Linear Companding Transform for the Reduction of Peak-to-Average Power Ratio of OFDM Signals

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ABSTRACT

A major drawback of orthogonal frequency-division multiplexing (OFDM) signals is their high peak-to-average power ratio (PAPR), which causes serious degradation in performance when a nonlinear power amplifier (PA) is used. Companding transform is a well-known method to reduce PAPR without restrictions on system parameters such as number of subcarriers, frame format and constellation type. Recently, a linear nonsymmetrical companding transform (LNST) that has better performance than logarithmic-based transforms such as μ-law companding is proposed. In this paper, we propose a new linear companding transform (LCT) with more design flexibility than LNST. Computer simulations show that the proposed transform has a better PAPR reduction and bit error rate (BER) performance than LNST with better power spectral density (PSD), an excellent BER performance can be achieved by the proposed transform with small input backoffs (IBO) to the power amplifier.

Key Words: Orthogonal frequency-division multiplexing (OFDM), peak-to-average power ratio (PAPR), linear companding transform (LCT), nonlinear power amplifier (PA).

1. Introduction

Orthogonal frequency-division multiplexing (OFDM) is a multicarrier modulation scheme that divides the incoming bitstream into parallel, lower rate substreams and transmits them over orthogonal subcarriers, so that the bandwidth of each subcarrier is much smaller than channel's coherence bandwidth and hence, each subcarrier will experience relatively flat fade [1]. It is a bandwidth efficient modulation scheme and has the advantage of mitigating inter-symbol interference (ISI) in frequency selective fading channels. OFDM today used in many wireless standards such as terrestrial digital video broadcasting (DVB-T), digital audio broadcasting (DAB-T), and adopted in wireless local area networks (WLANs) (IEEE 802.11a, ETSI Hiperlan2) and wireless metropolitan area networks (IEEE 802.16d). The main drawback of OFDM is its high peak-to-average power ratio (PAPR) which causes serious degradation in performance when nonlinear power amplifier (PA) is used. This high PAPR forces the transmit PA to have a large input backoff (IBO) in order to ensure linear amplification of the signal, which significantly reduces the efficiency of the amplifier, Furthermore, high PAPR requires high resolution for the receiver analog-to-digital converter (A/D). Since the dynamic range of the signal is much larger for high PAPR, a high-resolution quantizer is required to reduce quantization error, which requires more bits and places a complexity and power burden on the

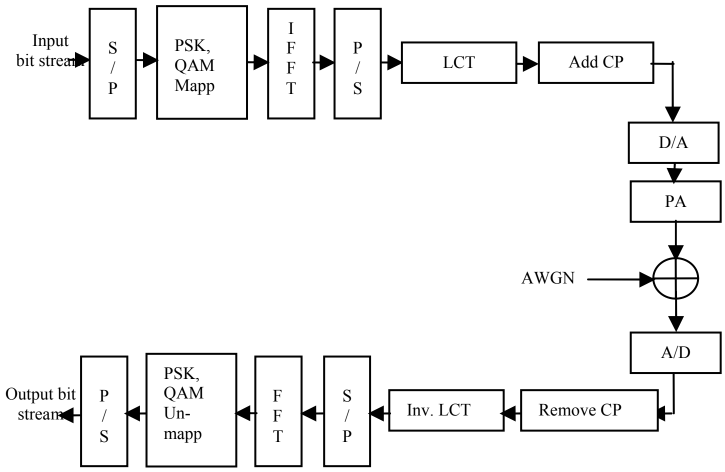


Figure1: Typical companded OFDM system.

receiver front end. In the literature, many solutions have been proposed to reduce PAPR such as block coding, selective mapping (SLM), partial transmit sequence (PTS), tone reservation and injection, [2, & reference therein]. However, most of these solutions have restrictions on system parameters such as number of subcarriers, frame format, and constellation type. Signal distortion solutions such as clipping [3][4] and companding [5-17] can be used without restriction on the system parameters but at the price of increased bit error rate (BER) and spectral regrowth. Although clipping performs very well with low modulation orders, clipping error becomes very significant with higher orders and seriously degrades performance [17], which makes companding more suitable for high data rates applications. The use of μ-law companding as PAPR reduction scheme for OFDM systems is firstly investigated in [5], where the authors did an elegant theoretical performance analysis of companded OFDM signals. However, their work only considered the effect of quantization noise and ignored PA nonlinearity. In [12], a general companding transform is proposed, where the performance of four typical companding schemes, namely, linear symmetrical transform (LST), linear nonsymmetrical transform (LNST), nonlinear symmetrical transform (NLST), and nonlinear nonsymmetrical transform (NLNST), is investigated. It shown that, LNST is the best among the proposed companding schemes in terms of PAPR reduction and BER. These performance gains were achieved by introducing an inflexion point in LNST so that small and large signal's amplitudes could be treated with different scales, which gives more flexibility and freedom in companding design to meet the system requirements such as PAPR reduction, required signal's average power, Power amplifier characteristics, and BER. However, when the input signal pass through the inflexion threshold, transformed signal will have abrupt jump, which degrade the power spectral density (PSD) of transformed signal. In [16], the authors proposed a linear transform that has one-to-one mapping between the input and the transformed signals, the companding form is designed so that the output signal has no abrupt jumps, which resulted in a better PSD. However, its PAPR reduction capability and BER performance are lower than LNST. Furthermore, the effect of PA nonlinearity is ignored. In this paper, we propose a new linear companding transform (LCT) with two inflexion points to give more design flexibility, the performance of the proposed transform and LNST is evaluated in AWGN channel with the presence of nonlinear amplification using computer simulations, results show that the proposed transform has a better PAPR reduction and BER than LNST with better PSD. Moreover, with small *IBO*, an excellent BER is achieved by the proposed transform. The rest of paper is outlined as follows. Section 2 addresses the PAPR of OFDM signals. Section 3 introduces the proposed linear transform and discusses the design criteria. Section 4 introduces the nonlinear power amplifier model that used in the simulation, while section 5 discusses simulation results. The paper is finally concluded in section 6.

2. PAPR Formulation

Figure 1 shows a typical companded OFDM system, where input bit stream is first converted into N parallel lower rate bit streams and then fed into symbol mapping to obtain symbols [$S_k = S_0, S_1, \ldots, S_{N-1}$]. In order to obtain a real output of IFFT, S_k is made conjugate symmetric [18]

$$S_{N/2+k} = S^*_{N/2-k},\ k = 1,\ldots,N/2-1 \quad (1)$$

Practically, carriers at DC and Nyquist frequency not used, so for even N,

$$S_0 = S_{N/2} = 0 \quad (2)$$

the output of IFFT can then be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j\frac{2\pi}{N}kn} \quad (3)$$
$$, n = 0,1,\ldots,N-1$$

The PAPR of the discrete OFDM signal may be expressed as

$$PAPR = \frac{\max\{|x(n)|^2\}}{E\{|x(n)|^2\}} \quad (4)$$

If OFDM signal oversampled by a factor ≥ 4, this PAPR is good approximate to the one of continuous OFDM signal [19]. Oversampling of a factor of L can be achieved by padding the symbols S_k with $(L-1)N$ zeros. After IFFT, the resultant symbols are converted to serial and companding is performed. A cyclic prefix (CP) then is inserted to the OFDM symbol interval to eliminate intersymbol interference (ISI).

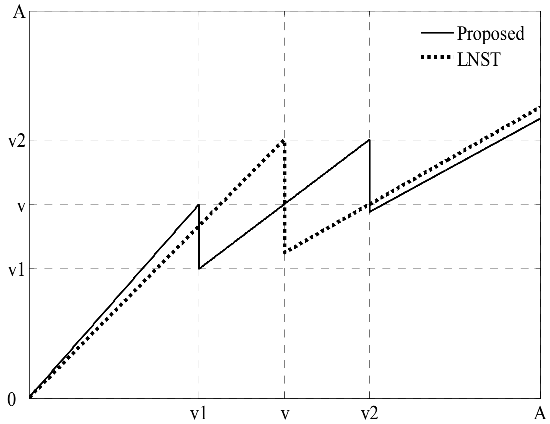


Figure2: Transforms profiles.

3. Proposed Transform

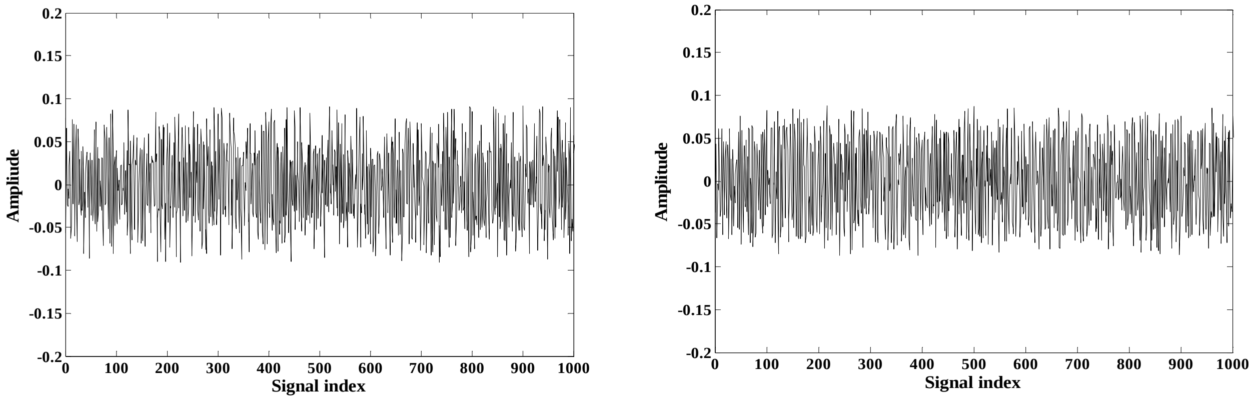
It shown that in [12], a linear companding transform with an inflexion point (LNST) can outperform logarithmic-based companding transforms such as μ-law companding. LNST can be expressed as [12]

$$y(n) = \begin{cases} \frac{1}{u} \cdot x(n) & |x(n)| \le v \\ u \cdot x(n) & |x(n)| > v \end{cases} \quad (5)$$

where $0 \le u \le 1$, and $0 \le v \le \max\{|x(n)|\}$. For a complex input $x(n)$, the companding transform should be applied to real and imaginary parts separately. At receiver, the original signal can be recovered according to

$$\tilde{x}(n) = \begin{cases} u \cdot R(n)\ , & n \in \varphi_1(v) \\ \frac{1}{u} \cdot R(n)\ , & n \in \varphi_2(v) \end{cases} \quad (6)$$
$$, R(n) = y(n) + w(n) + q(n)$$
$$\varphi_1(v) = \{n \forall |x(n)| \le v\}$$
$$\varphi_2(v) = \{n \forall |x(n)| > v\}$$

where $w(n)$ is noise component, $q(n)$ is quantization noise which is usually very small [5], $\varphi_{1(v)}$ and $\varphi_{2(v)}$ are the index sets of OFDM samples. It assumed that the receiver has the knowledge of the two sets. It is clear that due to the presence of the inflexion point v, small and large parts of the signal can be treated with different scales; enlarging small amplitudes by $1/u$ while compressing large amplitudes by u, which gives more flexibility and freedom in designing the companding form in order



(a) Companded signal by LNST (b) Companded signal by proposed

Figure 3: Companded OFDM signal by (a) LNST and (b) proposed transform.

to meet the given system requirements such as PAPR reduction, signal's average power, Power amplifier characteristics, and BER, and hence, leads to a better performance. However, taking into account the more accurate case that OFDM signal consists of three parts: small amplitudes, large amplitudes, and average amplitudes, more design flexibility and performance enhancement can be achieved if each one of these parts treated independently with a different scale. To satisfy this, we propose a new linear transform with two inflexion points, the new transform is

$$y(n) = \begin{cases} u_1 \cdot x(n) & |x(n)| < v_1 \\ u_2 \cdot x(n) & v_1 \le |x(n)| \le v_2 \\ u_3 \cdot x(n) & |x(n)| > v_2 \end{cases} \quad (7)$$

$$\tilde{x}(n) = \begin{cases} \frac{1}{u_1} \cdot R(n) & n \in \varphi_1(v_1) \\ \frac{1}{u_2} \cdot R(n) & n \in \varphi_2(v_{1,2}) \\ \frac{1}{u_3} \cdot R(n) & n \in \varphi_3(v_2) \end{cases}$$

$$, R(n) = y(n) + w(n) + q(n) \quad (8)$$
$$\varphi_1(v_1) = \{n \,\forall\, |x(n)| < v_1\}$$
$$\varphi_2(v_{1,2}) = \{v_1 \le n \,\forall\, |x(n)| \le v_2\}$$
$$\varphi_3(v_2) = \{n \,\forall\, |x(n)| > v_2\}$$

where $u_1>1$ and $u_3<1$. Regarding u_2, setting its value to unity can effectively reduce the undesired effect of noise expansion at the receiver since average amplitudes are scaled with unity and hence, no inverse scaling is required at the receiver. Figure 2 shows profiles of both transforms where

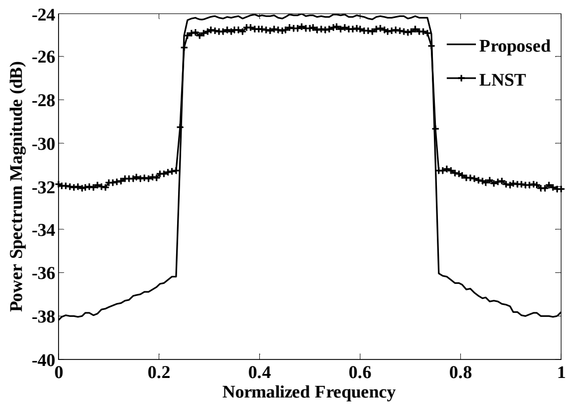


Figure 4: Power spectrums of LNST and proposed transform

$A = \max\{|x(n)|\}$, it is clear that with two inflexion points, more design flexibility is available and hence, a better tradeoff between PAPR and BER can be achieved. Figure 3 shows companded OFDM signal by LNST and proposed transform, where the two transform are designed to preserve the average power of the input signal for case study, for practical purpose, the average power of companded signal should be selected to best fit for specific PA characteristics included in the system [12]. The figure depicts the increased flexibility of the proposed transform, it allows for more reduction of PAPR by extra compression of maximum amplitudes -and by extra enlargement of small amplitudes- without affecting average amplitudes. Moreover, the flexibility of the proposed transform allows reducing abrupt jumps in the transformed signal, which leads to better power spectrum as depicted in figure 4. Since the receiver must have the knowledge of index sets, side information should be transmitted along with the signal. For LNST either $\varphi_1(v)$ or $\varphi_2(v)$ can be transmitted on a dedicated tones or imbedded in training sequences [12]. Specifically if v set to equal the square root of signal average power, then transmitting $\varphi_2(v)$ will be more efficient because it will contain smaller number of indices since the samples of large amplitudes are usually accruing with low probability. Regarding the proposed transform, advantages of the extra inflexion point come at the price of another index set that should be transmitted.

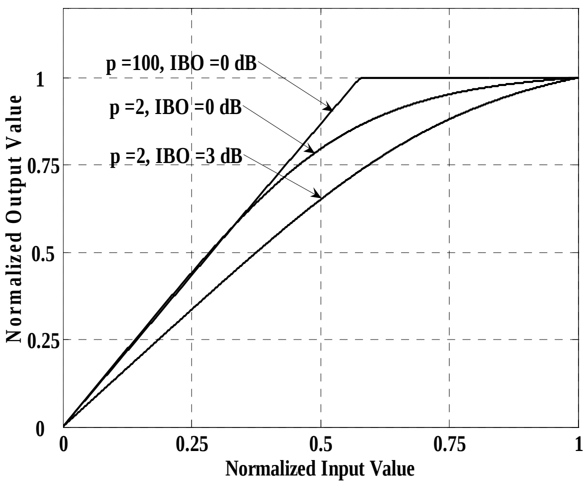


Figure 5: SSPA characteristics.

4. Nonlinear Power Amplifier

A widely accepted memoryless solid-state power amplifier (SSPA) model [20], which is extensively used in investigating PAPR of OFDM signals, is Rapp model [21], where a memoryless nonlinearity is assumed. Therefore, the PA has a frequency-nonselective response.

Representing the complex envelope of the input signal into the amplifier as

$$y(t) = |y(t)|\, e^{j\phi(t)} \qquad (9)$$

the transmitted output signal according to the model can be expressed as

$$y_{tx}(t) = \frac{a|y(t)|}{\left[1+\left(\frac{|y(t)|}{A_{sat}}\right)^{2p}\right]^{\frac{1}{2p}}}\, e^{j\phi(t)} \qquad (10)$$

where, a is the amplifier gain, A_{sat} is the saturation level, and p is a positive number to control nonlinearity characteristics of the amplifier. According to this model, SSPA introduces no phase distortion and only the AM/AM conversion is produced. Input power Backoff (*IBO*) can be expressed as

$$IBO = \frac{A_{sat}^2}{P_{in}} \qquad (11)$$

where P_{in} is the average power of the input signal. According to (11), the average power of the input signal should be scaled with the proper value for a given PA characteristics (A_{sat}, *IBO*). Figure 5 shows characteristics of SSPA model, as it shown, for large values of p, the model converges to a hard limiting amplifier that is exactly linear until it reaches its output saturation level. A good approximation of existing amplifiers is obtained by choosing p in the range of 2 to 3 [22]. In this paper, we selected $p = 2$.

Table 1: Transforms parameters and achieved PAPR reduction.

Transform	Parameters $A = \max\{\lvert x(n)\rvert\}$	PAPR reduction
LNST	$u = 0.488$ $v = \sqrt{P_{in}} = 20.58\%$ of A	45.8%
Proposed	$u_1 = 1.34$ $u_2 = 1$ $u_3 = 0.4$ $v_1 = 24.86\%$ of A $v_2 = 40.52\%$ of A	57.6%

5. Performance Evaluation

In order to evaluate and compare the performance of the two transforms and their impact on the system in the presence of nonlinear amplification, a MATLAB simulation is performed, assuming nonlinear AWGN channel and using randomly generated data bits.

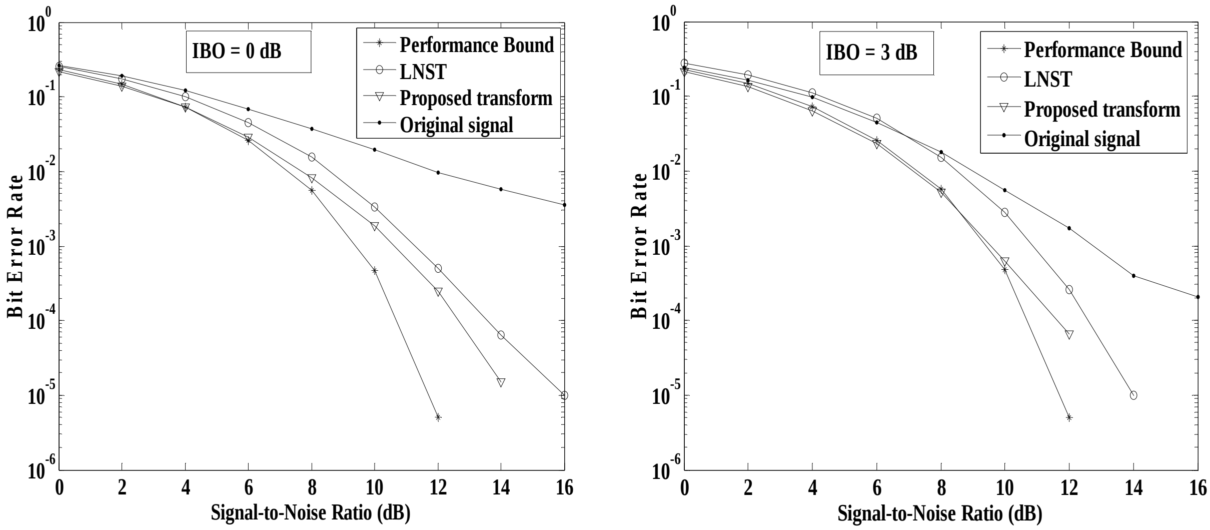


Figure 6: Performance of LNST and proposed transform in nonlinear AWGN channel.

The number of subcarriers N = 64 with 256-point IFFT/FFT (oversampling factor equal to 4) and DQPSK modulation. The transforms are designed to preserve the average power of the input signal; their parameters and achieved PAPR reduction are tabulated in table 1. Performance bound is obtained by disabling the power amplifier and transmitting original OFDM signal directly. The results are presented in figure 6. As it shown in the table, the proposed transform achieved PAPR reduction of 57.6% from original signal's PAPR with 11.8% more than LNST. Regarding BER, the proposed transform has a better performance than LNST. Specifically, for a target BER of 10^{-5} with *IBO* = 0 dB, the proposed transform requires a signal-to-noise ratio (SNR) of 14 dB while LNST requires about 16 dB. Moreover, an excellent BER performance that matches the performance bound achieved by the proposed transform with *IBO* of 3 dB.

6. Conclusion

In this paper, we proposed a new linear companding transform with two inflexion points to increase the flexibility of companding design, it shown that the proposed transform has a better PAPR reduction and BER performance than LNST proposed in [12] with better PSD. An excellent BER performance that matches the performance bound would be achieved with *IBO* of 3 dB. In general, with the aid of the two inflexion points, different signal levels can be scaled independently of each other. Thus, the proposed transform can be designed to meet the system requirements, power amplifier characteristics, and achieve a good tradeoff between PAPR reduction and BER. Furthermore, the proposed transform is simple to implement and has no limitations on the system parameters such as number of subcarriers modulation order, and constellation type.

References:

[1] Ahmad R. S. Bahai and Burton R. Saltzberg, *Multi-Carrier Digital Communications: Theory and Applications of OFDM*, Kluwer Academic Publishers, New York, 2002, pp. 14 - 15.

[2] C. Schurgers and M. Srivastava, “A systematic approach to peak-to-average power ratio in OFDM,” in Proc. SPIE, Vol. 4474, 2001, pp. 454 - 464.

[3] J. Armstrong, “Peak-to-average reduction for OFDM by repeated

clipping and frequency domain filtering," IEEE Electron. Lett., vol. 38, May 2002, pp. 246 - 247.

[4] X. Li and L. J. Cimini, Jr., "Effects of clipping and fltering on the performance of OFDM," IEEE Commun. Lett., Vol. 2, No. 5, May 1998, pp. 131 - 133.

[5] X. Wang, et al., "Reduction of peak-to-average power ratio of OFDM system using a companding technique," IEEE Trans. Broadcast., Vol, 45, No. 3, Sep. 1999, pp. 303 - 307.

[6] A. Mattsson, et al., "Comments on: reduction of peak-to-average Power ratio of OFDM system using a companding technique," IEEE Trans. Broadcast., Vol. 45, No. 4, Dec. 1999, pp. 418 - 419.

[7] X. Wang, et al., "Reply to the comments on reduction of peak-to-average power ratio of OFDM system using A companding technique," IEEE Trans. Broadcast., Vol. 45, No. 4, Dec. 1999, pp. 420 - 423.

[8] X. Wang, et al., "On the SER analysis of A-law companded OFDM system," in Proc. Global Telecommun. Conf., 2000. GLOBECOM '00. IEEE, Vol. 2, Dec. 2000, pp. 756 - 760.

[9] X. Huang, et al., "Reduction of peak-to-average power ratio of OFDM signals with companding transform," IEE Elec. Lett., Vol. 37, No.8, April 2001, pp. 506 - 507.

[10] Yuanbin Guo and Joseph R. Cavallaro, "Reducing peak-to-average power ratio in OFDM systems by adaptive dynamic range companding," 3G Wireless, World Wireless Congress, paper 159, San Francisco, CA, May 2002, pp. 536 - 541.

[11] T. Jiang and G. Zhu, "Nonlinear companding transform for reducing peak-to-average power ratio of OFDM signals," IEEE Trans. Broadcast., Vol. 50, No. 3, Sep. 2004, pp. 342 - 346.

[12] X. Huang, et al., "companding transform for reduction in peak-to-average power ratio of OFDM signals," IEEE Trans. Wireless Commun., Vol. 03, No. 6, Nov. 2004, pp. 2030 - 2039.

[13] T. Jiang, et al., "Exponential companding transform for PAPR reduction in OFDM systems," IEEE Trans. Broadcast., Vol. 51, No. 2, June 2005, pp. 244 - 248.

[14] Tao Jiang, et al., "Two novel nonlinear companding schemes with iterative receiver to reduce PAPR in multi-carrier modulation systems," IEEE Trans. Broadcast., Vol. 52, No. 2, June 2006, pp. 268 - 273.

[15] Thomas G. Pratt, et al., "OFDM link performance with companding for PAPR reduction in the presence of non-linear amplification," IEEE Trans. Broadcast., Vol. 52, No. 2, June 2006, pp. 261 - 267.

[16] Chin-Liang Wang and Sheng-Ju Ku, "A Low-Complexity Companding Transform for Peak-To-Average Power Ratio Reduction in OFDM Systems, " in Proc. IEEE Int. Conf. on Acoustics, Speech and Signal Processing ICASSP 2006 Vol. 4, 2006, pp. IV-329 - IV-332.

[17] S. A. Aburakhia, E. F. Badran, and D. A. E. Mohamed, "A comparison between signal clipping and μ-law companding schemes for the

reduction of peak-to-average power ratio of OFDM Signals," in Proc. 1st Int. Conf. on Digital Commun. & Computer Applications DCCA2007, March 2007, pp. 423 - 431.

[18] S. C. Thompson, "Generating Real-Valued OFDM Signals with the Discrete Fourier Transform," online: http://zeidler.ucsd.edu/↑ sct/pubs/t5.pdf.

[19] M. Sharif, M. G. Alkhansari, and B. H. Khalaj, "New results on the peak power of OFDM signals based on oversampling," in Proc. IEEE Int. Commun. Conf., vol. 2, Helsinki, Finland, April 2002, pp. 866 - 871.

[20] Celal DUDAK and Arzu Tuncay KOC, "Solid state power amplifier (SSPA) nonlinearity effects on quadri-phase shift keying modulation," in Proc. 7th European Conf. on Wireless Technology, 2004, pp. 237 - 240.

[21] C. Rapp, "Effects of HPA nonlinearity on a 4-DPSK/OFDM signal for a digital sound broadcasting system," in Proc. 2nd European Conf. on Satellite Commun., Li´ege, Belgium, Oct. 1991, pp. 179 - 184.

[22] R. Van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House, London, 2000, pp. 127 - 128.