# Linear Precoding for Multi-Pair Two-Way MIMO Relay Systems With Max-Min Fairness

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***Abstract*—Two-way relaying has demonstrated significant gain in spectral efficiency by applying network coding when a pair of source nodes exchange information via a relay node. This paper is concerned with the scenario where multiple pairs of users exchange information through a common relay node equipped with multiple antennas. We aim to design linear precoding at the relay based on amplify-and-forward strategy. The goal is to maximize the minimum achievable rate among all the users subject to a peak relay power constraint so as to achieve the max-min fairness. We first convert this nonconvex problem into a series of semidefinite programming problems using bisection search and certain transformation techniques. A quasi-optimal solution is then obtained by using semidefinite relaxation (SDR). To reduce the design complexity, we further introduce a pair-wise zero-forcing (ZF) structure that eliminates the interference among different user pairs. By applying this structure, the precoding design problem is simplified to a power allocation problem which can be optimally solved. A simplified power allocation algorithm is also proposed. Simulation results show that the proposed SDR-based precoding not only achieves high minimum user rate but also maintains good sum-rate performance when compared with existing schemes. It is also shown that the proposed pair-wise ZF precoding with simplified power allocation strikes a good balance between performance and complexity.**

***Index Terms*—Multiple-input multiple-output (MIMO), multiuser, nonregenerative relay, precoding, two-way relaying.**

## I. Introduction

A wireless propagation channel possesses two basic features: broadcasting and multiple-access. As a result, the transmit signal on a certain link can be detected by other neighboring nodes in the wireless network and these nodes then can act as relays and help to forward the message to the destination receiver. Such process leads to the basic idea of cooperative communications, which now has received great attention from both academia and industry. Unlike the design of traditional wireless communication systems where users compete with each other to access wireless resources, cooperative communications enable user cooperation and resource sharing at the physical layer and hence can significantly enhance the system performance. Network coding [1], on the other hand, as a significant technical breakthrough in network communication, allows a network router to perform coding on multiple data packets so that the information contained in multiple packets can be embedded in one coded packet. By doing this, the network throughput can be increased significantly. While network coding is initially proposed for wired networks, its application in wireless networks is more promising due to the broadcast nature of wireless channels. Naturally, the combination of cooperative communication and network coding is believed to be a key technique to dramatically enhance the performance of wireless networks and even change the network architecture of further wireless networks.

A building block of wireless networks where network coding meets cooperative communication is the two-way relay channel [2]–[4], where two source nodes exchange information with each other via a relay node. Two-way relaying applies the principle of network coding at the relay node so as to mix the signals received from the two source nodes and then employs at each destination the self-interference (SI) cancelation technique to extract the desired information.

While most of the existing works on two-way relaying in the literature focus on the simplest two-user model, two-way relaying protocol can be generalized to a more complex model where multiple users simultaneously exchange information through a relay node. A major challenge of designing a multiuser two-way relay system (MU-TWRS) arises from the inter-user interference which may dramatically degrade the system performance if not handled properly.

A simple method to avoid inter-user interference in MU-TWRS is to let different users transmit on orthogonal channels, for example, by using the code-division-multiple-access (CDMA) [5] or orthogonal-frequency-division-multiple-access (OFDMA) [6]. A more advanced method to suppress interuser interference is to apply the multiple-input multiple-output (MIMO) technique so that the multiuser transmission can be achieved over the same frequency domain, also referred to as space-division-multiple-access (SDMA) as in [7]–[13]. Therein, it is critical to design the transceiver or precoder at each multiantenna node, especially the relay node. In [7], [8], the linear relay precoding under decode-and-forward (DF) relay strategy is studied. Since the relay is assumed to fully decode the received signals in the first time slot, the relay precoding only affects the performance of transmission in the second time slot. Therefore, by using the conventional zero-forcing (ZF) based precoder, the relay precoding studied in [7], [8] reduces to the power allocation problem, which is then solved optimally from the overall performance perspective. The relay precoding under am-

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plify-and-forward (AF) strategy, however, differs considerably from the DF case as the transmission of the two time slots are tightly coupled and hence is much more challenging. Based on both ZF and minimum mean-square-error (MMSE) criteria, the authors in [9] derive the optimal relay precoders for AF based multi-pair two-way relay systems for fixed source beamformers. The authors further devise the optimal scheduling method to maximize the sum rate while ensuring fairness among users in [10]. The AF based MU-TWRS with multiple pairs of users has also been studied in [11], [12] where several ZF based relay precoder structures have been proposed. The work [12] further derives the explicit analytical results to evaluate the system performance. In [13], the authors consider an AF based MU-TWRS model with one base station and multiple mobile users. Similar with [12], explicit analytical results are derived based on ZF precoding. The same MU-TWRS model as [13] has also been studied in [14], where the authors propose a joint precoder design using combined block diagonalization (BD) and algebraic norm-maximization [15]. Note that the aforementioned ZF based precoder designs all impose certain constraints on the number of relay antennas which may not be feasible for some scenarios. Furthermore, no power allocation has been considered, which is important to further improve the system performance.

The goal of this work is to study the linear relay precoding design for multi-pair two-way relay systems where multiple pairs of single-antenna users exchange information through a common multi-antenna relay node with AF relay strategy. The AF relay strategy is considered for its simplicity of implementation. Such scenario often occurs in mobile ad-hoc networks where multiple pairs of users intend to establish bi-directional communication links via a common relay node. The relay precoding is designed to maximize the minimum achievable rate among all the users so as to achieve the max-min fairness. The considered problem is nonconvex and its optimal solution is difficult to obtain. Through bisection search and certain transformation techniques, we decouple the original precoding design problem into a series of semidefinite programming (SDP) problems. Then we obtain a quasi-optimal solution by applying semidefinite relaxation (SDR). This precoding method does not impose any constraint on the number of relay antennas. Therefore, it can be used more widely in practical systems than the precoders proposed in [11], [12]. Simulation results show that the proposed iterative SDR based precoding design outperforms dramatically the precoders in [9] and [12] in the minimum user rate performance with comparable sum-rate performance under the same system configurations.

We also propose a zero-forcing based precoding scheme, called pair-wise zero-forcing (ZF-P) precoding. It aims to eliminate the interpair interference using ZF criterion. Based on the ZF structure, the precoding design is reduced to a power allocation problem. By formulating it into standard convex optimization problem, the optimal power allocation is obtained. Meanwhile, a simplified power allocation algorithm is introduced to further reduce computational complexity with almost no performance loss. Simulation results show that this ZF-P precoding design with simplified power allocation strikes a good balance between performance and complexity.

The rest of paper is organized as follows. In Section II, we introduce the multi-pair two-way MIMO relay system model and present the problem formulation. The SDR based precoding design is included in Section III. The ZF based precoding design is presented in Section IV. Section V provides discussion on the design complexity and the relay antenna requirement of the proposed precoding schemes. Extensive simulation results are illustrated in Section VI. Finally, Section VII offers some concluding remarks.

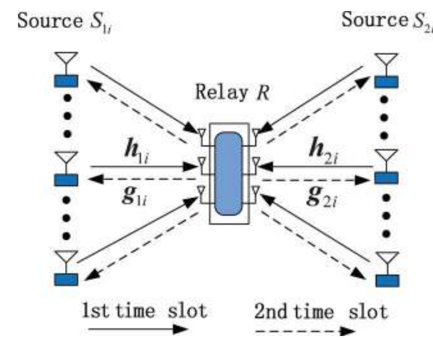

Fig. 1. Illustration of the multi-pair two-way relay system.

*Notations:* $\mathcal{E}[\cdot]$ denotes expectation over the random variables within the brackets. $\otimes$ and $\odot$ denote the Kronecker operator and Hadamard product, respectively. $vec(\cdot)$ signifies the matrix vectorization operator. $\mathrm{Tr}(\mathbf{A})$ and $\mathrm{Rank}(\mathbf{A})$ stand for the trace and the rank of matrix $\mathbf{A}$, respectively, and $\mathrm{Diag}(\mathbf{a})$ denotes a diagonal matrix with $\mathbf{a}$ being its diagonal entries. Superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote transpose, conjugate and conjugate transpose, respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $\|\mathbf{x}\|_2^2$ denotes the squared Euclidean norm of a complex vector $\mathbf{x}$ and $\|\mathbf{X}\|_F^2$ denotes the Frobenius norm of a complex matrix $\mathbf{X}$. $|z|$ denotes the norm of the complex number $z$, $\mathbb{C}^{x \times y}$ denotes the space of $x \times y$ matrices with complex entries. The distribution of a circular symmetric complex Gaussian vector with mean vector $\mathbf{x}$ and covariance matrix $\mathbf{\Sigma}$ is denoted by $\mathcal{CN}(\mathbf{x}, \mathbf{\Sigma})$.

## II. System Model and Problem Formulation

Consider a multi-pair two-way relay system as shown in Fig. 1, where multiple pairs of source nodes intend to exchange information within each pair, denoted as $S_{1k}$ and $S_{2k}$, for $k = 1, 2, \cdots, K$, under the assistance of a common relay node, denoted as $R$. Each source node is equipped with single antenna while the relay node is equipped with $M$ antennas. Due to the impairments such as multipath fading, shadowing and path loss of wireless channels, we assume that the direct-path link between the each source pair can be ignored. It is also supposed that the relay node operates in the half-duplex mode, i.e., it cannot transmit and receive simultaneously.

The bi-directional transmission of all user pairs are completed in two time slots by applying AF two-way relaying. In the first time slot, also referred to as multiple-access (MAC) phase, all the $2K$ sources transmit their signals to the relay node simultaneously over the same frequency band. Let $s_{jk}$ denote the signal transmitted from source $S_{jk}$ with $\mathcal{E}(s_{jk}s_{jk}^*) = P_{jk}$ where $j = 1, 2$ denotes the user index within each pair, and $k = 1, 2, \cdots, K$ denotes the pair index, $P_{jk}$ is the transmit power at source $S_{jk}$.

The received signal at the relay node in the first time slot can be expressed as

$$\mathbf{y}_R = \sum_{k=1}^{K}(\mathbf{h}_{1k}s_{1k} + \mathbf{h}_{2k}s_{2k}) + \mathbf{n}_R = \mathbf{H}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{s}_2 + \mathbf{n}_R, \tag{1}$$

where $\mathbf{h}_{jk} \in \mathbb{C}^{M\times 1}$ is the channel vector from $S_{jk}$ to relay node $R$ during the MAC phase, $\mathbf{H}_j = [\mathbf{h}_{j1}, \mathbf{h}_{j2}, \ldots, \mathbf{h}_{jK}]$ and $\mathbf{s}_j = [s_{j1}, s_{j2}, \ldots, s_{jK}]^T$. Here $\mathbf{n}_R \sim \mathcal{CN}(\mathbf{0}, \sigma_R^2\mathbf{I}_M)$ denotes the additive complex Gaussian noise vector at the relay node.

After receiving the signals transmitted from the sources, the relay node applies linear processing by multiplying it with a precoding matrix $\mathbf{F} \in \mathbb{C}^{M\times M}$. Then the $M \times 1$ signal vector to be transmitted from the relay node can be expressed as

$$\mathbf{x}_R = \mathbf{F}\mathbf{y}_R.$$

We assume that the maximum transmission power at the relay node is $P_R$, which yields

$$\mathrm{Tr}\left\{\mathbf{F}\left(\mathbf{H}_1\mathbf{P}_1\mathbf{P}_1^H\mathbf{H}_1^H + \mathbf{H}_2\mathbf{P}_2\mathbf{P}_2^H\mathbf{H}_2^H + \sigma_R^2\mathbf{I}_M\right)\mathbf{F}^H\right\} \le P_R, \tag{2}$$

where $\mathbf{P}_j = \mathrm{Diag}(\sqrt{P_{j1}}, \sqrt{P_{j2}}, \cdots, \sqrt{P_{jK}})$, $j = 1, 2$. In the second time slot, also referred to as broadcast (BC) phase, the relay node $R$ broadcasts $\mathbf{x}_R$ to all $2K$ user receivers. Then the received signals at $S_{jk}$ can be written as

$$\tilde{y}_{jk} = \sum_{l=1}^{K}\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}l}s_{\bar{j}l} + \sum_{l=1}^{K}\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{jl}s_{jl} + \mathbf{g}_{jk}^T\mathbf{F}\mathbf{n}_R + n_{jk}, \quad j = 1, 2, \quad k = 1, 2, \cdots, K \tag{3}$$

where $\bar{j} = 2$ if $j = 1$ and $\bar{j} = 1$ if $j = 2$, $\mathbf{g}_{jk} \in \mathbb{C}^{M\times 1}$ denotes the channel vector from the relay node $R$ to source $S_{jk}$ and $n_{jk} \sim \mathcal{CN}(0, \sigma_{jk}^2)$ denotes the additive complex Gaussian noise at $S_{jk}$.

In the considered multi-pair two-way relay systems, we assume that the channel characteristics of each link change slowly enough so that they can be perfectly estimated by using pilot symbols or training sequences. Besides that, the relay precoding design is conducted at the relay node due to its convenience to collect the channel state information (CSI). Moreover, we suppose that each node $S_{jk}$ estimates the combined channel coefficients $\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}$ and $\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{jk}$ by itself. Since the node $S_{jk}$ knows its own transmit signal $s_{jk}$, the back-propagating self-interference term $\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{jk}s_{jk}$ can be completely canceled from $\tilde{y}_{jk}$ in (3) before the demodulation, which yields

$$y_{jk} = \mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}s_{\bar{j}k} + \sum_{l\ne k}\left(\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}l}s_{\bar{j}l} + \mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{jl}s_{jl}\right) + \mathbf{g}_{jk}^T\mathbf{F}\mathbf{n}_R + n_{jk}, \tag{4}$$

for $j = 1, 2$ and $k = 1, 2, \cdots, K$. In (4) the first term is the desired signal, the second term is the interference from other pairs of users and the remaining terms represent the back-propagated noise from the relay node and the additive noise at the destination itself.

From (4), the performance of each user (with single-user detection) can be directly characterized by the signal-to-interference-plus-noise ratio (SINR), which is written as

$$\mathrm{SINR}_{jk} = \frac{P_{\bar{j}k}\left|\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}\right|^2}{\sum_{l\ne k}\left(P_{\bar{j}l}\left|\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}l}\right|^2 + P_{jl}\left|\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{jl}\right|^2\right) + \sigma_R^2\left\|\mathbf{g}_{jk}^T\mathbf{F}\right\|_2^2 + \sigma_{jk}^2}. \tag{5}$$

Accordingly, the achievable rate is expressed as $\gamma_{jk} = \frac{1}{2}\log_2(1 + \mathrm{SINR}_{jk})$ where the pre-log factor 1/2 is due to the fact that two time slots are used for one round of information exchange. Our objective is to maximize the minimum achievable rate among all the users, thereby, providing max-min fairness. Note that considering user fairness is more relevant than sum-rate performance in multi-pair two-way relay systems. The optimization problem is formulated as

$$\max_{\mathbf{F}} \min_{\forall j\in\{1,2\},\, k\in\{1,2,\ldots,K\}} \gamma_{jk} \quad \text{s.t.} \quad (2) \tag{6}$$

It is easy to verify that the relay precoding design problem (6) is nonconvex. The optimal solution is not easy to obtain in an efficient way. In Section III, we introduce the SDR based precoding design which does not impose any constraints on the relay antenna number. Then in Section IV, we introduce a precoding design with ZF structure, which has lower computational complexity but requires certain constraints on the relay antenna number.

## III. Precoding Design Based on Semidefinite Programming Relaxation

Since maximizing the minimum user rate is equivalent to maximizing the minimum user SINR, the max-min problem in (6) can be equivalently written as:

$$\max_{\mathbf{F}} \min_{\forall j,k} \frac{\mathrm{Tr}\left(P_{\bar{j}k}\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}\mathbf{h}_{\bar{j}k}^H\mathbf{F}^H\right)}{\mathbf{E}_{jk} + \sigma_R^2\mathrm{Tr}\left(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{F}^H\right) + \sigma_{jk}^2} \tag{7a}$$

$$\text{s.t.} \quad (2) \tag{7b}$$

where

$$\mathbf{E}_{jk} = \mathrm{Tr}\left(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\left(\sum_{l\ne k}\left(P_{\bar{j}l}\mathbf{h}_{\bar{j}l}\mathbf{h}_{\bar{j}l}^H + P_{jl}\mathbf{h}_{jl}\mathbf{h}_{jl}^H\right)\right)\mathbf{F}^H\right).$$

In obtaining (7a), the rules $|\mathbf{a}^T\mathbf{b}|^2 = \mathrm{Tr}(\mathbf{a}^*\mathbf{a}^T\mathbf{b}\mathbf{b}^H)$ and $\|\mathbf{a}^T\mathbf{B}\|_2^2 = \mathrm{Tr}(\mathbf{a}^*\mathbf{a}^T\mathbf{B}\mathbf{B}^H)$ have been used for the SINR expression in (5). Notice that due to the fractional structure of the objective function, (7) is similar to a quasi-convex problem according to [16, Chapter 4.2.5]. Hence we can rewrite (7) as the following optimization problem by introducing an auxiliary variable $t$

$$\begin{aligned}&\max_{\mathbf{F},t} t\\ s.t.\quad &\frac{\mathrm{Tr}\left(P_{\bar{j}k}\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}\mathbf{h}_{\bar{j}k}^H\mathbf{F}^H\right)}{\mathbf{E}_{jk}+\sigma_R^2\mathrm{Tr}\left(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{F}^H\right)+\sigma_{jk}^2}\geq t,\ \forall j,k\\ &\text{and}\quad (2)\end{aligned}\tag{8}$$

Then, solving (8) can be simplified by using bisection search. More specifically, we first fix the variable $t$ as $\hat{t}$ ($\hat{t}$ should be chosen from the specific interval containing the optimal $t$), and then solve the following problem

$$\min_{\mathbf{F}} \mathrm{Tr}\left\{\mathbf{F}\left(\mathbf{H}_1\mathbf{P}_1\mathbf{P}_1^H\mathbf{H}_1^H +\mathbf{H}_2\mathbf{P}_2\mathbf{P}_2^H\mathbf{H}_2^H+\sigma_R^2\mathbf{I}_M\right)\mathbf{F}^H\right\}\tag{9a}$$

$$s.t.\quad \frac{\mathrm{Tr}\left(P_{\bar{j}k}\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}\mathbf{h}_{\bar{j}k}^H\mathbf{F}^H\right)}{\mathbf{E}_{jk}+\sigma_R^2\mathrm{Tr}\left(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{F}^H\right)+\sigma_{jk}^2}\geq \hat{t},\ \forall j,k\tag{9b}$$

Problem (9) can be interpreted as minimizing the relay power consumption while guaranteeing that the SINR of each user is above a pre-determined threshold. Since SINR is a monotonically increasing function with respect to the relay power $P_R$, if the optimal solution of (9) for a given $\hat{t}$ is larger than $P_R$, it means that the given minimum SINR threshold $\hat{t}$ cannot be attained by all users in the system and hence should be reduced. Otherwise, if the optimal solution is smaller than $P_R$, we should increase $\hat{t}$. As such, problem (9) can be regarded as a feasibility check problem. Now, the optimal solution of (8) can be found if the optimal solution of subproblem (9) is obtained in each iteration of the bisection search.

In the following, we recast (9) into a suitable form such that efficient optimization tools can be applied to find its solution. To proceed, the SINR constraint (9b) is rewritten as

$$\mathrm{Tr}\left(\frac{P_{\bar{j}k}}{\hat{t}}\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{h}_{\bar{j}k}\mathbf{h}_{\bar{j}k}^H\mathbf{F}^H\right)-\mathrm{Tr}\left(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T\mathbf{F}\mathbf{Q}_{jk}\mathbf{F}^H\right)\geq\sigma_{jk}^2,\tag{10}$$

where $\mathbf{Q}_{jk}=\sum_{l\neq k}(P_{\bar{j}l}\mathbf{h}_{\bar{j}l}\mathbf{h}_{\bar{j}l}^H+P_{jl}\mathbf{h}_{jl}\mathbf{h}_{jl}^H)+\sigma_R^2\mathbf{I}_M$. By using the rule [17]

$$\mathrm{Tr}(\mathbf{ABCD})=\left(vec(\mathbf{D}^T)\right)^T(\mathbf{C}^T\otimes\mathbf{A})vec(\mathbf{B}),\tag{11}$$

the inequality (10) can be reexpressed as

$$\mathbf{f}^H\left(\mathbf{R}_{jk}^1(t)-\mathbf{R}_{jk}^2\right)\mathbf{f}\geq\sigma_{jk}^2,\tag{12}$$

where $\mathbf{f}=vec(\mathbf{F})$, $\mathbf{R}_{jk}^1(t)=\frac{P_{\bar{j}k}}{t}(\mathbf{h}_{\bar{j}k}\mathbf{h}_{\bar{j}k}^H)^T\otimes(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T)$ and $\mathbf{R}_{jk}^2=\mathbf{Q}_{jk}^T\otimes(\mathbf{g}_{jk}^*\mathbf{g}_{jk}^T)$. Again using (11), the objective function in (9a) can be equivalently denoted as

$$\mathrm{Tr}\left\{\mathbf{F}\left(\mathbf{H}_1\mathbf{P}_1\mathbf{P}_1^H\mathbf{H}_1^H+\mathbf{H}_2\mathbf{P}_2\mathbf{P}_2^H\mathbf{H}_2^H+\sigma_R^2\mathbf{I}_M\right)\mathbf{F}^H\right\}=\mathbf{f}^H\mathbf{R}\mathbf{f},\tag{13}$$

where $\mathbf{R}=\left(\sum_{j=1}^2\mathbf{H}_j\mathbf{P}_j\mathbf{P}_j^H\mathbf{H}_j^H+\sigma_R^2\mathbf{I}_M\right)^T\otimes\mathbf{I}_M$. According to (12) and (13), the optimization problem (9) is transformed into the quadratically constrained quadratic program (QCQP) form as

$$\begin{aligned}&\min_{\mathbf{f}}\mathbf{f}^H\mathbf{R}\mathbf{f}\\ s.t.\quad &\mathbf{f}^H\mathbf{R}_{jk}(\hat{t})\mathbf{f}\geq\sigma_{jk}^2,\ \forall j,k\end{aligned}\tag{14}$$

where $\mathbf{R}_{jk}(\hat{t})=\mathbf{R}_{jk}^1(\hat{t})-\mathbf{R}_{jk}^2$. By introducing $\tilde{\mathbf{F}}=\mathbf{f}\times\mathbf{f}^H\in\mathbb{C}^{M^2\times M^2}$, we further transform (14) into the following semidefinite programming (SDP) form:

$$\begin{aligned}&\min_{\tilde{\mathbf{F}}\succeq 0}\mathrm{Tr}(\mathbf{R}\tilde{\mathbf{F}})\\ s.t.\quad &\mathrm{Tr}\left(\mathbf{R}_{jk}(\hat{t})\tilde{\mathbf{F}}\right)\geq\sigma_{jk}^2,\ \forall j,k,\\ &\mathrm{Rank}(\tilde{\mathbf{F}})=1\end{aligned}\tag{15}$$

Due to the rank-one constraint, the optimization problem (15) is nonconvex. So we relax the rank-one constraint and solve the problem:

$$\min_{\tilde{\mathbf{F}}\succeq 0}\mathrm{Tr}(\mathbf{R}\tilde{\mathbf{F}})\tag{16a}$$

$$s.t.\quad \mathrm{Tr}\left(\mathbf{R}_{jk}(\hat{t})\tilde{\mathbf{F}}\right)\geq\sigma_{jk}^2,\ \forall j,k,\tag{16b}$$

The optimal solution of (16) can be readily obtained due to its convexity.

After the termination of bisection search, if the obtained $\tilde{\mathbf{F}}$, denoted as $\tilde{\mathbf{F}}^{opt}$, in (16) is rank-one, the optimal solution of (14), denoted as $\mathbf{f}^{opt}$, can be obtained by applying the eigenvalue decomposition (EVD) technique, i.e.,

$$\mathbf{f}^{opt}=\sqrt{d}\mathbf{u},\tag{17}$$

where $d$ and $\mathbf{u}$ are the eigenvalue and eigenvector of $\tilde{\mathbf{F}}^{opt}$, respectively. Otherwise, we apply the randomization technique [18] to obtain the final solution of (14) from (16). Namely, we first generate a random vector $\tilde{\mathbf{x}}=(\tilde{\mathbf{F}}^{opt})^{\frac{1}{2}}\mathbf{n}$ with $\mathbf{n}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_{M^2})$ and set $\tilde{\mathbf{X}}=\tilde{\mathbf{x}}\tilde{\mathbf{x}}^H$. If $\tilde{\mathbf{X}}$ satisfies $\mathrm{Tr}(\mathbf{R}_{jk}(\hat{t})\tilde{\mathbf{X}})>0,\forall j,k$, then one candidate of solution of (14) can be obtained as $\mathbf{x}=a\tilde{\mathbf{x}}$, where the scalar $a$ is chosen to minimize the power consumption while satisfying the constraint in (16a). It can be seen easily that $a=\max_{j,k}\sqrt{\frac{\sigma_{jk}^2}{\mathrm{Tr}(\mathbf{R}_{jk}(\hat{t})\tilde{\mathbf{X}})}}$. After generating a number of samples, the best candidate which obtains the minimum relay power is chosen to be the solution of (14). If the number of samples are large enough, the randomization technique can find the optimal solution of (14) on average. To obtain the final solution of (7), we also need to scale $\mathbf{x}$ to satisfy the relay power constraint.

Overall, the SDR based precoding design is summarized as:

**Algorithm 1:** (SDR-based relay precoding design)

- Find the upper bound of $t$ as $\bar{t}$ and set $t_{min}=0$, $t_{max}=\bar{t}$,
- **Repeat**
  1) Set $\hat{t}=\frac{(t_{min}+t_{max})}{2}$;
  2) Solve (16) and denote the optimal relay precoder as $\tilde{\mathbf{F}}^{opt}(\hat{t})$;
  3) Compute the consumed relay power $P=\mathrm{Tr}(\mathbf{R}\tilde{\mathbf{F}}^{opt}(\hat{t}))$. If $P>P_R$, set $t_{max}=\hat{t}$. Otherwise, set $t_{min}=\hat{t}$;
- **Until** $|t_{max}-t_{min}|\leq e$ (where $e$ is a preset error precision.)
- **Obtain rank-one solution**: If $\tilde{\mathbf{F}}^{opt}$ in (16) is rank-one, then obtain the optimal relay precoder of (14) by using EVD as in (17). Otherwise, apply the randomization approach to get the final solution;

*Remark 1:* In Algorithm 1, the upper bound $\bar{t}$ can be obtained by ignoring the interference from other users. More specifically, for each $S_{jk}$, we only consider the effective channel link $\mathbf{g}_{jk}$ and $\mathbf{h}_{\bar{j}k}$. Then as shown in [19], the optimal relay precoder should have a form as $\mathbf{F}_{jk} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$ where $\mathbf{U}$ is the right singular-vector matrix of $\mathbf{g}_{jk}$ and $\mathbf{V}$ is the left singular-vector matrix of $\mathbf{h}_{\bar{j}k}$. Since both $\mathbf{h}_{\bar{j}k}$ and $\mathbf{g}_{jk}$ have only one positive singular value, the optimal $\mathbf{F}_{jk}$ can be written as $\mathbf{F}_{jk} = \alpha_{jk}\mathbf{g}_{jk}\mathbf{h}_{\bar{j}k}^H$, where $\alpha_{jk} = \sqrt{\frac{P_R}{P_{\bar{j}k}\|\mathbf{g}_{jk}\|_2^2\|\mathbf{h}_{\bar{j}k}\|_2^4+\sigma_R^2\|\mathbf{g}_{jk}\mathbf{h}_{\bar{j}k}^H\|_F^2}}$ is used to scale the transmit signal at relay node to satisfy the power constraint. Then we obtain an upper bound of $t$ as

$$\bar{t} = \max_{j,k} \frac{\alpha_{jk}^2\|\mathbf{g}_{jk}\|_2^4\|\mathbf{h}_{\bar{j}k}\|_2^4}{\alpha_{jk}^2\sigma_R^2\|\mathbf{g}_{jk}\|_2^4\|\mathbf{h}_{\bar{j}k}\|_2^2 + \sigma_{jk}^2}.$$

*Remark 2:* The obtained $\hat{t}$ after the termination of bisection search and before extracting the rank-one solution in Algorithm 1 can be viewed as an upper bound of the optimal solution of the original problem (7). This is because relaxing the rank-one constraint enlarges the feasible region of the optimized variable in (15). This upper bound can be used to validate the optimality of the SDR-based precoding.

## IV. Precoding Design Based on Zero-Forcing

Although Algorithm 1 attains good performance as verified in Section VI, it usually results in relative high computational complexity. In this section, we apply the zero-forcing structure in the relay precoder design as in [11], [12], [14], [20] so that the precoding design for max-min fairness reduces to a simple power allocation problem. This method may lead to a sub-optimal solution, however, it has much lower computational complexity.

Generally, there are two ways to perform the zero-forcing precoding at the relay node. One way is to use the zero-forcing precoding to eliminate all the interference, including the self-interference, among the users as in [9]. The complete interference cancelation in such scheme requires that the relay node has enough relay antennas (satisfying $M \geq 2K$) to construct the null space for each user. In this case, no self-interference cancelation is needed at each destination receiver. The alternative way is to combine the techniques of relay zero-forcing and physical layer network coding such that both the relay and the destination nodes participate to cancel the interference. In this way, the relay precoder only needs to eliminate the inter-pair interference but not the intra-pair interference (thus we refer to our proposed ZF based scheme as pair-wise zero-forcing (ZF-P) relay precoding). For such ZF-P precoding scheme, the relay node needs at least $2K-1$ antennas instead of $2K$ antennas as required in the first scheme. A similar ZF-based precoding scheme is introduced in [12]. But that work does not study power optimization, neither user fairness.

The main idea of the proposed ZF-P precoding is to adopt the block-diagonalization (BD) technique to eliminate the inter-pair interference in both MAC and BC phases. The same idea has also been used for the cellular based MU-TWRS in [14]. For ease of presentation, we set the bidirectional transmission between $S_{1k}$ and $S_{2k}$ as example. We first define $\hat{\mathbf{H}}_k$ as

$$\hat{\mathbf{H}}_k = [\tilde{\mathbf{H}}_1, \tilde{\mathbf{H}}_2, \ldots, \tilde{\mathbf{H}}_{k-1}, \tilde{\mathbf{H}}_{k+1}, \ldots, \tilde{\mathbf{H}}_K], \quad (18)$$

where $\tilde{\mathbf{H}}_k = [\mathbf{h}_{1k}, \mathbf{h}_{2k}]$ is the channel matrix of the $k$-th pair in MAC phase. Applying the singular-value decomposition (SVD) technique to (18), we get

$$\hat{\mathbf{H}}_k = \left[\mathbf{U}_{\hat{\mathbf{H}}_k}^{(1)}, \mathbf{U}_{\hat{\mathbf{H}}_k}^{(0)}\right] \mathbf{\Sigma}_{\hat{\mathbf{H}}_k} \mathbf{V}_{\hat{\mathbf{H}}_k}^H$$

where $\mathbf{U}_{\hat{\mathbf{H}}_k}^{(1)} \in \mathbb{C}^{M\times(2K-2)}$ holds the first $2K-2$ left singular vectors, $\mathbf{U}_{\hat{\mathbf{H}}_k}^{(0)} \in \mathbb{C}^{M\times(M-2K+2)}$ holds the last $M-2K+2$ left singular vectors which forms the null space of $\hat{\mathbf{H}}_k^H$. To obtain the receive beamforming vector, we further define

$$\left[\mathbf{u}_{\mathbf{h}_k}^{(1)}, \mathbf{U}_{\mathbf{h}_k}^{(0)}\right] \mathbf{\Sigma}_{\mathbf{h}_k} \mathbf{V}_{\mathbf{h}_k}^H = \mathbf{U}_{\hat{\mathbf{H}}_k}^{(0)^H} \mathbf{h}_k, \quad (19)$$

where $\mathbf{h}_k = \mathbf{h}_{1k} + \mathbf{h}_{2k}$, $\mathbf{u}_{\mathbf{h}_k}^{(1)} \in \mathbb{C}^{(M-2K+2)\times 1}$ and $\mathbf{U}_{\mathbf{h}_k}^{(0)} \in \mathbb{C}^{(M-2K+2)\times(M-2K+1)}$ form the orthogonal basis of $\mathbf{h}_k^H\mathbf{U}_{\hat{\mathbf{H}}_{1k}}^{(0)}$ and its null space, respectively. Therefore, applying the receive beamforming vector as

$$\mathbf{t}_k^m = \mathbf{U}_{\hat{\mathbf{H}}_k}^{(0)} \mathbf{u}_{\mathbf{h}_{1k}}^{(1)}, \quad (20)$$

we can get zero inter-pair interference at the relay node for pair $\{S_{1k}, S_{2k}\}$ within the MAC phase. Similarly, we can also design the transmit beamforming vector for $S_{1k}$ and $S_{2k}$ in the BC phase by defining

$$\begin{aligned}\hat{\mathbf{G}}_k &= [\tilde{\mathbf{G}}_1, \tilde{\mathbf{G}}_2, \ldots, \tilde{\mathbf{G}}_{k-1}, \tilde{\mathbf{G}}_{k+1}, \ldots, \tilde{\mathbf{G}}_K]^T \\ &= \mathbf{U}_{\hat{\mathbf{G}}_k} \mathbf{\Sigma}_{\hat{\mathbf{G}}_k} \left[\mathbf{V}_{\hat{\mathbf{G}}_k}^{(1)}, \mathbf{V}_{\hat{\mathbf{G}}_k}^{(0)}\right]^H, \end{aligned} \quad (21)$$

where $\tilde{\mathbf{G}}_k = [\mathbf{g}_{1k}, \mathbf{g}_{2k}]$ denotes the channel matrix of the $k$-th pair in BC phase, and $\mathbf{V}_{\hat{\mathbf{G}}_k}^{(0)} \in \mathbb{C}^{M\times(M-2K+2)}$ denotes the null basis of $\hat{\mathbf{G}}_k$. Then we get

$$\mathbf{U}_{\mathbf{g}_k} \mathbf{\Sigma}_{\mathbf{g}_k} \left[\mathbf{v}_{\mathbf{g}_k}^{(1)}, \mathbf{V}_{\mathbf{g}_k}^{(0)}\right]^H = \mathbf{g}_k^T \mathbf{V}_{\hat{\mathbf{G}}_k}^{(0)}, \quad (22)$$

where $\mathbf{g}_k = \mathbf{g}_{1k} + \mathbf{g}_{2k}$, $\mathbf{v}_{\mathbf{g}_k}^{(1)}$ and $\mathbf{V}_{\mathbf{g}_k}^{(0)}$ are defined similarly as in (19). The inter-pair interference-free transmit beamforming vector for $S_{1k}$ and $S_{2k}$ within the BC phase is obtained as $\mathbf{t}_k^b = \mathbf{V}_{\hat{\mathbf{G}}_k}^{(0)} \mathbf{v}_{\mathbf{g}_k}^{(1)}$. Finally, we get the relay precoder which has following structure

$$\mathbf{T} = \sum_{k=1}^{K} \delta_k \mathbf{t}_k^b \mathbf{t}_k^{mH} = \mathbf{T}_b \mathbf{\Delta} \mathbf{T}_m^H. \quad (23)$$

where $\mathbf{T}_b = [\mathbf{t}_1^b, \mathbf{t}_2^b, \cdots, \mathbf{t}_K^b]$, $\mathbf{T}_m = [\mathbf{t}_1^m, \mathbf{t}_2^m, \cdots, \mathbf{t}_K^m]$ and $\mathbf{\Delta} = \mathrm{Diag}(\delta_1, \delta_2, \cdots, \delta_K)$ with $\delta_k, \forall k$ being the power allocation parameter for pair $\{S_{1k}, S_{2k}\}$. By using the relay precoder (23), the interpair interference for all other users is completely canceled and the received signal at each node can be denoted as

$$\begin{aligned} y_{jk} = \delta_k \mathbf{g}_{jk}^T \mathbf{t}_k^b \mathbf{t}_k^{mH} \mathbf{h}_{\bar{j}k} s_{\bar{j}k} + \delta_k \mathbf{g}_{jk}^T \mathbf{t}_k^b \mathbf{t}_k^{mH} \mathbf{h}_{jk} s_{jk} \\ + \delta_k \mathbf{g}_{jk}^T \mathbf{t}_k^b \mathbf{t}_k^{mH} \mathbf{n}_R + n_{jk}, \ \forall j,k \end{aligned} \quad (24)$$

where the first term is the desired signal, the second term is the self interference and the remaining terms represent the back-propagated noise from the relay node and the additive noise at the destination itself. Then after the self-interference cancelation in each node, the received signal-to-noise ratio (SNR) is written as

$$\mathrm{SNR}_{jk} = \frac{\delta_k^2 c_{jk}}{\delta_k^2 d_{jk} + \sigma_{jk}^2}, \ \forall j,k \quad (25)$$

where $c_{jk} = P_{\bar{j}k}|\mathbf{g}_{jk}^T \mathbf{t}_k^b \mathbf{t}_k^{mH} \mathbf{h}_{\bar{j}k}|^2$, $d_{jk} = \sigma_R^2 \|\mathbf{g}_{jk}^T \mathbf{t}_k^b \mathbf{t}_k^{mH}\|_2^2$.

By introducing a power allocation vector $\boldsymbol{\delta} = [\delta_1, \delta_2, \cdots, \delta_K]^T$, the original max-min problem expressed in (7) can be rewritten as the following power allocation problem:

$$\max_{\boldsymbol{\delta}, t} t \tag{26a}$$

$$s.t. \quad \frac{\delta_k^2 c_{jk}}{\delta_k^2 d_{jk} + \sigma_{jk}^2} \geq t, \ \forall j, k$$

$$\boldsymbol{\delta}^T (\mathbf{K} \odot \mathbf{L}^T) \boldsymbol{\delta} \leq P_R \tag{26b}$$

where $\mathbf{K} = \mathbf{T}_b^H \mathbf{T}_b$ and $\mathbf{L} = \mathbf{T}_m^H (\sum_{j=1}^2 \mathbf{H}_j \mathbf{P}_j \mathbf{P}_j^H \mathbf{H}_j^H + \sigma_R^2 \mathbf{I}_M) \mathbf{T}_m$. Here the constraint (26b) is obtained by using the rule $\mathrm{Tr}(\mathbf{AMB}^T \mathbf{M}) = \mathbf{m}^T (\mathbf{A} \odot \mathbf{B}) \mathbf{m}$ with $\mathbf{M} = \mathrm{Diag}(\mathbf{m})$ in [17] for the relay power constraint (2). Since both $\mathbf{K}$ and $\mathbf{L}$ are positive semidefinite, the matrix $\mathbf{K} \odot \mathbf{L}^T$ is also positive semidefinite [17]. Thus, the relay power constraint in (26b) is convex. Similar to Algorithm 1, we can solve (26) by converting it into a series of subproblems as follows for a fixed $\hat{t}$:

$$\min_{\boldsymbol{\delta}} \boldsymbol{\delta}^T (\mathbf{K} \odot \mathbf{L}^T) \boldsymbol{\delta}$$

$$s.t. \qquad \delta_k \geq \sqrt{\frac{\left(\sigma_{jk}^2 \hat{t}\right)}{(c_{jk} - d_{jk} \hat{t})}}, \ \forall j, k \tag{27}$$

where the optimal $\hat{t}$ can be obtained using bisection search. Note that, during the bisection search, the maximum value of $\hat{t}$ should be chosen such that $c_{jk} - d_{jk}\hat{t} > 0$ for $\forall j, k$. Therefore, the search region of $t$ is easy to be determined as $[0, \bar{t}]$ where $\bar{t} = \min_{j,k} \frac{c_{jk}}{d_{jk}}$.

Although the bisection approach can obtain the optimal power allocation, it still leads to high computational complexity. By taking a closer look at problem (26), we find that the main difficulty in solving (26) is due to the fact that the matrix $\mathbf{K} \odot \mathbf{L}^T$ is non-diagonal. Thus, we next propose a simplified power allocation algorithm by relaxing the matrix $\mathbf{L}$ as a diagonal matrix $\tilde{\mathbf{L}}$ which is given as

$$\tilde{\mathbf{L}} = \sigma_R^2 \mathbf{D} + \mathbf{T}_m^H \left( \mathbf{H}_1 \mathbf{P}_1 \mathbf{P}_1^H \mathbf{H}_1^H + \mathbf{H}_2 \mathbf{P}_2 \mathbf{P}_2^H \mathbf{H}_2^H \right) \mathbf{T}_m. \tag{28}$$

Here $\mathbf{D}$ is a diagonal matrix containing the diagonal entries of $\mathbf{T}_m^H \mathbf{T}_m$. The second term in the right hand side of (28) is diagonal because each column vector $\mathbf{t}_k^m$ as defined in (20) lies in the null space of $\hat{\mathbf{H}}_k^H$ defined in (18) and as a result, the matrix $\mathbf{T}_m^H$ diagonalizes $\mathbf{H}_1$ and $\mathbf{H}_2$ simultaneously.

Substituting $\tilde{\mathbf{L}}$ into (26), we get the following simplified problem

$$\max_{t, \delta_k, \forall k} \ t$$

$$s.t. \qquad \frac{\delta_k^2 c_{jk}}{\delta_k^2 d_{jk} + \sigma_{jk}^2} \geq t, \ \forall j, k$$

$$\sum_k \beta_k \delta_k^2 \leq P_R \tag{29}$$

where $\beta_k$ is the diagonal entry of $\mathbf{K} \odot \tilde{\mathbf{L}}$. Note that the introduced relaxation can be ignored if SNR in the MAC phase is large enough. To solve (29), we have the following lemma.

*Lemma 1:* For the optimal solution of (29), it has

$$\min \left\{ \frac{\delta_k^2 c_{1k}}{\delta_k^2 d_{1k} + \sigma_{1k}^2}, \frac{\delta_k^2 c_{2k}}{\delta_k^2 d_{2k} + \sigma_{2k}^2} \right\} = t, \ \forall k \tag{30}$$

and

$$\sum_k \beta_k \delta_k^2 = P_R.$$

*Proof:* It is easy to verify that $\mathrm{SNR}_{jk}$ in (25) is an increasing function with respect to $\delta_k$. Thus, we conclude that the optimal solution in (29) must consume all the power at the relay node, i.e., $\sum_k \beta_k \delta_k^2 = P_R$. If at the optimal solution, the minimum SNR of the pair $\{S_{1k}, S_{2k}\}$ is larger than the minimum SNR of the pair $\{S_{1l}, S_{2l}\}$ when $l \neq k$, we can always decrease the value $\delta_k$ and increase the value $\delta_l$ to further increase $\min_{j,k} \mathrm{SNR}_{jk}$. This contradicts the assumption that $\{\delta_k\}$ is the optimal solution. Thus, we conclude that the optimal power allocation in (29) should make the minimum SNR within all the user pairs equal to each other. ■

Applying Lemma 1, we obtain the following simplified method to conduct the power allocation. Namely, we first assume that for pair $\{S_{1k}, S_{2k}\}$, node $S_{j_k k}$ has the minimum SNR where $j_k$ can be chosen as 1 or 2. Then based on Lemma 1, we have

$$\delta_k^2 = \frac{\sigma_{j_k k}^2}{c_{j_k k}/t - d_{j_k k}}, \quad k = 1, 2, \cdots, K. \tag{31}$$

Substituting (31) into the power constraint, we obtain

$$\sum_k \frac{\beta_k \sigma_{j_k k}^2}{c_{j_k k}/t - d_{j_k k}} = P_R. \tag{32}$$

Then we can solve (32) to get the value of $t$ for the chosen $\{j_1, j_2, \cdots, j_K\}$. By trying all the possible cases of $\{j_1, j_2, \cdots, j_K\}$, the maximum $t$ is finally obtained as the optimal solution of (29) and the optimal power allocation can be further acquired through (31). If $K$ is not very large, the computational complexity can be significantly reduced by using the proposed simplified method. The design complexity of different precoding schemes shall be compared in the next section. Besides that, we also find that compared to the method of solving (26) using bisection search, no advanced software is needed to solve (29) by using Lemma 1.

Note that the solution obtained from Lemma 1 is based on relaxation in (28). The derived solution may violate the relay power constraint or do not consume all the relay power. Thus we need to scale the obtained relay precoder to satisfy the power constraint or to further improve the performance. The scalar can be computed as

$$\theta = \sqrt{\frac{P_R}{\mathrm{Tr}\left( \mathbf{T} \left( \mathbf{H}_1 \mathbf{P}_1 \mathbf{P}_1^H \mathbf{H}_1^H + \mathbf{H}_2 \mathbf{P}_2 \mathbf{P}_2^H \mathbf{H}_2^H + \sigma_R^2 \mathbf{I}_M \right) \mathbf{T}^H \right)}}$$

where $\mathbf{T} = \mathbf{T}_b \boldsymbol{\Delta} \mathbf{T}_m^H$ with $\boldsymbol{\Delta}$ being solved from (29). Finally, we obtain the relay precoder as $\mathbf{F} = \theta \mathbf{T}$.

TABLE I
COMPARISON DESIGN COMPLEXITY COMPARISON AND RELAY ANTENNA REQUIREMENT

| | Design Complexity | Relay Antenna Requirement |
|---|---|---|
| SDR-based precoding | $O((\max(M,2K)^4\sqrt{M}\log(1/\epsilon))\cdot\lceil\log_2(\bar{t}/e)\rceil+n_{rd})$ | No |
| Optimal ZF-P precoding | $O((3K+1)(K+1)^2(2K+1)^{0.5}\log(1/\epsilon)\cdot\lceil\log_2((\min_{j,k}c_{jk}/d_{jk})/e)\rceil)$ | $M\geq 2K-1$ |
| Simplified ZF-P precoding | $O(2^K)$ | $M\geq 2K-1$ |
| [9] | $O(1)$ | No |
| [12] | $O(1)$ | $M\geq 2K-1$ |

## V. DISCUSSION ON COMPLEXITY AND ANTENNA REQUIREMENT

In this section, we shall provide discussions on the design complexity and the relay antenna requirement for the proposed precoding designs and the existing methods in [9] and [12]. The overall comparisons are summarized in Table I, where "Optimal ZF-P precoding" means that the optimal power allocation method is applied and "Simplified ZF-P precoding" means to use the simplified relay power allocation method. In addition, since the designs proposed in [9] and [12] have the closed-form solutions, their complexities can also be simply viewed as $O(1)$.

According to [18], the complexity of solving the SDP problem (16) can be approximated as $O\left(\max(M,2K)^4\sqrt{M}\log\left(\frac{1}{\epsilon}\right)\right)$, where $\epsilon$ denotes the solution accuracy. The number of iterations for the bisection search used in Algorithm 1 is given by $\left\lceil\log_2\left(\frac{\bar{t}}{e}\right)\right\rceil$ where $\bar{t}$ and $e$ are the preset search bound and error precision, respectively. Therefore, the overall complexity of the SDR based precoding design is as shown in Table I, where $n_{rd}$ denotes the complexity of randomization.

For optimal ZF-P precoding, since the QCQP problem (27) can be transformed into the second-order cone programming (SOCP) problem, according to [21], its complexity can be approximated as $O\left((3K+1)(K+1)^2(2K+1)^{0.5}\ln\left(\frac{1}{\epsilon}\right)\right)$. Since the number of iteration is $\left\lceil\log_2\left(\left(\min_{j,k}\frac{c_{jk}}{d_{jk}}\right)/e\right)\right\rceil$, the overall complexity of the optimal ZF-P precoding is as shown in Table I. For simplified ZF-P precoding, since we only need to solve the (32) for each chosen $\{j_1,j_2,\cdots,j_K\}$, the design complexity can be simply viewed as $O(2^K)$. We can find that if $K$ is not very large, the simplified ZF-P precoding can save the computational complexity significantly. While in practice, the number of user pair $K$ cannot be very large in order to achieve effective transmission.

For the required relay antenna number, since the proposed ZF-P precoding design and [12] apply the block-diagonalization technique, they need at least $2K-1$ relay antennas. While for the proposed SDR precoding design and [9], there is no constraint imposed on the relay antenna number.

## VI. SIMULATION RESULTS

In this section, some examples are presented to evaluate the proposed precoding designs. The channel on each link is set to be normalized Rayleigh fading, i.e., the elements of each channel matrix are complex Gaussian random variables with zero mean and unit variance. We also assume $P_{jk}=P$ and $\sigma_{jk}^2=\sigma^2$ for $\forall j,k$. The average SNR for the MAC and BC phases are defined as $\rho_{MAC}=\frac{KP}{\sigma_R^2}$ and $\rho_{BC}=\frac{P_R}{\sigma^2}$, respectively.

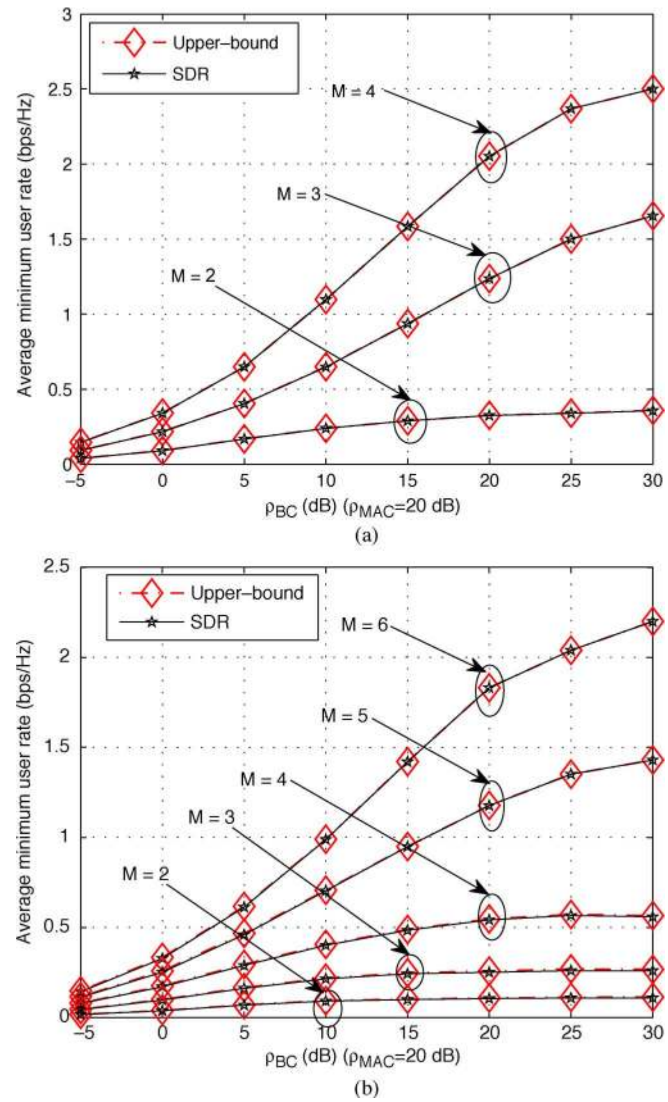


Fig. 2. The performance of the proposed SDR-based precoding design at different relay antenna number for $\rho_{BC}$ at $\rho_{MAC}=20$ dB. (a) $K=2$. (b) $K=3$.

In Fig. 2(a) and (b), we check the optimality of the proposed SDR based precoding design for $K=2$ user pairs and $K=3$ user pairs, respectively, by comparing with the performance upper bound. The upper bound is obtained by skipping the procedure to get the rank-one solution as claimed in *Remark 2*. It is observed that the proposed SDR based precoding design almost attains the performance upper bound, which confirms that the solution obtained from the SDR based method approaches the optimal solution. On the other hand, we find that the SDR based

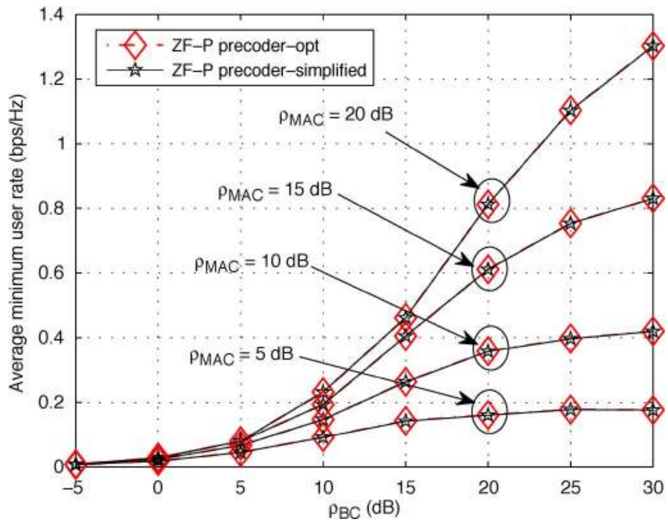


Fig. 3. The performance of the proposed simplified power allocation methods under different MAC SNR scenarios at $K = 3$ and $M = 6$.

precoding design works well for any number of relay antennas. The observation also shows that the performance of SDR based precoding design is sensitive to the relay antenna number and it can be enhanced significantly if the relay antenna number is increased.

Fig. 3 shows the optimality of the solution obtained by the proposed simplified power allocation method for the ZF-P precoding design. Although we have claimed that the applied relaxation can only be ignored under the high MAC SNR conditions, the observations verify that the proposed simplified power allocation method by *Lemma 1* also works well in the low SNR scenarios. Thus, without loss of generality, we next only use the simplified method for performance comparison.

In Fig. 4(a) and (b), we compare the proposed designs with the existing methods in [9] with MMSE design and in [12] with coherent combining of null-space vectors scheme in terms of minimum user rate. Here we consider $K = 3$ user pairs with $M = 5$, 6 antennas at the relay node. We observe that the SDR based precoding design significantly outperforms the methods in [9], [12] for both $M = 5$ and $M = 6$. For the simplified ZF-P precoding scheme, one can see that, at $M = 6$, it performs slightly better than [12] and performs worse than the methods in [9]. This indicates that if the number of relay antennas is large enough, using the relay precoder to completely eliminate the interference among all the users is more beneficial than the scheme with network coding to achieve the user fairness. However, when with less relay antennas, as shown in Fig. 4(b) where we set $M = 5$, the performance of [9] is significantly degraded and becomes worse than the simplified ZF-P precoding scheme when $\rho_{BC}$ is lager than 10 dB. Compared with [12], the proposed ZF-P is consistently better.

Besides the max-min fairness, we also present the sum-rate performance of the proposed designs in comparison with existing designs in Fig. 5(a) and (b). We observe that the SDR based precoding design not only performs well to achieve the user fairness, it can also obtain good performance in sum rate.

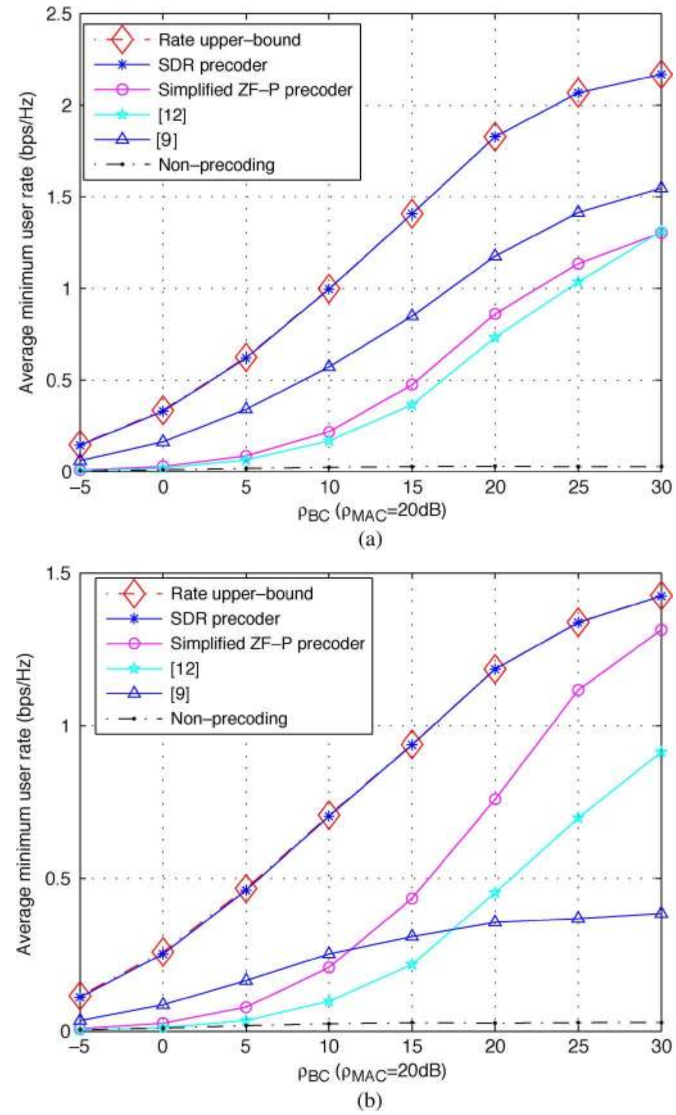


Fig. 4. The average minimum rate comparison for different relay precoding designs at $K = 3$. (a) $M = 6$. (b) $M = 5$.

Since the objective of the simplified ZF-P precoding design is to achieve the user fairness, it suffers gain loss in the sum rate when $M = 6$. However, when with less relay antennas, i.e., $M = 5$, the sum rate difference between [12] and the simplified ZF-P precoding design is significantly reduced and the simplified ZF-P precoding design significantly outperforms [9] when $\rho_{BC} \geq 10$ dB.

In Fig. 6, we further illustrate the system outage performance achieved by the proposed designs for $K = 3$. Since the considered system is a multiuser system, we define that the system is in outage if any of the users is in outage. The target transmission rate of each user is set to be $R = 0.5$ bits/s/Hz. In Fig. 6, we can observe similar results as in Fig. 4. Namely, the SDR based precoding always performs the best. Although the simplified ZF-P precoding is inferior to [9] when $M = 6$, it begins to significantly outperform [9] when with less relay antennas at $M = 5$. The curves in Fig. 6 also show that in general, the SDR based precoding obtains the highest diversity order among all the considered schemes and that the simplified ZF-P

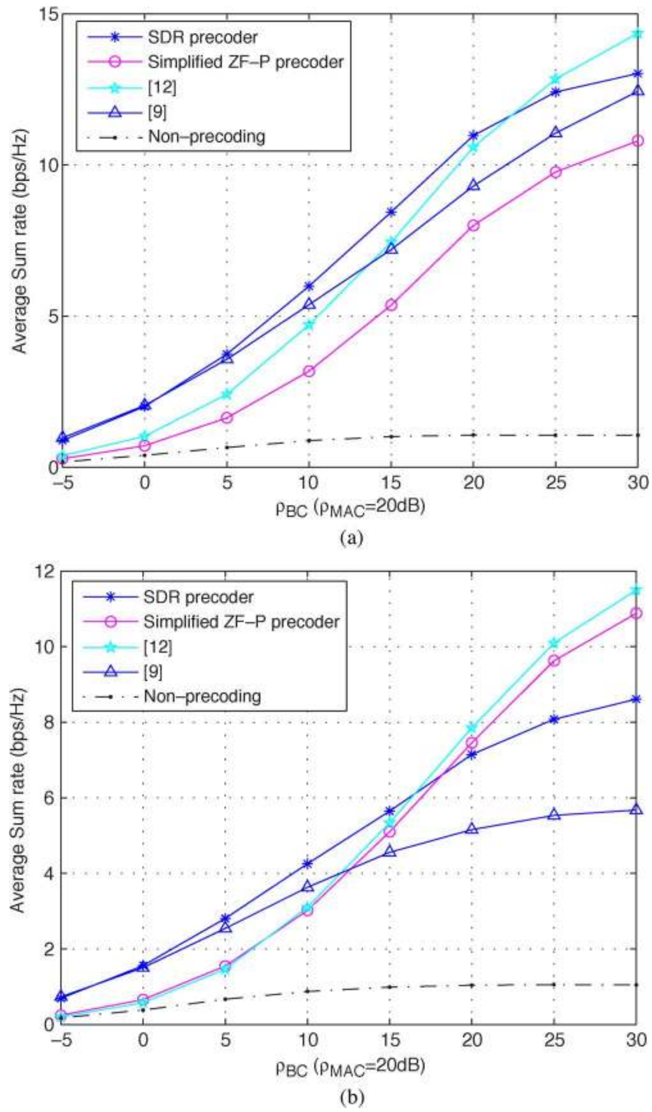


Fig. 5. The average sum-rate comparison for different relay precoding designs at $K = 3$. (a) $M = 6$. (b) $M = 5$.

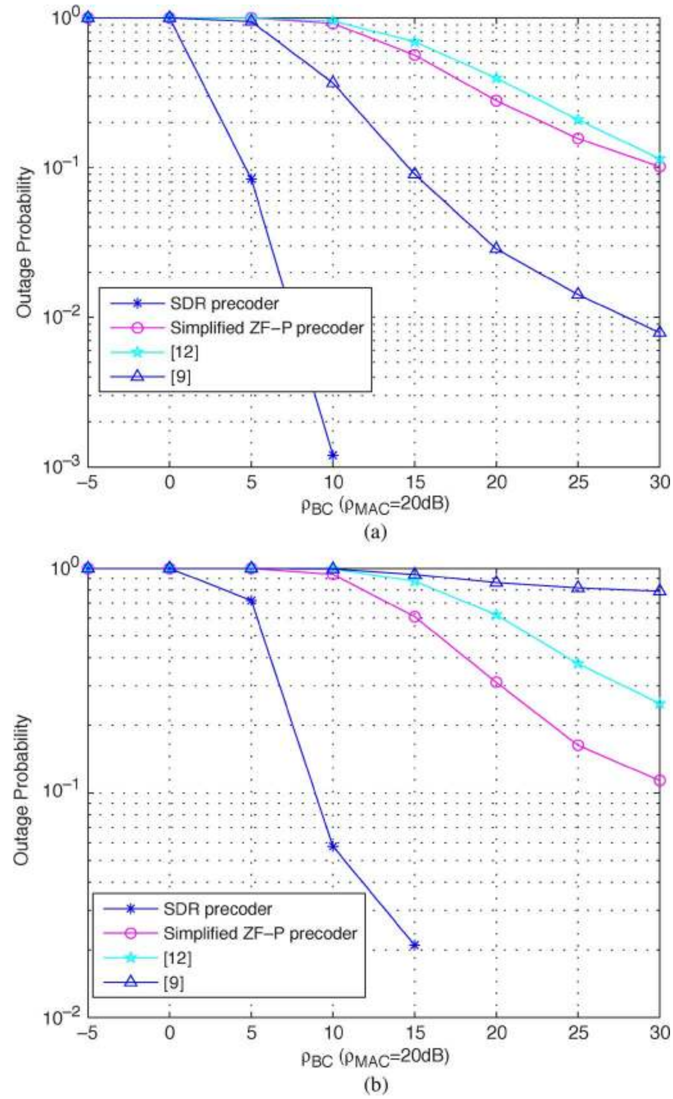


Fig. 6. The outage performance comparison for different relay precoding designs with $K = 3$ at target user rate $R = 0.5$ bits/s/Hz. (a) $M = 6$. (b) $M = 5$.

precoding almost achieves the same diversity order with [12]. When $M = 6$, [9] can achieve higher diversity order than the simplified ZF-P precoding. However, when with less relay antennas, the achievable diversity order of [9] is greatly reduced as shown in Fig. 6(b).

Based on these observations, we conclude that the proposed SDR based precoding can obtain the best performance in both minimum user rate and system outage among all the considered schemes. It also achieves good system sum-rate performance at any number of relay antennas. Although from Table I, we see that the SDR based precoding has the highest design complexity, its performance gain in both minimum user rate and system outage over the existing works is also dramatic. We believe that such performance gain is worthwhile in certain scenarios where the processing ability is not a limiting issue. When the number of relay antennas is just enough, the proposed simplified ZF-P precoding method can provide a good balance between performance and complexity.

## VII. Conclusions

In this paper, we investigated the linear relay precoding to achieve the max-min user fairness for multi-pair two-way MIMO relay systems. By using the bisection search, we first proposed to convert the primal precoding design into a series of SDP optimization problems and then obtained the final solution using semidefinite relaxation. This SDR-based precoding can achieve good performance and is suitable for any number of relay antennas. Simulation results demonstrated that it achieves significantly higher minimum user rate than existing precoding schemes while maintaining reasonable sum-rate performance. We further proposed the ZF-based relay precoding structure to eliminate inter-pair interference. The precoder optimization then reduces to a power allocation problem. We solved the optimal power allocation through standard optimization tools. Moreover, we presented a simplified power allocation method without performance degradation. Simulation results showed that this pair-wise ZF precoding scheme with simplified power allocation is promising when the number of relay antennas is just enough.

## REFERENCES


[1] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, no. 4, pp. 1204–1216, 2000.

[2] S. Zhang, S. C. Liew, and L. L. Lam, "Physical layer network coding," in *Proc. ACM SIGCOM'06*, Sep. 2006.

[3] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379–389, 2007.

[4] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "Xors in the air: Practical wireless network coding," *IEEE/ACM Trans. Netw.*, vol. 16, no. 3, pp. 497–510, 2008.

[5] M. Chen and A. Yener, "Multiuser two-way relaying: Detection and interference management strategies," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4296–4305, 2009.

[6] K. Jitvanichphaibool, R. Zhang, and Y.-C. Liang, "Optimal resource allocation for two-way relay-assisted OFDMA," *IEEE Trans. Veh. Technol.*, vol. 58, no. 7, pp. 3311–3321, 2009.

[7] C. Esli and A. Wittneben, "One- and two-way decode-and-forward relaying for wireless multiuser MIMO networks," in *Proc. IEEE Global Telecommun. Conf. (IEEE GLOBECOM 2008)*, 2008.

[8] C. Esli and A. Wittneben, "Multiuser MIMO two-way relaying for cellular communications," in *Proc. IEEE 19th Int. Symp. Pers., Indoor and Mobile Radio Commun. (PIMRC 2008)*, 2008.

[9] J. Joung and A. H. Sayed, "Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1833–1846, 2010.

[10] J. Joung and A. H. Sayed, "User selection methods for multiuser two-way relay communications using space division multiple access," *IEEE Trans. Wireless Commun.*, vol. 9, no. 7, pp. 2130–2136, 2010.

[11] E. Yilmaz, R. Zakhour, D. Gesbert, and R. Knopp, "Multi-pair two-way relay channel with multiple antenna relay station," in *Proc. IEEE Int. Commun. (ICC) Conf.*, 2010.

[12] C. Y. Leow, Z. Ding, K. K. Leung, and D. L. Goeckel, "On the study of analogue network coding for multi-pair, bidirectional relay channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 670–681, 2011.

[13] Z. Ding, I. Krikidis, J. Thompson, and K. K. Leung, "Physical layer network coding and precoding for the two-way relay channel in cellular systems," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 696–712, 2011.

[14] J. Zhang, F. Roemer, and M. Haardt, "Beamforming design for multi-user two-way relaying with MIMO amplify and forward relays," in *Proc. IEEE Int. Acoust., Speech Signal Process. (ICASSP) Conf.*, 2011.

[15] F. Roemer and M. Haardt, "Algebraic norm-maximizing (anomax) transmit strategy for two-way relaying with MIMO amplify and forward relays," *IEEE Signal Process. Lett.*, vol. 16, no. 10, pp. 909–912, 2009.

[16] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[17] X. Zhang, *Matrix Analysis and Applications*. Tsinghua, China: Tsinghua Univ. Press, 2004.

[18] Z.-Q. Luo, W. Kin Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, 2010.

[19] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1398–1407, 2007.

[20] F. Roemer, J. Zhang, M. Haardt, and E. Jorswieck, "Spectrum and infrastructure sharing in wireless networks: A case study with relay-assisted communication," in *Proc. Future Netw. MobileSummit Conf. IIMC Int. Inf. Manage. Corp.*, 2010.

[21] M. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, "Applications of second-order cone programming," *Linear Algebra and Its Appl.*, vol. 284, pp. 193–228, 1998.


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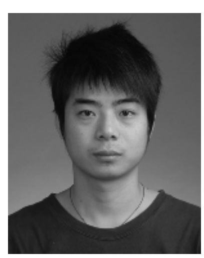

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