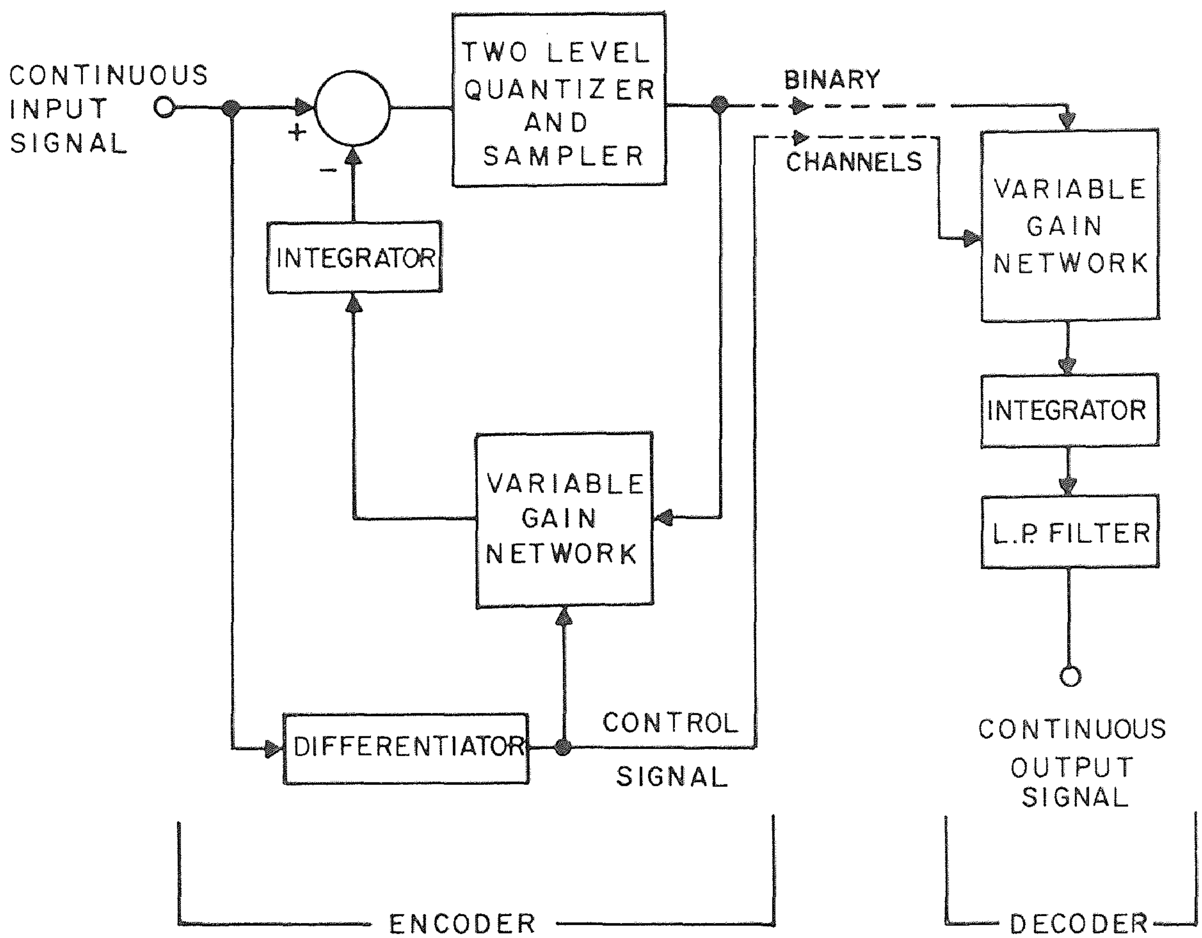


FIG. 4-2 CONTINUOUS ADAPTIVE DM SYSTEM

selects a gain K_i greater than K_{i-1}, such that the new larger step size is K_i multiplied by the smallest step size k, or simply $K_i k$. If the pulse polarity remains unchanged, the step size is incrementally increased to $K_{i+1}k$, $K_{i+2}k$, etc., until the largest value of $K_n k$ is reached. The step size incrementally decreases when polarity reversals occur. In the decoder, the same pulse sequences are sensed by a switch control identical to that in the encoder, and thus the step size changes are made synchronously and identically. Since the step size is changed at a rate equal to that of the sampling rate, the discrete adaptive DM system may be viewed as a linear DM into which instantaneous companding has been introduced.

Figure 4-3 illustrates possible waveforms of the discrete adaptive system. Note that from sampling intervals 1 through 9 inclusive, there are never more than two consecutive pulses of the same polarity; hence no slope overload. But at the 10th interval, a pulse of the same polarity as the previous two intervals appears indicating the beginning of overload. Detecting this condition, the control switches to the K_2 position making the new step size in the 10th interval equal to $K_2 k$. Similarly, the 11th interval step size is increased to $K_3 k$, where $K_3 > K_2$. At the 12th interval,

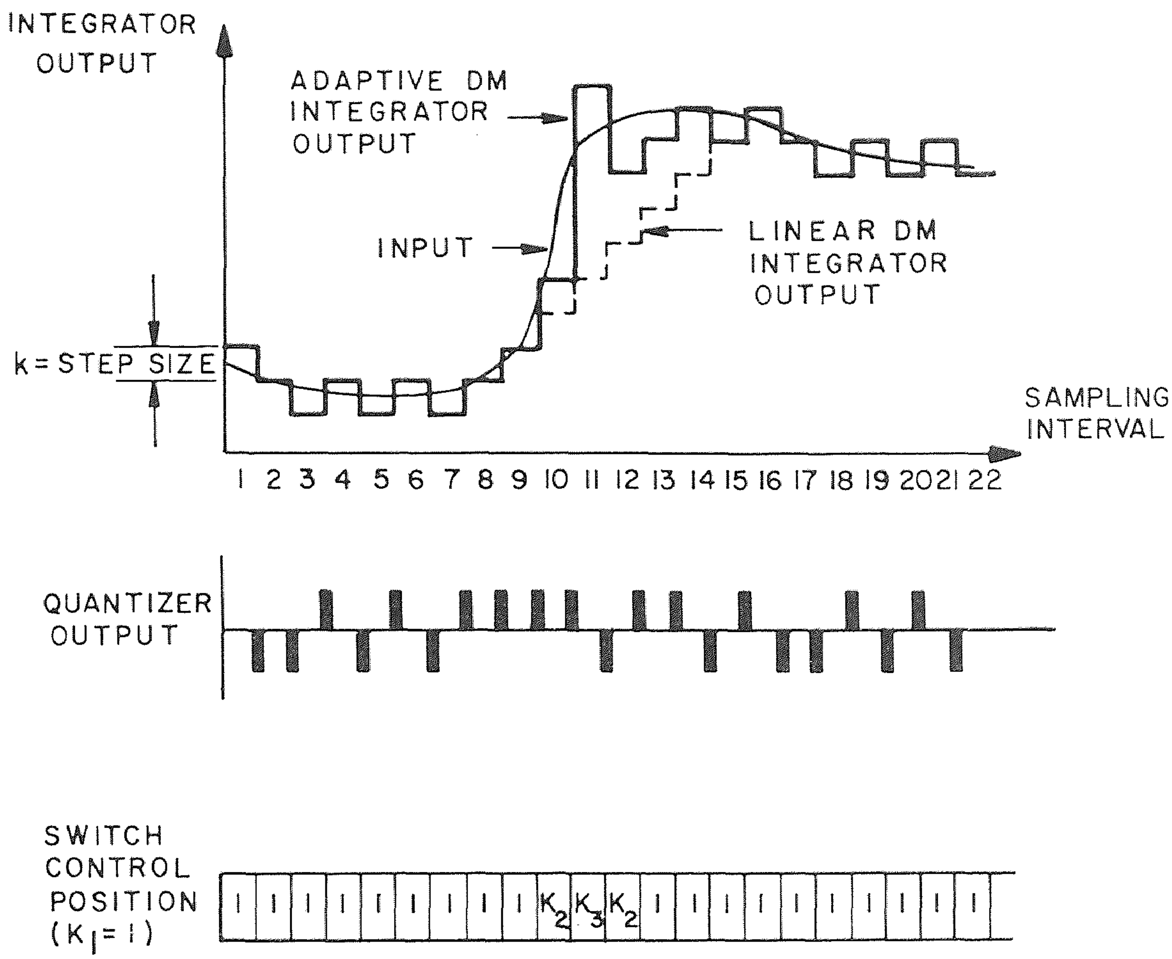


FIG. 4-3 WAVEFORMS OF DISCRETE ADAPTIVE DM SYSTEM.

the polarity reverses and the step size decreases to K_2k. Similarly, the polarity reverses again at the 13th interval and the smallest step size k is reached. From the 14th on, the pulse sequence indicates no slope overload. The dotted line illustrates the overload of a linear delta system.

Because the discrete adaptive system is able to change its step size as a function of the pulse sequence, it is thus capable of modifying its overload noise performance. As a result, the range over which it produces optimum performance is expanded, as shown in Figure 4-4. The amount and character of this expansion will be part of the subject of the quantitative discussion given in Section Five.

In the continuous adaptive system illustrated in Figure 4-2, the control signal is the continuous derivative of the input signal. Because the control signal must occupy some of the transmission channel frequency space, it must of necessity require only a fraction of the input signal bandwidth. As a result, the rate at which the step size is varied is very much smaller than the sampling rate. Thus the continuous adaptive DM system can be considered as the equivalent of a linear DM into which syllabic companding has been introduced.

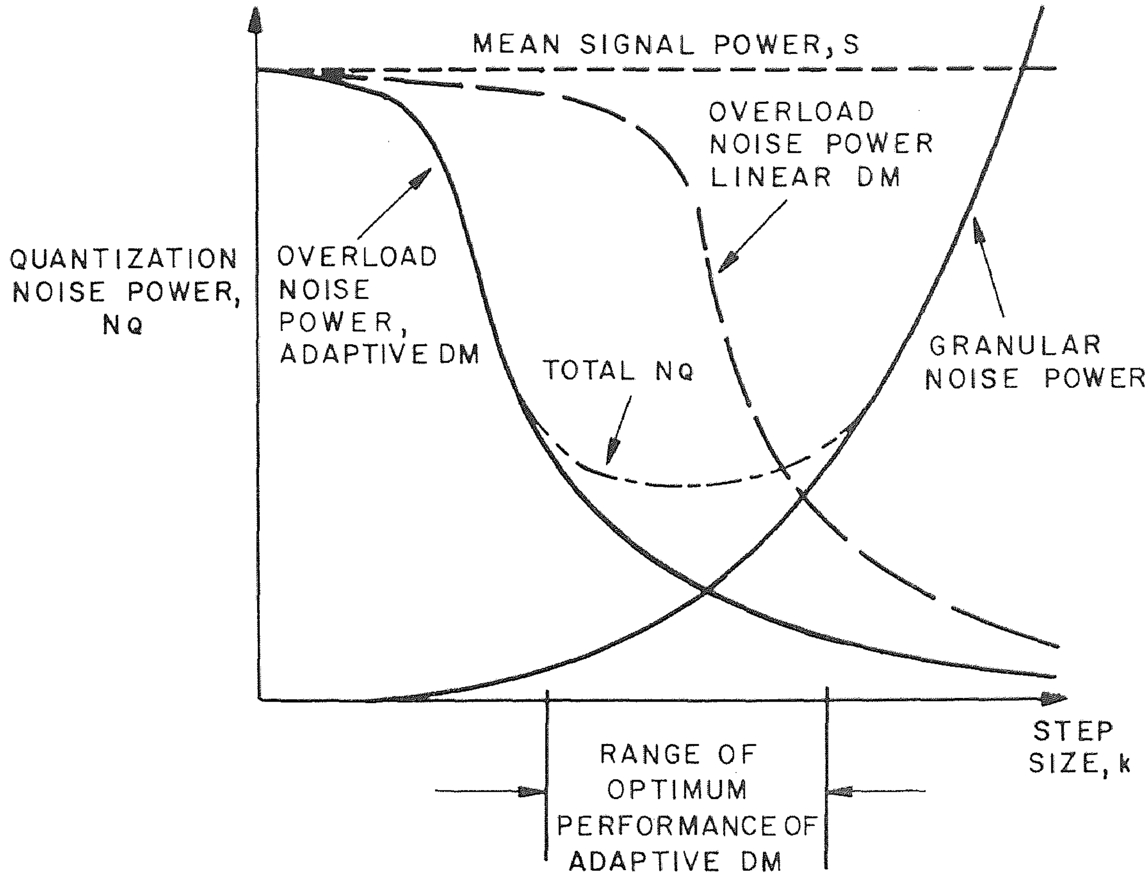


FIG. 4-4 DISCRETE ADAPTIVE DM QUANTIZATION NOISE POWER

Several configurations similar to those illustrated in Figures 4-1 and 4-2 appear in the literature.[6,12,40,42,43] Brown and Brolin[6] have quantitatively discussed a continuous adaptive DM system for speech application. Winkler[42,43] has given a qualitative description of a special case similar to that of discrete adaptive DM.

In Section Five, a quantitative account of the performance characteristics of discrete adaptive DM with television, speech, and broadband signals will be given. In Section Six, a quantitative comparison of the performances of linear DM, discrete adaptive DM, and PCM will be made.

5. DISCRETE ADAPTIVE DM

5.1 Normalized Slope Loading Factor Defined

Because the discrete adaptive DM system is able to increase its step size in an instantaneous manner at the sampling rate from the smallest value k to $K_2k, \ldots, K_nk$ in sequential increments, slope overload is not the controlling degradation until the derivative of the signal $f'(t)$ is greater than the maximum slope capability of the system, that is when

$$|f'(t)| > K_n k f_s . \qquad (5\text{-}1)$$

As a result, the maximum value of the slope loading factor for adaptive DM is greater than that given by Equation (3-2) for linear DM by the factor K_n, and is therefore

$$\text{maximum } \Delta \text{ (adaptive DM)} = \frac{K_n k f_s}{\sqrt{D}} . \qquad (5\text{-}2)$$

It is somewhat more convenient, for purposes of comparison with linear DM, to use a slope loading factor definition consistent with that of Equation (3-2). We therefore define what will be called the "normalized slope loading factor" (Δ') for adaptive DM. It is given by

$$\Delta' \equiv \frac{k' f_s}{\sqrt{D}} \tag{5-3}$$

where k′ is the product of K_n and k. The normalized slope loading factor thus has a value at each sampling instant given by one member of the sequence

$$\frac{1}{K_n}\Delta', \frac{K_2}{K_n}\Delta', \ldots, \frac{K_{n-1}}{K_n}\Delta', \Delta'.$$

That is, when the instantaneous derivative of the signal is and remains very small, the normalized slope loading factor value becomes

$$\frac{kf_s}{\sqrt{D}} ;$$

and when the derivative is and remains very large, the normalized slope loading factor value becomes

$$\frac{K_n k f_s}{\sqrt{D}} .$$

5.2 Quantization Noise Power

It is shown in Appendix B that the asymptotic bounds for discrete adaptive DM overload noise power N_O', granular noise power, N_G', and minimum quantization noise power, minimum N_Q, are given by

$$N_O' = \frac{8\pi^2}{27}\left(\frac{D}{\omega_m^2}\right)e^{-3s'}(3s'+1), \qquad \text{for} \quad s' < \ln 2B \tag{5-4}$$

$$N_G' = \frac{\pi^2}{6B^3K_n^2}\left(\frac{D}{\omega_m^2}\right)\left(s'\right)^2,$$

$$\text{for} \quad \begin{cases} s' < 8\,K_n \\ s' > K_n\sqrt{(\ln B)^2 + 2.06\ln B + 1.17} \end{cases} \tag{5-5}$$

$$N_G' = \frac{\pi^2}{48B^3K_n^3}\left(\frac{D}{\omega_m^2}\right)\left(s'\right)^3, \qquad \text{for} \quad s' > 8K_n \tag{5-6}$$

Minimum N_Q

$$= \frac{\pi^2}{6}\left(\frac{D}{\omega_m^2}\right)\left[\frac{(\ln B)^2 + 2.06\ln B + 1.17}{B^3}\right],$$

$$\text{for} \quad \begin{cases} s' > \ln 2B \\ s' < K_n\sqrt{(\ln B)^2 + 2.06\ln B + 1.17} \end{cases} \tag{5-7}$$

Equation (5-4) applies in the region of slope overload, that is the region defined by values of the slope loading factor which are less than that value representing the optimum value given by Equation (3-6), or $\Delta' < (\ln 2B)$. Equation (5-5) applies for values of Δ' greater than that obtained when Equations (5-5) and (5-7) are set equal, and less than $8K_n$. The former of these bounds states, in effect, that granular noise power must be equal to (or greater than) the minimum total quantization noise power given by Equation (5-7). The latter bound contains the factor K_n as a consequence of slope loading factor normalization.

Because the maximum value of the slope loading factor is given by Equation (5-2), and since Equations (3-2) and (5-3) are equivalent except for a change of variable, the asymptotic lower bound for adaptive DM overload noise power is the same as that for linear DM given by Equation (3-7) in which Δ is replaced by Δ'.

Since granular noise power has been decreased relative to that of linear DM by the factor $1/K_n^2$ as shown in Equations (5-5) and (5-6), and since it is subject to the constraint imposed by Equation (5-7), then the range of normalized slope loading factor over which quantization noise power is minimum has been extended. In other words, discrete adaptive DM does not produce optimum performance at only one particular value of the slope loading factor as is the case with linear DM but extends the range of optimum performance from that value given by Equation (3-6) to that value obtained when granular noise power $N_G(\Delta < 8K_n)$ is set equal to the minimum value of quantization noise power. As a result, adaptive DM performs what may be considered a companding operation, that is, it extends the useful performance range of the linear DM system.

Companding in a quantizing system refers to the process of signal compression and later expansion, the former in the encoder and the latter in the decoder.[39] The purpose of companding is to allow weak signals (i.e., small signal power) to be encoded with approximately the same quantizing noise as strong signals (i.e., large signal power). In PCM, companding can be obtained by using a nonuniform quantizer. In the discrete adaptive DM system, companding is thus achieved by changing the size of the quantum step in sequential increments. A quantitative comparison of adaptive DM companding with PCM logarithmic companding will be given in Section Six.

5.3 Selection of Final and Intermediate Gain Factors

An important problem in discrete adaptive DM is the selection of the final gain factor K_n. It is clear from Equations (5-5), (5-6), (3-4), and (3-5), that the amount of signal power variation that the adaptive system tolerates before performance falls substantially below that of maximum S/N_Q has been increased by the factor $[K_n^2]$. In the communication literature,[39] an increase of tolerable signal power variation without performance degradation has been referred to as companding improvement or simply the amount of companding, and is usually expressed in decibels. For discrete adaptive DM, the approximate companding improvement C expressed in decibels becomes

$$C = (20 \log K_n) \tag{5-8}$$

At $K_n = 1$, the special case of linear DM results and optimum performance occurs at only one value of mean signal power, or in other words one value of the slope loading factor [i.e., that value given by Equation (3-6)].

If the power of a given message signal varies from some smallest value S_1 to some largest value S_2, it is a simple matter to select the appropriate values of step size k and multiplier K_n to achieve the desired companding. From Equations (3-2) and (3-6), it is clear that the step size should be

$$k = \left(\frac{\sqrt{D}}{f_s} \ln 2B\right)\sqrt{S_1} \tag{5-9}$$

where D is the derivative power calculated on the basis of unity mean signal power, and $\sqrt{S_1}$ is the smallest standard deviation of the signal. Combining Equations (5-9) and (2-3), the adaptive DM encoder step size thus becomes

$$k = \pi\sqrt{\frac{D}{\omega_m^2}}\left(\frac{\ln 2B}{B}\right)\sqrt{S_1} \quad . \tag{5-10}$$

The gain multiplier K_n is simply the ratio of the standard deviations of the largest and smallest values of signal power, or

$$K_n = \sqrt{\frac{S_2}{S_1}} \quad . \tag{5-11}$$

Another problem in discrete adaptive DM is the selection of intermediate gain factors $K_2, K_3, \ldots, K_{n-1}$. The choice of final gain factor K_n is dictated by the amount of desired companding as discussed above. The effect of intermediate gain factors on S/N_Q performance was investigated by computer simulation, and typical results are illustrated in Figure 5-1, where $K_n = 4$ (i.e., the largest step size is four times that of the smallest step). Three cases are illustrated. Case I represents a two level (i.e., n = 2) adaptive system, that is a sequence of two consecutive pulses of the same sign causes the step size to increase from the smallest value k to its largest value $K_2k = 4k$, with no intermediate values. The performance of this method falls considerably below the predicted asymptotes illustrated. Case II represents exponential gain factor increments, that is $K_i = 2^{i-1}$, and is a three level adaptive system (i.e., n = 3). The sequence $k, K_2k, \ldots, K_nk$ becomes k,2k,4k. Although the results of Case II are significantly better than those of Case I, they still are somewhat less than expected.

Case III in Figure 5-1 represents linear gain factor increments, that is $K_i = i$, and is in this instance a four level adaptive system (i.e., n = 4). The results using linear increments show approximately a three decibel increase over exponential increments in companding improvement near maximum S/N_Q, and are closer to the asymptotes predicted by Equations (5-4) through (5-7). Computer simulation results using

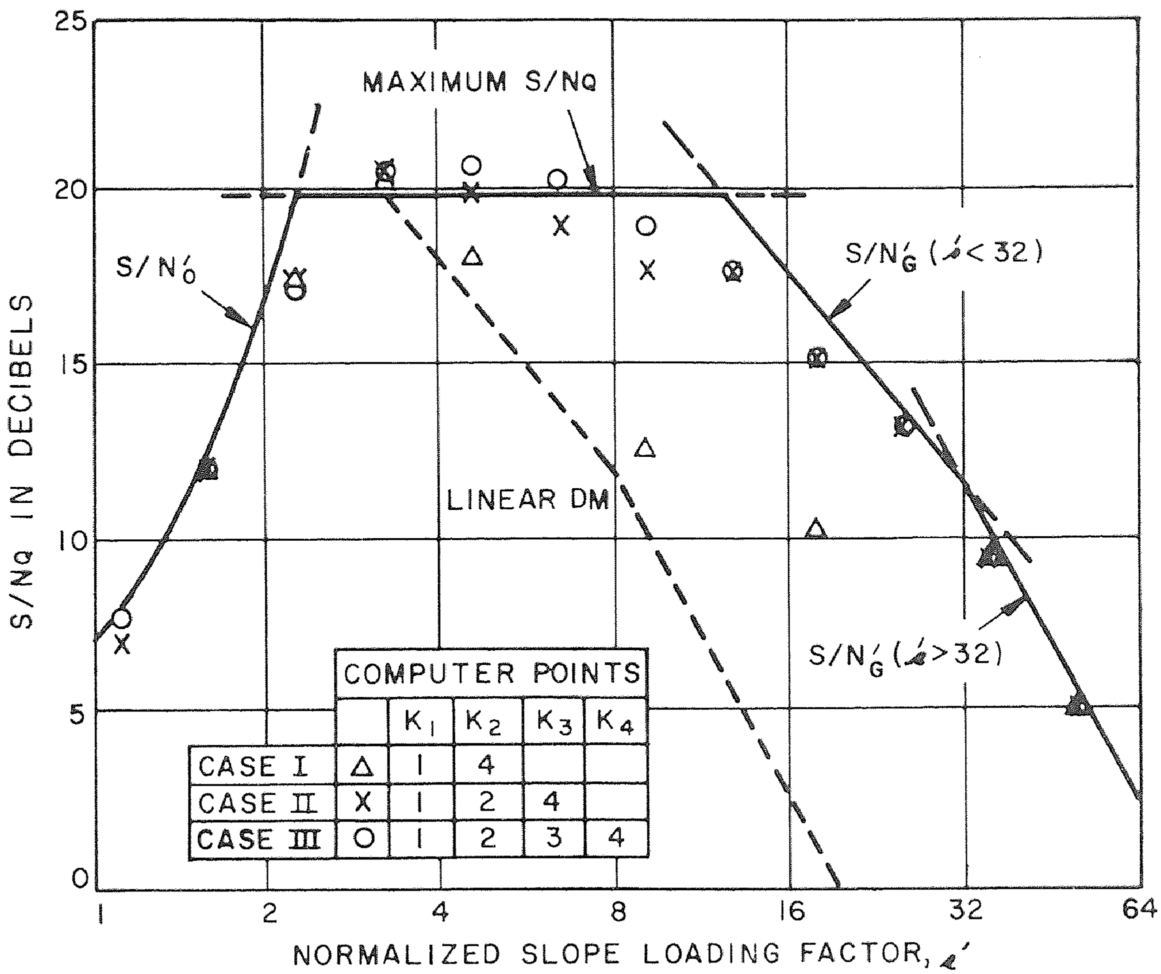


FIG. 5-1 S/Nq PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH UNIFORM SIGNAL SPECTRUM, B=8, K_n=4, AND VARIOUS COMPUTER SIMULATED VALUES OF K_i, GAUSSIAN DISTRIBUTION.

linear increments as in Case III will be given in the applications to follow.

5.4 Application to Television, Speech, and Broadband Signals

5.4.1 General

In this section, the application of the results of Sections 5.1, 5.2, and 5.3 to television, speech, and broadband signals will be given along with several numerical examples illustrating the performance of adaptive DM as a function of the normalized slope loading factor, using the gain multiplier K_n as a system variable. Table 5-1 summerizes the parameters that will be used to illustrate the adaptive DM system performance. In the illustrations to follow, bandwidth expansion factor values of four and eight will be used; other values of course can be substituted into the expressions given in Table 5-1.

5.4.2 Television Signal

Figures 5-2 and 5-3 illustrate television signal performance with a value of eight for the bandwidth expansion factor. Computer simulation points are illustrated with linear gain factor increments, and with both exponential and Gaussian signal densities given for comparison. This comparison is an important

TABLE 5-1

Adaptive DM Performance With

Television, Speech, and Broadband Signals

Parameter	From Equation	Television	Speech	Broadband
δ'	(5-3)	$3.8\ BK_n k$	$0.93\ BK_n k$	$0.55\ BK_n k$
$\frac{S}{N_O'}$ (in Decibels)	(5-4)	$[16.9 + 13\delta' - 10 \log_{10}(3\delta'+1)]$	$[4.6 + 13\delta' - 10 \log_{10}(3\delta'+1)]$	$[0.1 + 13.0\delta' - 10 \log_{10}(3\delta'+1)]$
$\frac{S}{N_G}$, $\delta' < 8\ K_n$ (in Decibels)	(5-5)	$[19.4 + 30 \log_{10}B + 20 \log_{10}K_n - 20 \log_{10}\delta']$	$[7.1 + 30 \log_{10}B + 20 \log_{10}K_n - 20 \log_{10}\delta']$	$[2.6 + 30 \log_{10}B + 20 \log_{10}K_n - 20 \log_{10}\delta']$
$\frac{S}{N_G}$, $\delta' > 8\ K_n$ (in Decibels)	(5-6)	$[28.4 + 30 \log_{10}B + 30 \log_{10}K_n - 30 \log_{10}\delta']$	$[16.2 + 30 \log_{10}B + 30 \log_{10}K_n - 30 \log_{10}\delta']$	$[11.6 + 30 \log_{10}B + 30 \log_{10}K_n - 30 \log_{10}\delta']$
k	(5-10)	$0.26\left(\frac{\ln 2B}{B}\right)\sqrt{S_1}$	$1.1\left(\frac{\ln 2B}{B}\right)\sqrt{S_1}$	$1.8\left(\frac{\ln 2B}{B}\right)\sqrt{S_1}$

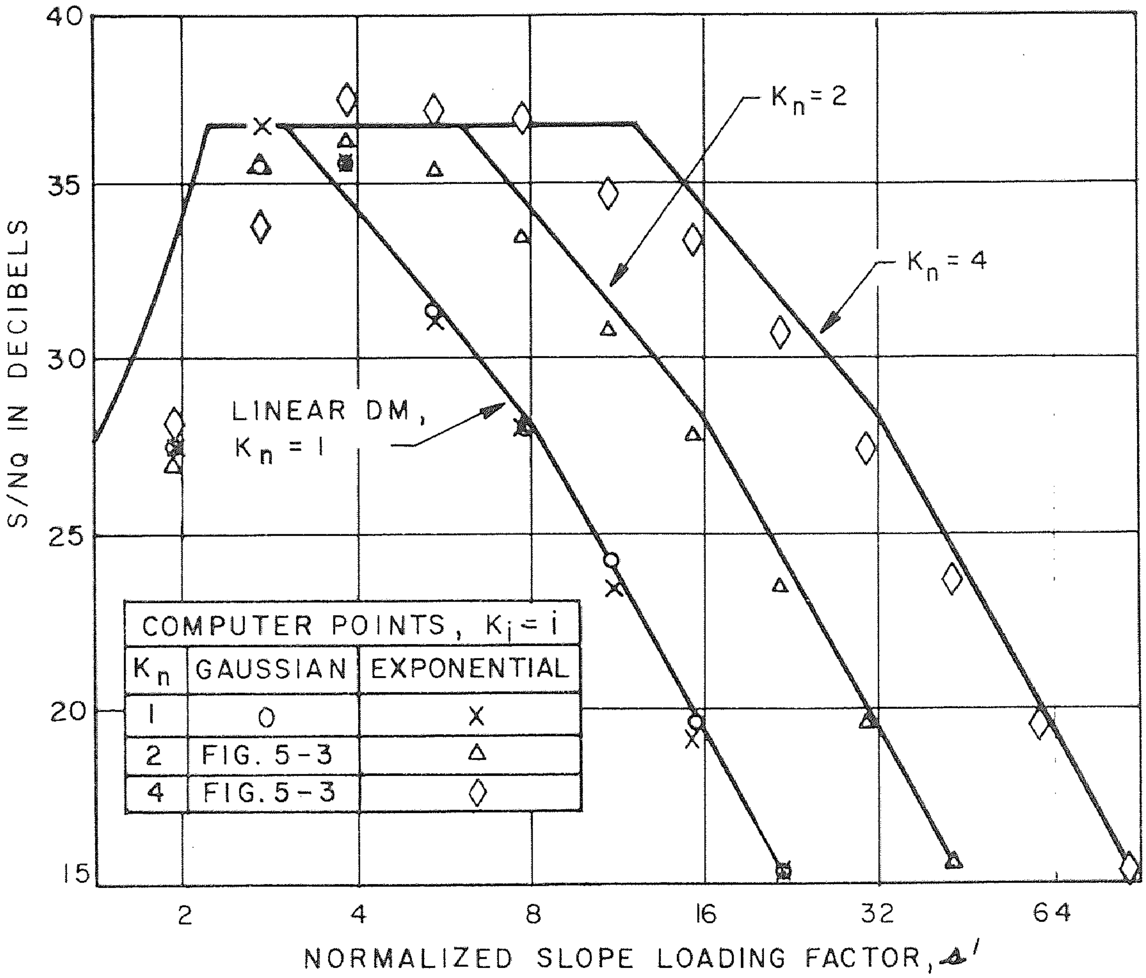


FIG. 5-2 S/NQ PERFORMANCE OF DISCRETE ADAPTIVE DM, TELEVISION SIGNAL SPECTRUM, B=8.

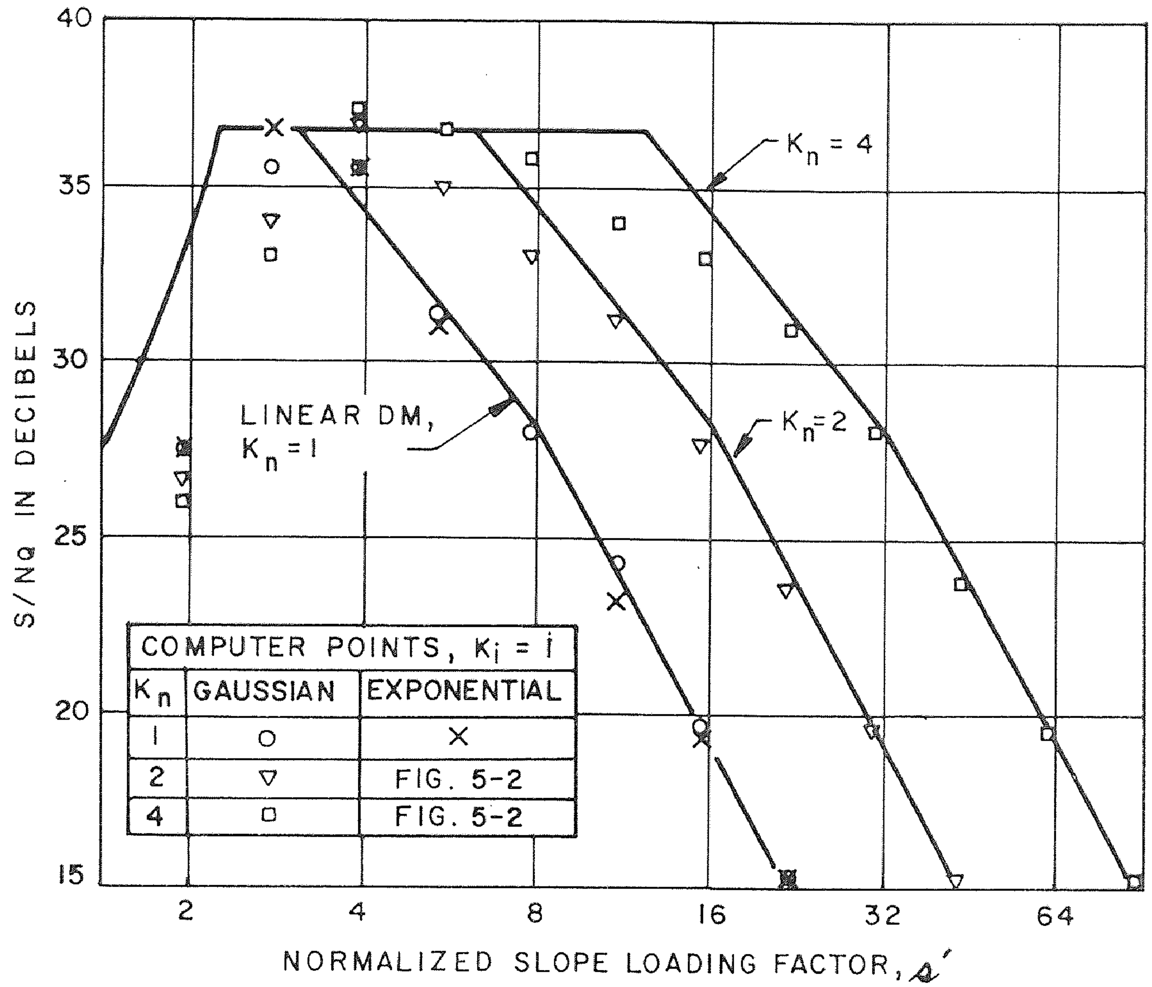


FIG. 5-3 S/Nq PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH TELEVISION SIGNAL SPECTRUM, B = 8.

one since the statistics of television signals are rarely stationary, as the studies of O'Neal[33] and Kretzmer[22] indicate. The computer results show that the performance of adaptive DM with an exponential signal density is essentially the same as that with a Gaussian density.

The granular noise power asymptotes illustrated in Figures 5-2 and 5-3 are given in Table 5-2. The asymptote for overload noise power is given in Table 5-1; that for maximum S/N_Q is determined from Equation (5-7) as 37 decibels for the case $B = 8$.

The power of a video signal varies considerably from line to line in a raster scanned field as well as from picture to picture over long periods of time. Since one would like to make K_n as large as possible to encompass as many different picture types as possible, but since equipment complexity increases as K_n increases, a reasonable compromise can be obtained by letting $K_n = 4$. This value of K_n represents the ratio of the standard deviations of two video signals, the first obtained from a picture which is half black and half white, and the second obtained from the measurements of O'Neal.[33] For some applications (e.g., closed circuit television, graphics display, etc.), other values of K_n may be more

TABLE 5-2

Adaptive DM Performance With a Television Signal,

B = 8 (Illustrated in Figures 5-2 and 5-3)

	$K_n = 1$	$K_n = 2$	$K_n = 4$
$\frac{S}{N'_G}$, $\Delta' < 8 K_n$ (in Decibels)	$[46.5-20 \log_{10}\Delta']$	$[52.5-20 \log_{10}\Delta']$	$[58.5-20 \log_{10}\Delta']$
$\frac{S}{N'_G}$, $\Delta' > 8 K_n$ (in Decibels)	$[55.5-30 \log_{10}\Delta']$	$[64.5-30 \log_{10}\Delta']$	$[73.5-30 \log_{10}\Delta']$
C (in Decibels)	0	6	12

appropriate. Figures 5-2 and 5-3 illustrate values of K_n of two and four. Other values can be obtained from the expressions given in Table 5-1.

As an example of how the parametric optimization of the adaptive DM system might be completed for the television signal case, we shall use a value of four for K_n, and assume an input termination at the encoder of one ohm. The value of S_1 can be obtained from the results of O'Neal[33] who, letting the peak-to-peak composite signal voltage of a raster scanned picture be unity, computed the rms video of three scenes to be approximately 0.1 volts. Using Equation (5-10) and Table 5-1, the step size in volts would then be

$$k = 0.026\left(\frac{\ln 2B}{B}\right) \qquad (5\text{-}12)$$

For the value of B illustrated (i.e., B = 8), the step size becomes 9.0 millivolts. For entertainment television having a bandwidth of 4.5×10^6 Hertz, the required sampling rate is then 72×10^6 Hertz. In this example, the adaptive DM system would yield a maximum S/N_Q of 36 decibels, and produce a companding improvement of 12 decibels. Had the linear DM system been used for a signal whose rms value varies over the range of four to one, a decrease of at least nine, and possibly as much as thirteen decibels from maximum S/N_Q would have been obtained. Thus, the performance advantage of adaptive DM is obvious.

5.4.3 Speech Signal

Figures 5-4 and 5-5 illustrate speech signal performance with bandwidth expansion factor values of four and eight respectively. Computer simulation points are illustrated with both exponential and Gaussian signal densities given for comparison. Again the computer results show that the performances with both densities are essentially the same. Table 5-3 gives the asymptotes illustrated. The overload noise power asymptote is given in Table 5-1.

The mean power of speech varies considerably with time as well as with individual characteristics (e.g., age, sex, inflections, etc.). A detailed treatment of such considerations can be found in the work of Fletcher.[14] At best, a companded system designed to process speech is a compromise between practical and theoretical considerations. In one widely used PCM system, for both theoretical[39] and practical[26] reasons a compandor has been found useful for speech which employs a logarithmic nonuniform quantizer. In Section Six, a quantitative comparison will be made of this PCM system with that of

TABLE 5-3

Discrete Adaptive DM Performance With a Speech Signal

	Figure 5-5	Figure 5-4
	$B = 4,\ K_n = 8$	$B = 8,\ K_n = 4$
$\frac{S}{N'_G}$, $\delta' < 8\ K_n$ (in Decibels)	$[43.3-20\ \log_{10}\delta']$	$[46.3-20\ \log_{10}\delta']$
$\frac{S}{N'_G}$, $\delta' > 8\ K_n$ (in Decibels)	$[61.4-30\ \log_{10}\delta']$	$[61.3-30\ \log_{10}\delta']$
Maximum $\frac{S}{N_Q}$ (in Decibels)	17	24
C (in Decibels)	18	12

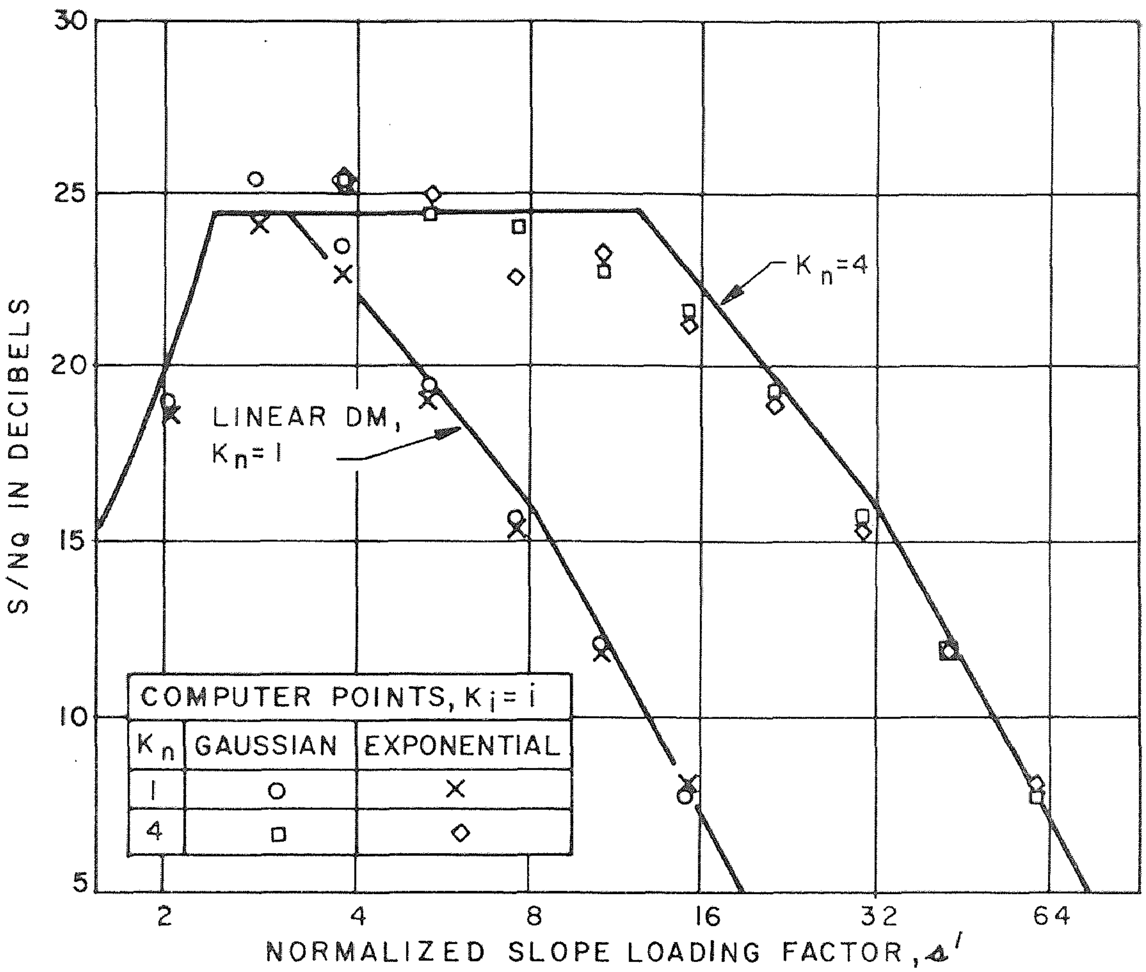


FIG. 5-4 S/NQ PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH SPEECH SIGNAL SPECTRUM, B = 8.

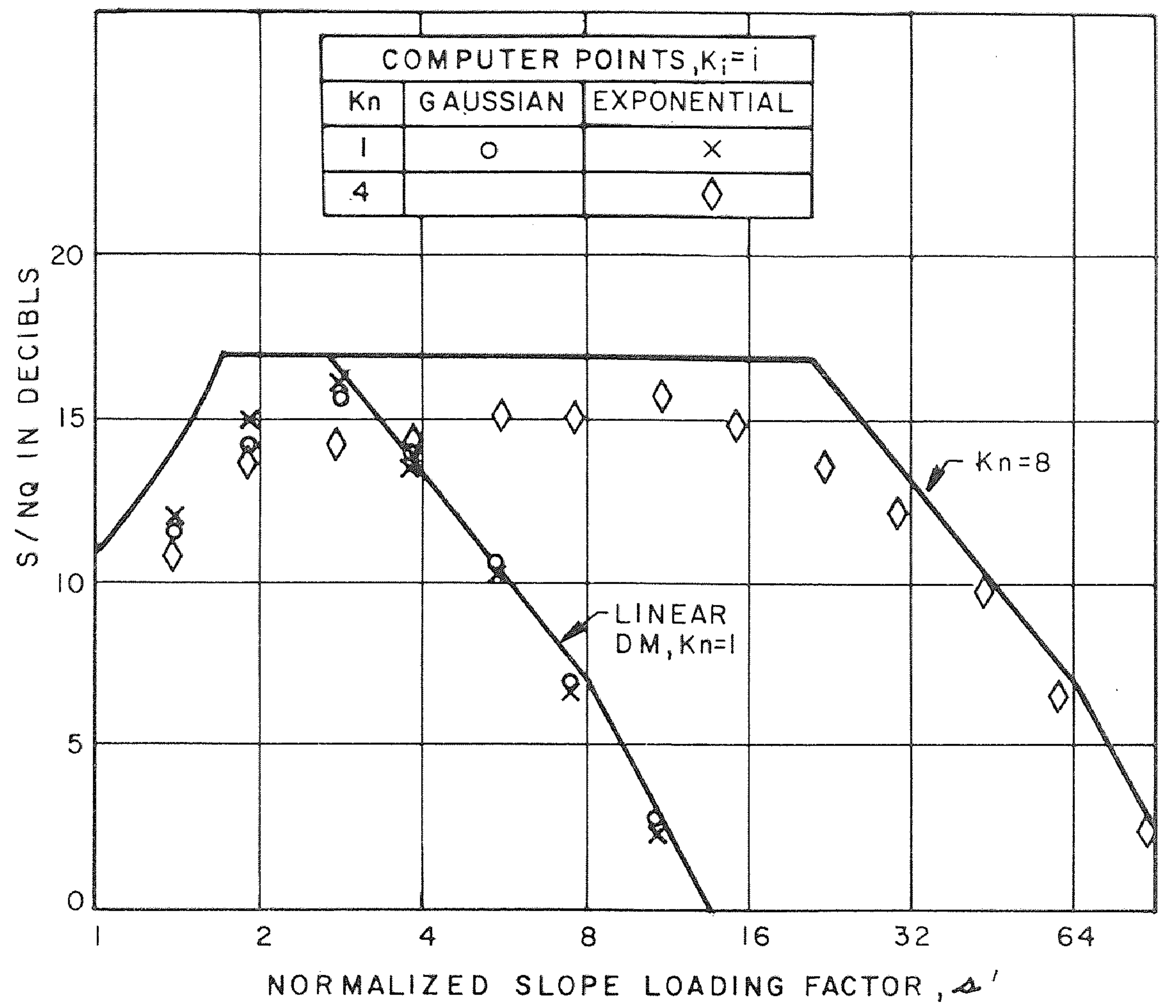


FIG. 5-5 S/NQ PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH SPEECH SIGNAL SPECTRUM, B = 4.

adaptive DM. It will suffice to say at this point that the PCM system yields a companding improvement which appears to be about the same as that of the adaptive DM system having a final gain factor K_n value of eight as illustrated in Figure 5-5. Although in the adaptive DM system, it is only necessary to increase the value of K_n to achieve a greater companding improvement, it is in practice very difficult to increase the companding improvement of PCM for reasons discussed by Mann, et al.[26] As a result, it appears that for speech application, adaptive DM may have some advantages not presently enjoyed by PCM. More will be said about comparisons with PCM in Section Six.

As an example of the optimum selection of parameter values of adaptive DM for speech application, a value of eight for K_n will be assumed. Let it be required that the quantization noise power be less than the signal power by approximately 25 decibels. From Equation (5-7) or by use of Figure 3-2, we find that a bandwidth expansion factor value of eight is needed. If we assume that S_2 is unity in Equation (5-11), then the value of step size from Equation (5-10), letting the input termination at the encoder be one ohm, becomes 0.047 volts.

5.4.4 Broadband Signal

Figures 5-6 and 5-7 illustrate broadband signal (i.e., uniform spectrum) performance, the former for $K_n = 1,2,4,8$ and the latter for $K_n = 1,16,32,64$. In general, computer results for the spectra considered show again that both Gaussian and exponential signal amplitude distributions yield substantially the same performance. For large values of K_n (i.e., $K_n \geq 16$), the results indicate that S/N_Q performance falls below that predicted by Equations (5-4) and (5-7), especially in the region $2 < s' < 8$, as shown in Figure 5-7. The companding improvement, however, for large K_n is not greatly decreased. For example, when $K_n = 64$ as in Figure 5-7, the companding improvement realized such that S/N_Q remains within three decibels of maximum S/N_Q, as shown by computer results, is approximately 32 decibels. This result differs from that predicted by Equation (5-8) by four decibels.

The granular noise power asymptotes illustrated in Figure 5-6 and 5-7 are given in Table 5-4. The overload noise power asymptote is given in Table 5-1. The maximum S/N_Q asymptote is obtained from Equation (5-7), and is 20 decibels for a bandwidth expansion factor value of eight.

TABLE 5-4

Discrete Adaptive DM Performance With a Broadband Signal, B = 8

	Figure 5-6		
	$K_n = 2$	$K_n = 4$	$K_n = 8$
$\frac{S}{N'_G}$, $\Delta' < 8\ K_n$ (in Decibels)	$[36-20\ \log_{10}\Delta']$	$[42-20\ \log_{10}\Delta']$	$[48-20\ \log_{10}\Delta']$
$\frac{S}{N'_G}$, $\Delta' > 8\ K_n$ (in Decibels)	$[48-30\ \log_{10}\Delta']$	$[57-30\ \log_{10}\Delta']$	$[66-30\ \log_{10}\Delta']$
C (in Decibels)	6	12	18

TABLE 5-4 (Cont)

Discrete Adaptive DM Performance With a Broadband Signal, B = 8

	Figure 5-7		
	$K_n = 16$	$K_n = 32$	$K_n = 64$
$\frac{S}{N'_G}$, $\Delta' < 8\ K_n$ (in Decibels)	$[54-20 \log_{10}\Delta']$	$[60-20 \log_{10}\Delta']$	$[66-20 \log_{10}\Delta']$
$\frac{S}{N'_G}$, $\Delta' > 8\ K_n$ (in Decibels)	$[75-30 \log_{10}\Delta']$	$[84-30 \log_{10}\Delta']$	$[93-30 \log_{10}\Delta']$
C (in Decibels)	24	30	36

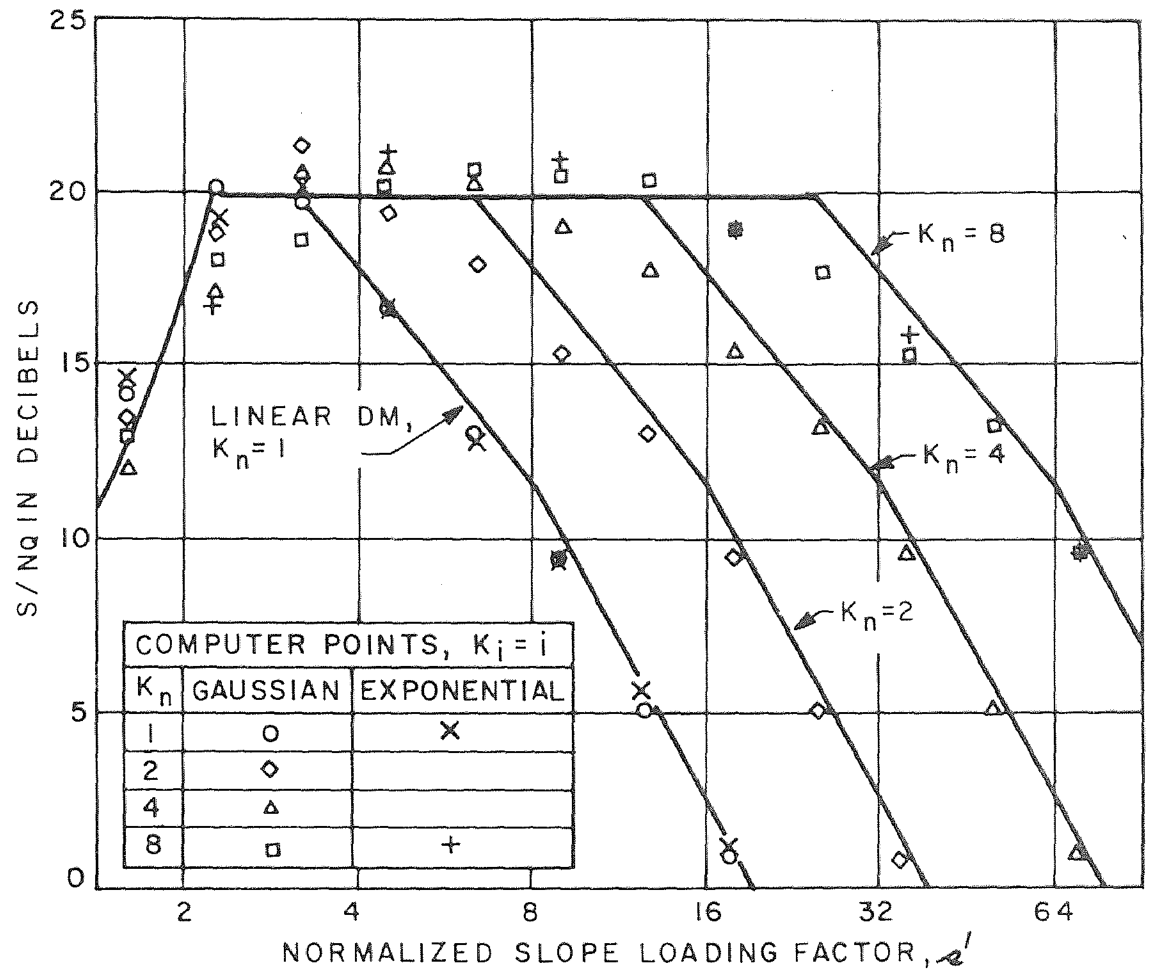


FIG. 5-6 S/NQ PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH UNIFORM SIGNAL SPECTRUM, B = 8.

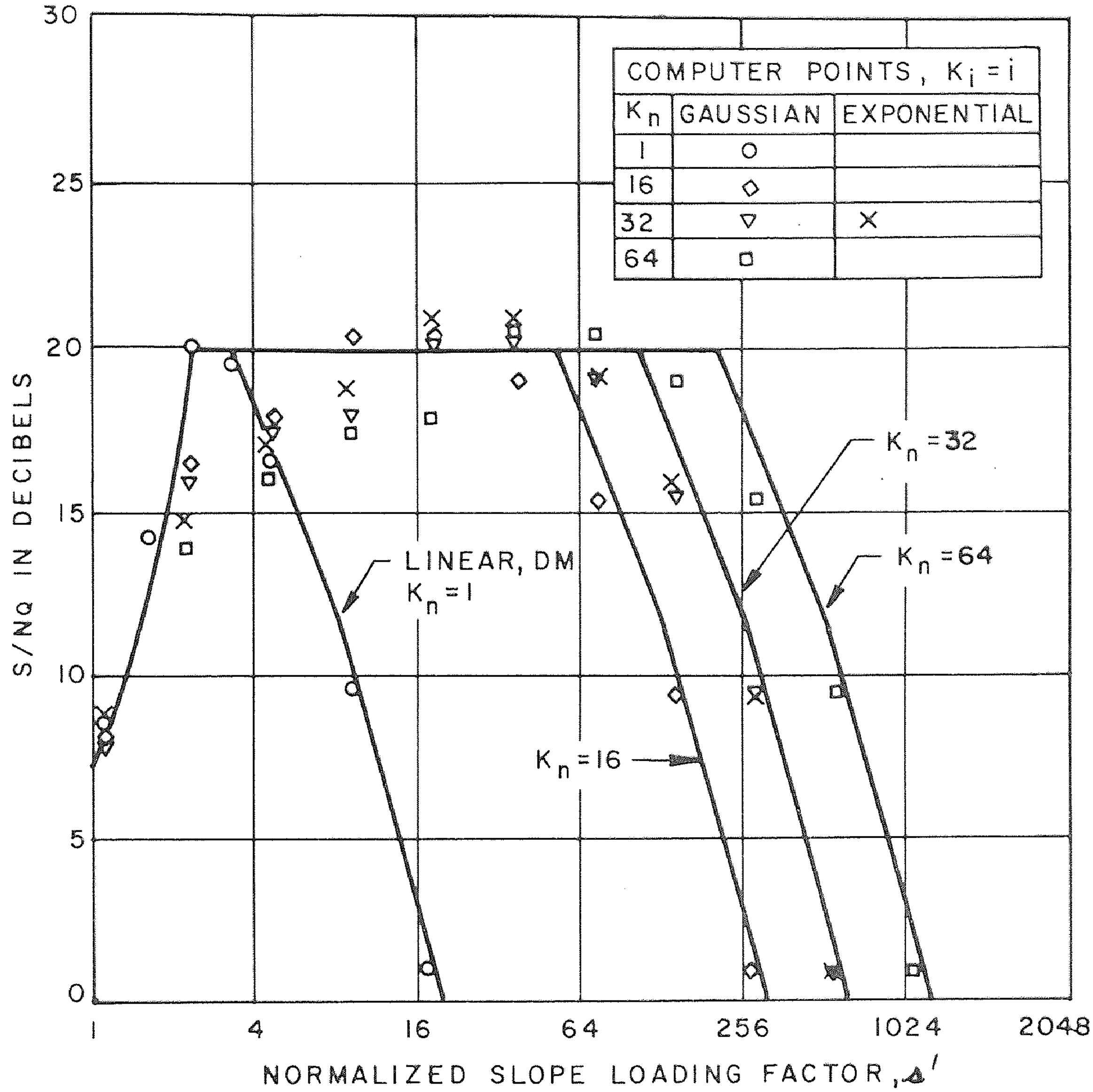


FIG. 5-7 S/NQ PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH UNIFORM SIGNAL SPECTRUM, B= 8.

5.5 Discussion of Results

In this section, it has been shown that the discrete adaptive system provides DM with a companding capability. Large values of companding improvement are possible. A comparison of adaptive DM companding with that of companded PCM will be made in the next section. Computer simulation results have verified that the maximum S/N_Q performance of adaptive DM remains essentially the same as that of linear DM. The use of linear rather than exponential increments for the intermediate gain factors $K_2, K_3, \ldots, K_{n-1}$ yields a performance substantially that of the predicted asymptotes. In all cases studied, the computer simulation results using a Gaussian signal density were essentially the same as those using an exponential density. The companding improvement afforded by the adaptive system is determined by the final gain factor K_n.

Because of the nonstationary nature of both television and speech signals, adaptive DM appears better suited than linear DM to such signals. For television, small values of the final gain factor (i.e., $K_n \approx 4$) should suffice; for speech, larger values would be recommended. More will be said about television and speech in the next section.

6. COMPARISONS WITH PCM

6.1 General

Quantization in PCM is a memoryless operation of converting the continuous message signal into a discrete signal that assumes only a finite number of levels. As in DM, the quantization noise in PCM manifests itself into two forms. The first is that resulting from the discrete quantization process, and will be called granular noise so as to draw an analogy with its DM counterpart. In the literature,[4,39] however, this is commonly known as quantizing noise, since the second form of noise is usually ignored. This second form of PCM quantization noise is caused by the limiting of the message signal to the maximum and minimum levels of the quantizer. This noise is similar to that produced by a linear device with saturation (i.e., an ideal limiter), and will be called overload noise. As opposed to DM overload noise, which is produced when the message signal slope exceeds the slope capability of the DM quantizer, PCM overload noise is produced when the message signal amplitude exceeds the maximum level of the PCM quantizer. Exact analytical expressions for both PCM granular and overload noise powers are derived in Appendix C as a function of the bandwidth expansion factor (which for PCM equals the number of digits of encoding), and a defined quantity called herein the "amplitude loading factor." The analogy of the amplitude loading factor with the slope loading factor of DM will become obvious.

When the number of quantizing levels is sufficiently large (i.e., when the PCM quantum step size is small compared to the standard deviation of the signal), PCM granular and overload noises are uncorrelated and their powers are additive. The sum will be referred to as the quantization noise power. It will be assumed that (1) the message signal is stochastic with zero mean, unit standard deviation, and bandlimited to ω_m; (2) the signal is sampled at the Nyquist rate (i.e., the sampling rate ω_s is twice ω_m); (3) errorless transmission exists in the digital channel.

In a PCM system, the quantizer levels or steps need not be uniformly spaced. There are two different reasons why a nonuniform quantizer may improve the performance of the PCM system. The first is that if the message signal statistics are both well known and stationary, then the quantizer design may be optimized for a given amplitude density by spacing the levels such that the mean square error (i.e., granular noise power) is minimized. Max[27] determined for a Gaussian distribution the optimum level spacing and computed the error. The results, however, are not dramatic, the improvement amounting to less than three decibels. Furthermore, changes in either the amplitude density or the mean value of the signal produce larger changes in the noise power than those of a uniform quantizer. The second reason for desiring a nonuniform quantizer is to achieve companding for nonstationary signals. Speech is a good example of a signal for which PCM companding has been usefully employed. Unfortunately, the nonuniform quantizing characteristic required for companding may not be similar to that of the nonuniform optimum quantizer characteristic discussed by Max.[27] As a result, a noise penalty may be paid if companding is used.

Smith[39] has described a logarithmic nonuniform quantizer which provides companding and has been found desirable when the message signal is speech. Using his result for granular noise in PCM with logarithmic companding, the optimum performance of PCM will be determined and compared with that of a uniform quantizer. Then, both of these will be compared with that of linear and adaptive DM.

6.2 Quantization Noise Power of PCM

6.2.1 Quantization Noise Power With Uniform Quantizer

Given that the PCM quantizer sorts the input into a finite number of ranges and produces uniformly spaced output of representative levels whose upper and lower saturation levels are α times the standard deviation of the signal, then it is shown in Appendix C that the granular noise power N_G and overload noise power N_O are given by

$$N_G = \frac{1}{3}\frac{\alpha^2}{2^{2B}} \qquad (6\text{-}1)$$

$$N_O = 2\int_{\alpha}^{\infty}(x-\alpha)^2 p(x)dx \qquad (6\text{-}2)$$

where p(x) represents the message signal amplitude probability density function, and B is again the bandwidth expansion factor of the transmission channel. The quantity α will be called the amplitude loading factor for PCM. It is analogous to the slope loading factor of linear DM since it represents the ratio of the quantizer maximum encoding level to the standard deviation of the signal. In general, as the amplitude loading factor increases in value, overload noise power decreases, and granular noise power increases. This is so since both the level at which saturation occurs and the size of the quantum step (i.e., spacing between levels) increases as α increases.

Table 6-1 summarizes the results from Equations (6-1) and (6-2) for message signals having Gaussian and exponential amplitude probability density functions. The results in terms of signal to quantization noise power ratio in decibels as a function of the amplitude loading factor is given for the Gaussian case in Figure 6-1 and the exponential case in Figure 6-2. The form and shape of the characteristic curves illustrated are shown to be similar to those of DM in Figure 3-3. The difference basically is that whereas DM performance is limited by slope overload; PCM performance is limited by amplitude overload. The dashed lines in Figures 6-1 and 6-2 illustrate the asymptotic bounds of overload noise power.

Figures 6-1 and 6-2 show that the optimum performance of PCM with uniform quantization is greater for a message signal having a Gaussian amplitude probability density than it is for one having an exponential density. These figures also show that PCM realizes its optimum performance at only one value of the amplitude loading factor. Thus, if the standard deviation of the signal

TABLE 6-1

PCM Performance With a Uniform Quantizer, and Gaussian and Exponential Signal Densitites

	Gaussian Density	Exponential Density
Signal Amplitude Probability Density Function $p(x)$	$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	$\frac{1}{\sqrt{2}} e^{-\sqrt{2}\,\lvert x \rvert}$
Granular Noise Power, N_G	$\frac{1}{3} \frac{\alpha^2}{2^{2B}}$	$\frac{1}{3} \frac{\alpha^2}{2^{2B}}$
Overload Noise Power, N_O	$\left(1+\alpha^2\right)\left(2\int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx\right) - \left(\sqrt{\frac{2}{\pi}}\, \alpha e^{-\frac{1}{2}\alpha^2}\right)$	$e^{-\sqrt{2}\,\alpha}$

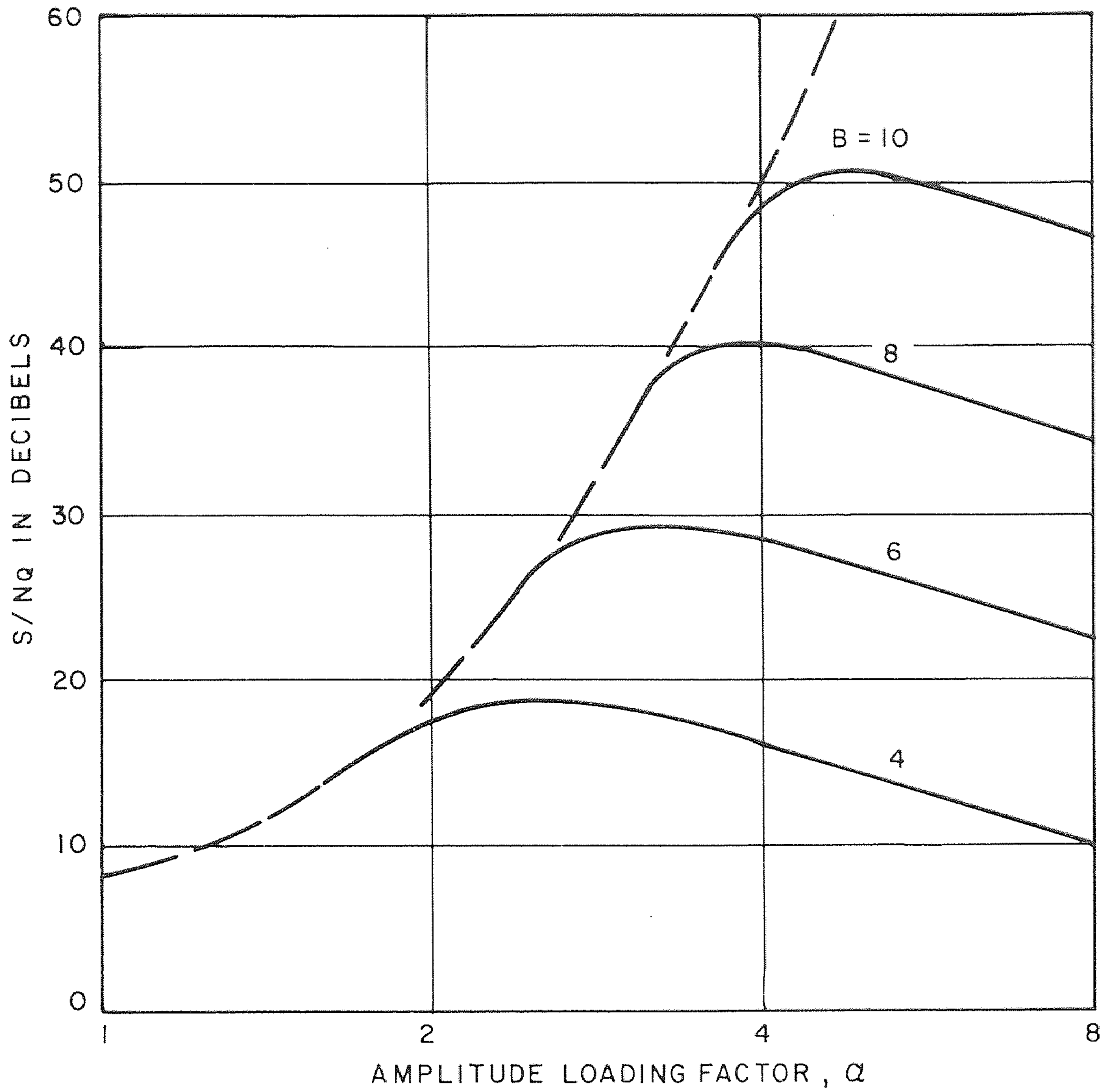


FIG. 6-1 S/Nq PERFORMANCE OF PCM WITH GAUSSIAN SIGNAL DENSITY AND UNIFORM QUANTIZER

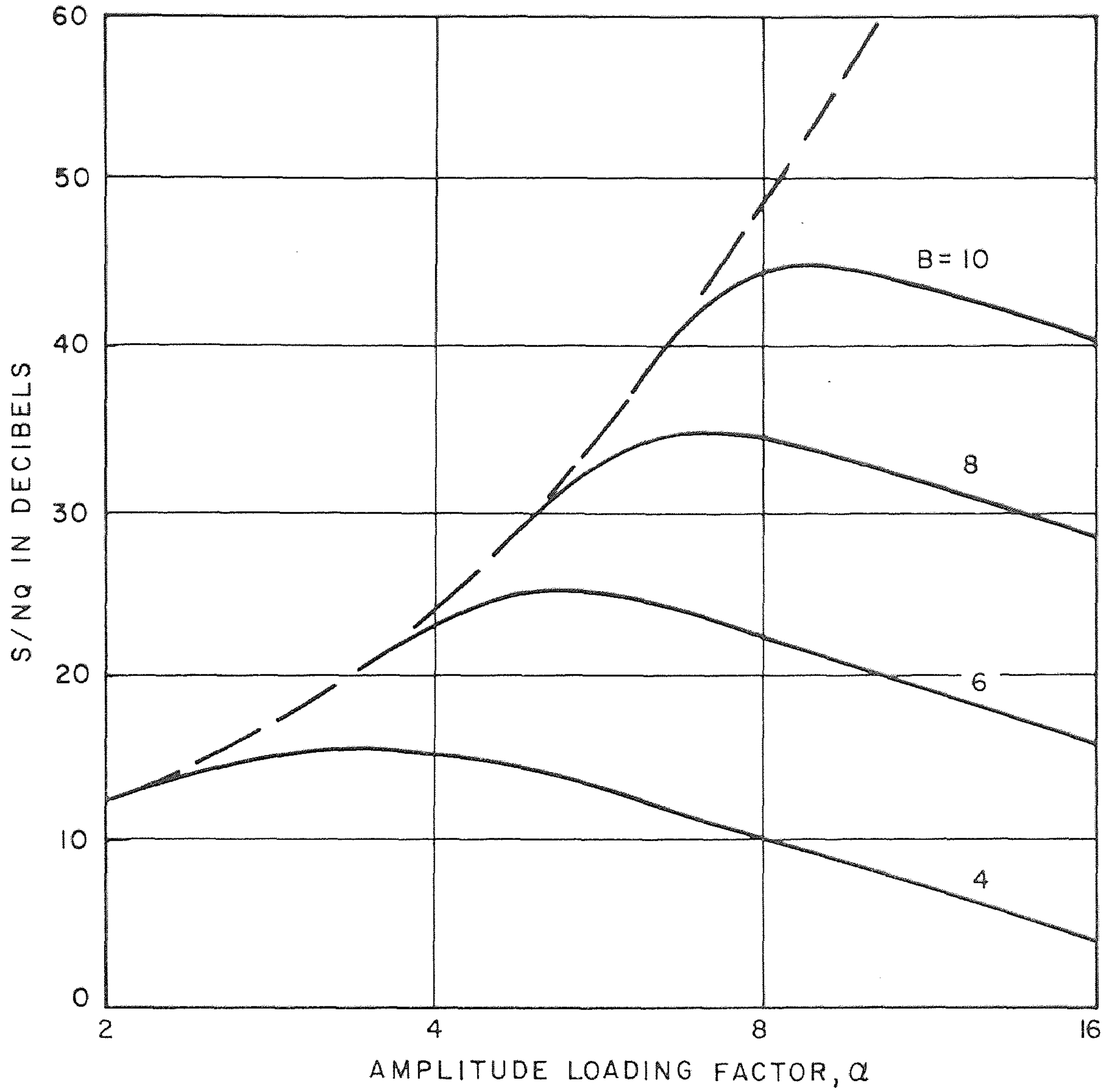


FIG. 6-2 S/Nq PERFORMANCE OF PCM WITH EXPONENTIAL SIGNAL DENSITY AND UNIFORM QUANTIZER.

changes, the performance of PCM is affected. In Section Three, it was found that the performance of linear DM was also sensitive to changes in the mean power of the signal. Adaptive DM, however, was able to provide the companding necessary for nonstationary ensembles. It will be shown next that the companding in PCM provided by a logarithmic quantizer does in fact extend the range of optimum performance, but by differing amounts for Gaussian and exponential signal densities.

6.2.2 Quantization Noise Power With Logarithmic Companding

If the PCM system employs the logarithmic companding reported by Smith,[39] then the granular noise power N_{GC} with such companding has been shown to be given by

$$N_{GC} = \frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2 \cdot \left[1 + \left(\frac{\alpha}{\mu}\right)^2 + 2A\left(\frac{\alpha}{\mu}\right)\right] \qquad (6\text{-}3)$$

where μ, a dimensionless quantity called the compression parameter, determines the companding improvement, and the quantity A is defined as

$$A = 2\int_0^\alpha xp(x)dx \quad . \qquad (6\text{-}4)$$

Table 6-2 summarizes the results of Equations (6-3) and (6-4) applied to the cases of Gaussian and exponential densities. When the PCM system contains no companding (i.e., $\mu = 0$), Equation (6-3) reduces to that of Equation (6-1). This corresponds to direct uniform quantization of the input signal. Overload noise power as given by Equation (6-2) is of course unchanged regardless of whether uniform or nonuniform quantization is employed.

Figures 6-3 and 6-4 illustrate the results given in Table 6-2 for the cases of bandwidth expansion factor values of 4, 6, 8, and 10 (i.e., 4, 6, 8, and 10 digits of encoding respectively) and a value of 100 for μ. This particular value of μ is chosen because it represents the largest value that has been found practicable. For PCM, a higher degree of compression (i.e., $\mu > 100$) is in practice very difficult to achieve for reasons explained by Mann, et al,[26] although Smith[39] had recommended for speech, values of $100 < \mu < 1000$. From Figures 6-2 and 6-4, it is shown that such companding improves the optimum performance (i.e., maximum S/N_Q) when the signal has an exponential distribution for values of the bandwidth expansion factor greater than four. This is caused by the matching of the quantizer logarithmic characteristic to the

TABLE 6-2

Granular Noise Power of PCM With Logarithmic Companding

	Signal Amplitude Probability Density	
	Gaussian	Exponential
A	$\sqrt{\frac{2}{\pi}}\,[1-e^{-\frac{1}{2}\alpha^2}]$	$\frac{1}{\sqrt{2}}\,[1 - e^{-\sqrt{2}\,\alpha}(\sqrt{2}\,\alpha+1)]$
N_{GC}	$\frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2 \cdot \left[1 + \left(\frac{\alpha}{\mu}\right)^2 + 2\sqrt{\frac{2}{\pi}}\left(1-e^{-\frac{1}{2}\alpha^2}\right)\left(\frac{\alpha}{\mu}\right)\right]$	$\frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2 \cdot \left[1 + \left(\frac{\alpha}{\mu}\right)^2 + \sqrt{2}\left(1 - e^{-\sqrt{2}\,\alpha}[\sqrt{2}\,\alpha+1]\right) \cdot \left(\frac{\alpha}{\mu}\right)\right]$
N_{GC}, $\alpha > 3$	$\frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2 \cdot \left[1 + \left(\frac{\alpha}{\mu}\right)^2 + 2\sqrt{\frac{2}{\pi}}\left(\frac{\alpha}{\mu}\right)\right]$	$\frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2 \cdot \left[1 + \left(\frac{\alpha}{\mu}\right)^2 + \sqrt{2}\left(\frac{\alpha}{\mu}\right)\right]$

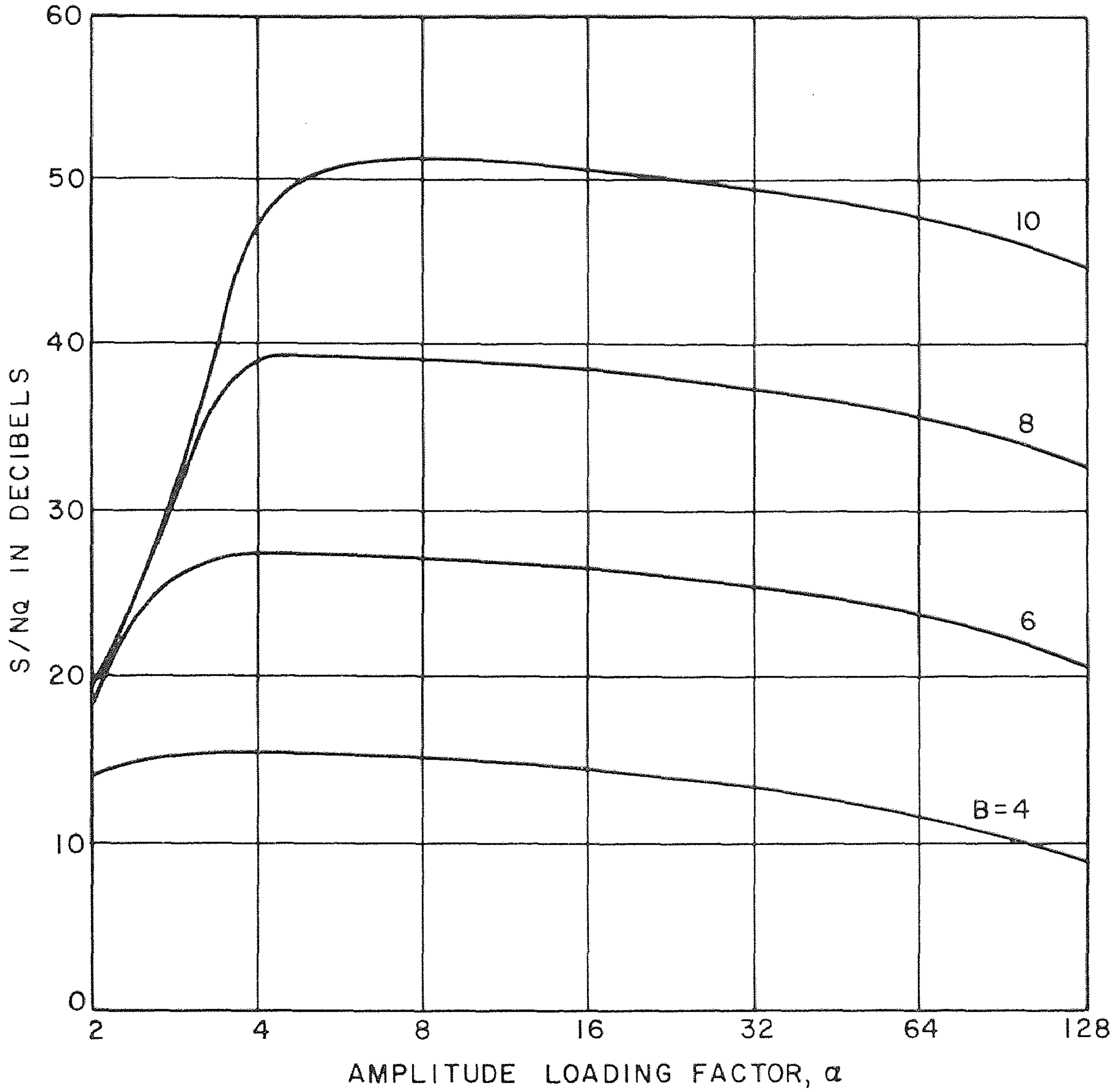


FIG. 6-3 S/NQ PERFORMANCE OF PCM WITH LOGARITHMIC COMPANDING, $\mu = 100$; GAUSSIAN SIGNAL DENSITY.

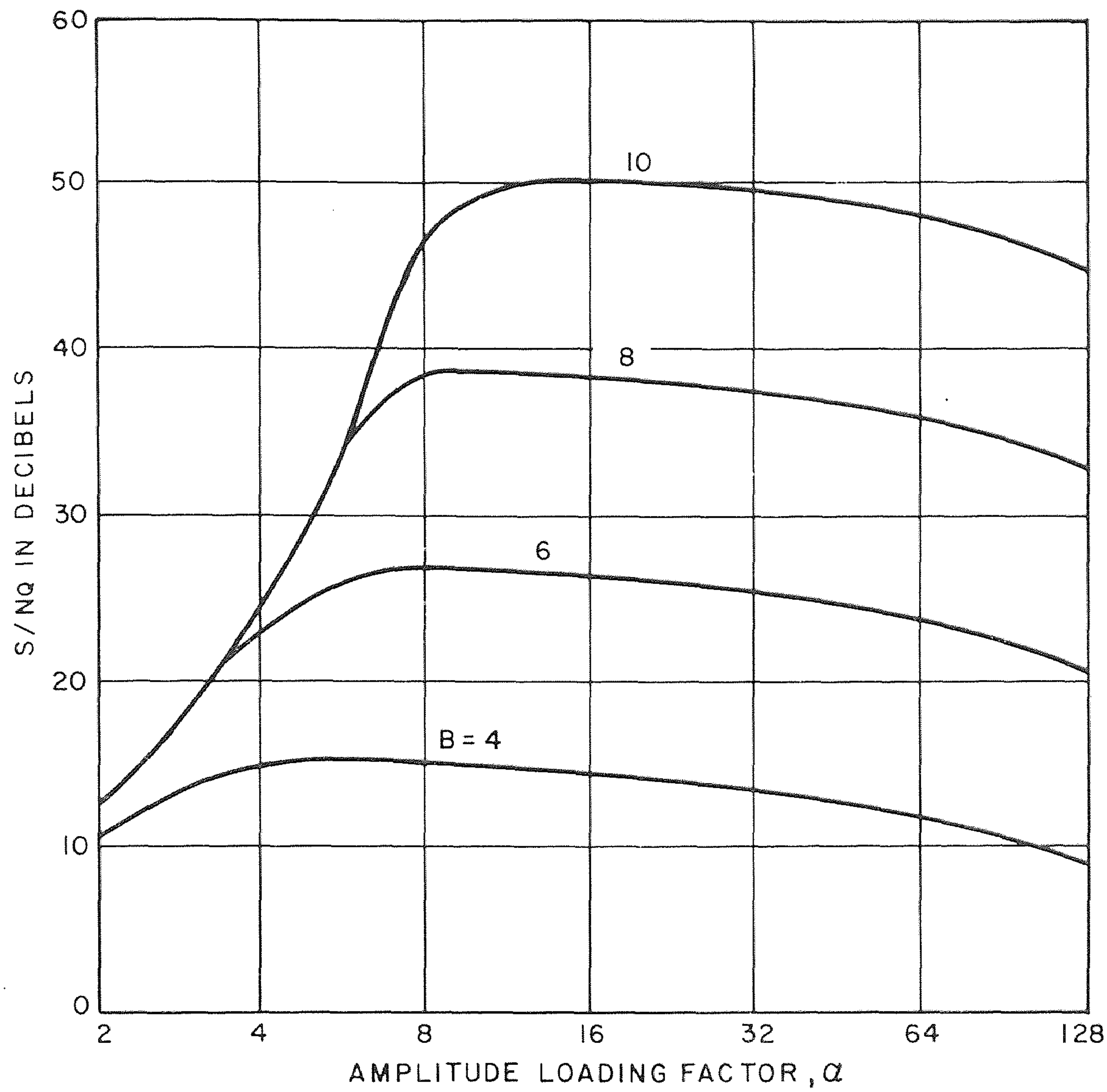


FIG. 6-4 S/NQ PERFORMANCE OF PCM WITH LOGARITHMIC COMPANDING, μ = 100; EXPONENTIAL SIGNAL DENSITY.

signal amplitude exponential probability density, thus reducing the minimum mean square error.

For the Gaussian case shown in Figures 6-1 and 6-3, however, it is shown that optimum performance for $B < 10$ is degraded when the quantizer uses a logarithmic companding characteristic. Thus a performance penalty must be paid if such companding is used.

Figures 6-3 and 6-4 show that the optimum performance with companding is approximately the same for both Gaussian and exponential densities. The companding improvement, however, is greater for a Gaussian signal density than it is for an exponential signal density.

6.2.3 Quantization Noise Power With Amplitude Limiting

In many practical applications, the message signal arriving at the encoder terminals has been limited or saturated in amplitude by one or more physical devices. Such saturation will be referred to herein as amplitude limiting. The amplitude probability density will be assumed zero beyond some value β multiplied by the signal standard deviation. With such peak limiting of the signal, Equation (6-2) is modified simply and becomes

$$N_{OA} = 2\int_{\alpha}^{\beta} (x-\alpha)^2 p(x)dx \qquad (6\text{-}5)$$

Equation (6-1) describes the granular noise power, which remains unchanged. If $\alpha \geq \beta$, then Equation (6-5) vanishes since by definition overload does not exist, and the granular noise power becomes the only source of degradation.

For the case of a signal having a Gaussian amplitude probability density, Equation (6-5) becomes

$$N_{OA} = \left[\left(1+\alpha^2\right)\left(2\int_{\alpha}^{\infty} p(x)dx\right) - \sqrt{\tfrac{2}{\pi}}\ \alpha e^{-\frac{1}{2}\alpha^2}\right]$$

$$- \left[\left(1+\alpha^2\right)\left(2\int_{\beta}^{\infty} p(x)dx\right) - \sqrt{\tfrac{2}{\pi}}\ (2\alpha-\beta)e^{-\frac{1}{2}\beta^2}\right] \qquad (6\text{-}6)$$

For the exponential signal density case, Equation (6-5) becomes

$$N_{OA} = e^{-\sqrt{2}\ \alpha}\left[1 - e^{-\sqrt{2}\ (\beta-\alpha)}(1 + [\beta-\alpha]\cdot[\beta-\alpha+\sqrt{2}])\right] \qquad (6\text{-}7)$$

Figures 6-5 and 6-6 illustrate the case of $\beta = 4$ and a uniform quantizer for Gaussian and exponential densities respectively. The effect of a $\beta \geq 4$ on PCM performance for the Gaussian case is small for any number of digits less than ten. For the exponential case, however, the effect on performance is more substantial since the overload noise power is significantly reduced. The effect on performance can be seen by comparing Figure 6-6 with 6-2, and Figure 6-5 with 6-1 for exponential and Gaussian densities respectively. The dashed lines in Figures 6-5 and 6-6 illustrate the asymptotic bounds of overload noise power.

Because overload noise power is reduced in the presence of signal amplitude limiting, PCM optimum performance is improved. The improvement can be observed by comparing Figures 6-1, 6-2, 6-5, and 6-6. Amplitude limiting in PCM can thus be viewed as the counterpart of slope limiting of DM. That is, the effect in both systems is one of reducing the overload noise power.

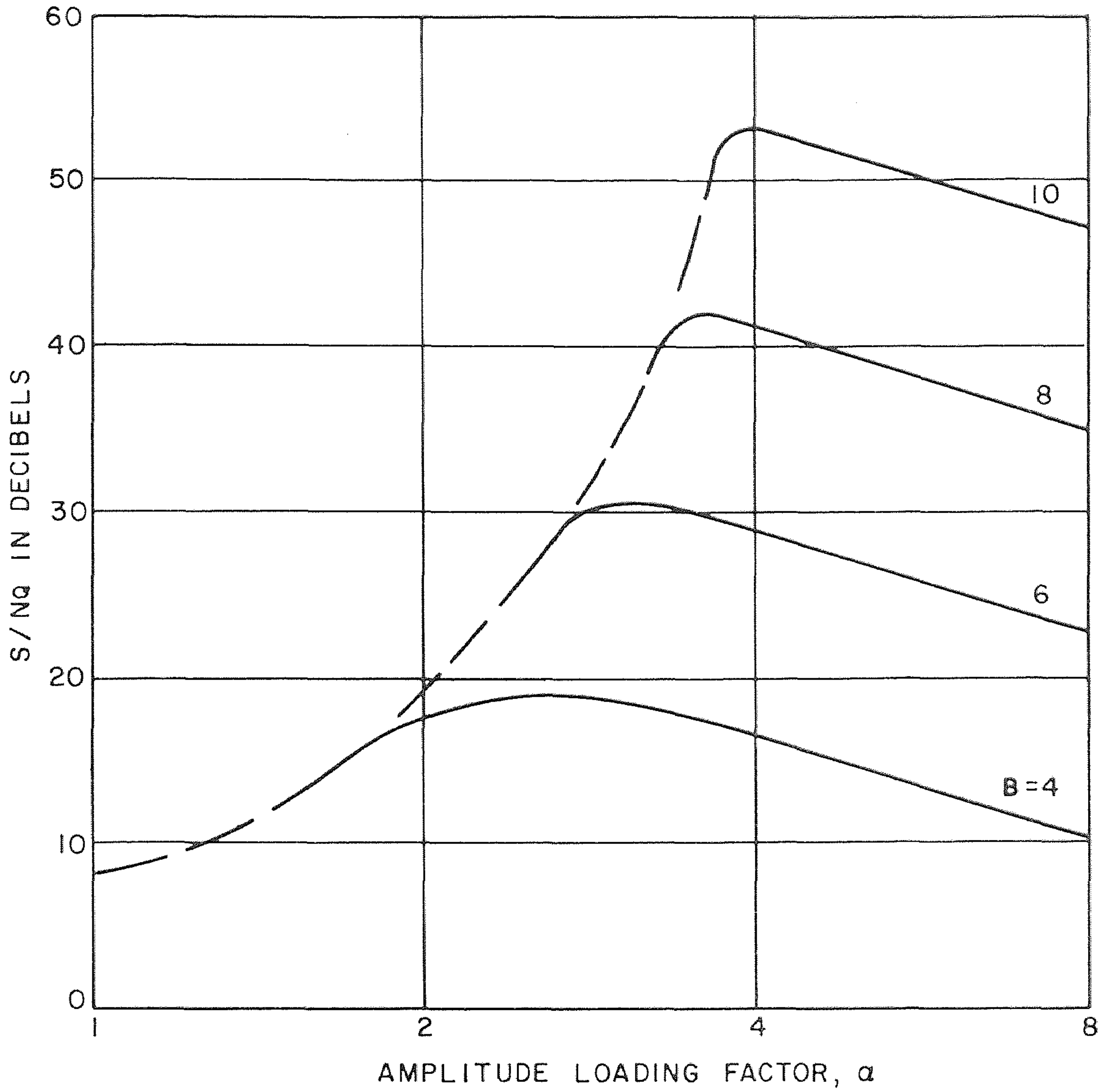


FIG. 6-5 S/NQ PERFORMANCE OF PCM WITH AMPLITUDE LIMITING, $\beta = 4$; GAUSSIAN SIGNAL DENSITY; UNIFORM QUANTIZER

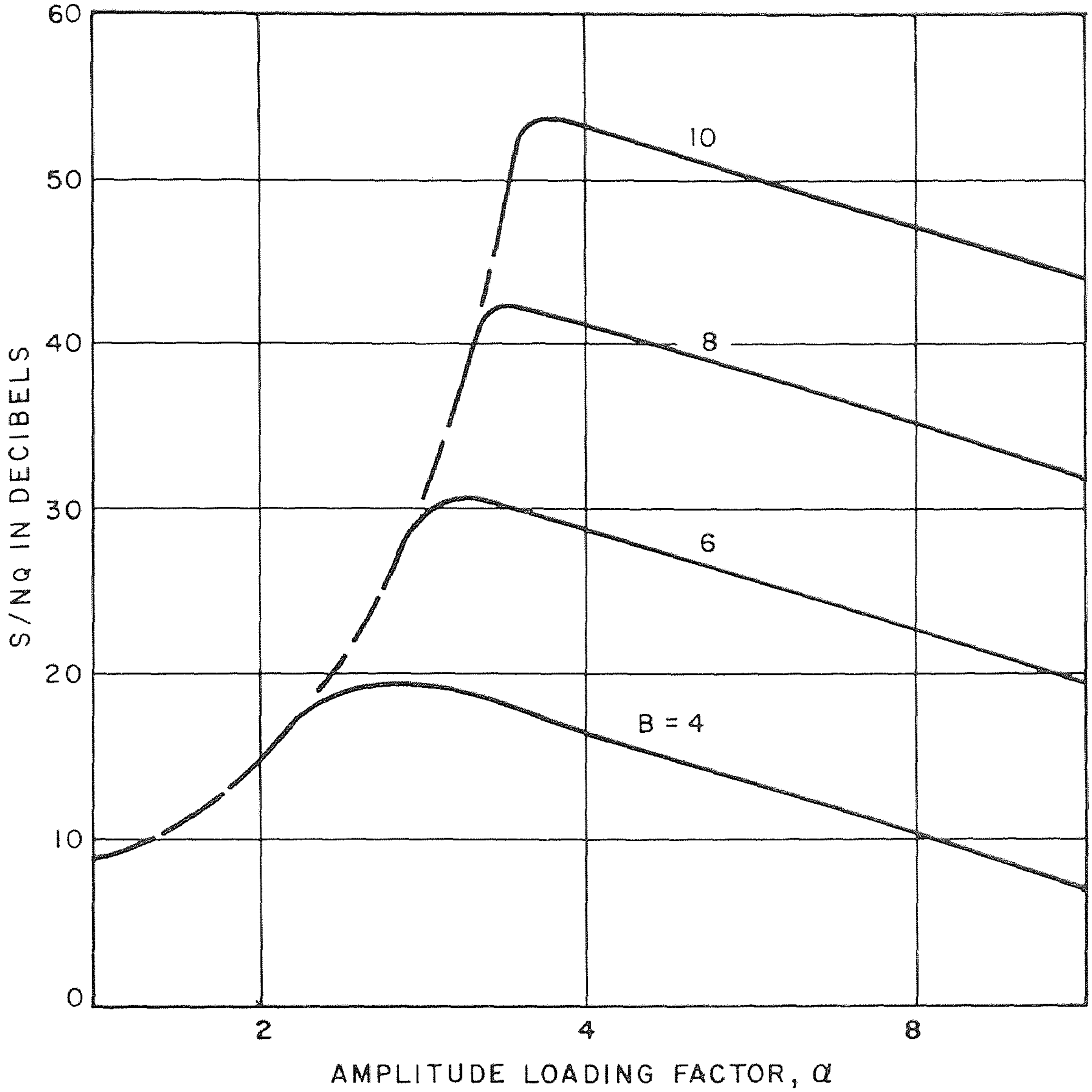


FIG. 6-6 S/NQ PERFORMANCE OF PCM WITH AMPLITUDE LIMITING, β = 4; EXPONENTIAL SIGNAL DENSITY; UNIFORM QUANTIZER.

6.3 Application to Television, Speech, and Broadband Signals

6.3.1 Television Signal

For a television signal, DM provides a greater maximum S/N_Q than PCM for values of B less than eight (i.e., eight digits of PCM encoding). For entertainment television, approximately six or seven digits of PCM encoding has been found to produce pictures of good quality.[15] Although the S/N_Q performance is not the only important criterion in characterizing picture quality, it provides a sound basis upon which to objectively compare and optimize promising encoding systems. A final comparison rests of course with a subjective test. Because of the nonstationary nature of television signals, adaptive DM and companded PCM appear better suited to such signals than linear DM and PCM with uniform quantizing.

The optimum performance of DM and PCM with uniform quantizing are illustrated in Figure 6-7 for the television signal case characterized by the integrated power spectrum given in Tables 3-2 and 3-3, and the exponential amplitude probability density given in Table 6-1. Adaptive DM produces the same optimum performance as linear DM, which was illustrated in Figures 5-2 and 5-3. Although PCM with nonuniform quantizing should produce

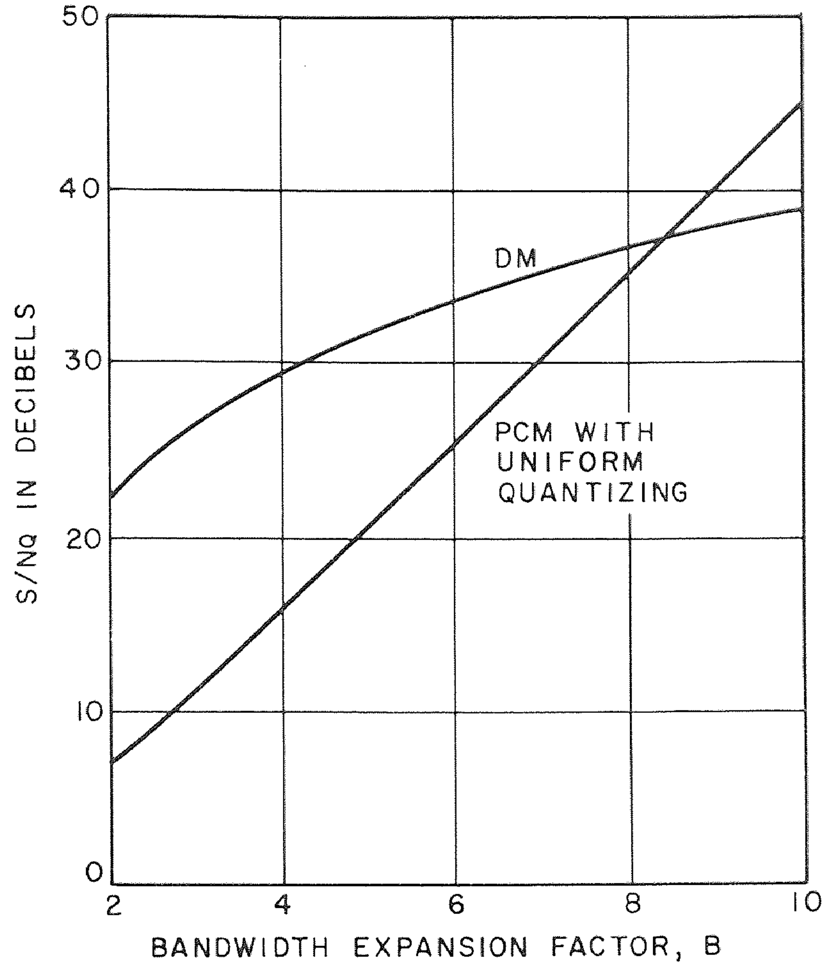


FIG. 6-7 COMPARISON OF DM AND PCM OPTIMUM PERFORMANCES AS A FUNCTION OF THE BANDWIDTH EXPANSION FACTOR, FOR A TELEVISION SIGNAL.

encoded television pictures of a quality superior to that of uniform quantizing, neither the degree of compression nor the optimum quantizer characteristic for video signals have been reported. A study of such optimization should include the results of subjective tests. This conclusion of course also applies to adaptive DM. The next task required toward the application of adaptive DM to television signals should be that of organizing and conducting subjective tests. These tests could determine, for example, the minimum value of the adaptive DM final gain factor as well as the optimum selection of intermediate gain factors. Using a selected ensemble of pictures, the subjective tests could also provide a measure of the relative acceptability of encoded pictures as a function of the bandwidth expansion factor.

Figure 6-8 illustrates the performance of linear DM, adaptive DM(K_n = 4), and PCM with an amplitude limiting factor of ten (i.e., β = 10), and a value of eight for the bandwidth expansion factor. The value of ten for β was found by O'Neal[33] to represent a video signal based on measurements of three different scenes. The performance asymptotes of linear DM (dashed line) and adaptive DM are obtained from the results of Sections 3.3 and 5.4.2 respectively. The abscissa values of

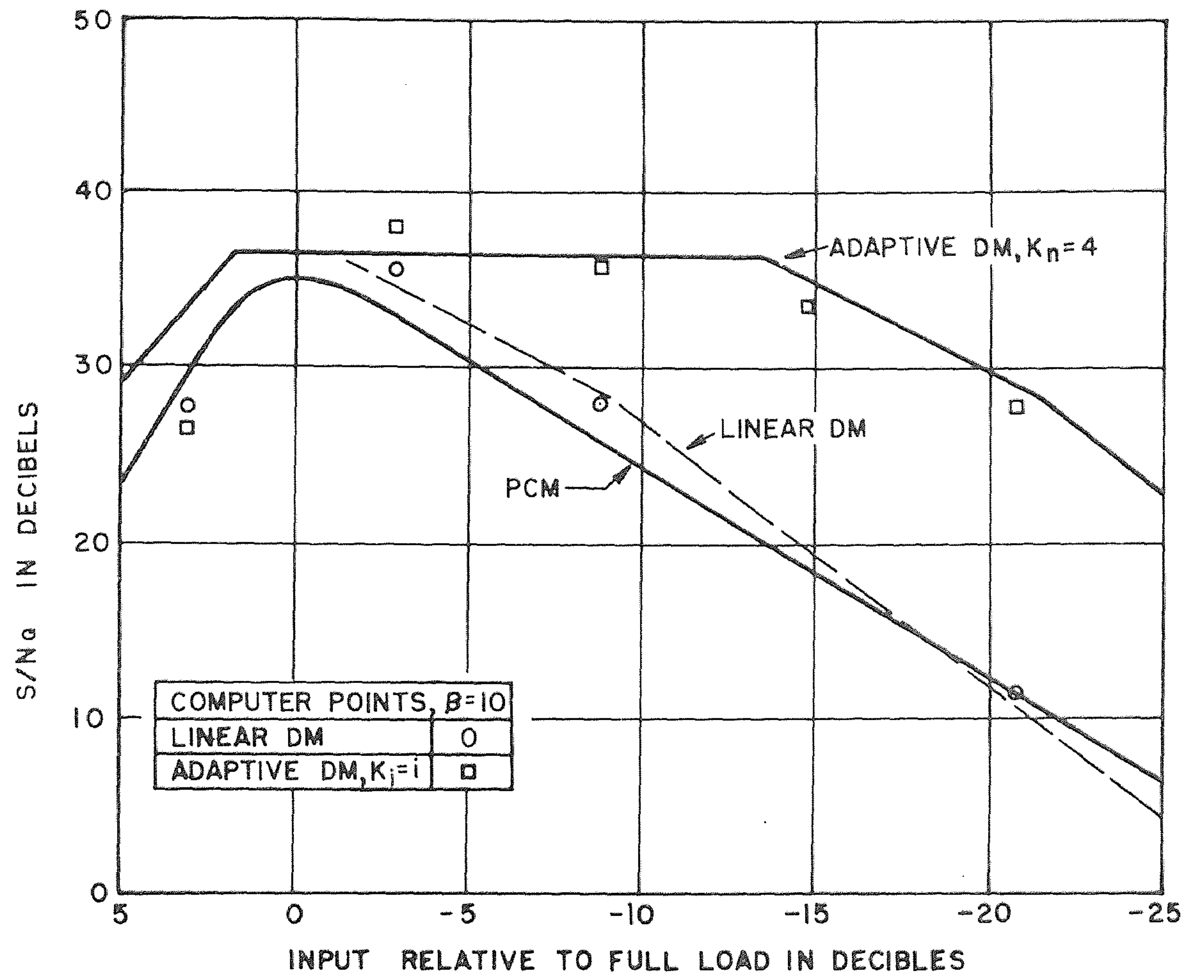


FIG. 6-8 S/Nq PERFORMANCE OF LINEAR DM, ADAPTIVE DM, AND PCM WITH UNIFORM QUANTIZER, ALL WITH AMPLITUDE LIMITING AT β=10, FOR TELEVISION SIGNAL, B=8.

Figure 6-8 are obtained by letting zero decibels correspond to that value of the slope loading and amplitude loading factors at which optimum performance is obtained. For DM, the slope loading factor at optimum performance is given by Equation (3-6) of Section 3.2, and, for B = 8, has the value $\Delta = 2.77$. The abscissa is related to the normalized slope loading factor by the expression

$$\left\{\begin{matrix}\text{Input Relative to Full Load} \\ \text{(in Decibels)}\end{matrix}\right\} = 20 \log_{10} \frac{\ln 2B}{\Delta'} \quad (6\text{-}8)$$

where ln 2B is equal to 2.77 for B = 8. For PCM, the abscissa is related to the amplitude loading factor by the expression

$$\left\{\begin{matrix}\text{Input Relative to Full Load} \\ \text{(in Decibels)}\end{matrix}\right\} = 20 \log_{10} \frac{7.0}{\alpha} \quad (6\text{-}9)$$

where the quantity 7.0 represents the value of amplitude loading factor at which maximum S/N_Q is achieved by PCM at B = 8 and $\beta = 10$.

6.3.2 Speech Signal

Companded PCM using the nonuniform quantizer reported by Smith[39] can now be compared with the discrete adaptive DM discussed in Section 5.4.3. McDonald[28] computer simulated the case of a speech message signal, and a four digit nonuniform quantizer having the logarithmic characteristic reported by Smith.[39] His results are illustrated in Figure 6-9 along with that for comparison of the discrete adaptive DM system having the same bandwidth expansion factor (i.e., for four digit PCM, B = 4), and a K_n of eight. This particular value of K_n was chosen because it yields approximately the same amount of companding as the logarithmic quantizer of Smith[39] with $\mu = 100$. The abscissa of Figure 6-9 corresponds to that given by McDonald,[28] and for the case of adaptive DM is related to the normalized slope loading factor by the expression given by Equation (6-8). The point zero decibels on the abscissa of Figure 6-9 corresponds to a value of 2.08 for the normalized slope loading factor at B = 4.

Figure 6-10 illustrates the results given in Tables 6-1 and 6-2 for the PCM system with a uniform quantizer (i.e., $\mu = 0$) and the logarithmic nonuniform quantizer (i.e., $\mu = 100$), together with the results from McDonald[28] illustrated by the dashed line, and adaptive DM performance points obtained by computer simulation. The zero decibel point of the abscissa of Figure 6-9 corresponds to the value $\alpha = 4$ on the abscissa of Figure 6-10.

For PCM, a higher degree of companding than that illustrated in Figures 6-9 and 6-10 (i.e., $\mu > 100$) is in

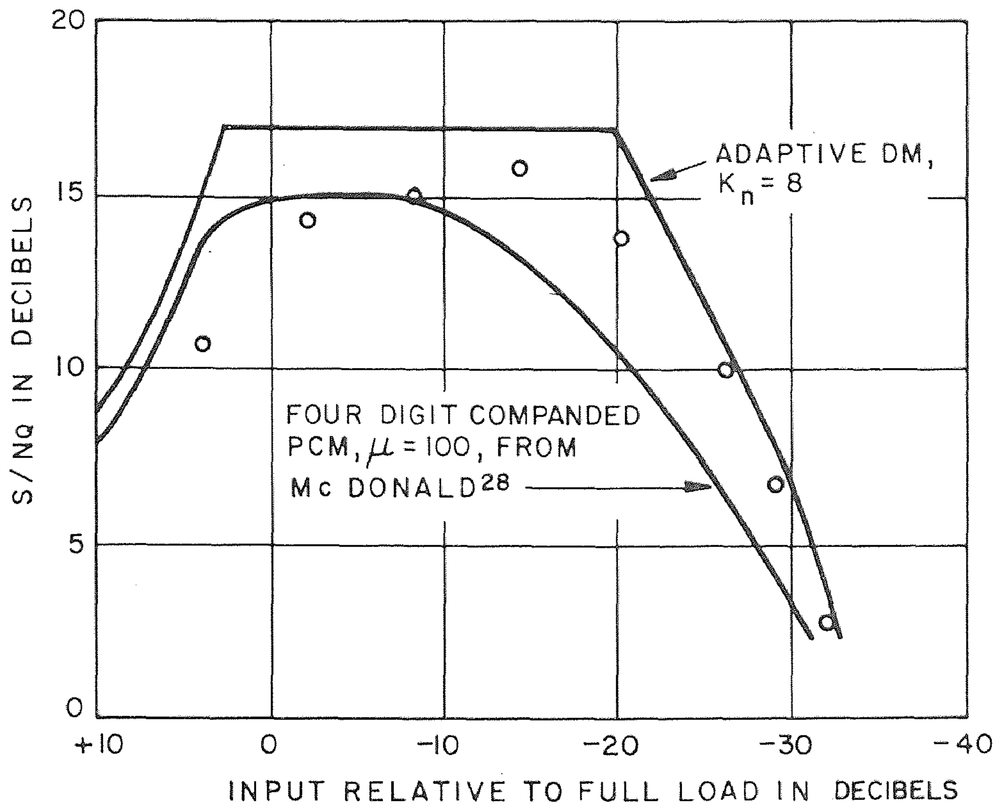


FIG. 6-9 COMPARISON OF COMPANDED PCM AND DISCRETE ADAPTIVE DM; SPEECH SIGNAL, B = 4; POINTS FROM ADAPTIVE DM COMPUTER SIMULATION $K_i = i$.

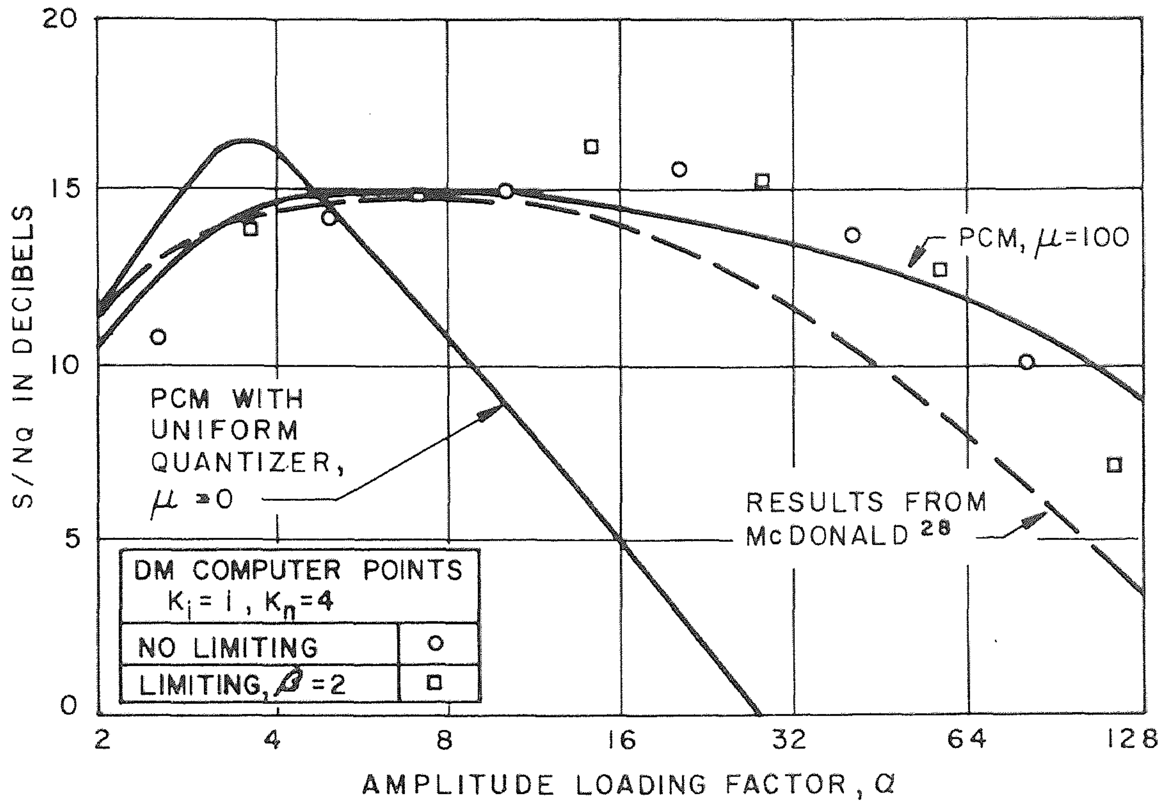


FIG. 6-10 S/NQ PERFORMANCE OF PCM WITH LOGARITHMIC COMPANDING, $\mu = 100$; EXPONENTIAL SIGNAL DENSITY, B=4; POINTS FROM ADAPTIVE DM COMPUTER SIMULATION, $K_i = i$, $K_n = 8$.

practice very difficult to achieve for reasons explained by Mann, et al.[26] For discrete adaptive DM, on the other hand, there appears to be no difficulty for either theoretical or practical reasons in extending the companding improvement to values much larger than that illustrated in Figures 6-9 and 6-10. Whether the additional companding capability that discrete adaptive DM offers could in fact improve speech communication is not known at this time. Because of the subjective nature of speech communication, further tests would be required before more conclusions regarding the possible benefits of discrete adaptive DM over PCM could be reached.

The effect of amplitude limiting on PCM performance is discussed in 6.2.3; the effect on DM performance is small as shown by the computer simulation results illustrated in Figure 6-10. For example, for values of $\beta \geq 2$, the maximum S/N_Q is increased with amplitude limiting by approximately one decibel for linear and adaptive DM when the signal is speech and the bandwidth expansion factor has a value of eight.

6.3.3 Broadband Signal

For broadband signals characterized by the uniform spectrum cited in Tables 3-1 and 3-2, and the Gaussian density in Table 6-1, it is clear that PCM provides superior S/N_Q performance to that of DM. Figure 6-11 illustrates the optimum performance of PCM and DM for a broadband signal as a function of the bandwidth expansion factor.

Figure 6-12 illustrates the performance of companded PCM and discrete adaptive DM, the former with a bandwidth expansion factor value of six, and the latter with a value of sixteen. The different values of B were selected so that the maximum S/N_Q produced by both systems would be approximately the same. The performance asymptotes of adaptive DM were obtained from Equations (5-4), (5-5), (5-6), and (5-7) and from Table 5-1; those for companded PCM from Equations (6-3) and (6-4) and from Table 6-2. Although the DM system illustrated in Figure 6-12 requires a greater transmission bandwidth, it is shown capable of achieving a higher degree of compression than the PCM system. This particular performance advantage of adaptive DM may be desirable for certain applications.

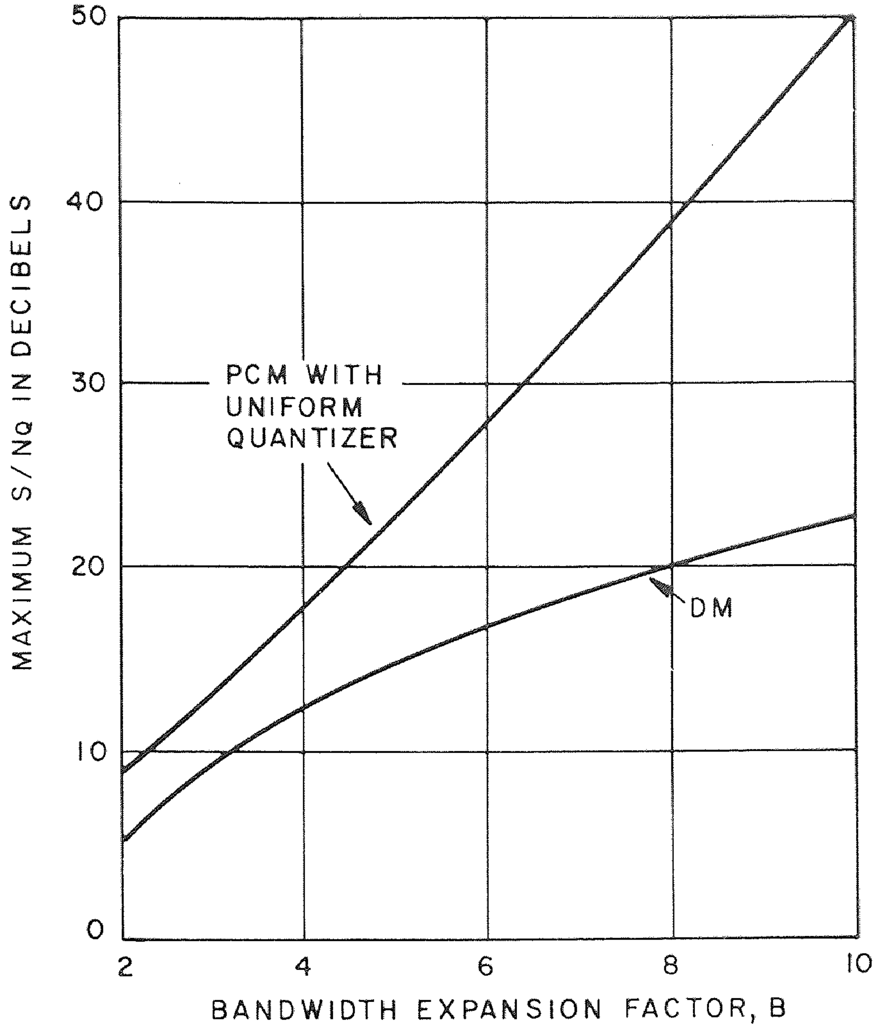


FIG. 6-11 COMPARISON OF DM AND PCM OPTIMUM PERFORMANCES AS A FUNCTION OF THE BANDWIDTH EXPANSION FACTOR, FOR A BROADBAND SIGNAL.

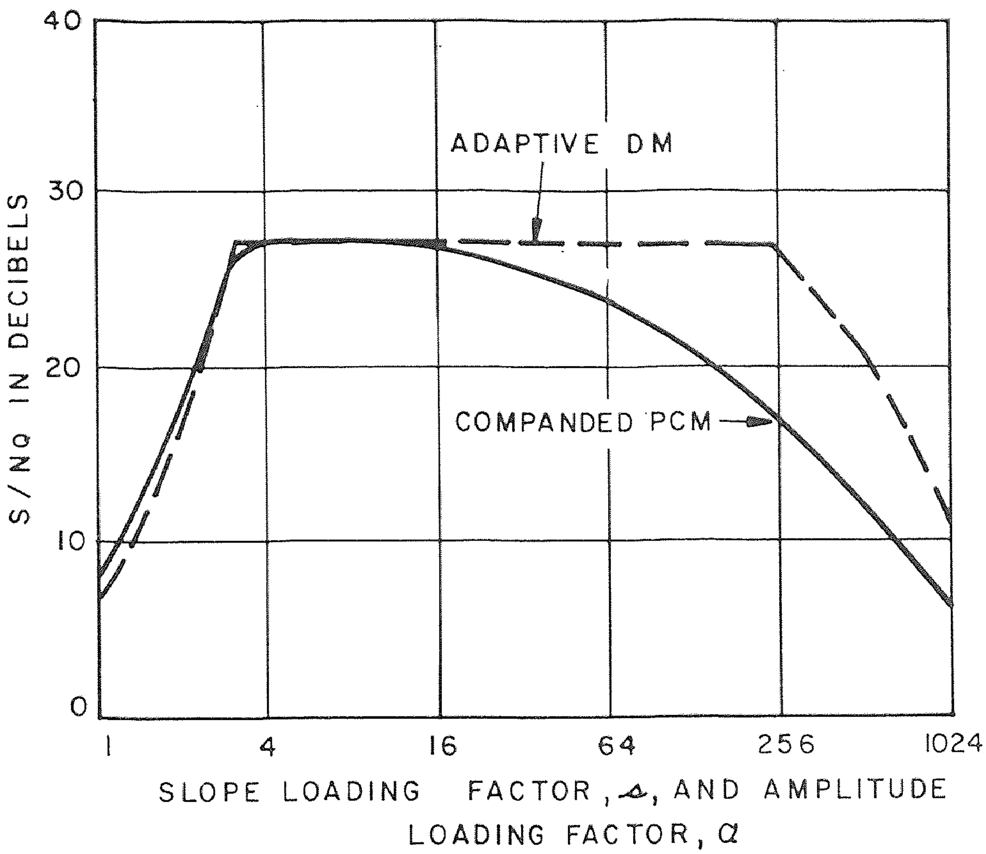


FIG. 6-12 S/NQ PERFORMANCE OF ADAPTIVE DM, B=16, K_n = 64, K_i = i, AND PCM WITH LOGARITHMIC COMPANDING, μ =100, B=6; FOR A BROADBAND SIGNAL.

7. CONCLUSIONS

The performances and optimizations of linear DM, adaptive DM, and PCM have been presented together with the results of computer simulations. The three important cases of television, speech, and broadband signals are treated in detail.

The results presented can be grouped into the following three categories. First, linear DM granular, overload, and minimum quantization noise powers are described by simple closed form solutions. From these expressions, and from computer simulations, the following have been found for linear DM.

(1) It is possible to predict with a simple expression the optimum performance obtainable from DM at various bandwidth expansion factor values.

(2) Minimum quantization noise power is proportional to the mean power of the signal derivative; as a result, S/N_Q performance with an integrated spectrum such as television or speech exceeds that with a uniform spectrum such as a broadband signal.

(3) A defined quantity called the slope loading factor is a useful parameter in characterizing DM performance. When used to describe S/N_Q

performance, the slope loading factor becomes a normalizing variable. The value of slope loading factor at which optimum performance occurs is dependent only on the bandwidth expansion factor.

(4) The S/N_Q performance with a Gaussian signal amplitude probability density is approximately the same as that with an exponential density.

(5) If the mean power of the signal changes by a relatively small amount, S/N_Q performance decreases; as a result, for signals such as speech and television, consisting of non-stationary message ensembles, companding is desirable.

Second, an adaptive DM system which seems promising for television and speech is evaluated. Quantization noise power asymptotes are presented which describe the expected performance of the adaptive system. From these results, and from those of computer simulations, the following findings were made.

(1) The adaptive system provides DM with a companding capability.

(2) The maximum S/N_Q performance of adaptive DM remains approximately the same as that of linear DM.

(3) The final gain factor K_n determines for adaptive DM the amount of companding improvement. Large values of companding improvement are possible.

(4) The intermediate gain factors $K_2, \ldots, K_{n-1}$ determine how well the companded S/N_Q performance meets the predicted asymptotes. The use of linear rather than exponential increments for the intermediate gain factors yields a performance substantially that of the asymptotes presented.

(5) The S/N_Q performance of adaptive DM is the same for both Gaussian and exponential signal densities.

(6) Because of the nonstationary nature of television and speech signals, adaptive DM appears better suited than linear DM to such signals.

Third, the performance of PCM with Gaussian and exponential signal densities is presented, and a comparison is made between PCM and linear and adaptive DM for television, speech, and broadband signals, with the following conclusions being reached.

(1) The characteristic form of the S/N_Q performance relationships of PCM with amplitude loading factor is similar to that of DM with slope

loading factor as the independent variable. For PCM with uniform quantization, a signal with Gaussian density yields a greater maximum S/N_Q performance than one with exponential density.

(2) When logarithmic companding is introduced in the PCM system, the optimum performance is approximately the same for both Gaussian and exponential densities. The companding improvement, however, is greater for a signal having a Gaussian density than it is for one having an exponential density.

(3) When the message signal is amplitude limited, the effect on PCM performance is one of decreasing the amplitude overload noise power. As a result, amplitude limiting in PCM is the counterpart of slope limiting in DM.

(4) For a television signal, DM provides a greater maximum S/N_Q performance than PCM for values of the bandwidth expansion factor less than eight. Alternatively, it could be stated that for the same S/N_Q performance, DM offers a bit rate or channel bandwidth reduction capability in comparison with PCM in the region $B < 8$.

(5) For a speech signal with a bandwidth expansion factor value of four, the performance of adaptive DM, with a final gain factor value of eight using linear increments for the intermediate gain factors, is approximately the same as that of companded PCM which uses a logarithmic quantizer with $\mu = 100$.

(6) Adaptive DM appears capable of realizing a larger companding improvement than PCM.

(7) For a broadband signal, the performance of PCM is superior to that of DM.

(8) Because of the complex nature of television and speech communication, subjective tests are required before further conclusions regarding the performance advantages of discrete adaptive DM can be reached.

8. RECOMMENDATIONS

In the final analysis, the merit of a communication system is determined by the effect of its distortions on the perception characteristics of the ear or eye. The mean square error or signal-to-noise power ratio criterion applied to a communication system is often helpful in making reasonable parametric choices and quantitative system evaluations, but with few exceptions the final fidelity test must involve listening or viewing the received signal. It is hoped that this investigation has provided the insight necessary to make reasonable judgments regarding the performance and optimization of linear DM, adaptive DM, and PCM for television and speech signals. But it is not claimed that the conclusions herein can be substituted for the results of definitive subjective tests. Thus, the first recommendation for future study is that of experimentally investigating and subjectively evaluating the effect of linear and adaptive DM quantizing distortions on speech and television communication.

It is clear that the study of adaptive systems for television and speech communication is in an early stage of development. One need only survey the literature to appreciate the sparsity of information available on the subject. A good deal of additional work remains to be done with DM and PCM systems. For example, in Section Four herein, a system referred to as continuous adaptive DM and illustrated in Figure 4-2 appears to have the potential of adapting to the statistics of the message signal. Brown and Brolin[6] have discussed a system similar to continuous adaptive DM for speech application. The system appears promising enough to warrant further investigation, particularly with respect to television signals.

There are, of course, a number of other forms of adaptive DM. For example, instead of controlling step size, one might choose to adaptively control the sampling rate, or perhaps the number of quantizing levels, or even the feedback network itself. The optimization of such systems, or even their effect on system performance is at present unknown in communication science. In general, it may be said that the study of the potential and performance of adaptive feedback quantizing systems for television and speech communication is a vast area providing considerable opportunity for exploration and research. It is obviously not expected that any one research effort would answer all questions, but it is believed that the investigation of such systems should continue, that original contribution to engineering science is possible, and that future graduate level research in this subject area will remain fertile for a long time.

APPENDIX A

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL OR ABBREVIATION	MEANING	INTRODUCED IN SECTION
α	Amplitude loading factor of PCM	6.2.2
β	Amplitude limiting factor	6.2.3
B	Bandwidth expansion factor	2.1
C	Companding improvement of adaptive DM	5.3
D	Mean power of signal derivative	3.1
DM	Delta modulation	1
DPCM	Differential pulse code modulation	1
E_j	Total quantization error sequence in computer simulation of DM	Appendix D
$f(t)$	Instantaneous value of input message signal	Appendix B
$f'(t)$	Instantaneous derivative of input message signal	3.1
f_D	Digital transmission channel bandwidth	2.1

LIST OF SYMBOLS AND ABBREVIATIONS (Cont)

SYMBOL OR ABBREVIATION	MEANING	INTRODUCED IN SECTION
f_m	Message signal bandwidth	2.1
f_N	Equivalent noise bandwidth of DM error power	Appendix B
f_s	Sampling rate	2.1
$F(\omega)$	One-sided power spectrum of input message signal	3.1
$g(t)$	Instantaneous value of output message signal	Appendix B
I_j	Input sequence of samples in computer simulation of DM	Appendix D
J	Number of input samples in computer simulation of DM	Appendix D
k	DM quantizer step size	2.1
k'	Normalized step size of adaptive DM	5.1
K_i	Intermediate gain factor of adaptive DM	4
K_m	Final gain factor of adaptive DM	4

LIST OF SYMBOLS AND ABBREVIATIONS (Cont)

SYMBOL OR ABBREVIATION	MEANING	INTRODUCED IN SECTION
μ	Compression parameter of PCM	6.2.2
n_j	Filtered noise sequence in DM computer simulation	Appendix D
$n(t)$	Instantaneous value of noise or error	Appendix B
N_G	Granular noise power	2.1
N_{GC}	PCM granular noise power with companding	6.2.2
N_O	Overload noise power	2.1
N_{OA}	PCM overload noise power with amplitude limiting	6.2.3
N_Q	Quantization noise power	2.1
$p(x)$	Probability density function of input signal	6.2.1
$p(n)$	Probability density function of granular noise	Appendix B
P	Binary transmission channel pulse rate	2.1
PCM	Pulse code modulation	1
Q	Number of PCM quantizing levels	Appendix C

LIST OF SYMBOLS AND ABBREVIATIONS (Cont)

SYMBOL OR ABBREVIATION	MEANING	INTRODUCED IN SECTION
Q_j	Quantizer output sequence in DM computer simulation	Appendix D
Δ	Slope loading factor of linear DM	3.1
Δ'	Normalized slope loading factor of adaptive DM	5.1
S	Mean signal power	2.1
S_j	Sequence consisting of sum of all previous values of Q_{j-1} in DM computer simulation	Appendix D
S/N_G	Signal to granular noise power ratio	Appendix B
S/N_O	Signal to overload noise power ratio	Appendix B
S/N_Q	Signal to quantization noise power ratio	3.2
ω	Angular frequency	3.1
ω_3	Corner (i.e., 3 decibel) frequency of integrated spectrum	1

LIST OF SYMBOLS AND ABBREVIATIONS (Cont)

SYMBOL OR ABBREVIATION	MEANING	INTRODUCED IN SECTION
ω_m	Message signal maximum angular frequency	1
ω_N	Error signal noise equivalent bandwidth	Appendix B

APPENDIX B

DM QUANTIZATION NOISE POWER DERIVATIONS

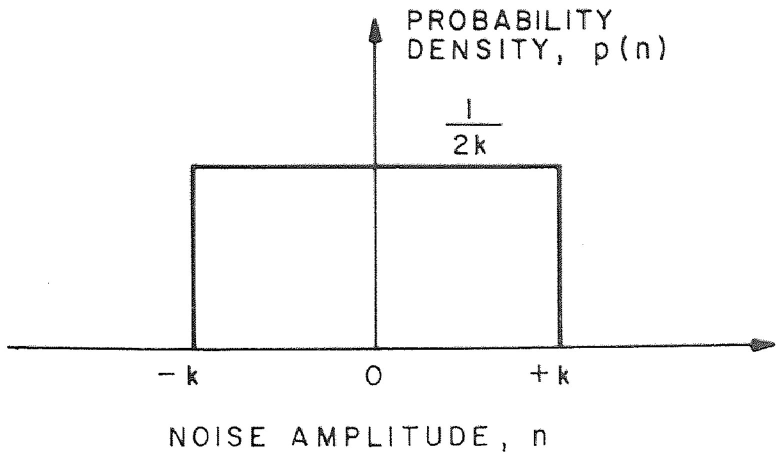
Granular noise in DM is similar to that of PCM. Bennett[4] and Bruce[5] have shown for PCM that both the spectrum and amplitude probability density function of the error are uniformly distributed. The error or noise n(t) is defined by the difference between the DM input signal f(t) and output signal g(t) or

$$n(t) = f(t) - g(t) \tag{B1}$$

In the granular noise region (i.e., no slope overload), the noise signal varies with time, resembling a series of straight lines of varying slopes extending over an interval between minus and plus k, the quantum step size, as illustrated in Figure 2-2 of Section 2.1. The probability density function p(n) of granular noise can therefore be approximated by

$$p(n) = \frac{1}{2k} \tag{B2}$$

This function is illustrated in Figure B1. The granular noise power N_G can then be obtained by calculating the mean square error of a signal uniformly distributed between minus and plus the DM step size k, and then letting



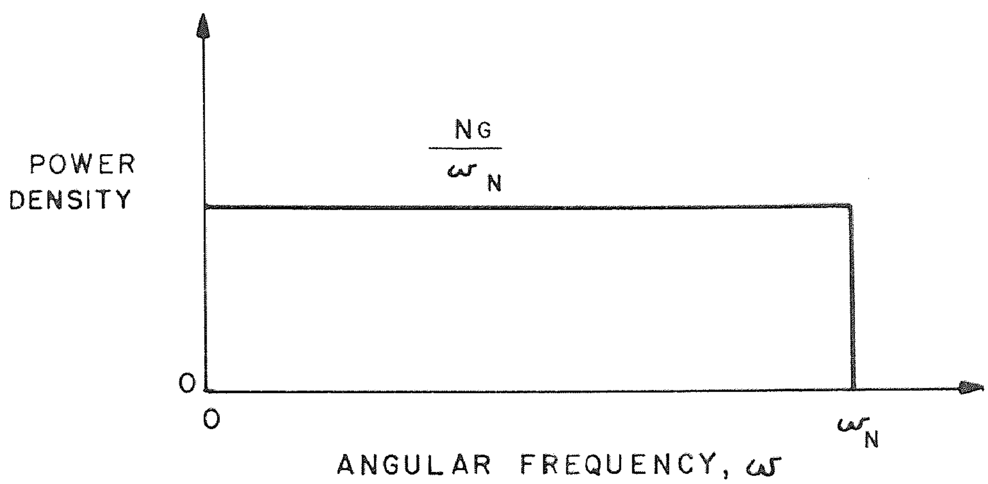


FIG. B1 DM GRANULAR NOISE PROBABILITY DENSITY FUNCTION; AND POWER DENSITY SPECTRUM.

the noise power in the bandwidth of the signal $(0 \rightarrow \omega_m)$ be proportional to the ratio of signal bandwidth to total error power bandwidth. Since the variance of the noise of uniform density p(n) is $\frac{1}{3} k^2$, then the granular noise power N_G within the signal bandwidth ω_m is given by

$$N_G = \frac{\omega_m}{\omega_N} \int_{-k}^{+k} n^2 p(n) dn = \frac{1}{3} k^2 \frac{\omega_m}{\omega_N} \qquad (B3)$$

where ω_N represents the rectangular noise equivalent bandwidth of the error power, as illustrated in Figure B1.

In Section Three, a quantity called the DM slope loading factor was defined as

$$\Delta = \frac{kf_s}{\sqrt{D}} , \qquad (B4)$$

where D represents the mean power of the signal derivative.

When the value of Δ is not large (e.g., $\Delta < 8$), the value of ω_N can be given approximately by ω_s. When the value of Δ is large (e.g., $\Delta > 8$, from either large step size or small slope), periodic patterns as mentioned in Section Two and illustrated in Figure 2-3 appear in the error waveform. These patterns tend to reduce the equivalent total error power bandwidth ω_N in proportion to the step size approximately as follows,

$$\omega_N = c \frac{\sqrt{D}}{k} \tag{B5}$$

where c is a constant of proportionality which can be determined empirically. This expression is equivalent to the statement that the period of a pattern is equal to the step size divided by the effective value of the input signal slope. Combining Equations (B4) and (B5) yields,

$$\omega_N = c \frac{f_s}{\Delta} \tag{B6}$$

Solving Equation (B4) for k, and substituting it into Equation (B3), we obtain after some manipulation the following expression for granular noise power,

$$N_G = \frac{\pi^2}{3} \left(\frac{D}{\omega_m^2}\right) \left(\frac{\omega_m}{\omega_N}\right) \frac{\Delta^2}{B^2} \tag{B7}$$

where the bandwidth expansion factor B is given in Section 2.1 by Equation (2-3). Substituting for ω_N in Equation (B7) the values given above in the two regions $\Delta < 8$ and $\Delta > 8$, we obtain

$$N_G = \frac{\pi^2}{6}\left(\frac{D}{\omega_m^2}\right)\frac{\Delta^2}{B^3}, \quad \text{for} \quad \Delta < 8 \tag{B8}$$

and

$$N_G = \frac{\pi^3}{3c}\left(\frac{D}{\omega_m^2}\right)\frac{\Delta^3}{B^3}, \quad \text{for} \quad \Delta > 8 \tag{B9}$$

Since at the value $\Delta = 8$, Equation (B8) and (B9) must be equal, we find that $c = 16\pi$, so that

$$N_G = \frac{\pi^2}{48}\left(\frac{D}{\omega_m^2}\right)\frac{\Delta^3}{B^3}, \quad \text{for} \quad \Delta > 8 \tag{B10}$$

For example, for a telvision spectrum, Equations (B8) and (B 10) become

$$N_G = 0.011\,\frac{\Delta^2}{B^3}, \quad \text{for} \quad \Delta < 8 \tag{B11}$$

$$N_G = 0.0014\,\frac{\Delta^3}{B^3}, \quad \text{for} \quad \Delta > 8 \tag{B12}$$

The corresponding signal-to-noise power ratios expressed in decibels become

$$\text{For } \Delta < 8,\ \frac{S}{N_G} \text{ (in Decibels)} = [19.6+30 \log_{10}B-20 \log_{10}\Delta] \tag{B13}$$

$$\text{For } \Delta > 8,\ \frac{S}{N_G} \text{ (in Decibels)} = [28.5+30 \log_{10}B-30 \log_{10}\Delta] \tag{B14}$$

A closed form expression for overload noise power N_O which is sufficiently accurate when the signal-to-noise power ratio is not too small can be derived quite simply with the aid of an empirical observation made from computer simulation results. It is observed that S/N_Q has a maximum value for each and every B at some value of the slope loading factor Δ. The relationship between B and Δ at maximum S/N_Q is illustrated in Figure 3-1. From the computer derived results, the relationship between the bandwidth expansion factor B and slope loading factor Δ at the maximum signal to quantization noise ratio can be given with reasonable accuracy as

$$e^{\Delta} = 2B \tag{B15}$$

Since the quantization noise power N_Q consists of the sum of granular N_G and overload N_O noise powers, and since at its minimum the derivative of quantization noise with respect to the slope loading factor must vanish, then for any given fixed value of the bandwidth expansion factor B, the quantization noise considered as a function of Δ only is a minimum when Equation (B15) is satisfied. In other words, since

$$N_Q = N_G + N_O, \tag{B16}$$

and

$$\frac{dN_Q}{d\Delta} = 0 \tag{B17}$$

when

$$e^{\Delta} = 2B, \tag{B18}$$

then for each value of B, at minimum N_Q (i.e., maximum S/N_Q), we have

$$\frac{dN_O}{d\Delta} = - \left. \frac{dN_G}{d\Delta} \right|_{\frac{1}{2}e^{\Delta}=B} \tag{B19}$$

and, therefore,

$$N_O = \int \left[- \left. \frac{dN_G}{d\Delta} \right|_{\frac{1}{2}e^{\Delta}=B} \right] d\Delta \tag{B20}$$

Since N_O does not depend on the choice of B, and N_G is given by Equation (B8) in the region of minimum N_Q, then the overload noise power becomes

$$N_O = \int - \frac{8\pi^2}{3} \left(\frac{D}{\omega_m^2} \right) \Delta e^{-3\Delta} d\Delta \tag{B21}$$

or,

$$N_O = \frac{8\pi^2}{27}\left(\frac{D}{\omega_m^2}\right)e^{-3\Delta}(3\Delta+1) \qquad \text{(B22)}$$

Thus, overload noise power is characterized as a function of the slope loading factor for a given signal power spectral density. As a numerical example, consider the case of a uniform signal spectrum. The overload noise power becomes

$$N_O = \frac{8\pi^2}{81}\, e^{-3\Delta}(3\Delta+1) \qquad \text{(B23)}$$

The signal to overload noise power ratio expressed in decibels is then,

$$\frac{S}{N_O} = [13.0\Delta - 10\,\log(3\Delta+1)] \qquad \text{(B24)}$$

Equation (B24) is illustrated in Figure B2 along with points obtained by computer simulation for $B = 8$, Gaussian signal density. The departure of computer derived results from that of Equation (B24) for values of Δ greater than 1.6 is caused by the influence of granular noise in this region of larger quantum step sizes. Figure 3-3 illustrates the composite effect of overload and granular noise powers.

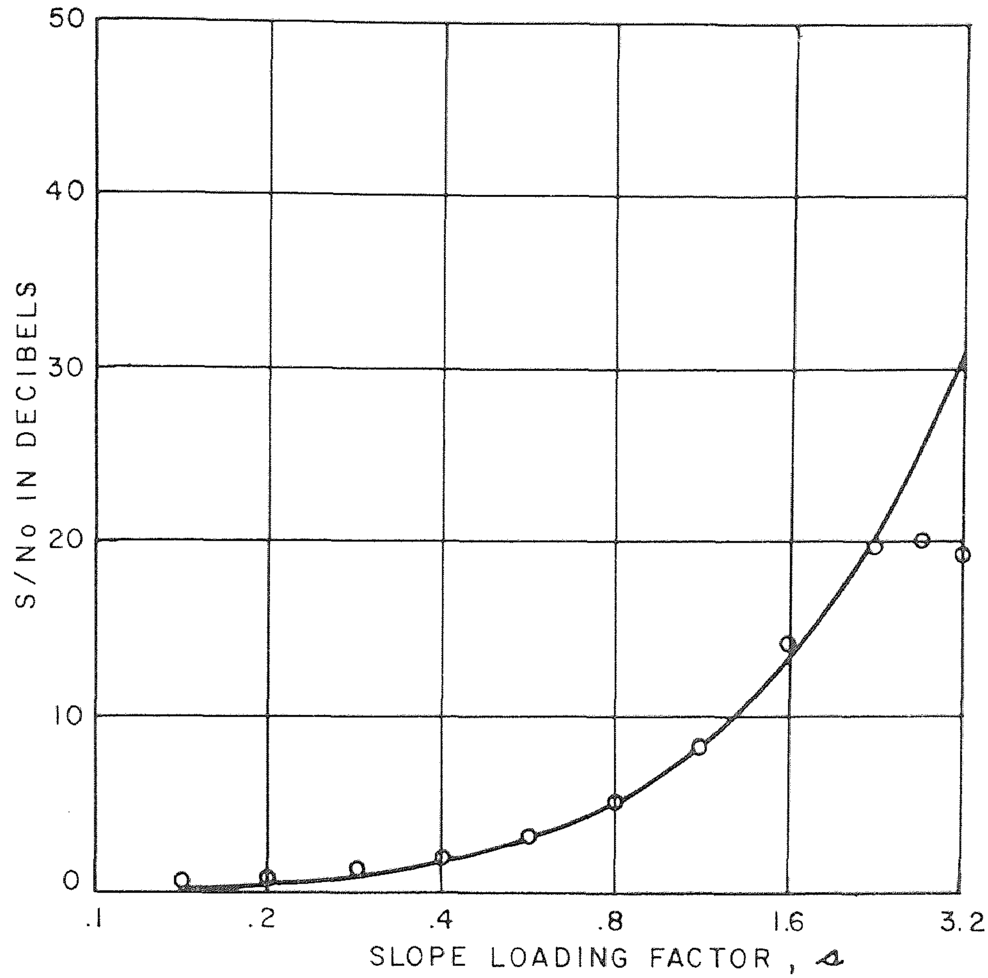


FIG. B2 S/No PERFORMANCE OF LINEAR DM WITH UNIFORM SIGNAL SPECTRUM; CURVE OBTAINED FROM EQUATION (B20), POINTS FROM COMPUTER SIMULATION, B=8 , GAUSSIAN SIGNAL DENSITY.

At optimum performance, the DM system produces its minimum total quantization noise N_Q. Since this total noise is the sum of granular N_G and overload N_O noise powers, and since peak performance occurs in cases of practical importance at values of the slope loading factor which are less than eight, then Equations (B8), (B15), and (B22) may be combined to yield the minimum total quantization noise N_Q as a function of the bandwidth expansion factor B. The result can be summarized as follows.

$$\text{minimum } N_Q = N_G\{(\Delta = \ln 2B) < 8\} + N_O\{\Delta = \ln 2B\} \tag{B25}$$

where,

$$N_G\{(s = \ln 2B) < 8\} = \frac{\pi^2}{6}\left(\frac{D}{\omega_m^2}\right)\frac{(\ln 2B)^2}{B^3} \tag{B26}$$

$$N_O\{\Delta = \ln 2B\} = \frac{\pi^2}{27}\left(\frac{D}{\omega_m^2}\right)\left(\frac{3 \ln 2B+1}{B^3}\right) \tag{B27}$$

Substitution of Equations (B26) and (B27) into (B25) yields

$$\text{minimum } N_Q = \frac{\pi^2}{6}\left(\frac{D}{\omega_m^2}\right)\left[\frac{(\ln B)^2 + 2.06 \ln B + 1.17}{B^3}\right] \tag{B28}$$

In the discrete adaptive DM system, since the instantaneous value of the step size varies from a minimum value k to the maximum $K_n k$, the instantaneous value of the slope loading factor will vary from a minimum value of $\frac{1}{K_n} \Delta'$ to a maximum of Δ'. The granular noise power N_G', therefore, can be no less than that given by Equations (B8) and (B10) into which is substituted the minimum value of the slope loading factor for adaptive DM in place of the slope loading factor for linear DM. The asymptotic bounds for discrete adaptive DM minimum granular noise are therefore given by the following.

$$N_G' = \frac{\pi^2}{6} \left(\frac{D}{\omega_m^2}\right) \frac{(\Delta')^2}{K_n^2 B^3}, \quad \text{for } \Delta' < 8\, K_n \qquad \text{(B30)}$$

$$N_G' = \frac{\pi^2}{48} \left(\frac{D}{\omega_m^2}\right) \frac{(\Delta')^3}{K_n^3 B^3}, \quad \text{for } \Delta' > 8\, K_n \qquad \text{(B31)}$$

Because the maximum value of the normalized slope loading factor is given by $k'\Delta$, the asymptotic lower bound for discrete adaptive DM overload noise power is the same as that for linear DM given by Equation (B22) in which the slope loading factor Δ is replaced by the normalized slope loading factor Δ'. The minimum value of total quantization noise for the DM system has been given by Equation (B28).

The minimum quantation noise asymptote extends over the range of normalized slope loading factor Δ' beginning at the value determined by equating N_O' and minimum N_Q given by Equations (B22) and (B28), and extending to the value determined by equating minimum N_Q and $N_G'(\Delta < 8\ K_n)$ given by Equations (B28) and (B30) respectively. The former is given approximately by Equation (B15); the latter is given by

$$\Delta' = K_n \sqrt{(\ln B)^2 + 2.06 \ln B + 1.17} \qquad \text{(B32)}$$

As a numerical example, consider the case of a uniform signal spectrum with $K_n = 8$, $B = 8$. The asymptotic bounds of granular, overload, and minimum quantization noise powers corresponding to Equations (B30), (B31), (B23), and (B28) respectively, are given in Table B1. These results are illustrated in Figure B3 in terms of signal-to-noise power ratio expressed in decibels, along with results obtained by computer simulation.

TABLE B1

Discrete Adaptive DM Performance With a Uniform Signal Spectrum ($K_n = 8, B = 8$)

Performance Parameter	From Equation	Result
$N_G(\Delta' < 64)$	(B30)	$1.67\times10^{-5}(\Delta')^2$
$N_G(\Delta' > 64)$	(B31)	$2.62\times10^{-7}(\Delta')^3$
N_O	(B23)	$e^{-3\Delta'}(3\Delta'+1)$
Minimum N_Q	(B28)	.010

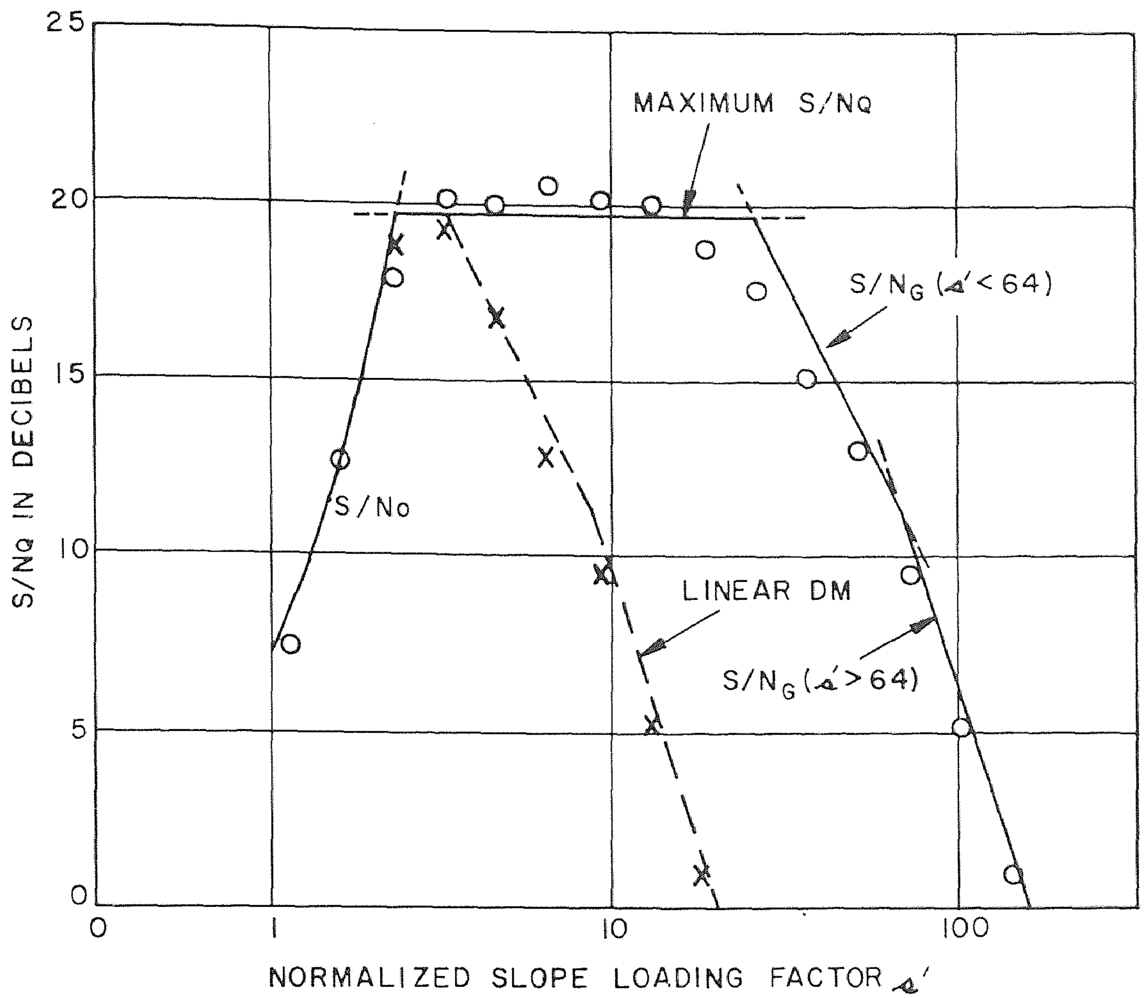


FIG B3 S/N_Q PERFORMANCE OF DISCRETE ADAPTIVE DM, WITH UNIFORM SIGNAL SPECTRUM $B=8, K_n=8$; ASYMPTOTES OBTAINED FROM TABLE B1, POINTS FROM COMPUTER SIMULATION, GAUSSIAN DENSITY, $K_i = i$

APPENDIX C

PCM QUANTIZATION NOISE POWER DERIVATIONS

The expression for granular noise power can be obtained from the work of Bennett,[4] who showed that

$$N_G = \frac{1}{12} k^2 \tag{C1}$$

where k is the size of a quantum step, assuming that the steps are of equal size (i.e., uniform quantizer). Given that the magnitude of the largest level is α times the root mean square value of the signal, and the quantizer produces a finite number of levels Q, the step size becomes

$$k = \frac{2\alpha}{Q} \tag{C2}$$

where again for convenience, the mean signal power, S, is assumed unity. Since the bandwidth expansion factor B is equal in PCM to the number of digits of encoding, then

$$Q = 2^B \tag{C3}$$

and the granular noise power becomes

$$N_G = \frac{1}{3} \frac{\alpha^2}{2^{2B}} \tag{C4}$$

An expression for the overload noise power, that is, the noise caused by limiting the signal to the largest quantum level α, can be obtained by writing the mean square value of the difference between the output of an ideal limiter and its input. Given an input signal with amplitude probability density function $p(x)$, a mean of zero and a unit variance, the mean square difference (i.e., overload noise power) was first reported by Shtein[38] as

$$N_O = 2\int_{\alpha}^{\infty}(x-\alpha)^2 p(x)dx \qquad (C5)$$

For the Gaussian density case, since

$$p(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2} \qquad (C6)$$

then,

$$N_O = \sqrt{\frac{2}{\pi}} \int_{\alpha}^{\infty}(x-\alpha)^2 e^{-\frac{1}{2}x^2} dx \qquad (C7)$$

or,

$$N_O = \sqrt{\frac{2}{\pi}} \int_{\alpha}^{\infty} x^2 e^{-\frac{1}{2}x^2} dx - 2\alpha\sqrt{\frac{2}{\pi}}\int_{\alpha}^{\infty} xe^{-\frac{1}{2}x^2} + \sqrt{\frac{2}{\pi}}\,\alpha^2\int_{\alpha}^{\infty} e^{-\frac{1}{2}x^2} dx \qquad (C8)$$

The first integral can be evaluated by parts. The result of calculating the integrals gives the following expression.

$$N_0 = (1+\alpha^2)\left[2\int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2} dx\right] - \sqrt{\frac{2}{\pi}}\, \alpha e^{-\frac{1}{2}\alpha^2} \qquad (C9)$$

For the exponential density case, since

$$p(x) = \frac{1}{\sqrt{2}}\, e^{-\sqrt{2}\,|x|} \qquad (C10)$$

then,

$$N_0 = \sqrt{2}\int_{\alpha}^{\infty} (x-\alpha)^2 e^{-\sqrt{2}\,x} \qquad (C11)$$

This integral is evaluated easily and becomes

$$N_0 = e^{-\sqrt{2}\,\alpha} \qquad (C12)$$

If the PCM system employs the logarithmic companding reported by Smith,[39] then the granular noise power N_{GC} was shown by Smith to be given by

$$N_{GC} = \frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2 \cdot \left[1 + \left(\frac{\alpha}{\mu}\right)^2 + 2A\left(\frac{\alpha}{\mu}\right)\right], \qquad (C13)$$

where the quantity A is defined by Smith as

$$A = 2\int_0^{\alpha} xp(x)dx. \tag{C14}$$

For the Gaussian density, the quantity A becomes

$$A = \sqrt{\frac{2}{\pi}}\,[1-e^{-\frac{1}{2}\alpha^2}], \tag{C15}$$

and the granular noise power is then

$$N_{GC} = \frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2\left[1 + \left(\frac{\alpha}{\mu}\right)^2 + 2\sqrt{\frac{2}{\pi}}\left(1-e^{-\frac{1}{2}\alpha^2}\right)\left(\frac{\alpha}{\mu}\right)\right]. \tag{C16}$$

In many applications, it is commonly found that $\alpha > 3$, in which case, then, Equation (C16) can be given approximately by

$$N_{GC} \doteq \frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2\left[1 + \left(\frac{\alpha}{\mu}\right)^2 + 2\sqrt{\frac{2}{\pi}}\left(\frac{\alpha}{\mu}\right)\right] \tag{C17}$$

For the exponential density case, the quantity A becomes

$$A = \frac{1}{\sqrt{2}}\,[1 - e^{-\sqrt{2}\,\alpha}(\sqrt{2}\,\alpha+1)], \tag{C18}$$

and the granular noise power is then

$$N_{GC} = \frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2\left\{1 + \left(\frac{\alpha}{\mu}\right)^2 + \sqrt{2}\left(\frac{\alpha}{\mu}\right)\left[1 - e^{-\sqrt{2}\,\alpha}(\sqrt{2}\,\alpha+1)\right]\right\} . \tag{C19}$$

Again, if $\alpha > 3$, then Equation (C19) can be given approximately by

$$N_{GC} \doteq \frac{1}{3}\left[\frac{\ln(1+\mu)}{2^B}\right]^2\left[1 + \left(\frac{\alpha}{\mu}\right)^2 + \sqrt{2}\left(\frac{\alpha}{\mu}\right)\right]. \tag{C20}$$

APPENDIX D

COMPUTER SIMULATION OF LINEAR AND ADAPTIVE DM

The computer simulations of linear and adaptive DM were accomplished using a Monte Carlo method reported by O'Neal.[32] Employing a FORTRAN program, he simulated linear DM, and used independent random numbers with a Gaussian distribution to simulate a flat bandlimited (i.e., uniform power spectrum) input signal. In this appendix, the method used by O'Neal is reviewed, the modifications necessary to simulate discrete adaptive DM are discussed, and the numerical results of all computer simulations are presented. The following were simulated as part of this investigation:

(1) Linear DM with variable step size and bandwidth expansion factor.

(2) Discrete adaptive DM with variable step size, intermediate gain factors, final gain factor, and bandwidth expansion factor.

(3) Message signals having Gaussian and exponential amplitude probability densities, with variable amplitude limiting.

(4) Message signals having uniform and integrated power spectra.

Figure D1 illustrates the block diagram simulated on the computer for the case of linear DM with single ideal integration. It is not necessary to simulate the DM system decoder since the logic in the encoder feedback path is identical to that of the decoder. The adder and delay element in the feedback path form an accumulator (i.e., the integrator illustrated in Figure 2-1). The delay time is simply that of one sample.

A message signal having a uniform power spectrum and a bandwidth of one half the sampling rate is easily simulated by using independent random numbers. This of course yields a value of unity for the bandwidth expansion factor, B. To obtain different values of the bandwidth expansion factor, it is simply necessary to filter the random samples with a digital low-pass filter whose cutoff frequency is the fraction 1/2B of the sampling rate. O'Neal[32] used a nonrecursive digital filter which obtained its low-pass characteristic by convoluting the input signal samples with a sequence of numbers representing the digital filter impulse response. Digital filtering techniques of this type have been reported by Kaiser.[23] A message signal having an integrated power spectrum is easily simulated by passing the random samples through a digital simulation

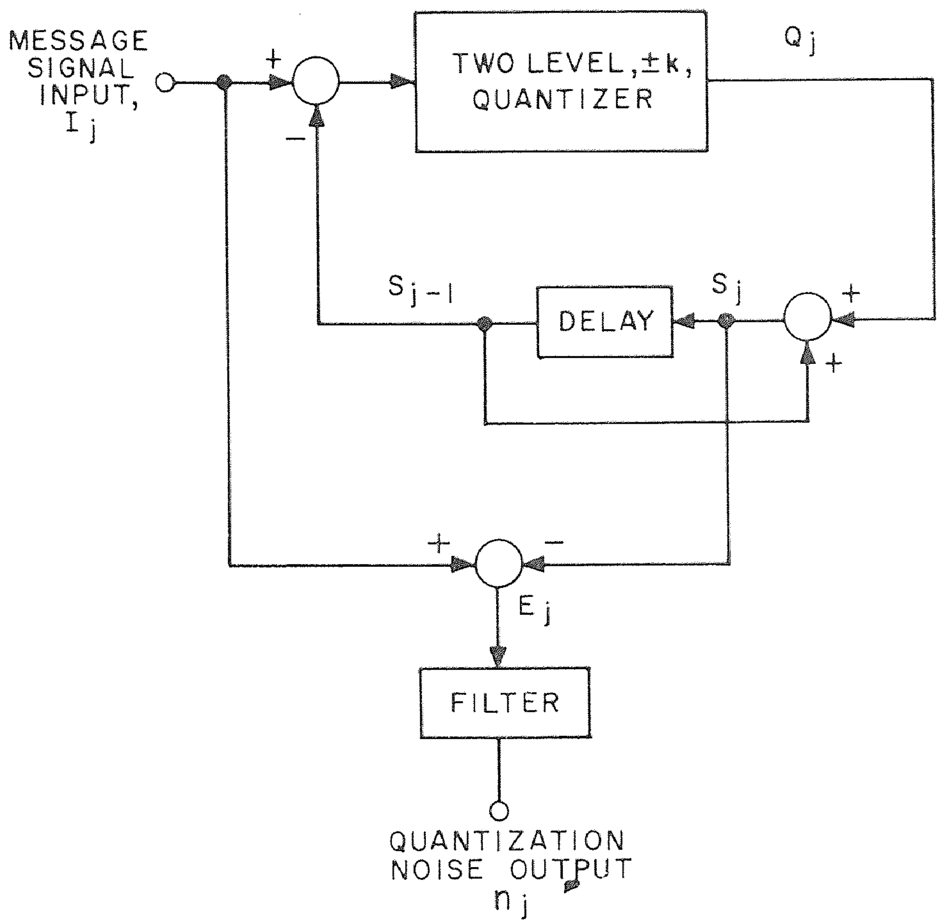


FIG. D1 BLOCK DIAGRAM OF LINEAR DM COMPUTER SIMULATION.

of a low-pass resistance capacitance network.[32] The input sequence of random samples is obtained from published tables, such as those of the RAND Corporation.[36]

The program required to accomplish the functions of linear DM operation is simple. Given the input sequence of numbers I_j with $j = 1,2,3,\ldots,J$ where the amplitude density and power spectrum of this sequence are those stated above, then the quantizer output Q_j in the interval j has a magnitude given by the step size k, and a sign given by the difference $(I_j - S_{j-1})$ or

$$Q_j = \{\text{Sign}(I_j - S_{j-1})\} \; |k| \tag{D1}$$

where S_{j-1} is the summation of all previous values of Q_{j-1}, or

$$S_{j-1} = Q_1 + Q_2 + \ldots + Q_{j-1}. \tag{D2}$$

The method by which these operations are accomplished is illustrated in Figure D1. The total quantization noise or error E_j in the interval j is simply

$$E_j = (I_j - S_j) \tag{D3}$$

To obtain the quantization noise power N_Q, it is necessary to filter the sequence E_j into the sequence n_j, and then average the sum of the squares of the sequence n_j over the number of samples used, or

$$N_Q = \frac{\sum_{j=1}^{J} n_j}{J} \qquad \text{(D4)}$$

The number of samples, J, used to represent the signal in the examples presented earlier for linear and adaptive DM is 5000. Although as few as 500 samples produced results which differed from those presented by less than 1.5 decibels, in all simulations given in this work 5000 samples were used. Figure D2 illustrates the effect on S/N_Q by the use of either 500, 1000, 2000, 4000, or 5000 samples for the case of a message signal having a uniform spectrum and Gaussian signal density, with a bandwidth expansion factor value of eight. The case of 5000 samples is represented in Figure D2 by dashed lines for the three values of the slope loading factor illustrated (i.e., Δ = 0.55, 2.2, and 8.8). The results of using 5000 samples for a broadband signal were illustrated in Figure 3-3 of Section 3.3. The results illustrated in Figure D2 show that the variation of S/N_Q with number of input samples is small. Table D1 presents the numerical results of S/N_Q expressed in decibels from

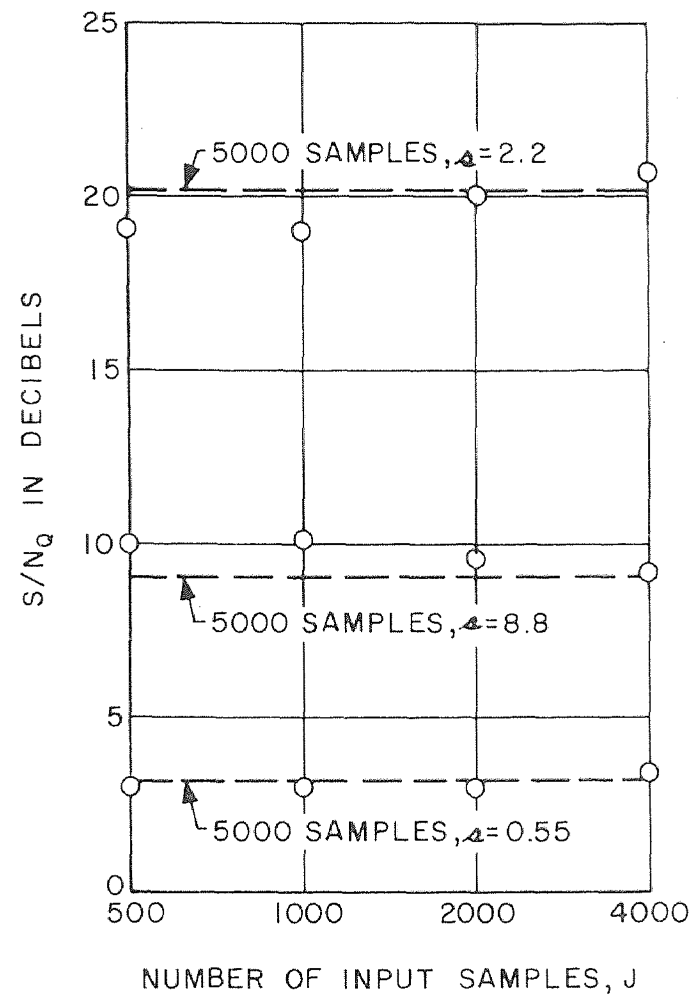


FIG. D2 S/N_Q PERFORMANCE OF LINEAR DM FROM COMPUTER SIMULATION RESULTS; UNIFORM SPECTRUM, GAUSSIAN DENSITY, B = 8.

TABLE D1

Computer Simulation Results for Linear DM, Gaussian Signal Density, Uniform Signal Spectrum, B = 8, with Several Input Sample Sizes.

Number of Input Samples	S/N_Q in Decibels for Following Step Sizes									
	0.090	0.125	0.18	0.25	0.36	0.50	0.72	1.0	1.4	2.0
500	1.8	3.0	4.6	7.7	12.8	19.0	19.8	18.2	13.6	10.1
1000	2.0	2.9	4.5	7.1	12.7	19.1	20.8	17.9	13.9	10.2
2000	1.9	3.0	4.7	8.0	14.1	20.1	20.5	17.3	13.4	9.7
4000	2.2	3.3	5.3	8.7	15.1	20.6	19.8	16.7	13.1	9.3
5000	2.0	3.2	5.1	8.4	14.2	20.1	19.5	16.6	12.7	9.0

computer simulating linear DM with a signal having a Gaussian density, unit variance, uniform spectrum, and a bandwidth expansion factor value of eight. The tabulations of S/N_Q are given in Table D1 for step sizes of 0.090, 0.125, 0.18, 0.25, 0.36, 0.50, 0.72, 1.0, 1.4, and 2.0. The slope loading factor corresponding to each of these step sizes can be calculated from the results given in Table 3-3 of Section 3.3.

The S/N_Q results from computer simulations of linear DM at various step sizes for television, speech, and broadband signals are given in Tables D2 through D5 inclusive. In all cases, the number of input samples is 5000. The slope loading factors corresponding to each of the step sizes given can be calculated from the results of Table 3-3 of Section 3.3.

Figure D3 illustrates the block diagram of discrete adaptive DM computer simulation. The accumulator is the same as that of linear DM. The sequence of two consecutive quantizer outputs of the same sign are sensed by the comparator, which in turn activates gain factor increments K_i (i.e., $1 \leq i \leq n$).

The modification of the linear DM program required to include the gain factor increments is simple.

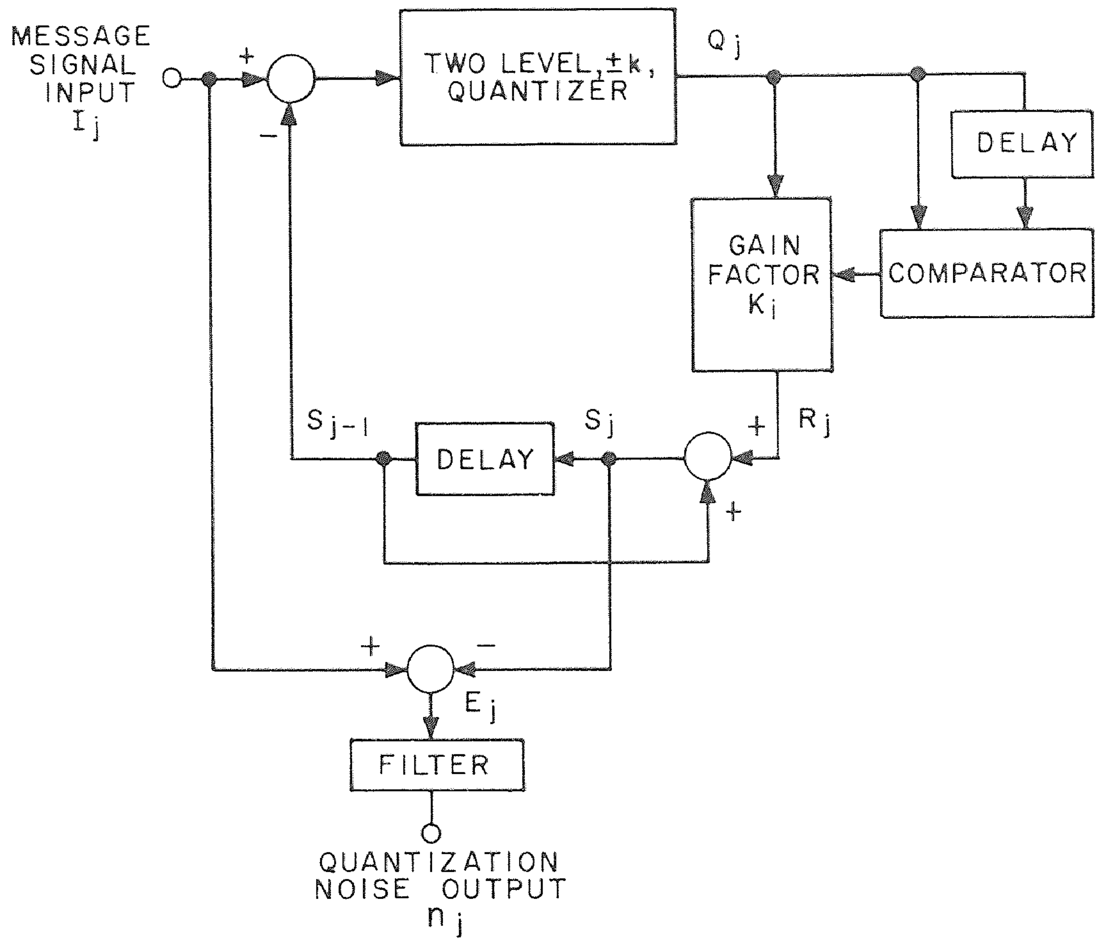


FIG. D3 BLOCK DIAGRAM OF DISCRETE ADAPTIVE DM COMPUTER SIMULATION.

Equations (D1), (D3), and (D4) remain the same. Equation (D2) is changed to read

$$S_{j-1} = R_1 + R_2 + \ldots + R_{j-1} \qquad (D5)$$

where the quantity R_j has the same sign as that of Q_j, but has a magnitude greater than Q_j by the gain factor K_i, or

$$R_j = \{Sign(I_j - S_{j-1})\} \mid K_i k \mid . \qquad (D6)$$

The gain factor K_i is obtained from an IF statement which reads, in effect, that if the signs of Q_{j-1} and Q_j are alike, increase K_i to K_{i+1}; if unalike, decrease K_i to K_{i-1}.

The S/N_Q results from computer simulations of discrete adaptive DM at various step sizes and gain factors for television, speech, and broadband signals are given in Tables D6 through D10. In all cases, the number of input samples is 5000. The normalized slope loading factor corresponding to each of the step sizes given can be calculated from the results of Table 5-1 of Section 5.4.

TABLE D2

Computer Simulation Results for Linear DM with a Television Signal Power Spectrum, for Several Values of the Bandwidth Expansion Factor, B.

Signal Density	B	S/N_Q in Decibels for the Following Step Sizes									
		.031	.045	.0625	.090	.125	.18	.25	.36	.50	.72
Gaussian	2	4.3	6.4	8.8	11.9	15.0	17.8	19.0	18.1	16.1	13.5
	4	7.7	11.2	14.7	19.8	24.7	27.6	26.0	22.7	19.4	15.4
	8	13.9	19.8	27.6	35.5	35.3	31.5	27.8	23.9	20.1	15.8
	16	26.9	36.7	43.9	41.1	37.7	33.1	29.9	24.8	21.0	16.3
	32	45.0	49.5	47.3	41.2	37.4	33.0	28.3	25.0	20.1	16.7
Exponential	2	5.3	7.6	9.8	12.8	16.2	19.0	20.1	18.7	16.7	13.6
	4	9.3	12.8	16.7	21.8	26.9	27.7	25.8	22.1	18.6	14.9
	8	15.7	21.6	27.8	36.3	35.5	31.2	27.9	23.7	19.8	15.5
	16	25.3	34.6	42.5	40.9	37.1	32.6	28.9	24.6	20.2	16.0
	32	53.9	50.8	46.8	42.6	39.1	33.6	29.6	24.8	20.6	16.8

TABLE D3

Computer Simulation Results for Linear DM with a Speech Signal Power Spectrum, for Several Values of the Bandwidth Expansion Factor, B.

Signal Density	B	S/N_Q in Decibels for the Following Step Sizes									
		.125	.18	.25	.36	.50	.72	1.0	1.4	2.0	2.8
Exponential	2	1.8	2.5	3.7	5.2	7.0	8.2	8.1	6.6	4.2	0.9
	4	3.3	5.3	7.9	11.8	15.0	15.9	13.9	10.5	6.7	2.7
	8	7.7	12.3	18.6	24.2	22.7	19.2	15.4	12.0	17.9	3.5
	16	18.0	30.1	32.0	27.7	24.6	19.7	15.9	12.1	8.1	3.7
	32	41.4	37.4	33.6	29.7	24.2	20.5	16.9	13.3	8.1	3.5
Gaussian	2	1.6	2.3	3.2	4.7	6.1	7.7	7.7	6.3	4.3	1.2
	4	3.1	5.0	7.3	11.3	13.8	15.8	14.0	10.8	6.9	3.0
	8	7.3	12.4	19.0	25.4	23.5	19.5	15.6	12.1	7.8	3.5
	16	21.3	34.4	31.9	28.5	24.8	20.5	15.9	12.1	8.1	3.7
	32	40.6	36.9	33.4	28.7	24.7	20.2	16.7	12.7	8.4	3.5

TABLE D4

Computer Simulation Results for Linear DM with a Broadband (i.e., Uniform) Signal Power Spectrum, for Several Values of the Bandwidth Expansion Factor, B.

Signal Density	B	S/N_Q in Decibels for the Following Step Sizes									
		.09	.125	.18	.25	.36	.50	.72	1.0	1.4	2.0
Gaussian	2	0.7	1.0	1.3	1.7	2.3	3.2	4.2	4.9	4.4	3.1
	4	1.2	1.6	2.3	3.3	5.1	7.3	10.0	12.1	10.7	7.7
	8	2.0	3.2	5.1	8.4	14.2	20.1	19.5	16.6	12.7	9.0
	16	5.4	9.1	15.6	27.8	29.2	25.7	21.9	18.7	14.9	9.7
	32	13.1	21.6	36.2	34.5	30.7	27.1	23.5	18.7	14.2	9.8
Exponential	2	0.9	1.1	1.5	1.8	2.7	3.4	4.5	4.9	4.3	2.9
	4	1.2	1.7	2.4	3.5	5.4	8.0	10.7	12.3	10.8	7.7
	8	2.2	3.2	5.2	8.7	14.4	19.0	19.7	17.0	13.4	9.6
	16	4.9	8.0	13.6	22.1	29.1	26.0	22.0	18.4	13.4	9.1
	32	18.5	35.3	38.4	35.2	32.3	28.2	23.7	19.5	14.1	9.6

TABLE D5

Computer Simulation Results for Linear DM with Television Speech, and Broadband Signals, for Several Values of the Amplitude Limiting Factor β

Signal	β	S/N_Q in Decibels for the Following Step Sizes									
		.004	.008	.016	.031	.0625	.125	.25	.50	1.0	2.0
Television (B = 8)	2.5	2.2	4.4	8.4	15.7	27.8	35.5	27.9	19.8	12.0	4.9
	5.0	2.2	4.4	8.4	15.7	27.8	35.5	27.9	19.8	12.0	4.9
	10.0	2.2	4.4	8.4	15.7	27.8	35.5	27.9	19.8	12.0	4.9
Speech (B = 4)	2.0	0.1	0.2	0.5	0.8	1.7	3.6	8.8	16.3	14.1	6.6
	4.0	0.1	0.2	0.4	0.8	1.7	3.6	8.6	15.8	13.9	6.7
	8.0	0.0	0.2	0.4	0.8	1.7	3.6	8.6	15.8	13.9	6.7
Broadband (B = 16)	2.0	0.6	0.7	1.0	1.6	3.4	9.4	27.1	26.5	18.5	10.5

TABLE D6

Computer Simulation Results for Discrete Adaptive DM With a Television Signal Power Spectrum, for Several Intermediate Gain Factor Values K_i, and Several Final Gain Factor Values K_n, at B = 8.

Signal Density	K_i	K_n	S/N_Q in Decibels for the Following Step Sizes									
			.022	.031	.045	.0625	.09	.125	.18	.25	.36	.50
Exponential	i	2	20.4	27.1	35.6	36.0	35.1	33.2	30.8	27.9	23.7	19.8
		3	28.2	33.6	37.4	36.0	34.8	33.2	30.8	27.9	23.7	19.8
		4	33.7	38.0	37.5	35.9	34.7	33.2	30.8	27.9	23.7	19.8
		8	36.5	37.3	37.4	36.0	34.7	33.2	30.8	27.9	23.7	19.8
		16	36.7	37.3	37.4	36.0	34.7	33.2	30.8	27.9	23.7	19.8
	2^i	4	35.2	36.7	36.1	34.6	34.5	33.1	30.6	27.9	23.7	19.8
	None	4	33.7	35.0	33.1	30.8	28.6	26.2	25.1	25.3	23.7	19.8

TABLE D6 (Cont)

Computer Simulation Results for Discrete Adaptive DM
With a Television Signal Power Spectrum, for Several Intermediate Gain
Factor Values K_i, and Several Final Gain Factor Values K_n, at B = 8.

Signal Density	K_i	K_n	S/N_Q in Decibels for the Following Step Sizes									
			.022	.031	.045	.0625	.09	.125	.18	.25	.36	.50
Gaussian	i	2	19.2	26.2	33.9	36.7	35.0	33.2	31.1	27.8	23.9	20.1
		3	27.8	33.2	36.7	35.7	34.1	32.9	31.1	28.0	23.9	20.1
		4	32.6	37.0	36.5	35.5	34.0	32.9	31.1	27.8	23.9	20.1
	2^i	4	32.3	35.9	35.5	34.1	33.8	32.8	31.1	27.8	23.9	20.1

TABLE D7

Computer Simulation Results for Discrete Adaptive DM with a Speech Signal Power Spectrum, for Several Intermediate Gain Factor Values K_i, and Several Final Gain Factor Values K_n.

Signal Density	B	K_i	K_n	S/NQ in Decibels for the Following Step Sizes									
				.045	.0625	.09	.125	.18	.25	.36	.50	.72	1.0
Exponential	4	i	4	5.3	7.8	10.6	13.6	15.8	16.2	16.0	14.9	14.0	12.6
			8	10.7	12.6	14.2	14.4	15.1	15.1	15.7	14.9	13.6	12.4
		2^i	4	5.0	7.2	10.7	13.7	15.5	15.9	14.7	13.9	13.4	12.4
			8	9.6	12.9	13.9	14.7	14.0	13.1	12.9	12.9	12.8	12.1
		None	4	4.8	6.9	10.3	13.9	15.1	13.6	11.5	9.3	6.9	6.1
	8	i	4	-	-	-	25.2	24.9	22.7	23.3	21.3	18.9	15.4
		2^i	4	-	-	-	25.2	23.6	22.8	22.2	21.2	18.9	15.4
Gaussian	8	i	4	-	-	-	25.3	24.4	24.0	22.8	21.6	19.3	15.7
		2^i	4	-	-	-	24.7	23.7	22.5	22.2	21.5	19.1	15.7

TABLE D8

Computer Simulation Results for Discrete Adaptive DM With a Broadband Signal (i.e., Uniform Power Spectrum, Gaussian Density), for Several Intermediate Gain Factor Values K_i, and Several Final Gain Factor Values K_n, at B = 8.

K_i	K_n	S/N_Q in Decibels for the Following Step Sizes																	
		.004	.008	.016	.031	.045	.0625	.09	.125	.18	.25	.36	.50	.71	1.0	1.4	2.0	2.8	4.0
i	2	-	-	-	1.2	1.8	2.8	4.6	7.6	13.6	18.7	21.6	19.4	17.9	15.3	13.3	9.6	5.2	1.0
	3	-	-	-	2.0	3.2	5.2	8.2	12.9	18.1	20.6	20.7	19.0	17.8	15.3	13.3	9.6	5.2	1.0
	4	-	-	-	2.9	4.7	7.6	12.0	17.1	20.5	20.8	20.3	19.0	17.7	15.3	13.3	9.6	5.2	1.0
	8	-	-	-	7.5	12.8	18.0	18.5	20.2	20.7	20.3	20.3	19.0	17.7	15.3	13.3	9.6	5.2	1.0
	16	1.3	3.2	8.2	16.5	-	17.7	-	20.4	-	20.3	-	19.0	-	15.3	-	9.0	-	1.0
	32	3.2	8.1	15.7	17.3	-	17.7	-	20.4	-	20.3	-	19.0	-	15.3	-	9.0	-	1.0
	64	8.2	13.7	16.1	17.3	-	17.7	-	20.4	-	20.3	-	19.0	-	15.3	-	9.0	-	1.0

TABLE D8 (Cont)

Computer Simulation Results for Discrete Adaptive DM
With a Broadband Signal (i.e., Uniform Power Spectrum, Gaussian Density),
for Several Intermediate Gain Factor Values K_i, and Several
Final Gain Factor Values K_n, at B = 8.

K_i	K_n	S/N_Q in Decibels for the Following Step Sizes																	
		.004	.008	.016	.031	.045	.0625	.09	.125	.18	.25	.36	.50	.71	1.0	1.4	2.0	2.8	4.0
2^i	4	-	-	-	2.8	4.5	7.2	12.3	17.8	20.8	20.2	19.0	17.7	17.8	15.3	13.3	9.6	5.2	1.0
	8	-	-	-	7.1	12.1	16.7	20.7	19.9	18.3	18.5	17.7	17.6	15.8	15.3	13.3	9.6	5.2	1.0
	16	1.2	2.8	7.5	16.6	-	17.4	-	17.2	-	17.7	-	17.6	-	15.4	-	9.0	-	1.0
	32	2.9	7.4	16.2	20.0	-	17.4	-	17.4	-	17.7	-	17.6	-	15.4	-	9.0	-	1.0
	64	7.4	16.4	19.2	18.0	-	17.4	-	17.4	-	17.7	-	17.6	-	15.4	-	9.0	-	1.0
None	4								17.5	-	18.0	-	12.6	-	10.2	-	9.5	-	1.0

TABLE D9

Computer Simulation Results for Discrete Adaptive DM With a Signal Uniform Power Spectrum and Exponential Density, for Several Intermediate Gain Factor Values K_i, and Several Final Gain Factor Values K_n, at B = 8.

K_i	K_n	S/N_Q in Decibels for the Following Step Sizes										
		.002	.004	.008	.016	.031	.125	.25	.50	.625	1.0	2.0
i	8	0.6	0.9	1.5	3.2	8.0	21.1	21.2	18.9	18.2	15.9	9.6
	32	1.5	3.3	8.8	14.6	16.9	21.2	21.2	18.9	18.2	15.9	9.6
2^i	8	0.6	0.8	1.4	2.9	7.5	19.6	18.4	17.7	17.5	15.9	9.6
	32	1.4	3.0	7.7	16.6	19.3	17.6	16.9	17.7	17.6	15.9	9.6

TABLE D10

Computer Simulation Results for Discrete Adaptive DM With Television and Speech Signals, for Several Values of the Amplitude Limiting Factor β.

Signal	β	S/N_Q in Decibels for the Following Step Sizes									
		.004	.008	.016	.031	.0625	.125	.25	.50	1.0	2.0
Television ($B = 8$, $K_i = i$, $K_n = 4$)	2.5	8.3	15.8	27.1	38.0	36.0	33.2	27.9	19.8	12.0	4.9
	5.0	8.3	15.8	27.1	38.0	36.0	33.2	27.9	19.8	12.0	4.9
	10.0	8.3	15.8	27.1	38.0	36.0	33.2	27.9	19.8	12.0	4.9
Speech ($B = 4$, $K_i = i$, $K_n = 8$)	2.0	0.8	1.7	3.5	7.8	14.0	15.0	16.4	15.2	12.7	6.7
	4.0	0.9	1.7	3.6	7.9	13.9	14.9	16.1	15.0	12.6	6.7
	8.0	0.9	1.7	3.6	7.9	13.9	14.9	16.1	15.0	12.6	6.7

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