Linear Precoding for Multiuser MIMO-OFDM Systems

Hassen Karaa, Raviraj S. Adve and Adam J. Tenenbaum
Dept. of Electrical and Computer Engineering, University of Toronto
10 King's College Road, Toronto, Ontario, M5S 3G4, Canada
Email: {hkaraa,rsadve,adam}@comm.utoronto.ca

***Abstract*— This paper develops linear precoding schemes for the downlink in multiuser multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems with multiple data streams per user. We extend an existing multiuser MIMO algorithm, that jointly optimizes the power allocation and the transmit and receive filters, to MIMO-OFDM systems. One extension is to solve the resulting problem of joint power allocation across OFDM subcarriers. This paper also presents efficient methods to reduce the computational load of the algorithm by interpolating the precoding and decoding matrices corresponding to different OFDM subcarriers. The simulations show that the proposed interpolation scheme outperforms previously known schemes, but requires that the precoder for each subcarrier be tailored to the interpolated receiver.**

I. Introduction

It is now widely accepted that multiple input multiple output (MIMO) systems increase the link reliability and/or spectral efficiency of multiuser wireless communications [1]. Moreover, when channel state information (CSI) is available at the transmitter, linear precoding can be used to further improve system performance by tailoring the transmission to the instantaneous channel conditions [2]–[5] while retaining the benefits of all-linear processing. CSI at the transmitter is mandatory in the multiuser downlink, where a base station attempts to communicate simultaneously with multiple users.

The literature contains various linear precoding schemes for multiuser communications. Most recently, Khachan *et al.* in [4] consider a multiuser MIMO system with multiple data streams per user and present an algorithm that jointly optimizes the power allocation and transmit and receive filters (precoders and decoders) for all users. Given a total power budget, the algorithm minimizes the sum mean squared error (min SMSE) between the transmitted and received signals. The same problem is also considered in [5] where uplink-downlink duality is used to cast the problem as a semi-definite programming convex optimization problem.

On a different front, orthogonal frequency division multiplexing (OFDM) is a simple, and now well-accepted, technique to mitigate the effects of intersymbol interference in frequency selective channels [6]. OFDM converts a broadband frequency selective channel to a series of narrowband channels by transmitting data in parallel over many subcarriers.

Combining OFDM with MIMO, producing so called MIMO-OFDM, significantly reduces receiver complexity in wireless multiuser broadband systems [7], thus making it a competitive choice for future broadband wireless communication systems. Since OFDM uses multiple subcarriers, optimal linear precoding for MIMO-OFDM can be implemented by deriving linear precoders for each subcarrier independently. However, due to the generally large number of subcarriers, the computational load is excessive, and this approach is probably impossible to implement in practice. Furthermore, this approach is computationally inefficient since the MIMO channels associated with adjacent subcarriers are highly correlated; the precoder and decoders are correlated as well. For a channel with L_t resolvable channel taps and N_c subcarriers, a rule of thumb would be that N_c/L_t subcarriers are correlated. This leads us to consider computational saving techniques for deriving the precoding and decoding filters corresponding to different subcarriers.

In this paper we extend the linear precoding algorithm presented in [4] to MIMO-OFDM. We first formulate and solve the joint power allocation problem over all subcarriers. Then, we present methods to reduce the computational load of MIMO-OFDM by exploiting the correlation between the pre/decoding matrices corresponding to adjacent OFDM subcarriers. This work can be viewed as a step towards a true multiuser orthogonal frequency division multiple access (OFDMA) system that allows different quality of service (QoS) constraints for each data stream of a given user.

There is very little work considering linear precoding in multiuser MIMO-OFDM systems. Duplicy *et al.* extend available algorithms and compare the complexity and performance of three iterative schemes to minimize the system bit error rate (BER) subject to a power constraint [8]. However, they do not consider any methods to save on computational load.

Computational and feedback saving methods were explored in [9] and [10] in the context of a single-user system. The authors propose a scheme to limit the feedback requirements for a MIMO-OFDM system: a fraction of the precoding matrices for chosen subcarriers are obtained at the receiver, quantized and fed back to the transmitter. The complete set of matrices is then recovered using interpolation while assuming the precoding matrices are unitary. The interpolator's parameters are optimized based on a mean square error (MSE) or mutual information criterion. The proposed method only applies to unitary matrices and requires the design of a unitary matrix codebook used to align the available matrices before interpolation. Our results show that this scheme is not as effective in the multiuser case.

On the other hand, Colieri *et al.* in [11] use interpolation in the context of OFDM channel estimation. They propose estimating the channels for a subset of the subcarriers, then using one of several interpolation schemes to obtain the channels for the remaining subcarriers. However, while these methods are effective in estimating channels, we show that they do not work as well in MIMO-OFDM precoding.

This paper develops an alternative interpolation scheme, optimal in the MMSE sense, for the case of multiuser precoding. This interpolator is shown to be particularly effective when the number of channels to be interpolated over approaches the limit of N_c/L_t.

This paper is organized as follows: Section II presents the system model and gives an overview of the MIMO algorithm. Section III extends the presented algorithm to MIMO-OFDM and solves the joint power allocation problem. Section IV presents interpolation and complexity reduction methods. Finally, Section V wraps up the paper drawing some conclusions.

II. System Model and MIMO Algorithm

This section presents a flat fading multiuser MIMO model and briefly summarizes the single-carrier minimum SMSE algorithm of [4] that jointly optimizes the pre/decoding matrices and the power allocation. It also presents the extension of the MIMO model to the MIMO-OFDM case.

A. Multiuser MIMO System Model

We consider the same setup used in [4]: a single base station equipped with M antennas transmitting to K decentralized users. User k is equipped with N_k antennas and $N = \sum_{k=1}^{K} N_k$. User k receives L_k data streams from the base station and $L = \sum_{k=1}^{K} L_k$. Thus we have M transmit antennas transmitting a total of L data streams to K users, who have a total of N receive antennas. The symbols of each user are collected in the data vector $\mathbf{x}_k = [x_{k1}, x_{k2}, \ldots, x_{kL_k}]^T$ and the overall data vector is $\mathbf{x} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_K^T\right]^T$.

User k's data streams are processed by the transmit filter $\mathbf{U}_k \in \mathcal{C}^{M\times L_k}$ before being transmitted over the M antennas. These individual precoders together form the global transmitter precoder matrix $\mathbf{U}_{M\times L} = [\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_K]$. Let the downlink transmit power vector for user k be $\mathbf{p}_k = [p_{k1}, p_{k2}, \ldots, p_{kL_k}]^T$, with $\mathbf{p} = \left[\mathbf{p}_1^T, \ldots, \mathbf{p}_K^T\right]^T$, and define $\mathbf{P}_k = diag\{\mathbf{p}_k\}$ and $\mathbf{P} = diag\{\mathbf{p}\}$. The channel between the transmitter and user k is assumed flat and is represented by the $N_k \times M$ matrix $\mathbf{H}_k^H$. The resulting $N \times M$ channel matrix is $\mathbf{H}^H$, with $\mathbf{H} = [\mathbf{H}_1,\, \mathbf{H}_2,\, \ldots, \mathbf{H}_K]$. The transmitter is assumed to know $\mathbf{H}$.

Based on this model, user k receives a length N_k vector

$$\mathbf{y}_k = \mathbf{H}_k^H \mathbf{U}\sqrt{\mathbf{P}}\mathbf{x} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{n}_k$ represents the additive white Gaussian noise (AWGN) at the user's receive antennas with power σ^2; that is, $\mathbb{E}[\mathbf{n}_k\mathbf{n}_k^H] = \sigma^2\mathbf{I}_{N_k}$, where $\mathbb{E}[\cdot]$ represents the expectation operator. To estimate its L_k symbols $\mathbf{x}_k$ in the downlink, user k processes $\mathbf{y}_k$ with its $L_k \times N_k$ decoder matrix $\mathbf{V}_k^H$ resulting in

$$\hat{\mathbf{x}}_k^{DL} = \mathbf{V}_k^H \mathbf{H}_k^H \mathbf{U}\sqrt{\mathbf{P}}\mathbf{x} + \mathbf{V}_k^H \mathbf{n}_k. \tag{2}$$

The global receive filter $\mathbf{V}^H$ is a block diagonal decoder matrix of dimension $L \times N$, $\mathbf{V} = diag\left[\mathbf{V}_1,\, \mathbf{V}_2, \cdots, \mathbf{V}_K\right]$.

The MIMO algorithm presented in the next section exploits the duality between the uplink and downlink of the system. We construct a virtual uplink where the uplink transmit power vector for user k is $\mathbf{q}_k = [q_{k1}, q_{k2}, \ldots, q_{kL_k}]^T$, with $\mathbf{q} = [\mathbf{q}_1^T, \ldots, \mathbf{q}_K^T]^T$. We define $\mathbf{Q}_k = diag\{\mathbf{q}_k\}$ and $\mathbf{Q} = diag\{\mathbf{q}\}$. The transmit and receive filters for user k become $\mathbf{V}_k$ and $\mathbf{U}_k^H$ respectively. The received vector at the base station and the estimated uplink symbol vector for user k are

$$\mathbf{y} = \sum_{i=1}^{K} \mathbf{H}_i \mathbf{V}_i \sqrt{\mathbf{Q}_i}\mathbf{x}_i + \mathbf{n}, \tag{3}$$

$$\hat{\mathbf{x}}_k^{UL} = \sum_{i=1}^{K} \mathbf{U}_k^H \mathbf{H}_i \mathbf{V}_i \sqrt{\mathbf{Q}_i}\mathbf{x}_i + \mathbf{U}_k^H\mathbf{n}. \tag{4}$$

The transmitted symbols are assumed to be independent with unit power, i.e., $\mathbb{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_L$. The noise, $\mathbf{n}$, is modelled as AWGN with $\mathbb{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2\mathbf{I}_M$. To ensure resolvability, in the uplink and downlink, $L \le M$ and $L_k \le N_k$, $\forall k$.

B. MIMO SMSE Minimization Algorithm

As presented in [4], we consider the problem of minimizing the SMSE. Let $\mathbf{E}_k^{DL}$ be the $L_k \times L_k$ error covariance matrix of user k in the downlink, where

$$\mathbf{E}_k^{DL} = \mathbb{E}\left[(\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^H\right]. \tag{5}$$

The diagonal entries of $\mathbf{E}_k^{DL}$ are the MSEs of the L_k substreams of user k and thus $\text{SMSE}_k^{DL} = \text{tr}[\mathbf{E}_k^{DL}]$, where $\text{tr}[\cdot]$ is the trace operator. The SMSE minimization problem is

$$\min_{\mathbf{p},\mathbf{U},\mathbf{V}} \sum_{k=1}^{K} \text{tr}[\mathbf{E}_k^{DL}] \tag{6}$$
$$\text{subject to} : \|\mathbf{p}\|_1 \le P_{max}.$$

Using uplink-downlink duality we can solve this problem in the uplink and transfer the result to the downlink. In the uplink, the optimal minimum MSE (MMSE) receiver is

$$\mathbf{U}_k^{MMSE} = \mathbf{J}^{-1}\mathbf{H}_k\mathbf{V}_k\sqrt{\mathbf{Q}_k}, \tag{7}$$

$$\text{where} \qquad \mathbf{J} = \mathbf{H}\mathbf{V}\mathbf{Q}\mathbf{V}^H\mathbf{H}^H + \sigma^2\mathbf{I}_M. \tag{8}$$

The sum MSE of the whole system is therefore

$$\text{SMSE} = \sum_{k=1}^{K} \text{tr}[\mathbf{E}_k^{UL,MMSE}] = L - M + \sigma^2\text{tr}[\mathbf{J}^{-1}]. \tag{9}$$

The SMSE expression in (9) is a function of two variables; uplink power allocation $\mathbf{Q}$ and uplink global transmit filter $\mathbf{V}$. We first assume that $\mathbf{V}$ is fixed. Therefore, minimizing SMSE is equivalent to minimizing the trace of $\mathbf{J}^{-1}$. The resulting optimization problem is convex in $\mathbf{Q}$ [12]:

$$\mathbf{Q}^{opt} = \arg\min_{\mathbf{Q}} \text{tr}[\mathbf{J}^{-1}], \text{ subject to } \text{tr}[\mathbf{Q}] = P_{max}. \tag{10}$$

The next step is to optimize $\mathbf{V}$ for a fixed power allocation $\mathbf{Q}$. The optimal $\mathbf{v}_{kj}$, which minimizes SMSE for a given power allocation when the beamforming vectors of all other streams are fixed, is the dominant generalized eigenvector of the matrix pair $(\mathbf{H}_k^H\mathbf{J}_{kj}^{-2}\mathbf{H}_k,\ \mathbf{I}/q_{kj}+\mathbf{H}_k^H\mathbf{J}_{kj}^{-1}\mathbf{H}_k)$, where $\mathbf{J}_{kj}=\mathbf{J}-q_{kj}\mathbf{H}_k\mathbf{v}_{kj}\mathbf{v}_{kj}^H\mathbf{H}_k^H$ [4]. Note that while each step of the iteration is optimal, it is not guaranteed that the algorithm will converge to the globally optimal solution. The single-carrier MIMO algorithm is summarized as follows:

TABLE I

SMSE MINIMIZATION ALGORITHM

Initialization: $\mathbf{V_k}=\text{SVD}(\mathbf{H}_k)$ and $\mathbf{q}=(P_{max}/L)\,[1,\ldots,.1]^T$
Iteration:
1- *Virtual Uplink Transmit Beamforming* (for $k=1:K, j=1:L_k$)
$\mathbf{v}_{kj}=\hat{\mathbf{e}}_{max}(\mathbf{H}_k^H\mathbf{J}_{kj}^{-2}\mathbf{H}_k,\mathbf{I}/q_{kj}+\mathbf{H}_k^H\mathbf{J}_{kj}^{-1}\mathbf{H}_k)$
$\mathbf{v}_{kj}=\mathbf{v}_{kj}/\Vert\mathbf{v}_{kj}\Vert$
2- *Virtual Uplink Power Allocation*
$\mathbf{q}=\arg\min_{\mathbf{q}}\text{tr}[\mathbf{J}^{-1}]$, subject to $q_{kj}\geq 0$, $\Vert\mathbf{q}\Vert_1=P_{max}$
3- *Repeat 1-2 until* $oldSMSE-newSMSE<\epsilon$
Update:
4- *Downlink Transmit Beamforming* (for $k=1:K$)
$\mathbf{U}_k=\mathbf{J}^{-1}\mathbf{H}_k\mathbf{V}_k\sqrt{\mathbf{Q}_k}$
5- *Set target SINR to actual SINR* (for $k=1:K, j=1:L_k$)
$\gamma_{kj}=SINR_{kj}^{UL}$
6- *Downlink Power Allocation*
$\mathbf{p}=\sigma^2(\mathbf{D}^{-1}-\mathbf{\Psi})^{-1}\mathbf{1}$

In the initialization step above, SVD refers to singular value decomposition. The structure of the matrices $\mathbf{D}$ and $\mathbf{\Psi}$ and further details of the algorithm are given in [4].

C. MIMO-OFDM System Model

OFDM combats intersymbol interference by converting a broadband frequency selective channel into a series of parallel narrowband flat fading channels. This is done by applying the inverse discrete Fourier transform (IDFT) at the transmitter and the discrete Fourier transform (DFT) at the receiver.

We consider a multiuser MIMO-OFDM system with possibly multiple data streams per user that employs linear precoding. This system can be seen as a series of parallel MIMO systems presented in Section II-A, each with a flat fading channel resulting from the length-N_c DFT operation applied to the multi-tap frequency selective channel. The discrete time domain channel impulse response has L_t taps with uniform profile and independent and identically distributed (i.i.d.) complex Gaussian distribution. We further assume that each user places data on every subcarrier and has the same number of streams on every subcarrier. A MIMO-OFDM system is illustrated in Fig. 1 where just one user k is shown.

The system has M transmit antennas and uses N_c subcarriers. The base station transmits L_k data streams to user k on every subcarrier, resulting in a total of N_cL_k data streams per user. The transmit filter $\mathbf{U}(n)$, corresponding to the n-th subcarrier, produces M outputs which are to be input to the corresponding antennas. However, the N_c such outputs for each antenna are first converted to the "time domain" using an IDFT, then converted to serial form and finally augmented with a cyclic prefix. At the receiver, the k-th user has N_k antennas and DFT blocks. The user attempts to decode its own L_k streams on subcarrier n by first removing the cyclic prefix, converting to parallel form, applying the DFT, and filtering using the decoder matrix $\mathbf{V}_k(n)$. The result is N_c decoded data vectors $\hat{\mathbf{x}}_k(n)$, each of length L_k. The goal is to minimize the SMSE over all K users and N_c subcarriers between $\hat{\mathbf{x}}_k(n)$ and $\mathbf{x}_k(n)$.

III. JOINT POWER ALLOCATION FOR MIMO-OFDM

The fact that OFDM makes a frequency selective channel effectively flat makes the extension of MIMO precoding to OFDM theoretically trivial. For example, the linear precoding algorithm in Table I can be executed for each subcarrier separately. The only interaction between subcarriers is due to the fact that the *total power* budget for the system is specified and not a per-carrier budget. The performance of MIMO-OFDM precoding is therefore optimized by *jointly* allocating the available power across both users and subcarriers.

Using (9), the SMSE across the streams of all the subcarriers can be written as

$$\begin{aligned}\text{SMSE} &= \sum_{n=1}^{N_c}\sum_{k=1}^{K}\text{tr}[\mathbf{E}_k(n)^{UL,MMSE}] \\ &= N_c(L-M)+\sigma^2\sum_{n=1}^{N_c}\text{tr}[\mathbf{J}(n)^{-1}]. \quad (11)\end{aligned}$$

where $\mathbf{J}(n)$ corresponding to the n-th subcarrier is calculated using (8). The SMSE expression in (11) is a function of two sets of variables; uplink power allocations $\mathbf{Q}(n)$ and uplink global transmit filters $\mathbf{V}(n)$ for $n=1,\ldots,N_c$. We first assume that all matrices $\mathbf{V}(n)$s are fixed. Therefore, minimizing SMSE is equivalent to minimizing $\sum_{n=1}^{N_c}\text{tr}[\mathbf{J}(n)^{-1}]$. Let $\mathbf{\Phi}=diag\{[\mathbf{q}(1)^T,\ldots,\mathbf{q}(N_c)^T]\}=diag\{\phi\}$ be the power allocation matrix for all the streams of all the subcarriers.

The resulting optimization problem is a sum of N_c convex problems in $\mathbf{Q}(n)$ respectively; therefore it is convex in $\mathbf{\Phi}$, thus allowing for a relatively easy solution [13]:

$$\begin{aligned}\mathbf{\Phi}^{opt} = \quad & \arg\min_{\mathbf{\Phi}}\sum_{n=1}^{N_c}\text{tr}[\mathbf{J}(n)^{-1}] \quad (12)\\ & \text{subject to } \sum_{n=1}^{N_c}\text{tr}[\mathbf{Q}(n)]=P_{tot}.\end{aligned}$$

where P_{tot} is the total power allocated to all the subcarriers.

The next step is to optimize the $\mathbf{V}(\mathbf{n})$s for a fixed power allocation $\mathbf{\Phi}$. We propose to optimize $\mathbf{V}(n)$ for minimum SMSE given the power allocation determined for subcarrier n using (12). The algorithm used for joint power allocation is presented in Table II.

The initialization of the algorithm in Table II is the same as the one used for the MIMO algorithm of Table I. In the iteration phase the algorithm determines $\mathbf{V}(n)$ for $n=1,\ldots,N_c$ based on the power allocation given to subcarrier n. Then, using the calculated $\mathbf{V}(n)$s, the power allocation over

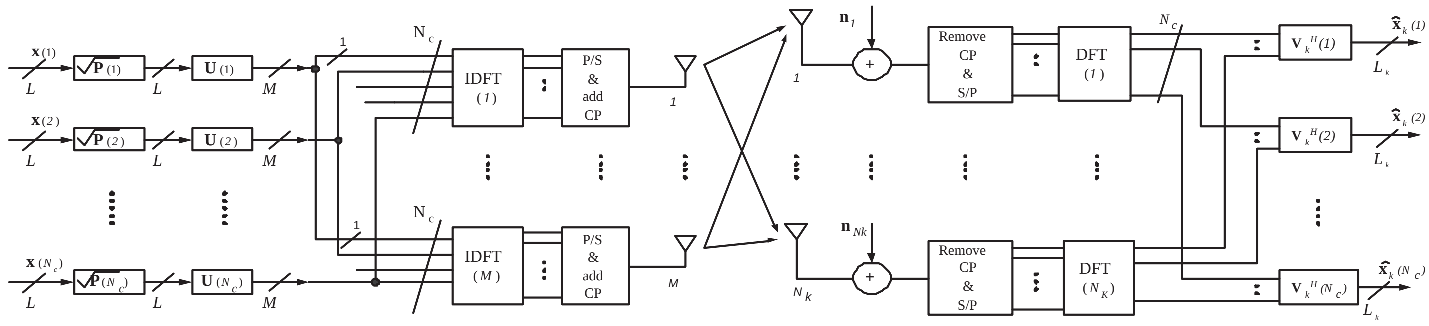

Fig. 1. MIMO-OFDM system with with K users, M transmit and N_K receive antennas and N_c subcarriers. The base station and one user k are shown.

TABLE II
JOINT POWER ALLOCATION ALGORITHM

Initialization:
$\mathbf{V_k(n)} = \text{SVD}(\mathbf{H}_k(n))$ and $\phi = \frac{P_{tot}}{LN_c}[1,\ldots,1]^T$
Iteration:
1- *Virtual Uplink Transmit Beamforming*
for $n = 1 : N_c, k = 1 : K, j = 1 : L_k$
$\mathbf{v}_{kj}(n) = \hat{\mathbf{e}}_{max}(\mathbf{H}_k(n)^H\mathbf{J}_{kj}(n)^{-2}\mathbf{H}_k(n),$
$\mathbf{I}/q_{kj}(n) + \mathbf{H}_k(n)^H\mathbf{J}_{kj}(n)^{-1}\mathbf{H}_k(n))$
$\mathbf{v}_{kj}(n) = \mathbf{v}_{kj}(n)/\|\mathbf{v}_{kj}(n)\|$
2- *Virtual Uplink Power Allocation*
$\phi = \arg\min_\phi \sum_{n=1}^{N_c} \text{tr}[\mathbf{J}(n)^{-1}],$
subject to $\phi_{nkj} \geq 0$, $\|\phi\|_1 = P_{tot}$
3- *Repeat 1-2 until* $oldSMSE - newSMSE < \epsilon$
Update:
for $n = 1 : N_c$
4- *Downlink Transmit Beamforming* (for $k = 1 : K$)
$\mathbf{U}_k(n) = \mathbf{J}(\mathbf{n})^{-1}\mathbf{H}_k(n)\mathbf{V}_k(n)\sqrt{\mathbf{Q}_k(n)}$
5- *Set target SINR to actual SINR* (for $k = 1 : K, j = 1 : L_k$)
$\gamma_{kj}(n) = SINR_{kj}^{UL}(n)$
6- *Downlink Power Allocation*
$\mathbf{p}(n) = \sigma^2(\mathbf{D}(n)^{-1} - \mathbf{\Psi}(n))^{-1}\mathbf{1}$

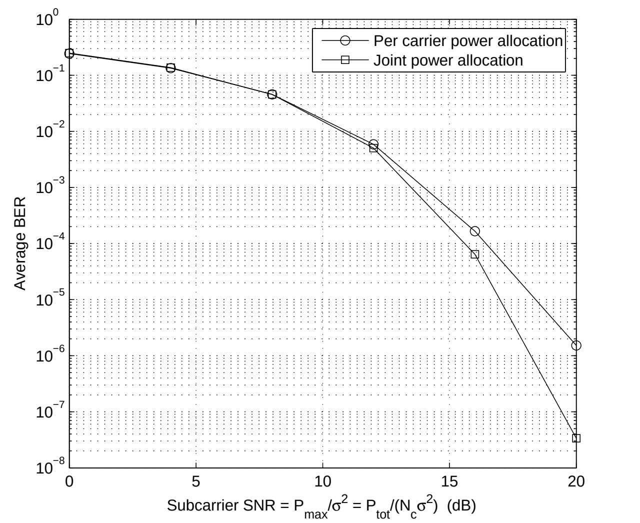

Fig. 2. Comparison of BER performance of joint power allocation vs. per subcarrier power allocation for MIMO-OFDM with $M = 4, K = 2, N_k = 2$ and $L_k = 1 \; \forall \; k, L_t = 6, N_c = 64$.

all streams of all the subcarriers is determined by solving the convex optimization problem in (12). This repeats until the change in the overall SMSE is negligible. The update phase repeats steps (4-6) of Table I for each subcarrier.

Figure 2 presents the impact of using optimal power allocations across both users and subcarriers. The figure illustrates the results of Monte Carlo simulations comparing the performance of the joint power allocation algorithm in Table II with the simpler case of allocating equal power $P_{max} = \frac{P_{tot}}{N_c}$ to each subcarrier. In each case the power allocated to each user is obtained optimally. The simulations consider a MIMO-OFDM system with $N_c = 64, M = 4, K = 2, N_k = 2$ and $L_k = 1$ for both users. The channel has $L_t = 6$ taps with uniform power profile and i.i.d. complex Gaussian distribution. The noise is assumed to be AWGN. We also assume that a cyclic prefix of appropriate length is used to avoid intersymbol interference. The plot shows the average BER versus average SNR per subcarrier. The results are the average of 2000 channel realizations with 10000 BPSK symbols per user per realization.

Figure 2 shows that, as expected, allocating power optimally across subcarriers provides some performance gains. However, the interesting indicator from this plot is that for all the additional complexity, this optimal power allocation does not provide significant gains at reasonable levels of BER. For example, at a BER = 1×10^{-3}, the power gains are approximately 0.2dB. Since complexity reduction is a central theme of this work, the rest of this paper uses an equal-power allocation across all subcarriers.

IV. COMPLEXITY REDUCTION USING INTERPOLATION

Since equal power allocation across subcarriers appears close to optimal in multiuser MIMO-OFDM systems, it is possible to execute the precoding algorithm of Table I independently for each subcarrier. However, since practical MIMO-OFDM systems typically have a large number of subcarriers, this is computationally prohibitive. The key to savings in computational load is that the fading on closely spaced subcarriers is correlated [11], [14]. This implies the corresponding precoder, decoders and power allocations are also correlated. Using an argument of degrees of freedom, we expect approximately N_c/L_t subcarriers to be well correlated. This can be exploited in various ways to reduce the computational load of obtaining the pre/decoding matrices and power allocation vectors.

We group the subcarriers into clusters each containing L_{cl} adjacent subcarriers. The simplest and most intuitive method to reduce the computations is *clustering*: obtaining the precoding matrices and power allocations for the center subcarrier of each cluster and using those parameters for all the subcarriers in that cluster [9], [14]. Clustering is therefore equivalent to piecewise constant interpolation. A more sophisticated interpolation scheme should outperform this method.

A. Proposed Interpolation Algorithm

We propose an interpolation method inspired by OFDM channel interpolation for the purpose of estimation presented in [11]. We interpolate the entries of the precoding matrices and power allocation vectors over subcarriers using a low pass interpolation algorithm presented by Oetken *et al.* in [15]. To achieve an interpolation factor of r, the algorithm uses a length $2rL + 1$ FIR interpolating filter with unit pulse response $h(n),\ n = -rL, \ldots, rL$. $h(n)$ is designed so that $\|(x \cdot f(r)) * h - x\|^2$ is minimized for bandlimited signals $x(n)$. The symbol $*$ represents convolution and $\cdot$ is multiplication. $f(n)$ is the sampling function defined as $f(n) = 1,$ if $n = 0\ mod\ r$, and 0 otherwise.

The interpolation scheme is shown in Table III. It first finds the virtual uplink precoding matrices $\mathbf{V}(n)$ and power allocation $\mathbf{q}(n)$, where n is the index of the subcarriers at the clusters' boundaries, by executing steps (1-3) of the MIMO algorithm in Table I. The $\mathbf{V}$ and $\mathbf{q}$ for the remaining subcarriers in each cluster are then found by interpolating these precoding matrices and power allocation. Finally, the downlink precoding matrices and power allocations are computed using closed form expressions by executing steps (4-6) in Table I for all subcarriers. This is essential because, for coherent detection, the decoder must match the encoder.

TABLE III
PROPOSED INTERPOLATION ALGORITHM

1-	for $n = 1 : L_{cl} : N_c$
	Find $\mathbf{V}(n)$ and $\mathbf{q}(n)$ using steps (1-3) of Table I.
2-	for all entries of $\mathbf{V}$ and $\mathbf{q}$
	Interpolate the samples obtained in step (1) along the frequency dimension.
3-	for $n = 1 : N_c$
	Normalize all the columns of $\mathbf{V}(n)$
	Scale $\mathbf{q}(n)$ to satisfy P_{max} requirement
4-	for $n = 1 : N_c$
	Compute $\mathbf{U}(n)$ and $\mathbf{p}(n)$ using steps (4-6) of Table I.

The interpolation steps of this algorithm tackle the most computationally intensive operations of the MIMO algorithm in Table I: the iterations finding $\mathbf{V}$ and $\mathbf{q}$, the SVD to obtain $\mathbf{V}$, and solving the convex optimization problem to obtain $\mathbf{q}$. Step 4 of the algorithm obtains the precoders, $\mathbf{U}(n)$, and power allocations, $\mathbf{p}(n)$, for each subcarrier, thereby matching the precoding with the decoding. The availability of closed form solutions makes these computations fairly efficient.

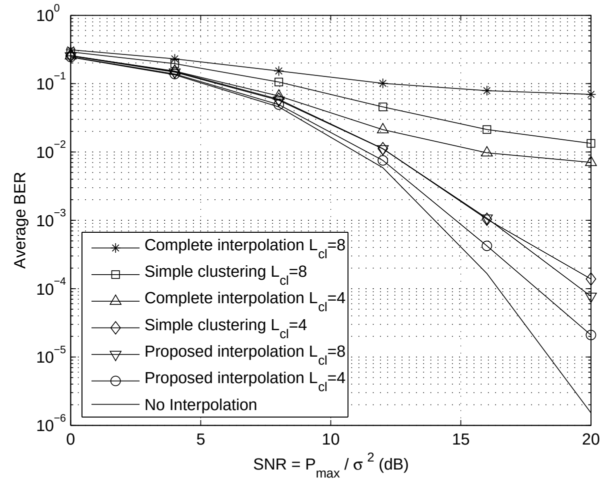

Fig. 3. Comparison of BER performance of proposed interpolation algorithm vs. clustering and complete interpolation for MIMO-OFDM with $M = 4, K = 2, N_k = 2$ and $L_k = 1\ \forall\ k, L_t = 6, N_c = 64$.

Figure 3 illustrates the performance of the proposed algorithm. The system parameters are the same as in Fig. 2. The figure compares the performance of the proposed interpolation algorithm to that of the simple clustering approach and to the complete system implementation (executing the MIMO algorithm for every subcarrier). To illustrate the importance of matching the precoder to the decoder, the figure also shows the results of interpolating *all* parameters ($\mathbf{V}, \mathbf{U}, \mathbf{p}$, and $\mathbf{q}$). The simulations were performed for cluster sizes of $L_{cl} = 4$ and 8. Note that $N_c/L_t = 64/6 \simeq 10$.

The figure shows that the proposed interpolation algorithm easily outperforms the simple clustering approach, especially for $L_{cl} = 8$. The performance of the algorithm approaches that of the complete system implementation (without interpolation) for both cluster sizes. A cluster size of $L_{cl} = 4$ results in a penalty of 1dB at a BER of 1×10^{-3}, while $L_{cl} = 8$ results in a penalty of 2dB.

B. Enhanced Clustering

One reason the clustering approach described earlier results in poor performance is that all parameters are clustered (equivalently, using piecewise constant interpolation). The precoders and decoders are therefore the same for all subcarriers in a cluster, but the channel changes, making them mismatched. For a fair comparison, in this section we investigate the matching of the downlink precoder, $\mathbf{U}(n)$, and power allocation, $\mathbf{p}(n)$, to the clustered decoder $\mathbf{V}$ and uplink power allocation $\mathbf{q}$. The algorithm is detailed in Table IV. This algorithm is more computationally efficient than the proposed interpolation algorithm as it avoids the interpolation step.

We verify the performance of this algorithm via Monte Carlo simulations. The results, based on the same system as in the previous examples, are shown in Fig. 4. As expected, this algorithm outperforms the simple clustering approach. The noticeable result here is that the enhanced clustering

TABLE IV

ENHANCED CLUSTERING ALGORITHM

1-	for $n = L_{cl}/2 : L_{cl} : N_c$
	Find $\mathbf{V}(n)$ *and* $\mathbf{q}(n)$ *using steps (1-3) of Table I.*
2-	for $n = 1 : N_c$
	Compute $\mathbf{U}(n)$ *and* $\mathbf{p}(n)$ *from steps (4-6) of Table I using*
	$\mathbf{V}(c)$ *and* $\mathbf{p}(c)$; $c =$ *index of center subcarrier of current cluster.*

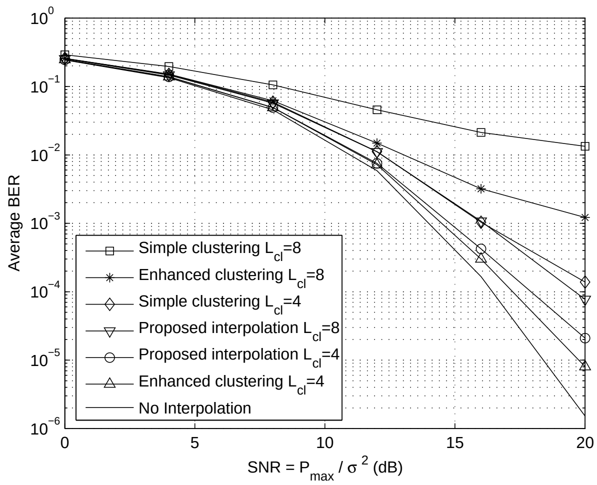

Fig. 4. Comparison of BER performance of enhanced clustering algorithm vs. interpolation with $M = 4, K = 2, N_k = 2$ and $L_k = 1 \; \forall \; k, L_t = 6, N_c = 64$.

approach performs slightly better than the proposed interpolation approach for $L_{cl} = 4$. This is counter-intuitive since the interpolation scheme is MSE optimal. However, it should be noted that being MSE optimal in terms of decoder matrices and uplink power allocation *does not* imply optimality in terms of BER. However, for $L_{cl} = 8$ the interpolation approach has significantly improved performance. The clustering approach, though enhanced, does not appear to be robust.

A central theme of this paper is the need for computationally efficient solutions to the multiuser MIMO-OFDM precoding problem. Toward this end, we present here the execution times of the various schemes[1]. The results in Table V are the average, over 1000 realizations, of the times required to execute the signal processing algorithms for a given channel.

TABLE V

EXECUTION TIMES FOR VARIOUS SCHEMES

Approach	Time taken (s)	Savings factor
Without interpolation	4.373	1
Enhanced clustering $L_{cl} = 4$	0.567	7.71
Interpolation $L_{cl} = 4$	0.784	5.57
Enhanced clustering $L_{cl} = 8$	0.351	12.45
Interpolation $L_{cl} = 8$	0.474	9.23

[1]Execution times based on program implementations are inherently unreliable and an "order of" comparison is generally required. However, this appears to be very difficult in our problem. The timing results presented here do provide some indication of the computational loads involved.

As expected, there is a trade-off between performance and execution time. Enhanced clustering is efficient, but is effective only up to a cluster size of $L_{cl} = 4$. The proposed interpolation scheme is more complex, but is also more robust to cluster size. With a penalty of 2dB, the interpolation scheme provides a good tradeoff between savings and performance loss.

V. CONCLUSION

This paper attempts to make practical the extension of multiuser MIMO linear precoding to MIMO-OFDM systems. The focus here, for convenience, is the algorithm of [4]. We solved the problem of optimal power allocation across subcarriers and showed that it does not provide significant performance gains. We then presented an enhanced clustering and an interpolation technique, which is MSE optimal, to interpolate across subcarriers the downlink decoders and uplink powers. We have shown that it is crucial to match the precoders to the decoders. The interpolation technique is robust to cluster size and appears to provide a good tradeoff between savings in computational load and performance.

REFERENCES

[1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, Mar. 1998.

[2] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. on Signal Processing*, vol. 50, no. 5, pp. 1051–1064, May 2002.

[3] A. Bourdoux and N. Khaled, "Joint TX-RX optimisation for MIMO-SDMA based on a null-space constraint," in *Proc. of IEEE VTC*, Vancouver, Canada, Sept. 2002, pp. 171–174.

[4] A. M. Khachan, A. J. Tenenbaum, and R. S. Adve, "Linear processing for the downlink in multiuser MIMO systems with mutiple data streams," in *Proc. of IEEE ICC - To appear*, June 2006.

[5] M. Schubert, S. Shi, E. A. Jorswieck, and H. Boche, "Downlink Sum-MSE transceiver optimization for linear multi-user MIMO systems," in *Proc. 39 Asilomar Conf. on Signals, Systems and Computers*, Oct. 2005, pp. 1424–1428.

[6] A. R. S. Bahai, B. R. Saltzberg, and M. Ergen, *Multi-carrier digital communications theory and applications of OFDM*. New York: Springer, 2004.

[7] H. Bolcskei, D. Gesbert, and A. Paulraj, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Trans. on Communications*, vol. 50, no. 2, pp. 225–234, Feb. 2002.

[8] J. Duplicy, J. Louveaux, and L. Vandendorpe, "Optimization of linear pre/decoders for multi-user closed-loop MIMO-OFDM," in *Proc. of IEEE ICC*, June 2005, pp. 2026–2030.

[9] J. Choi and R. W. Heath Jr., "Interpolation based unitary precoding for spatial multiplexing for MIMO-OFDM with limited feedback," in *Proc. of IEEE GLOBECOM*, Vancouver, Canada, Dec. 2004, pp. 214–218.

[10] J. Choi, B. Mondal, and R. W. Heath Jr., "Interpolation based unitary precoding for spatial multiplexing for MIMO-OFDM with limited feedback," *IEEE Trans. on Signal Processing - To appear*.

[11] S. Colieri, M. Ergen, A. Puri, and A. Bahai, "A study of channel estimation in OFDM systems," in *Proc. of IEEE VTC*, 2002, pp. 894–898.

[12] S. Shi and M. Schubert, "MMSE transmit optimization for multi-user multi-antenna systems," in *Proc. of IEEE ICASSP*, Philadelphia, PA, Mar. 2005.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York: Cambridge University Press, 2004.

[14] K. Wong, R. Cheng, K. Letaief, and R. Murch, "Adaptive antennas at the mobile and base stations in an OFDM/TDMA system," *IEEE Trans. on Communications*, vol. 49, no. 1, pp. 195–206, Jan. 2001.

[15] G. Oetken, T. W. Parks, and H. W. Schüssler, "New results in the design of digital interpolators," *IEEE Trans. on Acoust., Speech and Signal Processing*, vol. ASSP-23, no. 3, pp. 301–309, 1975.