# Linear and nonlinear damage detection using a scanning laser vibrometer

Steve Vanlanduit<sup>a,\*</sup>, Patrick Guillaume<sup>a</sup>, Johan Schoukens<sup>b</sup> and Eli Parloo<sup>a</sup> <sup>a</sup>Department of Mechanical Engineering (WERK), Brussel, Belgium <sup>b</sup>Department of Electrical Engineering (ELEC), Brussel, Belgium

Abstract: Because a Scanning Laser Vibrometer (SLV) can perform vibration measurements with a high spatial resolution, it is an ideal instrument to accurately locate damage in a structure. Unfortunately, the use of linear damage detection features, as for instance FRFs or modal parameters, does not always lead to a successful identification of the damage location. Measurement noise and nonlinear distortions can make the damage detection procedure difficult. In this article, a combined linear-nonlinear strategy to detect and locate damage in a structure with the aid of a SLV, will be proposed. To minimize the effect of noise, the modal parameters will be estimated using a Maximum Likelihood Estimator (MLE). Both noise and nonlinear distortion levels are extracted using the residuals of a two-dimensional spline fit. The validation of the technique will be performed on SLV measurements of a delaminated composite plate.

# 1. Introduction

The detection of structural faults in an early stage is required in different engineering areas: civil, aerospace, .... Traditional techniques used for this goal (X-ray observation, eddy current, ultrasonic scan, infrared tomography, coherent optical analysis, liquid penetration, ...) either require bulky instrumentation and need a large measurement time [5] or are limited to certain materials and specific damage situations (e.g superficial damage). Furthermore the approximate location of the damage should be known a priori and should be accessible.

From the early seventies on different attempts were made to detect damage in a structure from vibration measurements (see [11] for an overview). The main idea is that a change in a structure should imply a change in the modal parameters of the structure (resonance frequencies, mode shapes, mode shape curvatures). Because different damage scenarios, like for instance cracks, result in a nonlinear vibration behavior, the modal parameters are not always suitable as a damage detection feature. Only recently nonlinear damage detection methods were introduced in literature. The majority of the nonlinear techniques, however, uses an analytical model of the structure or damage case, which limits their applicability [1,8,32,45]. In [40] the authors suggested to use higher order FRFs as a sensitive technique to locate the position and depth of a crack. Because higher order FRFs are measured using stepped sine excitation, the measurement time is rather high, especially when one performs high spatial resolution measurements. In this article we propose a fast method which uses a broadband excitation signal to estimate linearized FRFs with an uncertainty level due to the nonlinear distortions. This enables the use of both linear and nonlinear features in the damage detection process without implying excessive measurement times.

Section 2 contains a list of maintained performance criteria to evaluate different damage detection schemes that are compared and applied to a case study further on. In Section 3 we briefly review some classical linear (Section 3.1) and nonlinear (Section 3.3) damage detection techniques. In Section 4 a novel nonlinear damage detection procedure will be elaborated. Damage detection results from scanning laser vibrometer

<sup>\*</sup>Corresponding author: S. Vanlanduit, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium. Tel.: +32 2 629 28 05; Fax: +32 2 629 28 65; E-mail: steve.vanlanduit@vub.ac.be.

(SLV) measurements of a delaminated composite panel are discussed in Section 5. Conclusions drawn from the research can be found in Section 6.

# 2. Performance criteria

Depending on the information that is gained from a damage detection scheme, the existing health monitoring methods can be classified, after Rytter [41], in the following levels :

- Level 1: detection of the presence of damage
- Level 2: determination of the location of the damage
- Level 3: quantification of the severity of the damage
- *Level 4:* estimation of the remaining life of the structure

The contribution in this paper is concentrated on Level 1 and Level 2 damage detection schemes. Level 3 methods usually require more elaborated tests or the availability of a physical model. The determination of the estimated operational time until total structural failure (Level 4) is usually performed with the aid of fracture mechanics, where the growth of the damage is studied.

To evaluate the different linear and nonlinear damage detection schemes included in this study, we use the following criteria:

- **Criterion 1:** *Sensitivity to damage:* is the method capable of detecting damage in an early stage of the deterioration process?
- **Criterion 2:** *Localization:* can the damage be located within sensor or sub-sensor resolution?
- **Criterion 3:** *Damage type identification:* is a separation between different damage scenarios (e.g. cracks, delaminations, ...) possible?
- **Criterion 4:** *Robustness towards changes in the measurement setup:* is the rebuilding of the measurement setup possible without disturbing the damage detection process? Does the method work in operating conditions?
- Criterion 5: *Robustness towards measurement noise:* does the technique work in noisy environments?
- **Criterion 6:** *Simplicity of the experimental setup:* can the measurements be done without complex measurement setups or large measurement times?
- **Criterion 7:** *Analytical model needed:* is a Finite Element Model of the structure required?

**Criterion 8:** *Necessity of reference measurements:* can damage be detected without information about the undamaged state of the structure?

In the current paper the aim is to perform fault detection on complex plate-like structures with the aid of a scanning laser vibrometer. For this specific application some criteria in the above mentioned list are more important compared to the classical damage detection case (i.e. a well known material measured with classical vibration sensors).

A large amount of the published research on structural health monitoring uses a Finite Element Model to detect and localize damage. To some extent the availability of a physical model of both healthy and damaged structure also enables the identification of the severity of the damage (Level 3). For complex real-life structures the demand for a FEM is much too stringent, and the modelling errors in the physical model can be unacceptable to enable a reliable detection of faults. Therefore, all damage detection techniques that need a Finite Element Model (i.e. fail on Criterion 7) are discarded from the present comparison. A literature survey on these techniques can be found in [11].

Criterion 6 is considered to be important because of the high spatial resolution nature of scanning laser vibrometer measurements, which gives rise to a large measurement time (typically a few hours). Increasing the measurement time with an order of magnitude would therefore make the damage detection unfeasible. SLV measurements can be quite noisy, especially if not enough laser light is reflected from the structures surface, which is typically the case for dark non-treated object. Therefore, Criterion 5 is also relevant.

# 3. Traditional damage detection techniques

# 3.1. Linear damage detection

From the seventies on a large amount of publications related to the detection of damage from changes in the resonance frequencies and damping values have been published (see [11] for references). Frequently they are used as a benchmark for testing newly developed techniques.

The main disadvantage of the natural frequency and damping techniques is that they are not very sensitive to small local faults. Depending on the application different researchers report on the lack of sensitivity of system poles to detect damage:

- For large structures, like platforms and buildings, it was noted repeatedly that local damage did not result in important changes of the system poles or the changes were comparable or even smaller than the change due to environmental conditions [3,14, 15,34]. Other authors found that for large structures the natural frequency was sensitive, but the damping value remained unaffected [27].
- The cracked beam is one of the most used test structures to evaluate a newly developed damage detection technique. In [19] it is reported that the resonance frequency change is a poor indicator of structural health of a slotted beam, which is often used to simulate a cracked beam in laboratory conditions. Gudmundson [22] and Ismail et al. [25] remarked that the behavior of the slotted beam is different from a true breathing crack. In their comparison they found that the resonance frequency has a reduced sensitivity with respect to breathing cracks compared to slotted cracks. The small sensitivity of resonance frequencies for beams was also discussed for concrete beams [47].
- For plates, Wolff et al. [56] concluded that a local damage (at the border of the plate) could not be detected with frequency changes. The inspection of forty car doors with spot welds allowed no separation between damaged and intact ones based on the resonance frequencies [42].
- In [48] shell structures with an introduced notch gave no resonance frequency changes exceeding the frequency resolution.
- Different authors report that damage scenarios like fatigue cracks [44] and delaminations [38] in composite materials, do not cause important resonance frequency changes.

Also, since the system poles are global parameters, they can not be used straightforwardly to locate faults. In [6] the authors present a method to find damaged locations from the correlation of different mode pairs. Their method does however not account for multiple damage locations.

Notwithstanding the lower sensitivity of resonance frequencies compared to damping values, the former are sometimes preferred because of their higher accuracy in the presence of high noise levels [13,16,30].

The recent research on accurate modal parameter estimators with computed confidence intervals for the modal parameters, has resulted in resonance frequency and damping techniques which are quite robust with respect to random measurement errors [4,12,36]. Brincker [4] introduced a so-called significance indicator, defined by the ratio of the resonance frequency deviation divided by the standard deviation of the resonance frequency. In this way resonances with a higher confidence are given a higher weight in the indicator function. In [36] it was shown that a proper excitation combined with an accurate modal parameter estimation technique is crucial to obtain reliable damage detection results.

Still, even if random errors are eliminated, it remains very difficult to quantify if a shift in the resonance frequency of a structure is due to damage or changing operating conditions (rebuilding of the test setup, change of the mass due to fuel consumption, change in temperature, humidity, ...).

# 3.2. Mode shapes

Mode shapes have been used as a damage detection feature by different authors and with varying success In order to compare mode shapes, different variations of the Modal Assurance Criterion (MAC) values were proposed in literature [2]. To locate the damage the socalled Coordinate MAC (COMAC) is often used [31].

Other authors use the mode shape curvature as a much more sensitive damage detection technique [35]. With the aid of a cantilevered plate containing a hole Chance et al. [7] show that mode shape curvatures are a better indicator of damage location than mode shape displacements. The main disadvantage of computational schemes to derive mode shape curvatures based on measurement differences [46], is their sensitivity to measurement noise. Because SLV measurements can be quite noisy in real-life situations the use of computed curvatures is not practical. The alternative of using special strain transducers provides a more accurate, but experimentally more demanding solution [9].

# 3.3. Nonlinear damage detection techniques

#### 3.4. Introduction

The main part of the existing nonlinear damage detection techniques use a nonlinear finite element model to correlate the vibration response of a structure with the analytical model of a damaged structure. The overview in [11] contains references to papers which analytically model the vibration of a cracked beam. Several authors discuss the choice of the model which can be used to describe the behavior of a breathing crack. Friswel et al. [20] simulated the nonlinear motion of a closing crack with a bilinear stiffness. In [8], a closed form solution of a crack – vibrating as a bilinear oscillator – is determined. The majority of the researchers consider the crack to be either open or closed (e.g. depending on the sign of the bending moment of the crack location as discussed in [1]). [29], however, models the crack with a torsional spring, allowing a continuously varying stiffness near the crack location. Although the models of damage scenarios like cracks get more and more complex, the true physical behavior can still not be modelled accurately. Moreover, by using an analytical model one of the strengths of nonlinear detection techniques – the operation without prior models or reference data – is abolished. Therefore, the use of experimental techniques is necessary.

#### 3.5. Experimental approaches

During the last decade more researchers investigated the possibility of using experimental nonlinear features to detect structural faults. Most application examples concentrate on the detection of cracks, which typically exhibit nonlinear behavior as was remarked by Gudmundson [22].

Research has been done on the detection of faults by quantifying the distortion introduced in the response of the structure under sinusoidal excitation. Several publications use a stepped sine procedure to measure higher order FRFS (HOFRF) as presented in [49]. Results of HOFRFs from a numerical simulation of a cracked beam are discussed in [40]. Tsyfansky et al. [51] show that for the particular example of fatigue cracks, the stepped sine nonlinear technique obtains a better sensitivity compared to classically applied linear techniques. Another approach involving stepped sine distortion measurements uses high frequency acoustic measurements at selected frequencies [52,53]. The method is called Single Mode Nonlinear Resonance Acoustic Spectroscopy (SIMONRAS) by the authors.

Zaitsev et al. [57] uses the inter-frequency modulation between a low and a high frequency sine to detect faults with the aid of vibro-acoustical diagnostic tools. The same technique has been introduced, earlier, in [28], and was applied with success to plexiglass, sandstone objects and on a cracked engine [26].

In [24] higher order statistics are applied to detect faults in a structure excited with random excitation. George et al. [21] apply bi-spectral analysis to a damaged concrete column and Rivola et al. [39] use similar ideas for detecting cracks.

Prime et al. [37] applied a variety of signal processing techniques to locate nonlinearities in a device consisting of 3 pieces of polycarbonate bonded together to simulate and opening and closing crack. One of their approaches is to estimate the instantaneous frequency with the Wigner-Ville transform, as opposed to the Hilbert transform which was used earlier for this goal [18]. The merits of another signal processing techniques, the wavelet transform, are discussed in more detail in [50].

Although many damage scenarios for particular structures result in nonlinear vibration, the location of the nonlinearity does not always indicate the position of the damage. In [32] the authors showed that the introduction of a localized nonlinearity in a rectangular frame also resulted in nonlinear vibrations in other parts of the frame.

On of the main disadvantages of the nonlinear detection techniques which are currently used as a damage detection technique, is their large measurement time. With the introduction of adapted multisine excitation signals to detect nonlinear distortions, faster experimental alternatives are available. An example of this class of excitation techniques is the so-called oddodd multisine, which contains energy only at a selected set of frequency lines. In [54] the oddodd multisine was used with success for damage detection purposes. Although the oddodd multisine technique is faster than methods based on sinusoidal excitation, it is still several times slower than a classical linear FRF measurement performed with a full spectrum excitation signal. In the next section a method to estimate FRFs together with noise levels and nonlinear distortion levels will be outlined. The measurement time will appear to be approximately the same as a classical linear FRF measurement.

# 4. The proposed technique

#### 4.1. Theoretical background

The proposed technique will make use of a so-called 'random multisine'. This is a broadband excitation signal with an arbitrary, user-defined amplitude spectrum and a random phase spectrum, i.e.:

**Definition 1** A signal is a random multisine if

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=-N}^{N} X_k e^{\frac{j2\pi f_{\max}kt}{N}}$$

with  $X_k = X_{-k}^* = X(k)e^{j\phi_k}$ ,  $f_{\max}$  the maximum frequency,  $N \in \mathbf{N}$  and  $\phi_k$  are a realization of an independent uniformly distributed random process on  $[0, 2\pi]$ .

In the modal analysis community this excitation signal is often called 'pseudo-random noise' when all amplitudes are equal. Because we use the random multisine signal in the broader context of periodic excitation signals with an arbitrary amplitude spectrum we will maintain the 'multisine' terminology which originates from system identification. In [43] it was shown that under certain assumptions the FRF of the system can be written as:

$$G(\omega_k) = G_0(\omega_k) + G_B(\omega_k)$$
$$+ G_S(\omega_k) + N_G(\omega_k)$$

with  $G_0(\omega_k)$  the underlying linear system,  $G_B(\omega_k)$ the systematic error due to the nonlinear contribution,  $G_S(\omega_k)$  the stochastic nonlinear contribution (SNLD) and  $N_G(\omega_k)$  the errors due to the output noise. The paper [43] also shows that averaging the FRF of the system over different realizations of the random multisine with the same amplitude spectrum, results in convergence to a FRF of a linear system which is called the Related Linear Dynamic System (RLDS) and  $G_{\text{RLDS}}(\omega_k) = G_0(\omega_k) + G_B(\omega_k)$ . The variance  $\sigma^2_{\rm BLDS}$  of this averaging process can be used as a measure of the uncertainty due to both noise and stochastic nonlinear distortions. The uncertainty due to noise only can be estimated using the variance  $\sigma_N^2$  of an averaging process over several equal realizations. In this manner noise uncertainty  $\sigma_N$  and nonlinear distortions  $\sigma_{\text{SNLD}} = \sqrt{\sigma_{\text{RLDS}}^2 - \sigma_N^2}$  can be separated (although in practice this equation is only valid for  $\sigma_{\text{RLDS}}^2 \ge \sigma_N^2$ ).

For measurements on complex plate-like structures where typically thousands of outputs are measured using a scanning laser vibrometer, these averaging processes would require a large measuring time. In the following section a process will be described to estimate noise information and nonlinear distortions from spatial averages of the residual errors of a spline fit of the high resolution vibration patterns.

#### 4.2. A nonlinear distortion estimation algorithm

Because the technique we propose uses residual errors of a spline fit to estimate nonlinearities we will denote the method Residual Estimation of Nonlinear Distortions (or in short REND). Although different versions of the method can be used, only the spatial domain variant is elaborated in this paper (this means that the spline fit is applied onto vibration patterns at different frequencies). Furthermore, a random selection out of  $N_{\text{reals}}$  different random multisine realizations is taken

55 55 10 10 10 10 180 180

Fig. 1. Dimensions (in mm) of the delaminated polyester plate. The dotted line denotes the delaminated area.



Fig. 2. Photo of the measurement setup.

to excite the structure each time a different location is measured. The algorithm contains the following steps: Procedure: Random scan spatial domain REND

- 1. Generate a vector R of  $N_o$  discrete random numbers between 1 and the number of desired different multisine realizations  $N_{\text{reals}}$ .
- 2. For  $r = 1, ..., N_{\text{reals}}$  measure the forces and responses for all outputs with a value r in the vector R generated in Step 1. while exciting the system with a single multisine realization.
- 3. Calculate the FRFs  $H(\omega_k, x_r, y_r)$  at frequency  $\omega_k$  and output location  $(x_r, y_r)$  for  $k = 1, \ldots, N_f$  and  $r = 1, \ldots, N_o$ .
- Compute the two-dimensional approximating splines s<sub>ωk</sub>(x, y) of H(ω<sub>k</sub>, x, y) for k = 1,..., N<sub>f</sub> [10].
- 5. Compute the spline fit residuals:

$$e_{\omega_k}(x,y) = s_{\omega_k}(x,y) - H(\omega_k, x, y) \quad (1)$$



Fig. 3. Relative change of resonance frequencies (left) and relative change of damping values (right) between damaged and undamaged states of a cracked plate. The estimates are computed using the classical LSCE estimator.



Fig. 4. MAC values between 10 selected modes for damaged and undamaged states, estimated with the LSCE-LSFD estimator.

for  $k = 1, ..., N_f$ .

- 6. Generate matrices  $I_1$  and  $I_2$ , with  $N_o$  rows and  $N_{\rm filt}$  columns. In each row i in  $I_1$ , store the  $N_{\rm filt}$  output numbers  $r_1, \ldots, r_{N_{\rm filt}}$  with the closest positions with respect to  $(x_i, y_i)$  and which have the same realization number as output i (the realization numbers were stored in the vector R). In each row j in  $I_2$ , store the  $N_{\rm filt}$  closest output numbers which have a realization number different from output j.
- Estimate the variances 
   <sup>2</sup>
   <sup>2</sup>
   <sub>N</sub>(ω<sub>k</sub>, x<sub>i</sub>, y<sub>i</sub>) and 
   <sup>2</sup>
   <sub>RLDS</sub>
   <sub>(ω<sub>k</sub>, x<sub>i</sub>, y<sub>i</sub>) of output i at position (x<sub>i</sub>, y<sub>i</sub>), using
   local residuals:
  </sub>

$$\hat{\sigma}_{N}^{2}(\omega_{k}, x_{i}, y_{i}) = \alpha \sum_{m=1}^{N_{\text{filt}}} \left| e_{\omega_{k}}(x_{\mathbf{I}_{\mathbf{I}}(i,m)}, y_{\mathbf{I}_{\mathbf{I}}(i,m)}) -\beta \sum_{n=1}^{N_{\text{filt}}} e_{\omega_{k}}(x_{\mathbf{I}_{\mathbf{I}}(i,n)}, y_{\mathbf{I}_{\mathbf{I}}(i,n)}) \right|^{2}$$
(2)

$$\sigma_{\text{RLDS}}^{\circ}(\omega_k, x_i, y_i) = \alpha \sum_{m=1}^{N_{\text{filt}}} \left| e_{\omega_k}(x_{\mathbf{I}_2(i,m)}, y_{\mathbf{I}_2(i,m)}) \right|$$
(3)

~ 2



Fig. 5. COMAC values between damaged and undamaged states, for LSCE-LSFD mode shapes.

$$-\beta \sum_{n=1}^{N_{\text{filt}}} e_{\omega_k}(x_{\mathbf{I}_2(i,n)}, y_{\mathbf{I}_2(i,n)}) \Big|^2$$
for  $k = 1, \dots, N_f$ , with  $\alpha = \frac{1}{N_{\text{filt}} - 1}$  and  $\beta = \frac{1}{N_{\text{filt}}}$ .

When  $N_o$  spatial locations have to be measured with a frequency resolution of  $f_0$ , the total measurement time is equal to  $T = \frac{N_o + 10N_{\text{reals}}}{f_0}$  (where ten periods are used before a measurement is taken to assure steady state conditions every time a new realization is used). When  $N_{0}$  is large (which is the case for scanning laser vibrometer measurements) the measurement time will be comparable with a classical linear FRF measurement, where no noise and nonlinear distortion levels are available. A large number of realizations will result in better estimates of the NLD, at the price of worse noise level estimates and a larger measurement time. As a compromise 25 realizations usually gives both acceptable noise and NLD level estimates, with a typical accuracy around 15%. This value is quite good compared with the 99% confidence interval  $[0.48\sigma, 1.55\sigma]$ for estimating the variance  $\sigma$  of a standard normal distribution from say 10 repeated measurements.

# 4.3. A combined linear-nonlinear damage detection scheme

The spline approximation computed in Step 4 of the REND algorithm results in the linearized FRF  $G_{\text{RLDS}}(\omega_k)$ . Moreover, the values  $\sigma_{\text{RLDS}}^2$  represent uncertainties on these linearized FRFs. A first part in the proposed linear-nonlinear approach consists of using the FRFS and the uncertainties as inputs of an ML modal parameter estimation algorithm as described in [23]. The result is an accurate set of modal parameters with confidence bounds. Moreover, because of the smoothing effect of the spline approximation, the SNR will be larger than for the raw measurements, and therefore in general averaging is needed to obtain a reasonable measurement quality. Based on the modal parameters and the uncertainties it is then possible to quantify if a change in modal parameters is due to a structural fault or if is merely attributed to the measurement noise.

In a second part of the combined linear-nonlinear method, the nonparametric estimates of the nonlinear distortions are used to detect the presence and the location of damage. Currently this qualification is done visually but an automatization by means of statistical classification algorithms is possible [17].

#### 5. Measurements

# 5.1. Experimental Setup

To validate the combined linear-nonlinear damage detection scheme, a case study was performed. The test object was a glass fiber composite plate with a polyester matrix. To produce a delamination the last layer was applied while the delaminated zone was already hardened (see Fig. 1 for dimensions of the test specimen). The undamaged state was obtained afterwards, by inserting polyester between the layers in the delaminated zone.



Fig. 6. Relative change of resonance frequencies (left) and damping values (right) between damaged and undamaged states (top) and uncertainty on resonance frequency and damping value estimates (bottom). Both estimates and uncertainties are computed using the proposed MLE procedure (Section 4.3).

The measurements were performed with an Ometron VPI sensor, and an LMS controller was used for positioning the laser mirrors. The controller was operated from MATLAB through the RS232 serial port. The acquisition of the measurements was done with a 16 channel 12 bit National Instruments measurement system, with external DIFA anti-alias filters. The instrumentation was driven using MATLAB software. The plates were excited with a Bruël&Kjær shaker (see Fig. 2).

# 5.2. Damage detection results

# 5.2.1. Modal parameter changes

In Fig. 3 the relative deviation between damaged and undamaged state of the resonance frequencies and damping values, computed using the well-known Least Squares Complex Exponential method (LSCE) [55], are shown. It can be observed that the damping values undergo a large change, while the resonance frequencies only vary about one percent. However, since no uncertainty on the modal parameters is available, it cannot be determined if this change is due to damage or if it is merely the consequence of the statistical variation of the parameters.

The same is true for the mode shapes which are estimated using the Least Squares Frequency Domain method [33], and compared using the MAC values (see Fig. 4). At first glance it would be concluded from Fig. 4 that the mode shapes at higher frequencies (i.e. mode numbers four to ten) enable to distinguish between damaged and undamaged states. However, from the COMAC values (see Fig. 5) it is clear that the estimated mode shapes are very noisy and could lead to wrong conclusions.



Fig. 7. MAC values between damaged and undamaged states, for MLE mode shapes.



Fig. 8. COMAC values between damaged and undamaged states, for MLE mode shapes.

From the results of the ML estimator in the proposed combined linear-nonlinear scheme (Fig. 6) it can be observed that the deviation of the damping values is a lot smaller than in the LSCE case in Fig. 3. More important, it can be seen from the uncertainties on the damping values in Fig. 6 that a large portion of the damping value change is due to measurement noise. The resonance frequencies are much more reliable, although they are not very sensitive to damage.

# 5.2.2. The combined linear-nonlinear approach

The ML MAC values for mode shapes at higher frequencies (Fig. 7) indicate that damage is present. In contradiction to the LSFD case discussed before, the low MAC values are not cause by measurement noise. Indeed, from the COMAC values in Fig. 8 the location of the delamination can be identified (compare Figs 1 and 8). When only the first six mode shapes are used to compute the COMAC values, the location of the damage is not possible anymore (see Fig. 9). When instead of the mode shapes the Laplacian of the mode shapes is used, the lower frequency mode shapes also indicate the presence of the damage location (the Laplacian or  $\Delta$  operator of a function f(x, y) is given by:  $\Delta f = \frac{\delta^2 f(x,y)}{\delta x^2} + \frac{\delta^2 f(x,y)}{\delta y^2}$ ). Indeed, from Fig. 10 it can be seen that in the Laplacian of the fifth mode



Fig. 9. COMAC values between damaged and undamaged states, for MLE. Only the first six mode shapes are used to calculate the COMAC values.



Fig. 10. Laplacians of the fifth mode shape extracted from measurements of an undamaged (left) and damaged (right) delaminated plate. The Laplacian is computed using differences of the spline approximation coefficients.

shape, the area which contains the delamination is visible. The Laplacian can be computed efficiently using differences of the spline approximation coefficients which are calculated in the combined linear-nonlinear approach. Unfortunately, the Laplacian is very sensitive to noise.

For more complex structures and larger inter-test time intervals (summer-winter) the test variability will be significant. Although the test variability can be reduced using adapted test procedures (non-contacting excitation, controlled environmental conditions,  $\ldots$ ), it cannot be completely eliminated and will form one of the weak points of linear damage detection techniques.

Since structural variations resulting from environmental changes are usually small, they will in general still result in linear behavior. Therefore, the authors believe that the use of nonlinear techniques could be much more generally applicable. Moreover, if it is assumed that the sound structure does not show any nonlinear behavior at a specific excitation level, no reference measurements are needed to compare the damaged and undamaged states of a device.

Using the technique proposed in Section 4.2 the nonlinear distortions can be estimated without significant effort. In this particular case the measurement took about 4.4 minutes (to measure  $N_o = 1500$  outputs with a frequency resolution of  $f_0 = 5.86$  Hz and 50 dif-



Fig. 11. Mean FRFs and nonlinear distortions extracted with the proposed spatial residual technique. Top: near the location of the delamination, bottom: opposite to the location of the delamination.



Fig. 12. Spatial view of the FRF nonlinear distortion extracted with the proposed spatial residual technique, for undamaged (left) and damaged (right) states of the device under test.

ferent realizations). The results show that the nonlinearity is higher for the damaged plate at locations near the delamination (see Fig. 11). Because the NLDs are compared with the noise levels (which are not shown here for clarity of the figure), the technique also works in the presence of high noise levels. Also, the location of the delamination can be found back from the spatial view (i.e. surface plot of the NLDs at all outputs and one particular frequency) of the nonlinear distortions in Fig. 12.

# 6. Conclusions

In this article we have shown by means a case studies that the use of nonlinear features can indeed give additional information on the presence and the location of damage in a structure. Furthermore we have introduced a technique which allows the determination of both linear and nonlinear features without adding measurement time compared to classical linear FRF measurements. In addition, uncertainties on the computed modal parameters are available to quantify the stochastic variability, and consequently to decide if a change in the modal parameters is due to a structural fault or measurement noise.

All conclusions concerning the performance of the damage detection techniques based on the current research are summarized in Table 6. Is is important to remark that the value of using nonlinear features in the damage detection process is case dependent. However,

Table 1
Overview of performance criteria for different techniques (see Section 2 for the criteria descriptions), based on the test included in the current
research. $++:$ very good, $+:$ good, 0: moderate, $-:$ bad, $:$ very bad

Method versus Criterion	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6	Criterion 7	Criterion 8	Average
Classical Approaches									
Resonance freqs. (LSCE)	0			_	_	++	++		-/0
Damping values (LSCE)	+			_		++	++		-/0
MAC values (LSFD)	+		_	_		++	++		-/0
COMAC values (LSFD)	+	+	—	_		++	+ +		0
Proposed Approach (using spline fit + MLE + REND, see Section 4.3)									
Resonance freqs.	0			_	+	++	++		0
Damping values	+			_	0	++	++		0
MAC values	+		_	_	0	++	++		0
COMAC values	+	+	_	_	0	++	++		0
Laplacian	++	++	_	_	_	++	++		0/+
REND, Section 4.2	+	+	0	+	+	+	++	0	+

with the introduction of the proposed technique both linear and nonlinear features are available, and thus conclusions on the state of a structure can be drawn even of no nonlinearities are introduced in the damaged structure.

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