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LINEAR OSCILLATIONS OF A DROP IN UNIFORM ALTERNATING ELECTRIC FIELDS

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ABSTRACT

Oscillations of a conducting drop immersed in a dielectric fluid in an alternating electric field has been modelled in order to understand the enhancement of the transport processes by the electric field. Numerical solutions for oscillation amplitude, velocity distribution, resonant frequency and streamlines were obtained. The effects of viscosity and density on the resonant frequency and the velocity distribution were investigated. It was found that the resonant frequency of viscous fluids was always smaller than the free oscillation frequency of the same droplet. The predicted scanning frequency response curve and the streamlines agree well with the experimental observations.

Introduction

Applications of electric force in direct contact heat transfer process or liquid extraction can result in a better energy efficiency than the conventional agitation methods (Thornton 1968). For liquid systems, which have insulating continuous phases and conducting dispersed phases, an imposed electric field will disturb the interface by interacting with induced electric charge, and thus enhance mass or heat transfer rate. In a static field, the electric force can reduce drop size, increase drop velocity, therefore, increase the interfacial area and the transport coefficient. An alternating electric field can also reduce the drop size effectively (Kawalski and Ziolkowski, 1981). Experiments on direct contact heat transfer showed that an alternating electric field with a proper frequency was more efficient than a static field in enhancing the heat transfer coefficient (Kaji et al., 1980, 1985). The experiments also showed that the enhancement of the heat transfer coefficient was directly related to the drop oscillations induced by the electric force. To understand the effects of the electric fields on the transport processes, we need knowledge of the hydrodynamics of the electrically forced drop oscillations.

Studies on free oscillations of a drop (Miller and Scriven 1968; Prosperetti 1980; Marston 1980) have revealed that a drop oscillates at its characteristic frequencies no matter how the oscillations are excited. For viscous fluids, the oscillation amplitudes decay gradually due to viscous dissipation. Drop oscillations under the sustained action of an external alternating force are different. The oscillation frequencies are the same as those of the external force (Lamb 1945), but the amplitudes are functions of the external forces as well as properties of the fluid system. A quasi-steady external force maintains the amplitudes constant, therefore, the decay factors are zero. Torza et al. (1971) and Sozou (1972) have investigated oscillations of an uncharged drop in alternating electric fields. The attention of their work was paid to the effects of electric properties (resistivity and dielectric constant) on the drop deformation and the flow patterns. They did not discuss the effects of the hydrodynamic properties, which are more essential to the study of the transport processes. Drop oscillations driven by acoustic waves have been studied by Marston (1980). Explicit solutions for oscillation amplitudes and phase shift angles were obtained for low viscosity systems. For solutions in a wide viscosity range, numerical methods must be used.

In this paper, we present a model for the hydrodynamics of small amplitude oscillations of a conducting drop immersed in a dielectric fluid in an alternating electric field. Expressions of velocity distributions are obtained analytically and boundary condition equations are solved numerically. Resonant frequencies are predicted and effects of viscosity and density on flow fields are discussed.

Model

Consider a charged fluid sphere immersed in another immiscible fluid. Both fluids are assumed incompressible and Newtonian, and properties in each phase are uniform. The interface is presumed to be free from any contamination by surfactants. It is also assumed that the effect of gravity on drop deformation can be neglected ($gR^2\Delta\rho/\sigma$ is small). Only small amplitude oscillations are considered, therefore the nonlinear term in the Navier-Stokes equation can be neglected (Levich 1962). It is further assumed that electric field far from the drop is uniform, the drop phase is conductive and the continuous phase dielectric so that electric equilibrium can be reached instantly and charge loss negligible. This study is limited to the axisymmetric flow, which has been frequently observed in experiments. Spherical coordinates (r, θ, ϕ) will be used and the origin is at the drop center. $\theta = 0$ is taken as the symmetry axis (the electric field is in this direction). The momentum equation is

$$\frac{\partial \mathbf{v}}{\partial t} = \nu \nabla^2 \mathbf{v} - \frac{1}{\rho} \nabla p \tag{1}$$

The continuity equation for an incompressible fluid is

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

By taking the divergence of the momentum equation and subtracting the continuity equation, the following pressure equation is obtained (Chandrasekhar 1961).

$$\nabla^2 p = 0 \tag{3}$$

The boundary conditions include: continuity of the normal and tangential velocities at the interface; normal stress balance at the interface: $\tau_{ri} + \tau_s = \tau_{ro}$; continuity of the tangential stress at the interface; normal velocity at the interface matches with the rate of displacement of the interface: $\partial\zeta/\partial t = v_r|_{r=\zeta}$; finite pressure and velocities at the drop center as well as at infinity. When the drop center moves, the origin of the coordinates moves with it. Consequently, it seems that the surrounding fluid moves while the drop position keeps unchanged. In the normal stress balance, τ_s is surface stress including the effects of surface tension and surface charge.

$$\tau_s = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \tau_e \qquad (4)$$

where σ is interfacial tension, R_1 and R_2 are principal radii of curvature of the drop surface, τ_e is stress produced by electric charge on the interface.

For free oscillations of a drop with net charge on its surface, the primary effect of net charge on the oscillations is induced by redistribution of the charge on deformed drop surface. A first order perturbation model (weighted in terms of deformation amplitude) is necessary to describe this charge redistribution (Rayleigh 1882, Hendricks and Schneider 1963). When an external alternating electric field exists, direct interaction between the charge and the field is the primary effect on drop motion and oscillations. This effect can be described by a zero order perturbation model. The effect of charge redistribution on the deformed surface becomes secondary. Under ordinary conditions, the secondary effect is much smaller than the primary effect. Only the primary effect will be considered in this paper, therefore, a zero order perturbation model is used. Theoretically, the boundary conditions should be satisfied on the deformed interface. As a zero order perturbation model, the boundary conditions will be satisfied on the undeformed spherical interface, and the normal and tangential components of vectors will be replaced by r and θ components, respectively.

In zero order perturbation, the electric charge distribution for a slightly deformed drop is approximated by that on a spherical conductor (Reitz 1967).

$$q = q_{net} + 3\epsilon E\cos\theta \tag{5}$$

where q_{net} is average net charge density, ϵ is permittivity of the surrounding fluid, E is field strength, θ is polar angle measured from the positive direction of the electric field. For an alternating electric field, $E = E^{\circ}\cos(\beta_e t)$, where E° is amplitude and β_e is frequency of the electric field. The normal stress generated by the charge is

$$\tau_e = \frac{q^2}{2\epsilon} = \left(\frac{q^2_{net}}{2\epsilon} + \frac{3}{2}\epsilon E^2\right) + 3q_{net}E^{\circ}\cos(\beta_e t)P_1 + \frac{3}{2}\epsilon E^{\circ 2}[\cos(2\beta_e t)+1]P_2 \tag{6}$$

where $P_n = P_n(\cos\theta)$, is a nth order Legendre function. There is no electric tangential stress on a conducting drop surface (Taylor 1966). The terms in the first parenthesis represent average pressure produced by the net and induced charge. For incompressible fluid, this average pressure only alters pressure distribution but does not contribute to any drop movement. The second and third terms contain only first and second order Legendre functions, respectively. According to the linear oscillation theory (Lamb 1945), the above electric forces will only affect the first mode (linear translation) and the second mode (prolate-oblate oscillation) motions, respectively. Oscillation frequencies are β_e and $2\beta_e$ for the first and second modes, respectively. The time-independent part in the last term will cause a static prolate deformation.

If higher order corrections were included in the model, secondary effects of drop deformation would induce oscillations of some other modes, but amplitudes of those oscillations would be much smaller than the primary motions considered here, as long as the assumption of small amplitude is satisfied.

Solution procedure for the above equations is similar to that described by Chandrasekhar (1961). The expressions of the solution are in complex forms, which involve half order Bessel and Hankel functions.

$$p_i = p_i^\circ + \rho_i \sum_{n=1}^{2} a_{1n} \exp(-\omega_n t) r^n P_n \tag{7}$$

$$p_o = p_o^\circ + \rho_o \sum_{n=1}^{2} a_{2n} \exp(-\omega_n t) r^{-n-1} P_n \tag{8}$$

$$v_{ri} = \sum_{n=1}^{2} \exp(-\omega_n t)\left[\frac{a_{1n} n r^{n-1}}{\omega_n} + \frac{a_{3n}}{r^{3/2}} J_{n+\frac{1}{2}}(x_i)\right] P_n \tag{9}$$

$$v_{ro} = \sum_{n=1}^{2} \exp(-\omega_n t)\left[-\frac{a_{2n}(n+1)}{\omega_n r^{n+2}} + \frac{a_{4n}}{r^{3/2}} H_{n+\frac{1}{2}}(x_o)\right] P_n + \frac{a_{51}}{\omega_1}\exp(-\omega_1 t) P_1 \tag{10}$$

$$v_{\theta i} = \sum_{n=1}^{2} \exp(-\omega_n t)\left(\frac{a_{1n} r^{n-1}}{\omega_n} + \frac{a_{3n}}{r^{3/2}}\left[\frac{J_{n+\frac{1}{2}}(x_i)}{n} - \frac{x_i J_{n+\frac{3}{2}}(x_i)}{n(n+1)}\right]\right)\frac{dP_n}{d\theta} \tag{11}$$

$$v_{\theta o} = \sum_{n=1}^{2} \exp(-\omega_n t)\left(\frac{a_{2n}}{\omega_n r^{n+2}} + \frac{a_{4n}}{r^{3/2}}\left[\frac{H_{n+\frac{1}{2}}(x_o)}{n} - \frac{x_o H_{n+\frac{3}{2}}(x_o)}{n(n+1)}\right]\right)\frac{dP_n}{d\theta} + \frac{a_{51}}{\omega_1}\exp(-\omega_1 t)\frac{dP_1}{d\theta} \tag{12}$$

$$\zeta = R + [a_{52}\exp(-\omega_2 t) + b_2] P_2 \tag{13}$$

Where p and p° are pressure and its time-independent part, respectively; v is velocity; subscripts i and o denote inner and outer fluids, subscripts r and θ denote r and θ components, respectively; n is mode number; $\omega_n = \pm i n \beta_e$, is a complex number corresponding to oscillation frequency of nth mode; $x = \sqrt{\omega_n/\nu}\, r$; a_{kn} (k=1,2,3,4,5) and b_2 are unknown constants; $J_{n+\frac{1}{2}}$ and $H_{n+\frac{1}{2}}$ are half order

Bessel and Hankel functions of the first kind, respectively; ζ represents deformed drop surface; R is the undeformed spherical drop radius. The last terms in Equations (10) and (12) represent fluid velocity far from the drop relative to the drop center.

Because the problem is linear, we can use the complex form in the boundary conditions and find out the unknown constants numerically, then take the real parts of the results at the end of the calculations. Substituting the above expressions into the boundary conditions and after some algebraic manipulation, a system of linear algebraic equations for each oscillation mode can be obtained. The equations can be solved numerically with the Gauss elimination method to get values of the unknown constants.

It is more efficient to present the results in dimensionless form. The variables are nondimensionalized as follow.

$$r^* = r/R,\ v^* = v/(\beta_e R),\ a^* = |a_{5n}|/R,\ p^* = p/(\rho_i \beta_e^2 R^2),\ \psi^* = \psi/(\beta_e R^3)$$

where r, v, a, p, and ψ are radial coordinate, velocity, amplitude, pressure, and streamfunction, respectively; the superscript * denotes dimensionless variables. Dimensionless groups that affect the dimensionless solution of the second mode oscillation are

$$e_i\ ,\ e_o,\ \beta^*,\ \rho_o/\rho_i,\ f$$

where $e_i = \nu_i\sqrt{\rho_i/\sigma R}$ and $e_o = \nu_o\sqrt{\rho_o/\sigma R}$, may be regarded as dimensionless viscosities; $\beta^* = \beta_e\sqrt{\rho_i R^3/\sigma}$, is a dimensionless frequency; ρ_o/ρ_i is the density ratio; $f = 3\epsilon E^{*2}R/2\sigma$ is the ratio of electric stress to the surface tension stress. $f = 0.1$ is used in this paper. The time-independent part of the normal stress balance gives solution for b_2: $(b_2/R) = f/4$.

Results and Discussion (for Second Mode)

Although first mode motion is also oscillatory and may play an important role in enhancing transport processes, it only involves linear translation but no shape deformation. Solution to the first mode can be obtained by the same method used for the second mode. The following discussions are emphasized on the second mode shape oscillation.

Frequency scanning response and resonant frequency

Amplitudes of forced oscillations are functions of the electric field and properties of the fluid system. The linear theory predicts that the amplitude is proportional to the external force. Effects of other parameters are complicated. By measuring the oscillation amplitude while varying the frequency of the force, a scanning frequency response curve can be obtained. Figure 1 shows calculated response curves. The dimensionless amplitude a^* is proportional to f, and only its relative significance is needed here, therefore, its scale is arbitrary. All the curves in Figure 1 have the same value when β^* approaches zero. At this point, the drop is in a quasi-equilibrium state and the only parameter that affects a^* is f. Therefore, a constant f gives a constant a^*. When β^* approaches ∞, a^* approaches zero. In between, the curve may pass through a peak, which indicates resonance. The shape of the peak is determined by e_i, e_o and ρ_o/ρ_i. When e_i or e_o increases, the response curve becomes flattened and the dimensionless resonant frequency decreases. When ρ_o/ρ_i increases, the dimensionless resonant frequency decreases, but the resonant amplitude first decreases then increases. Figure 2 compares a predicted scanning frequency response curve with that measured by Trinh et al. (1982) for a drop driven by an acoustic force. The predicted resonant frequency is tuned to coincide with the measured one by adjusting the interfacial tension (calculated at 0.029 N/m compared to 0.035 - 0.04 N/m given in their paper). The shapes of the curves are similar, indicating that electric force and acoustic force have similar effects on drop deformation.

The resonant frequencies can be obtained from the frequency response curves. Figure 3 compares the resonant frequencies calculated by this model with the free oscillation frequencies (Prosperetti 1980). When calculating dimensionless frequency for free oscillation, the oscillation frequency should be divided by the mode number in order to be consistent with the definition for the forced oscillation. The effects of viscosities on free and resonant frequencies are similar. As viscosities increase, the frequencies decrease. For inviscid fluids, the resonant frequency coincides with the natural frequency. For viscous fluids, the resonant frequency is always smaller than the free oscillation frequency. The larger the viscosities, the greater the difference between them.

Velocity distribution

Variations of velocities in the θ direction are described by Legendre functions. Only the variations in the r direction will be discussed. For convenience, $\theta = 0$ is chosen for v_r^* and $\theta = \pi/4$ for v_θ^* because in these directions the velocities are maximum. Other factors that affect the velocities are e_o, e_i, ρ_o/ρ_i, β^*, and time. It is clear from the velocity expressions that the velocities change with time periodically. The effect of β^* on the velocities is similar to that on the amplitude. Therefore, these two factors will not be further discussed. In the following, the resonant frequency and the time for maximum radial velocity at the interface are used. Figures 4 and 5 show the radial and tangential velocity profiles with $e_i = e_o = e$ as a parameter. The velocities decrease with increasing e. When e is small, the drop phase velocities are proportional to the radius except near the interface and the tangential velocity has very large gradients on both sides of the interface. The gradients increase with decreasing e, but no slip of velocity occurs as long as e is not zero. Consequently, the viscous dissipation is concentrated in a thin boundary layer near the interface for small viscosity fluids, as has been predicted by Miller and Scriven (1968) for a free oscillating drop. If e_i or e_o changes individually, the velocity gradient in the corresponding side of the interface will be mostly affected.

Effect of density ratio on the tangential velocity is illustrated in Figure 6 for $e_i = e_o = 0.001$. Of the profiles shown in the figure, $\rho_o/\rho_i = 1$ gives the smallest velocity. The velocity profiles shift outward if $\rho_o/\rho_i < 1$, and inward if $\rho_o/\rho_i > 1$. In either of the cases, the larger tangential velocity gradient is always in the less dense side of the interface.

Streamlines

Since the flow field of the oscillating drop is axisymmetric, the stream functions can be easily obtained from the integration of the velocity expressions. Flow patterns for mode 2 are illustrated in Figure 7. The effects of viscosity and density ratio are illustrated in different quadrants. The circle in the middle indicates the interface of the drop. In all the conditions the internal and external streamlines meet at the interface and form closed cycles. During oscillation the fluids move back and forth once in a period along the streamlines. The centers of the cycles indicate the positions of stagnant rings where the radial and tangential velocities are zero. The θ position of the rings are determined by $v_r = 0$, which gives $\theta = \cos^{-1}\sqrt{1/3}$. $v_\theta = 0$ determines the radial position of the rings, which turns out to be a function of many parameters. When the densities are the same ((a) and (b)), the stagnant rings move to the less viscous side of the interface, while when the viscous effects are the same ((a), (c) and (d)), the rings move to the less dense side of the interface. The internal streamlines observed by Trinh et al. (1982) qualitatively agree with the predicted results.

Conclusions

The proposed oscillation model for a drop in an alternating electric field is able to predict velocity field, scanning frequency response curve, resonant frequency, and streamlines. The predicted scanning frequency response curve and the streamlines are in good agreement with experimental observations. It is found that the resonant frequency of the forced oscillation is equal to the freoscillation frequency for inviscid fluids, but always smaller than the free oscillation frequency for viscous fluids. The viscosities and the density ratio have significant influence on the velocity profiles.

The enhancement of a transport process by an alternating electric field depends on how much the systems are disturbed. Operation at a resonant frequency is desirable because it produces maximum disturbance for the same energy input. The predicted scanning frequency response curves can give best operation frequency range for any fluid system. The velocity distribution is very useful for the modelling of a mass or heat transfer process.

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Notation

a^* = dimensionless oscillation amplitude

a_{kn} = constants in expressions of solutions, k=1, 2, 3, 4, 5

b_2 = second mode static deformation

E, E^* = alternating electric field strength, amplitude of E, V/m

e = $\nu\sqrt{\rho/\sigma R}$

f = $3\epsilon E^{*2}R/(2\sigma)$

$H_{n+\frac{1}{2}}$ = half order Hankel function

i - $\sqrt{-1}$

$J_{n+\frac{1}{2}}$ - half order Bessel function

n - oscillation mode number

P_n - Legendre function

p, p^* - pressure, static part of pressure, N/m^2

q, q_{net} - surface charge density, net surface charge density, $coul/m^2$

R - equivalent spherical drop radius, m

R_1, R_2 - principal radii of curvature of drop surface, m

r - radial coordinate, m

t - time, s

v - velocity, m/s

x - $r\sqrt{\omega/\nu}$

X - $R\sqrt{\omega/\nu}$

Greek letters

β_e - angular frequency of electric field, s^{-1}

ϵ - permittivity of the continuous phase, farad/m

ζ - radial position of deformed drop surface, m

θ - coordinate

ν - kinematic viscosity, m^2/s

ρ - density, kg/m^3

σ - interfacial tension, N/m

τ - stress, N/m^2

ψ - streamfunction, m^3/s

ω - $\pm in\beta_e$

Subscripts

e = electric field

i – inside the drop

n – 1, 2, oscillation mode number

o – outside the drop

r – radial direction

s – on the interface

θ – θ direction

Superscripts

* – dimensionless variables

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Figure Captions

Figure 1. Predicted scanning frequency response curves for 2nd mode oscillation. The line passing the peaks of the response curves indicates the effect of density ratio on the resonant amplitude for $e_i = e_o = 0.01$.

Figure 2. Comparison of predicted and observed (Trinh et al. 1982) 2nd mode scanning frequency response curves for a silicone/CCl_4 drop immersed in distilled water. The smooth curve is predicted. Drop volume = 1.5 cm^3, $\rho_i = \rho_o = 990$ kg/m^3, $\nu_i = 3.2\times10^{-6}$ m^2/s, $\nu_o = 1.01\times10^{-6}$ m^2/s.

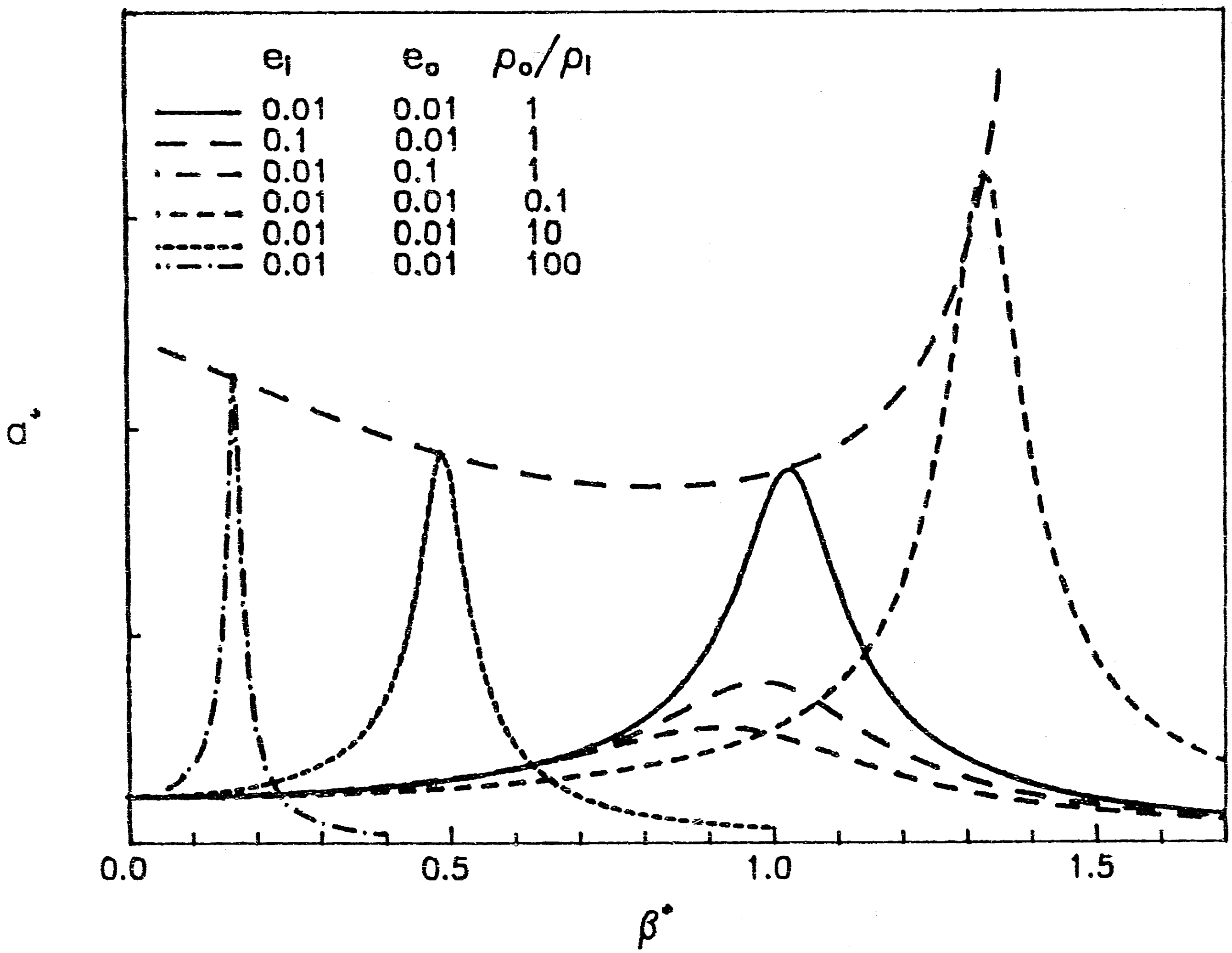
Figure 3. Comparison of resonant frequency of forced oscillation and free oscillation frequency for 2nd mode.

Figure 4. Radial velocity distribution for a drop undergoing 2nd mode oscillation with $e_i = e_o = e$ as a parameter.

Figure 5. Tangential velocity distribution for a drop undergoing 2nd mode oscillation under the same conditions as those of Figure 4.

Figure 6. Tangential velocity distribution for a drop undergoing 2nd mode oscillation with ρ_o/ρ_i as a parameter.

Figure 7. Streamlines for a drop undergoing 2nd mode oscillation.
(a): $e_i = e_o = 0.001$, $\rho_o/\rho_i = 1$; (b): $e_i = 0.01$, $e_o = 0.001$, $\rho_o/\rho_i = 1$;
(c): $e_i = e_o = 0.001$, $\rho_o/\rho_i = 100$; (d): $e_i = e_o = 0.001$, $\rho_o/\rho_i = 0.01$.



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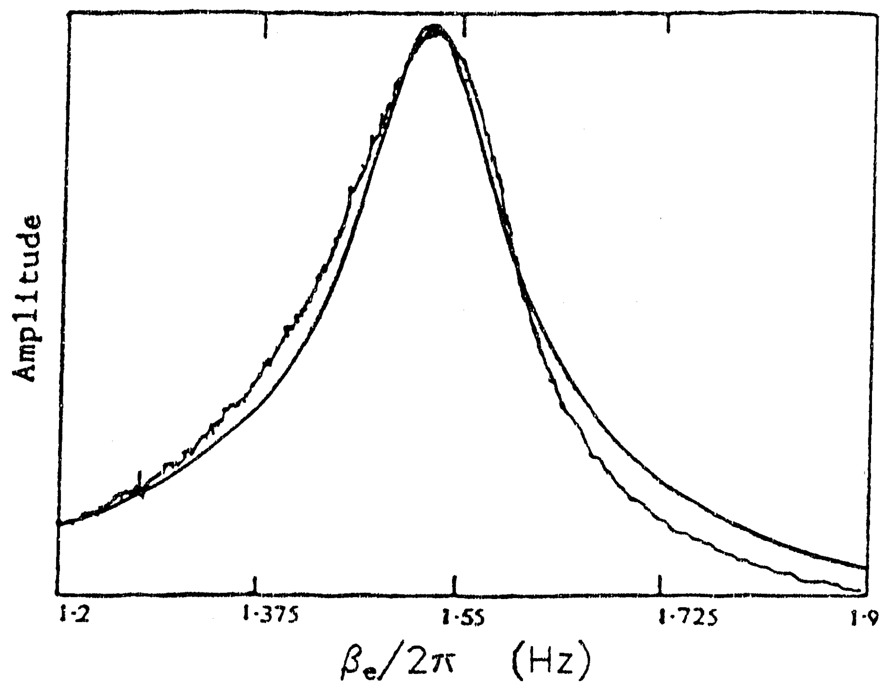

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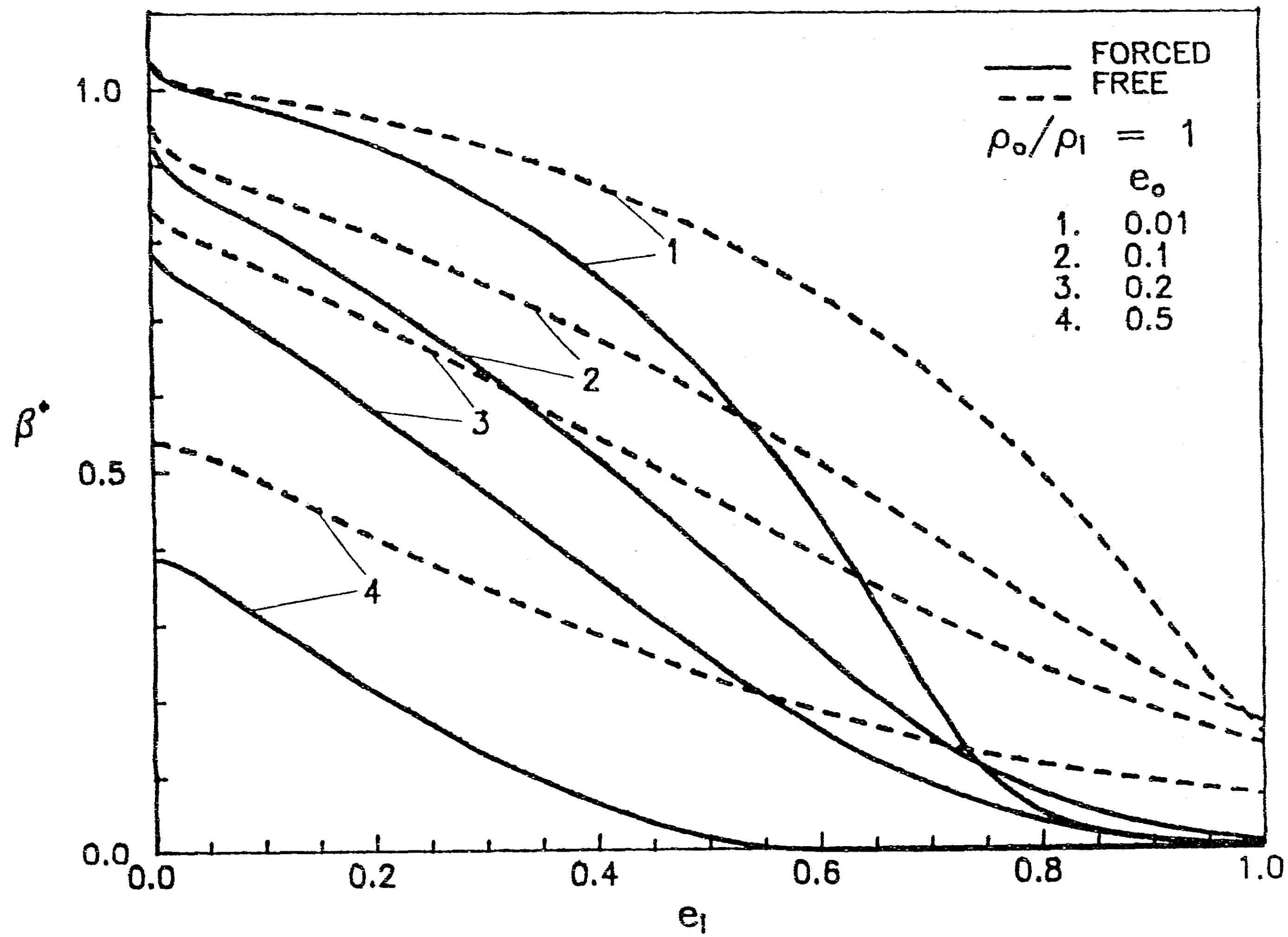

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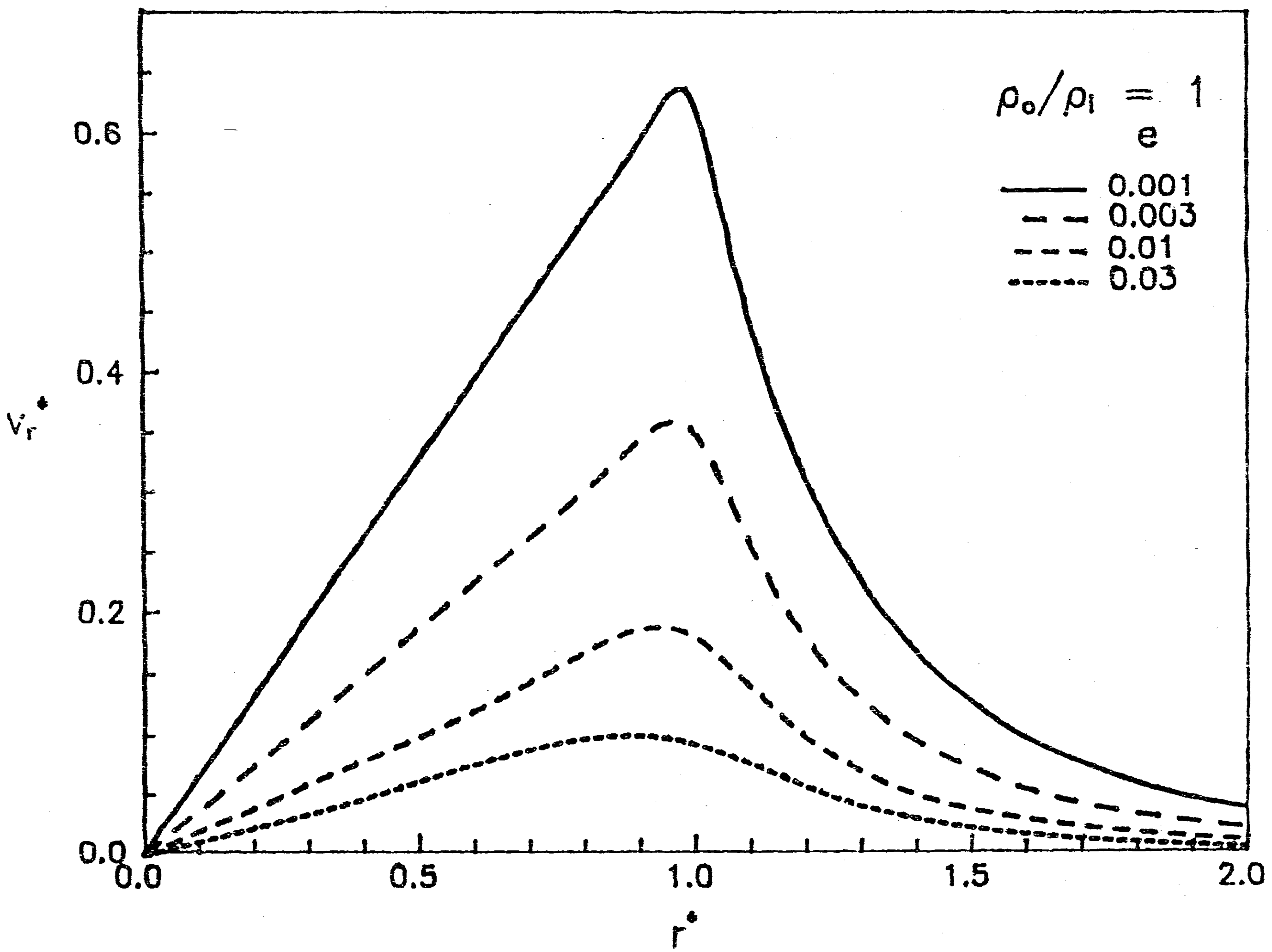

Figure 4. Radial velocity distribution for a drop undergoing 2nd mode oscillation with $e_I = e_o = e$ as a parameter.

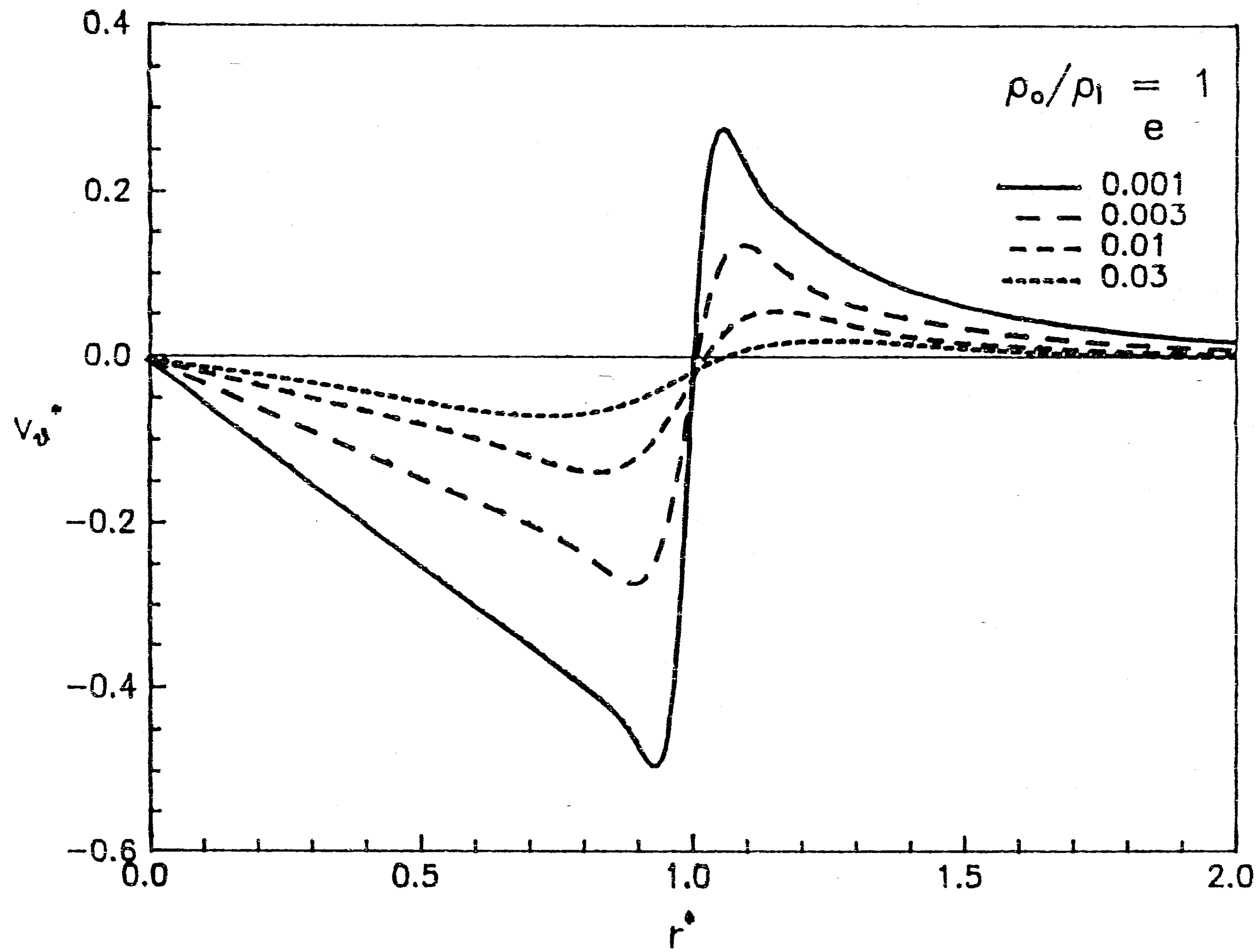

Figure 5. Tangential velocity distribution for a drop undergoing 2nd mode oscillation under the same conditions as those of Figure 4.

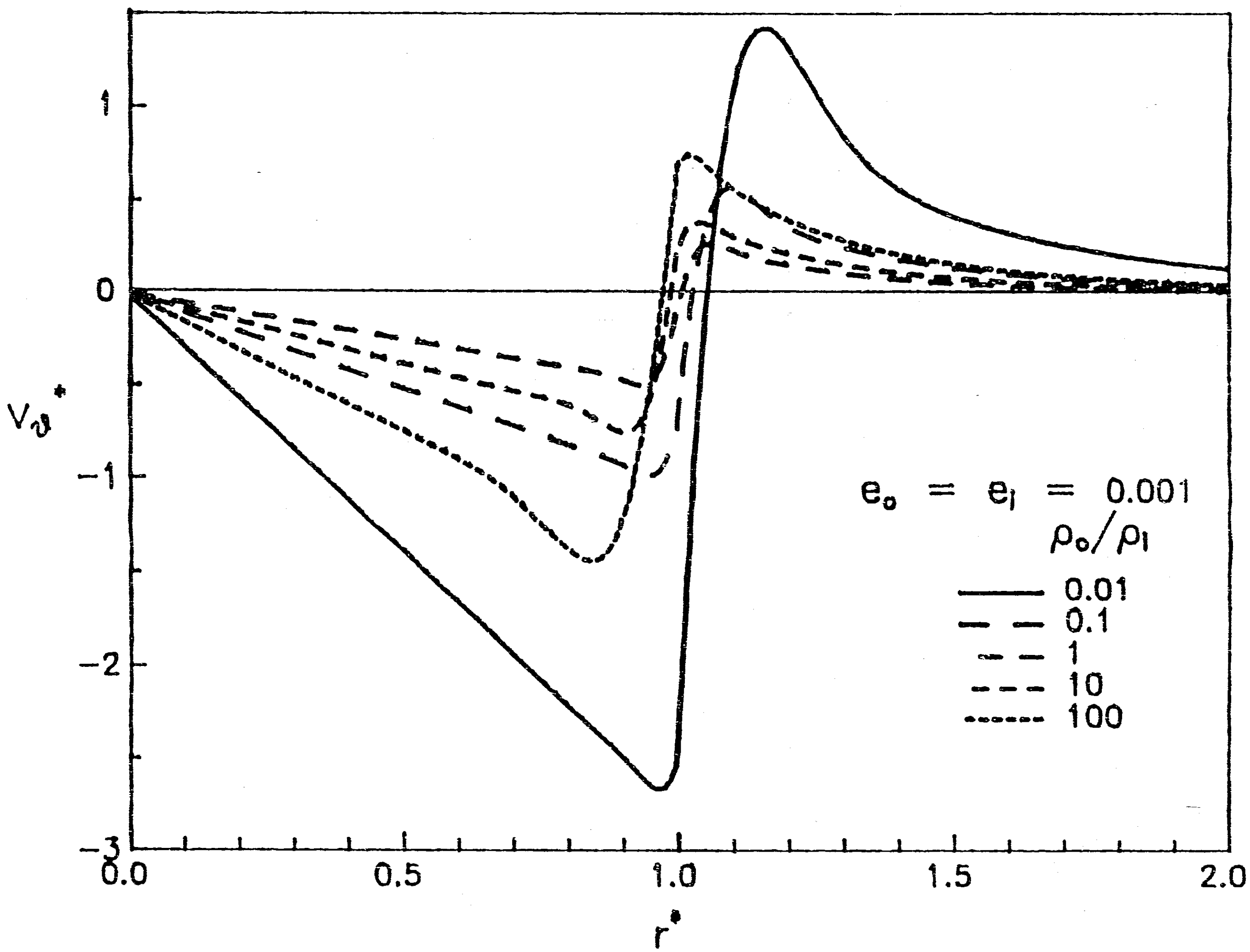

Figure 6. Tangential velocity distribution for a drop undergoing 2nd mode oscillation with ρ_o/ρ_i as a parameter.

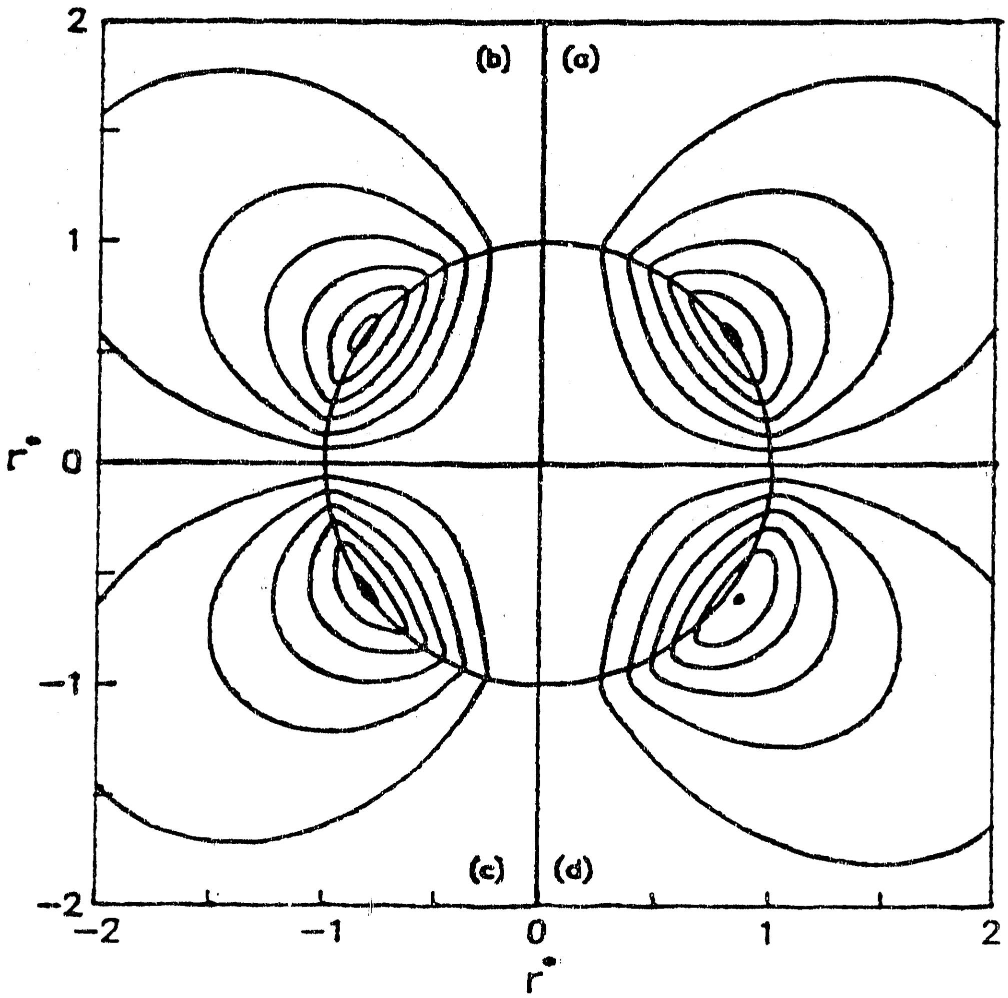

Figure 7. Streamlines for a drop undergoing 2nd mode oscillation.

(a): $e_i = e_o = 0.001$, $\rho_o/\rho_i = 1$; (b): $e_i = 0.01$, $e_o = 0.001$, $\rho_o/\rho_i = 1$;

(c): $e_i = e_o = 0.001$, $\rho_o/\rho_i = 100$; (d): $e_i = e_o = 0.001$, $\rho_o/\rho_i = 0.01$.

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