Linear Receivers for Frequency-Selective MIMO Channels with Redundant Linear Precoding Can Achieve Full Diversity

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***Abstract*— Since the introduction of the Diversity-Multiplexing Tradeoff (DMT) by Zheng and Tse for ML reception in frequency-flat MIMO channels, some results have been obtained also for the DMT of frequency-selective MIMO channels and for the DMT of suboptimal receivers such as linear (LEs) and decision-feedback equalizers (DFEs) for frequency-selective SIMO channels or frequency-flat MIMO channels. We have recently extended these results to the case of linear receivers for frequency-selective MIMO channels. However, the diversity properties of linear receivers turn out to be fairly catastrophic. In this paper we show that full diversity can be restored by the introduction of a convolutive linear MIMO precoding scheme that we showed earlier to allow to attain the optimal DMT for ML or DFE detection (in the frequency-flat case). The precoder needs to be used with a moderate amount of redundancy in the form of zero-padding, and with a MMSE design for the linear equalizer. A MMSE-ZF design also benefits substantially from the precoding. The proposed scheme is a significant extension of an earlier SISO result by Tepelenlioglu to the MIMO case.**

***Index Terms*— diversity, Multiple Input Multiple Output, linear equalization, linear precoding, zero padding**

I. Introduction

Consider a linear modulation scheme and single-carrier transmission over a Multiple Input Multiple Output (MIMO) linear channel with additive white noise. The multiple inputs and outputs will be mainly thought of as corresponding to multiple antennas. After a receive (Rx[1]) filter (possibly noise whitening), we sample the Rx signal to obtain a discrete-time system at symbol rate[2]. When stacking the samples corresponding to multiple Rx antennas in column vectors, the discrete-time communication system is described by

$$\underbrace{\boldsymbol{y}_k}_{n_r\times 1} = \underbrace{\mathbf{h}[q]}_{n_r\times n_t}\ \underbrace{\boldsymbol{a}_k}_{n_t\times 1} + \underbrace{\boldsymbol{v}_k}_{n_r\times 1} = \sum_{i=1}^{n_s} \underbrace{\boldsymbol{h}_i[q]}_{n_r\times 1}\ \underbrace{a_{i,k}}_{1\times 1} + \underbrace{\boldsymbol{v}_k}_{n_r\times 1} \qquad (1)$$

where k is the symbol (sample) period index, n_r and n_t are the number of Rx and Tx antennas respectively. The noise power spectral density matrix is $S_{\boldsymbol{vv}}(z) = \sigma_v^2 I_{n_r}$, q^{-1} is the unit sample delay operator: $q^{-1} a_k = a_{k-1}$, and $\mathbf{h}[z] = \sum_{i=0}^{L-1} \mathbf{h}_i z^{-i} = [\boldsymbol{h}_1[z] \cdots \boldsymbol{h}_{n_t}[z]]$ is the MIMO channel transfer function in the z domain. The channel delay spread is L symbol periods. In the Fourier domain we get the vector transfer function $\mathbf{h}(f) = \mathbf{h}[e^{j2\pi f}]$.

We introduce the vectors containing the SIMO impulse response coefficients[3] $\boldsymbol{h}_i = [\boldsymbol{h}_{i,0}^T \cdots \boldsymbol{h}_{i,L-1}^T]^T$ and the overall coefficient vector $\underline{\boldsymbol{h}} = [\boldsymbol{h}_1^T \cdots \boldsymbol{h}_{n_t}^T]^T$. Assume the energy normalization $\text{tr}\{R_{\underline{\boldsymbol{hh}}}\} = n_r$ with $R_{\underline{\boldsymbol{hh}}} = \text{E}\,\{\underline{\boldsymbol{h}}\underline{\boldsymbol{h}}^H\}$. By default we shall assume the i.i.d. complex Gaussian channel model: $\underline{\boldsymbol{h}} \sim \mathcal{CN}(0, \frac{1}{(L+1)n_t} I_{n_r n_t (L+1)})$ so that spatio-temporal diversity of order $n_r n_t L$ is available (which is the case from the moment $R_{\underline{\boldsymbol{hh}}}$ is nonsingular). The average per Rx antenna SNR is $\rho = \frac{\sigma_a^2}{\sigma_v^2}$. In this paper we consider full channel state information at the Rx (CSIR) and usually none (otherwise antenna selection) at the Tx (CSIT).

Whereas in non-fading channels, the probability of error P_e decreases exponentially with SNR, for a given symbol constellation, in fading channels the probability of error taking channel statistics into account behaves as $P_e \sim \rho^{-d}$ for large SNR ρ, where d is the diversity order. Also, at high SNR P_e is dominated by the outage probability P_o and has the same diversity order for a well-designed system. On the other hand, at high SNR the channel capacity increases with SNR as $\log \rho$, which can be achieved with adaptive modulation and coding (AMC) on the basis of the long-term SNR (slow feedback), not to be confused with the instantaneous SINR (fast feedback). In [1] it was shown however that both benefits at high SNR cannot be attained simultaneously and a compromise has to be accepted: the "diversity-multiplexing tradeoff" (DMT). In [1] the frequency-flat MIMO channel was considered. These results were extended to the frequency-selective SISO channel in [3] and the frequency-selective MIMO channel in [4], see also [5],[6]. In [7], it was shown for the frequency-selective SIMO channel that a Zero Forcing (ZF) or Minimum Mean Squared Error (MMSE) Decision-Feedback Equalizer (DFE) with unconstrained feedforward filter allows to attain the op-

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[1] In this paper, "Rx" stands for "receive" or "receiver" or "reception" etc., and similarly for "Tx" and "transmit", ...

[2] In the case of additional oversampling with integer factor w.r.t. the symbol rate, the Rx dimension would get multiplied by the oversampling factor.

[3] In this paper, $.^*$, $.^T$, and $.^H$ denote complex conjugate, transpose and Hermitian (complex conjugate) transpose respectively, and $\mathbf{h}^\dagger[z] = \mathbf{h}^H[1/z^*]$ denotes the paraconjugate (matched filter). Note that $\mathbf{h}^\dagger[e^{j2\pi f}] = \mathbf{h}^H(f)$.

timum diversity and similar results for the MIMO frequency-flat channel case, with a linear MIMO prefilter and a MMSE MIMO DFE appears in [8].

In practice also the Linear Equalizer (LE) is often used since its settings are easier to compute and there is no error propagation. Also in practice, for both LE and DFE, only a limited degree of non-causality (delay) can be used and the filters are usually of finite length (FIR). Analytical investigations into the diversity for SISO with LEs are much more recent, see [9],[11] for linearly precoded OFDM and [12] for Single-Carrier with Cyclic Prefix (SC-CP). The use of the DFE appears in [13] (FIR) and [14],[15] (SC-CP) where in the last two references diversity behavior is investigated through simulations. The DMT for various forms of LE and DFE with SIMO channels is investigated in [16]. In particular, the DMT for LE turns out to be fairly catastrophic since all benefits from frequency selectivity are lost, and this both for MMSE and MMSE-ZF designs, except for the case of fixed rate, for which is was noted that the MMSE design benefits from full diversity. With hindsight, this result needs to be qualified further since it turns out to apply only for sufficiently low rates. The DMT for frequency-flat MIMO with LEs has been derived in [17], see also [18] for the diversity in the fixed rate case (no AMC).

We have recently extended these results to the case of linear receivers for frequency-selective MIMO channels [19]. However, the diversity properties of linear receivers turn out to be fairly catastrophic: instead of a product of three diversity sources ($n_r n_t L$), only a single reduced (receive) source is left ($n_r - n_t + 1$). So, all frequency-selective diversity is lost, even in SISO systems. In [9], it was shown that the introduction of redundant linear precoding in OFDM allows a MMSE-ZF linear block receiver to regain full diversity in the SISO (or SIMO) case. In [10], it was observed that SC transmission with Zero Padding (ZP) is a particular case of such approach. The ZP introduces redundancy in the time (delay) dimension which allows a LE of inter-symbol interference (ISI) to maintain full diversity: every input symbol can be recovered linearly unless the whole channel impulse response becomes zero.

II. Linear Convolutional Space-Time Coder (STC)

In the frequency-selective MIMO (or MISO) case, the LE has to suppress not only ISI but also multi-stream (multi-input) interference and hence requires more redundancy to maintain full diversity. The redundancy can be increased accordingly so that it can be exploited with ZP, by introducing a convolutive linear MIMO precoding (space-time coding) scheme that we showed earlier [8] to allow to attain the optimal DMT for ML or DFE detection (in the frequency-flat case). We show that a MMSE-ZF design benefits substantially from the precoding (in particular allows to recover frequency selective diversity), but also that to recover full diversity, a MMSE design is required in the proposed scheme. Note that in this paper we focus on the diversity order of fixed rate transmission (with possibly rate-dependent results) and not on the DMT. The per symbol period mutual information for the frequency-selective MIMO channel with white Gaussian input, infinite block length is

$$C = \int_{-\frac{1}{2}}^{\frac{1}{2}} \log\det(I_{n_t} + \rho\, \mathbf{h}^H(f)\, \mathbf{h}(f))\, df$$

in which the equivalent system spectrum $\det(I_{n_t} + \rho\, \mathbf{h}^H(f)\, \mathbf{h}(f)) = \det(I_{n_r} + \rho\, \mathbf{h}(f)\, \mathbf{h}^H(f))$ corresponds to a system memory of $\min\{n_r, n_t\}\, L-1$. This implies that if the LE diversity should be maximized through ZP as in [10], an explicit input-output system memory of $\min\{n_r, n_t\}\, L-1$ is required. Since the channel memory is only $L-1$, it needs to be increased by linear precoding. This can be accomplished with the convolutive STC introduced in [8] (and earlier references therein). This convolutive STC is a structured form of linear dispersion codes. As precoding can only be done at the Tx side, the system memory gets actually increased to $n_t L-1$ for maximal diversity. As at high SNR, $C \approx \min\{n_r, n_t\}\, \log(\rho)$, it suffices (esp. for a LE Rx) to limit the number of symbol streams to $n_s = \min\{n_r, n_t\}$ (instead of n_t). Motivated by these considerations, we propose to use the paraunitary prefilter $\mathbf{T}[z^L]$ where

$$\begin{aligned} \mathbf{T}[z] &= \mathbf{D}[z]\, Q\ ,\ \ Q^H Q = n_t\, I_{n_s}\, , |Q_{ij}| = 1\, , \\ \mathbf{D}[z] &= \operatorname{diag}\{1,\, z^{-1}, \ldots, z^{-(n_t-1)}\} \end{aligned} \tag{2}$$

where Q is a (constant) tall unitary matrix with equal magnitude elements (e.g. a submatrix of a DFT or a Walsh-Hadamard matrix). Hence the signal appearing at the Tx antennas is of the form $\underbrace{\boldsymbol{a}_k}_{n_t\times 1} = \underbrace{\mathbf{T}[q^L]}_{n_t\times n_s}\ \underbrace{\boldsymbol{x}_k}_{n_s\times 1}$ with $S_{\boldsymbol{xx}}[z] = \sigma_x^2\, I_{n_s}$ and hence the signal $\boldsymbol{a}_k$ is no longer spatiotemporally white if $n_r < n_t$. So, symbol stream n ($x_{n,k}$, $n = 1, \ldots, n_s$) passes through the equivalent SIMO channel

$$\boldsymbol{g}_n[z] = \sum_{i=1}^{n_t} z^{-(i-1)L} \boldsymbol{h}_i[z]\, Q_{i,n} \tag{3}$$

which has delay spread Ln_t, due to the stream-specific delay diversity. The Matched Filter Bound (MFB) is the maximum attainable SNR for symbol-wise detection, when the interference from all other symbols has been removed. From (3), stream n has MFB

$$\text{MFB}_n = \rho\ ||\underline{\boldsymbol{h}}||^2\ \ ,\ \ \rho = \frac{\sigma_x^2}{\sigma_v^2} \tag{4}$$

where ρ has been redefined. Hence the proposed $\mathbf{T}[z^L]$ provides the same MFB and full diversity $n_t n_r L$ for all streams. Larger diversity order leads to larger outage capacity.

III. SINR of Block Receivers and Outage

Now consider the cascaded prefilter-channel system $\mathbf{g}[z] = \mathbf{h}[z]\, \mathbf{T}[z^L] = \sum_{i=1}^{n_t L-1} \mathbf{g}_i z^{-i}$ for which we introduce ZP of length $n_t L-1$ symbol periods in a block of length N. This leads to the banded block Toeplitz system matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_0 & 0 & \cdots & 0 \\ \mathbf{g}_1 & \mathbf{g}_0 & & \vdots \\ \vdots & & \ddots & \\ \mathbf{g}_{n_t L-1} & & & \mathbf{g}_0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & \mathbf{g}_{n_t L-1} \end{bmatrix} = \underbrace{[\mathbf{G}_0 \cdots \mathbf{G}_{N-n_t L}]}_{N n_r \times (N-n_t L+1) n_s} \tag{5}$$

For the purpose of joint description of MMSE-ZF and MMSE designs, consider $\underline{\underline{\mathbf{G}}} = \left[\mathbf{G}^H \;\; \frac{\delta}{\sqrt{\rho}} I\right]^H$ where $\delta = \left\{ \begin{array}{ll} 0 & \text{, MMSE-ZF design,} \\ 1 & \text{, MMSE design.} \end{array} \right.$ With the streamwise channel vectors, $\underline{\underline{\mathbf{G}}}_i = [\underline{\underline{\mathbf{G}}}_{i,1} \cdots \underline{\underline{\mathbf{G}}}_{i,n_s}]$, consider $\underline{\underline{\mathbf{G}}}'_{i,n} = P^{\perp}_{\overline{\underline{\underline{\mathbf{G}}}}_{i,n}} \underline{\underline{\mathbf{G}}}_{i,n}$ where $\overline{\underline{\underline{\mathbf{G}}}}_{i,n}$ is $\underline{\underline{\mathbf{G}}}$ with column $\underline{\underline{\mathbf{G}}}_{i,n}$ removed, and $P^{\perp}_{\mathbf{G}} = I - P_{\mathbf{G}}$, $P_{\mathbf{G}} = \mathbf{G}(\mathbf{G}^H\mathbf{G})^{\#}\mathbf{G}^H$. We get the following results for the SINR of block LEs

- $\text{MFB}_{i,n} = \rho \, \|\underline{\underline{\mathbf{G}}}_{i,n}\|^2 - \delta = \rho \, ||\underline{\boldsymbol{h}}||^2$
- $\text{SINR}^{LE,\delta}_{i,n} = \rho \, \|\underline{\underline{\mathbf{G}}}'_{i,n}\|^2 - \delta$.

For a rate R per stream symbol, the precoded system outage probability is $P_o = \text{Prob}\{\log(\det(I + \rho \mathbf{G}\mathbf{G}^H)) \; < \; N_{\text{tot}} R\}$ where $N_{\text{tot}} = n_s(N - n_t L + 1)$ is the total number of symbols in the block. For a LE, the outage is

$$P^{LE,\delta}_{o,i,n} = \text{Prob}\{\log(1 + \text{SINR}^{LE,\delta}_{i,n}) < R\} \; .$$

For a ZF design, a perfect outage of symbol (i,n) occurs when $\text{SINR}_{i,n} = 0$. For the MFB this can only occur if $\underline{\boldsymbol{h}} = 0$. For a suboptimal Rx however, the SINR can vanish for any $\underline{\boldsymbol{h}}$ on the *Outage Manifold* $\mathcal{M}_{i,n} = \{\mathbf{h} : \text{SINR}_{i,n}(\underline{\boldsymbol{h}}) = 0\}$. At fixed rate R, the diversity order is the codimension of (the tangent subspace of) the outage manifold. Two (extreme) cases can be considered, depending on whether the symbols in the block are the result of joint encoding ($j-enc$ schemes) or separate encoding ($s-enc$ schemes), with ensuing joint or separate decoding. The outage probabilities of $j-enc$ and $s-enc$ schemes are respectively

$$\begin{aligned} P^{j-enc}_o(R) &= \text{Prob}\left(\textstyle\sum_{i,n} \log(1 + \text{SINR}_{i,n}) \; < \; N_{\text{tot}} R\right) \\ P^{s-enc}_o(R) &= \text{Prob}\left(\textstyle\bigcup_{i,n} \{\log(1 + \text{SINR}_{i,n}) \; < \; R\}\right) \; . \end{aligned}$$

The outage manifold for a $s-enc$ scheme, which we consider here (knowing that $j - enc$ can only do better), is given by $\mathcal{M} = \bigcup_{i,n} \mathcal{M}_{i,n}$. $P^{s-enc}_o = P^{LE,\delta}_{o,i,n}$ for a symbol position i in the middle of the block. The outage analysis for MMSE designs is more tricky than for MMSE-ZF designs.

IV. Outage of Block Tx vs. the Jensen Channel

Outage for a ZF LE will only occur if $\mathbf{G}$ in (5) loses full column rank. Now, due to the block triangular edge of a banded block Toeplitz matrix, $\mathbf{G}$ can only lose rank if the block column $\mathbf{G}_0$, or its nonzero part $\underline{\mathbf{g}}$, loses column rank. For easy reference, we shall refer to $\underline{\mathbf{g}}$ as the "Jensen" channel (of $\mathbf{g}[z]$), as in [6], the concept of which was introduced in [4]. Here $\underline{\mathbf{g}}$ corresponds to $\mathbf{G}$ for a minimal block length with only one symbol per stream. Hence for ZF, it suffices to analyze the case $\mathbf{G} = \underline{\mathbf{g}}$. This can be further argued by extending the reasoning of [7] to $\det(I_{n_s} + \rho \, \mathbf{g}^{\dagger}[z] \, \mathbf{g}[z])$. For the MMSE case, the detailed diversity behavior may depend on the block length though.

V. Outage Analysis of the Jensen Channel

Note that $\underline{\mathbf{g}}^H \underline{\mathbf{g}} + \frac{\delta}{\rho} I_{n_s} = Q^H(D + \frac{\delta}{\rho n_t} I_{n_t})Q$ where $D = \text{diag}\,\{\|\boldsymbol{h}_1\|^2, \ldots, \|\boldsymbol{h}_{n_t}\|^2\}$.

A. MMSE-ZF Design $\delta = 0$

Outage occurs when $\underline{\mathbf{g}}^H \underline{\mathbf{g}}$ becomes singular. Let $d_1 \geq \cdots \geq d_{n_t}$ be the ordered diagonal elements of D. Then outage occurs when $d_{n_s} = 0$. Since d_{n_s} is the maximum of $n_t - n_s + 1$ instances of $\|\boldsymbol{h}_i\|^2$, we get the diversity order:

Theorem 1: For a LE, $d^{ZF} = (n_t - n_s + 1) n_r L$, $N \geq n_t L$. Note that the diversity is full ($n_t L$) for the case $n_r = 1$, in which case, after applying delay diversity $\mathbf{T}[z]$, the result from [10] applies immediately.

B. MMSE Design $\delta = 1$

Consider first $n_r \geq n_t = n_s$. Then we have an eigendecomposition and $(\underline{\mathbf{g}}^H \underline{\mathbf{g}} + \frac{1}{\rho} I_{n_s})^{-1} = Q^H(D + \frac{1}{\rho n_t} I_{n_t})^{-1} Q$ and from its diagonal elements we get $\text{SINR}^{LE,1}_{1,n} = (\sum_{i=1}^{n_t} \frac{1}{1+\rho\|\boldsymbol{h}_i\|^2})^{-1} - 1$.

Theorem 2: For a MMSE LE, $N = n_t L$, $n_r \geq n_t = n_s$, $\text{Prob}\{\log(1 + \text{SINR}^{LE,1}_{1,n}) < R\} = \text{Prob}\{\sum_{i=1}^{n_t} \frac{1}{1+\rho\|\boldsymbol{h}_i\|^2} > 2^{-R}\} = P^{LE,1}_{o,1,n} \doteq \frac{1}{\rho^d}$ with $d = \min\{n_t, \lfloor 2^{-R} \rfloor + 1\}$.

Proof: At high SNR, the i.i.d. $1/(1 + \rho\|\boldsymbol{h}_i\|^2)$ become either 0 or 1. Now, $\text{Prob}\{1/(1 + \rho\|\boldsymbol{h}_i\|^2) = 1\} = \text{Prob}\{\|\boldsymbol{h}_i\|^2 < 1/\rho\} \doteq \rho^{-n_r L}$. And $\sum_i 1/(1 + \rho\|\boldsymbol{h}_i\|^2) > 2^{-R}$ if $\lfloor 2^{-R} \rfloor + 1 \leq n_t$ terms are equal to 1, and hence the $(\lfloor 2^{-R} \rfloor + 1)$-largest term becomes equal to 1. Q.E.D.

Full diversity requires $R \leq -\log_2(n_t - 1)$ ($= \infty$ for $n_t = 1$).

For $n_s = n_r < n_t$, we remark that decreasing the number of streams improves the SINR. Hence at least the diversity of the case $n_s = n_t$ is recovered.

VI. MMSE Analysis in the Low/High Rate Limit

Note: $\text{diag}\,(\mathbf{G}^H\mathbf{G}) = \|\underline{\mathbf{h}}\|^2 I = (\text{MFB}/\rho) I$. With $k = (i,n)$,
$\text{SINR}^{LE,1}_k = \rho \left(\|\mathbf{G}_k\|^2 - \mathbf{G}^H_k \overline{\mathbf{G}}_k (\overline{\mathbf{G}}^H_k \overline{\mathbf{G}}_k + \frac{1}{\rho} I)^{-1} \overline{\mathbf{G}}^H_k \mathbf{G}_k \right) = \rho\|\underline{\mathbf{h}}\|^2 \left(1 - \rho\|\underline{\mathbf{h}}\|^2 \, \widetilde{\mathbf{G}}^H_k \overline{\widetilde{\mathbf{G}}}_k (I + \rho\|\underline{\mathbf{h}}\|^2 \overline{\widetilde{\mathbf{G}}}^H_k \overline{\widetilde{\mathbf{G}}}_k)^{-1} \overline{\widetilde{\mathbf{G}}}^H_k \widetilde{\mathbf{G}}_k \right)$
for any N, where $\widetilde{\mathbf{G}} = \|\underline{\mathbf{h}}\|^{-2}\mathbf{G}$, $\text{diag}\,(\widetilde{\mathbf{G}}) = I$. Hence $\text{MFB}(1 - \text{MFB}\|\overline{\widetilde{\mathbf{G}}}^H_k \widetilde{\mathbf{G}}_k\|^2) \leq \text{SINR}^{LE,1}_k \leq \text{MFB}$. Outage occurs when $\text{SINR} < 2^R - 1$.

a) Low Rate Limit: For very small $2^R - 1$, outage occurs when SINR and hence MFB are small and hence $\text{SINR} \approx \text{MFB}$. Hence full diversity (even MMSE LE and MFB curves identical).

b) High Rate Limit: Now for outage: $\text{MFB} > 2^R - 1 \gg 1$ and $I + \rho\|\underline{\mathbf{h}}\|^2 \overline{\widetilde{\mathbf{G}}}^H_k \overline{\widetilde{\mathbf{G}}}_k \approx \rho\|\underline{\mathbf{h}}\|^2 \overline{\widetilde{\mathbf{G}}}^H_k \overline{\widetilde{\mathbf{G}}}_k$ and hence $\text{SINR}^{LE,1}_k \approx \text{SINR}^{LE,0}_k$ and $d^{MMSE} = d^{ZF}$.

VII. Simulations

A number of simulations have been performed that all confirm the analysis results; the ones shown here are for the parameters $n_t = n_r = 2$, $L = 2$, $N = 16$. Outage probabilities have been computed for Gaussian symbols assuming separate encoding for the LEs, for which hence the results for symbols at the edge or in the middle of the block are shown separately. The best curve in the figures corresponds to a fictitious outage, with the MFB as SINR. The reason why these curves are considered is that they exhibit unambiguously the full diversity. The second best

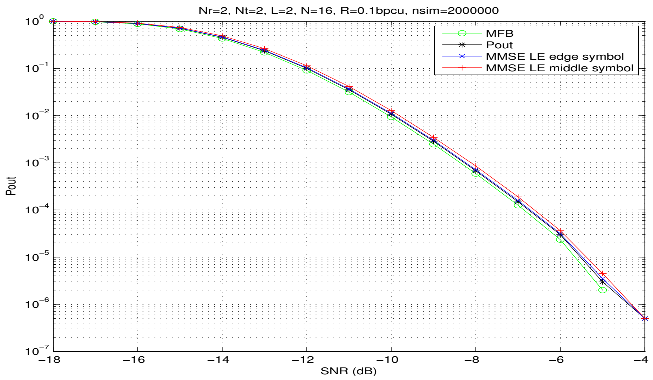

Fig. 1. Outage probability vs. SNR for MFB, block outage, and MMSE LE for the case $n_t = n_r = 2$, $L = 2$, $N = 16$, $R = 0.1$bpcu, and 2.10^6 Monte Carlo runs.

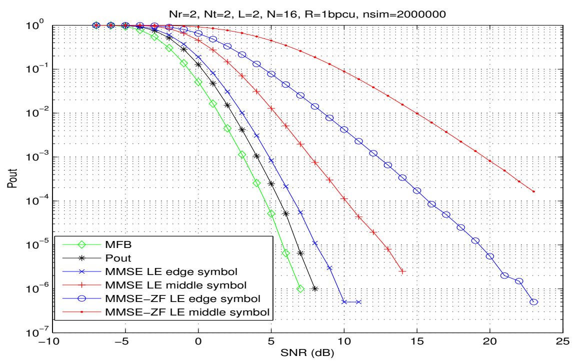

Fig. 2. Outage probability vs. SNR for MFB, block outage, MMSE LE and MMSE-ZF LE for the case $n_t = n_r = 2$, $L = 2$, $N = 16$, $R = 1$bpcu, and 2.10^6 Monte Carlo runs.

curve corresponds to the block outage, which would be attained by a block DFE with joint encoding. The simulations show that the block outage exhibits full diversity, since the curves are parallel to those of the MFB. The results in Fig. 1 show that for low rate ($R = 0.1$bpcu (bits per symbol)), a MMSE LE performs indeed identical to the MFB, as the analysis showed. At a medium rate ($R = 1$bpcu), the results in Fig. 2 show that with a MMSE LE, the edge symbols still benefit virtually of full diversity, whereas the middle symbols are starting to suffer. The MMSE LE performs in any case much better than the MMSE-ZF LE, the slope of which is insensitive to the rate transmitted. The slopes of the outage curves of the MMSE-ZF LE in Fig. 2 are identical to those of the MMSE LE in Fig. 3 where a high rate ($R = 4$bpcu) is used, as predicted by the analysis.

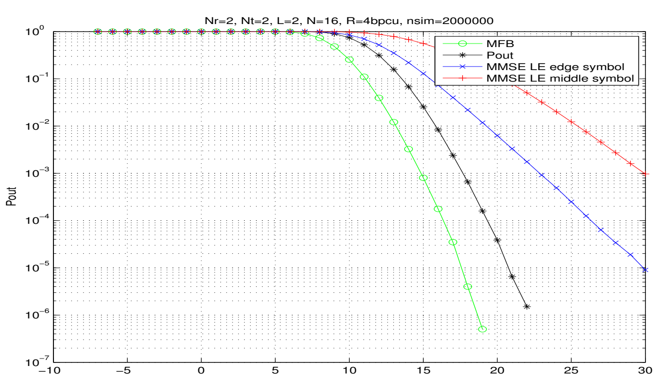

Fig. 3. Outage probability vs. SNR for MFB, block outage, and MMSE LE for the case $n_t = n_r = 2$, $L = 2$, $N = 16$, $R = 4$bpcu, and 2.10^6 Monte Carlo runs.

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