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LINEAR FILTERING APPLIED TO SAFEGUARDS OF NUCLEAR MATERIAL

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Recently there has been widespread publicity on the problems of Nuclear materials theft or diversion. Due to the proliferation of nuclear reactors throughout the world, the concern about theft or diversion of nuclear materials at various points in the fuel cycle has greatly increased. Steps are being taken to improve the accountability systems; however, there is still a need for more powerful statistical techniques to rapidly detect theft or diversion.

Of particular concern, is the problem of detecting continual thefts of relatively small amounts of material. This paper suggests using Kalman Filtering techniques as a powerful method of detecting this problem.

It is possible to develop a linear model of the material balance area. If:

$X_1(t)$ = on-hand inventory at the beginning of period t.

$X_2(t)$ = material removed from the balance area in period t.

$U_1(t)$ = total material into the facility

- total material shipped from the facility during the time interval (t-1, t).

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Then, in matrix form the system model is:

$$\underline{X}(t+1) = \underline{A}(t)\ \underline{X}(t) + \underline{U}(t) \tag{1}$$

where

$$\underline{A}(t) = \begin{bmatrix} 1 & -1 \\ A_{21} & A_{22} \end{bmatrix} \qquad \underline{X}(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \qquad \text{and } \underline{U}(t) = \begin{bmatrix} U_1(t) \\ 0 \end{bmatrix}$$

By specifying the constants, A_{21}&A_{22}, various scenarios can be modeled. For example, if $A_{21}=0$, and $A_{22}=1$, the model specifies the situation where a constant quantity of material is being removed from the facility each period. Another interesting situation which can be modeled is the case where a fixed fraction α of the on-hand inventory is being removed each period. In this later case, $A_{21}=\alpha$, and $A_{22}=0$.

It is assumed that the state vector $\underline{X}^T(t) = (X_1(t),X_2(t))$ is not completely observable. In fact, only noisy observations of $X_1(t)$ are available. If Y(t) denotes the observation of $\underline{X}(t)$ at time t then:

$$Y(t) = \underline{H}(t)\underline{X}(t) + V(t) \tag{2}$$

where $\underline{H}(t) = [1,0]$ and V(t) is a zero mean random variable with known variance R(t). The model defined by equation (1) and (2) models the situation where we have imperfect estimates of the on-hand inventory. Contributions to the error term V(t) are due primarily to material balance accounting errors. These errors are random, tend to average to zero, are somewhat correlated and represent only temporary misplacement of material. The specific objective of this study was to investigate the feasibility of estimating $X_2(t)$ from imperfect observations of $X_1(t)$. Various scenarios were considered, each scenario simulating a "realistic" situation where material was being removed from the material balance area.

The Kalman Filter[1-3] is a technique for obtaining an estimate of $\underline{X}(t)$, $\hat{\underline{X}}(t)$, from noisy observations of $\underline{X}(t)$.

The Kalman Filter assumes the sequence {V(t): t=1,2,...} is uncorrelated in time. In this situation the filter produces an optimal-unbiased-linear estimate of $\underline{X}(t)$. That is, $\hat{\underline{X}}(t)$ minimizes the trace of $E[\underline{e}(t),\underline{e}(t)^T]$ over the class of unbiased linear estimators. It can be shown[1] that if V(t) is normally distributed, $\hat{\underline{X}}(t)$ is the optimal unbiased estimate of $\underline{X}(t)$ over the class of all estimators.

In addition to producing estimates of $\underline{X}(t)$ each period the filter updates the initial estimate of the error covariance matrix $\underline{G}(o)$ with a matrix $\underline{G}(t)$. The matrix $\underline{G}(t)$ can be used to place tolerances on the elements of $\hat{\underline{X}}(t)$. The tolerances, in turn, can be used to conduct tests to determine if elements of $\hat{\underline{X}}(t)$ are significantly different from zero.

We have postulated a material flow situation since actual figures on material balances for large reprocessing plants are not available. The hypothetical case presented here assumes an initial material inventory of N kg and the amount of temporarily misplaced material was normally distributed with mean 0 and variance .004 N. Net changes in inventory were assumed to have a range of .28 N/month. Both material diversion models discussed earlier were used to model losses. Different amounts of material diversion ranging from 0 to .017 N/day were simulated. Assuming an initial unaccounted material loss of zero and complete lack of knowledge of the covariance matrix, the filter detected the loss in as few as 5 months.

Details of these examples will be presented along with decision rules for determining if an estimate of material loss ($X_2(t)$) is significantly different from zero.

References

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