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Linear theory and velocity correlations of clusters

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ABSTRACT

Linear theory provides a reasonable description of the velocity correlations of biased tracers both perpendicular and parallel to the line of separation, provided one accounts for the fact that the measurement is almost always made using pair-weighted statistics. This introduces an additional term which, for sufficiently biased tracers, may be large. Previous work suggesting that linear theory was grossly in error for the components parallel to the line of separation ignored this term.

Key words: galaxies: clusters: general – cosmology: observations – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Experiments will soon measure correlations in the peculiar velocity field traced by massive galaxy clusters. The linear theory description of these correlations as traced by dark matter particles, whose motions are driven by gravitational instability from an initially Gaussian random field, has been available for about 20 years (Gorski 1988). The extension of this formalism to biased tracers, within the framework of perturbation theory, is in Fisher (1995). Sheth et al. (2001b) derive similar results within the context of the Halo model (see Cooray & Sheth 2002 for a review), and Regös & Szalay (1995) study this problem in the context of peaks (see Desjacques 2008 for corrections to their expressions). All these analyses yield consistent results.

However, measurements of the velocity correlations of massive haloes in simulations have given the impression that although biased linear theory provides a reasonable description of the correlations between the velocity components that are perpendicular to the line of separation $\Psi_\perp$, it is wildly discrepant for the components which are parallel to the line of separation $\Psi_{||}$ (Croft & Efstathiou 1995; Peel 2006), except on scales larger than about 100 Mpc. This discrepancy has been attributed to non-linear effects, such as those described by Colberg et al. (2000) and Sheth & Diaferio (2001).

The main purpose of this short note is to show that, in fact, linear theory does indeed provide a good description of $\Psi_{||}$, provided one accounts for the fact that the measurement is actually pair weighted and so the mean streaming motions may not be negligible. For dark matter, this mean is sufficiently small that it can be ignored, but ignoring the mean streaming of massive haloes towards one another is a very bad approximation. We show that keeping this term in the theoretical calculation provides substantially better agreement with the measurements.

Section 2 provides the linear theory expressions for the velocity statistics of interest. Section 3 presents a comparison of these expressions with simulations, and a final section summarizes our results. An Appendix provides some technical details of the calculation.

2 BIASED LINEAR THEORY

In linear theory for dark matter, the mean approach velocity of particle pairs along the line of separation is given by

$$\frac{v_{12}^{\rm dm}(r)}{Hr} = -\frac{2f(\Omega)}{1+\xi(r)}\int\frac{{\rm d}k}{k}\,\frac{k^3\,P(k)}{2\pi^2}\,\frac{j_1(kr)}{kr}, \tag{1}$$

where H is the Hubble constant, $f(\Omega) \approx \Omega^{5/9}$ for flat Λ cold dark matter (ΛCDM) models,

$$\xi(r) = \int\frac{{\rm d}k}{k}\,\frac{k^3\,P(k)}{2\pi^2}\,j_0(kr) \tag{2}$$

and $P(k)$ is the linear theory power spectrum of the density fluctuation field. This is often called the streaming motion; the term $1+\xi$ in the denominator reflects the fact that the average is over all pairs with separation r. For what is to follow, it will be convenient to write this as $v_{12}^{\rm dm}(r) \equiv \langle v_{||}^{\rm dm}(r)\rangle$ to emphasize the fact that the motion is parallel to the separation vector. In linear theory, the mean motion perpendicular to the line of separation is zero: $\langle v_\perp^{\rm dm}(r)\rangle = 0$.

Dark matter haloes are biased tracers of the underlying density field. If this bias is linear, but the velocities are unbiased, then

$$[1+b^2\,\xi(r)]\,\langle v_{||}(r)\rangle = b\,v_{12}^{\rm dm}(r)\,[1+\xi(r)], \tag{3}$$

where b is the linear bias factor (see Appendix).

The linear theory velocity dispersion is

$$\sigma_v^2 = \frac{[f(\Omega)H]^2}{3}\int{\rm d}k\,\frac{P(k)\,W^2(kR)}{2\pi^2}, \tag{4}$$

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where the smoothing window is related to halo mass by setting $W(x) = (3/x)\, j_1(x)$ with $R = (3M/4\pi\bar{\rho})^{1/3}$, where $\bar{\rho}$ is the comoving number density. The factor of 3 makes this a one-dimensional velocity dispersion. The presence of W makes the velocity dispersion decrease slightly with increasing halo mass (Sheth & Diaferio 2001). Note that there is no additional dependence on halo bias factor.

The two-point velocity correlation tensor is

$$\Psi_{ij}(\boldsymbol{r}) = \langle v_i(\boldsymbol{x})\, v_j(\boldsymbol{x}+\boldsymbol{r})\rangle = \Psi_\perp(r)\,\delta_{ij} + [\Psi_{||}(r) - \Psi_\perp(r)]\,\hat{\boldsymbol{r}}_i \hat{\boldsymbol{r}}_j, \quad (5)$$

where $r = |\boldsymbol{r}|, \hat{\boldsymbol{r}} = \boldsymbol{r}/r$ and $\Psi_{||}(r)$ and $\Psi_\perp(r)$ are the radial and transverse correlation functions (Monin & Yaglom 1975). In linear theory, the velocity field is potential, so $\Psi_{||}(r) = \mathrm{d}r\Psi_\perp(r)/\mathrm{d}r$ (e.g. Gorski 1988). For Gaussian initial conditions, the linear theory correlation between velocity components perpendicular to the line of separation is

$$\Psi_\perp(r) = f(\Omega)^2 H^2 \int \mathrm{d}k\, \frac{P(k)\, W^2(kR)}{2\pi^2}\, \frac{j_1(kr)}{kr}, \quad (6)$$

whereas the linear theory correlation between velocity components parallel to the line of separation is

$$\Psi_{||}(r) = f(\Omega)^2 H^2 \int \mathrm{d}k\, \frac{P(k)\, W^2(kR)}{2\pi^2}\, j_0(kr) - 2\Psi_\perp(r) - \frac{\langle v_{||}(r)\rangle^2\, [1 + b^2\, \xi(r)]}{4}. \quad (7)$$

A number of previous analyses have ignored the final term in this expression. The Appendix shows why, if the velocity correlations are estimated using pairs of tracer particles (as is commonly done), it should be included.

3 COMPARISON WITH SIMULATIONS

We compare the above expressions with measurements made in the Hubble Volume simulation (Evrard et al. 2002). The background cosmology for this simulation is flat ΛCDM with $\Omega_0 = 0.3$, $\sigma_8 = 0.9$ and $H = 100\,h$ km s^{-1} with $h = 0.7$. We present results for haloes within a very narrow mass bins centred on $\log_{10} M/h^{-1}\,\mathrm{M}_\odot = 14, 14.5$ and 15. There were 28 956, 21 461 and 9352 haloes in each bin.

The measured one-dimensional velocity dispersions of the haloes are $\sigma_\mathrm{v} = 313$, 303 and 283 km s^{-1}, whereas the values predicted by equation (4) are slightly smaller: 289, 276 and 258 km s^{-1}. This may be an indication that the non-linear effects of the sort discussed by Colberg et al. (2000) and Sheth & Diaferio (2001) have affected halo velocities. If we were to set $W \to 1$ in equation (4) as a crude way of accounting for non-linear evolution, then this would make $\sigma_\mathrm{v} = 322$ km s^{-1}.

Fig. 1 shows the mean streaming motions of massive haloes along the line of separation. Note that these velocities can be large: the important point in the present context is that, on small scales, $v_{12} \gg \sigma_\mathrm{v}$.

The smooth curves show $v_{12}^{\mathrm{dm}}(r)\,[1 + \xi(r)]$ of equation (1), multiplied by bias factors of $b = 1.2$, 2 and 3. They provide a reasonably accurate description of the measurements. One might have thought that a more appropriate model would not multiply by $1 + \xi$, and would include a factor of $W^2(kR)$ in the integral of equation (1) for the same reason that it is included in the expressions for σ_v, $\Psi_{||}$ and $\Psi_\perp$. We have found that including such smoothing produces too little streaming (compared to the simulations) on small scales.

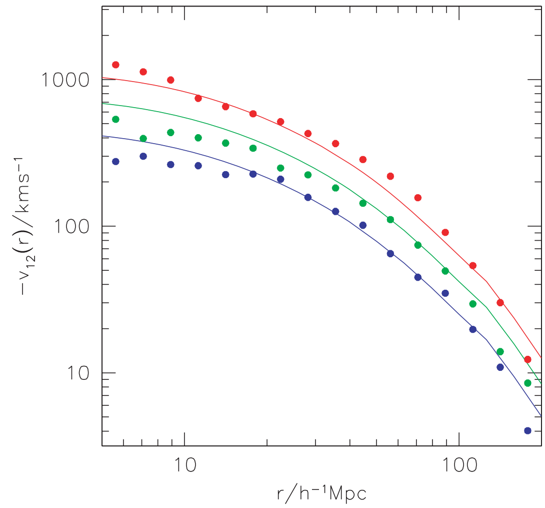

Figure 1. Mean streaming motions of haloes with masses $\log_{10}(M/h^{-1}\,\mathrm{M}_\odot) = 14$, 14.5 and 15 (bottom to top). Smooth curves show the linear bias prediction, $b\,v_{12}(r)$, with $b = 1.2, 2$ and 3.

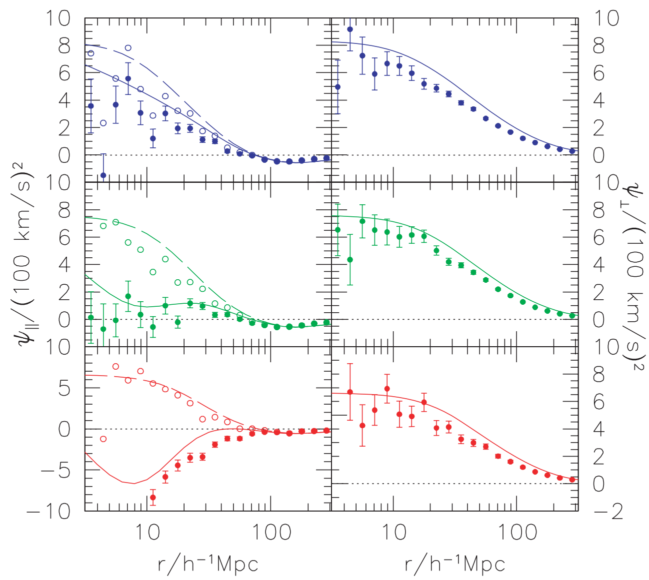

Figure 2. Velocity correlations of haloes with mass $\log_{10}(M/h^{-1}\,\mathrm{M}_\odot) \approx 14$, 14.5 and 15 (top to bottom) along (left) and perpendicular (right) to the line of separation. Filled circles show the measured correlations, open circles in the panels on the left show the result of adding the contribution of the mean streaming motions, $\langle v_{12}(r)\rangle^2/4$, to the filled circles. Solid curves show the linear theory predictions; dashed curves in the panels on the left show the linear theory prediction when the mean streaming is assumed to be negligible.

If the larger streaming on small scales is a consequence of non-linear effects, then it may be that removing the smoothing term is a convenient way to approximately account for non-linear evolution (e.g. setting $W \to 1$ in equation 4 brings σ_v into slightly better agreement with the measurements). Multiplying by $1 + \xi$ serves a similar purpose.

Fig. 2 compares the predicted velocity correlations for these haloes (curves) with the measurements (symbols). The solid curves show the predictions of equations (6) and (7). The dashed curves in the panels on the left show the result of ignoring the final term in equation (7); note how the dashed curves grossly overestimate the measured signal (filled symbols). This is the apparent failure of linear theory noted by Croft & Efstathiou (1995) and Peel (2006). However, they are in good agreement with the measurements if, in the measurements, we add back the contribution from the mean streaming term (open symbols). This, and the reasonable agreement between the solid curves and solid symbols, indicates that linear theory is, in fact, not so badly flawed.

4 DISCUSSION

The mean streaming motions of massive haloes can be as large as 1000 km s^{-1} on scales of the order of $10\,h^{-1}$ Mpc. In contrast, the one-dimensional velocity dispersions of these haloes are of the order of 300 km s^{-1}. Nevertheless, these motions are rather well described by biased linear theory. The fact that the mean streaming motions of haloes can be substantially larger than the halo velocity dispersions has an important consequence: they must not be ignored when making the linear theory estimate of velocity correlations $\Psi_{||}$ and $\Psi_{\perp}$ for pair-weighted statistics. Ignoring this contribution leads one to predict that $\Psi_{||}$ is positive; i.e. objects stream along with one another, whatever their mass. The measurements show that while this is true for the lower mass haloes (they are still quite massive!), $\Psi_{||}$ is negative for the most massive haloes. This means that the most massive haloes move towards, rather than along with, one another; this is consistent with linear theory provided one accounts for the fact that the measurements are pair-weighted, so one should not ignore the contribution of the mean streaming motions.

We note that, although they do not say so explicitly, halo model analyses of halo motions (Sheth et al. 2001a) have correctly included the effects of this mean streaming. It is the appearance of more recent studies which ignore this effect (e.g. we had communicated the importance of this term to Peel 2006, but his figures do not include it) which prompted our work. Because our analysis shows that linear theory can be used down to scales which are significantly smaller than previously thought, we hope that our analysis will aid in the interpretation of data from the next generation of surveys.

For deep surveys, the gain in accuracy is likely to be modest. This is because sky surveys typically estimate the correlation between radially projected velocities, separated by some angle θ on the sky. This quantity is related to ours by

$$\Psi_{12} = \Psi_{\perp} \cos\theta + (\Psi_{||} - \Psi_{\perp})\, f(\theta, r_1, r_2), \tag{8}$$

where $\Psi_{||} \equiv \sigma_v^2 \psi_{||}$, $\Psi_{\perp} \equiv \sigma_v^2 \psi_{\perp}$, both are evaluated at scale $r = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}$ and

$$f(\theta, r_1, r_2) = \frac{\left(r_1^2 + r_2^2\right)\cos\theta - r_1 r_2 (1 + \cos^2\theta)}{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}. \tag{9}$$

This can also be written as

$$\Psi_{12}(r) = \Psi_{\perp}(r)\,\frac{r_1 r_2}{r^2}\,\sin^2\theta + \Psi_{||}(r)\left(\cos\theta - \frac{r_1 r_2}{r^2}\,\sin^2\theta\right), \tag{10}$$

showing that our extra term is most important for pairs which are close in angle but lie at different distances along the line of sight. For very deep surveys (r_1 and r_2 greater than 100 Mpc, say) the observable is dominated by $\Psi_{\perp}$ for all scales where the addition of our extra term is important. Nevertheless, we emphasize that the term is present, and its inclusion greatly improves comparison with simulations, thus strengthening ones confidence in the validity of linear theory.

Although accounting for mean streaming motions results in substantially better agreement with the simulations, there remains room for improvement. Our treatment of how $\langle v_{||}(r)\rangle$ should depend on halo mass on small scales is rather ad hoc. Methods motivated by perturbation theory are beginning to provide more detailed prescriptions for this term (e.g. Smith, Scoccimarro & Sheth 2008). Also, in perturbation theory, the power spectrum of peculiar velocities is expected to be slightly suppressed relative to the linear theory prediction, even on relatively large scales (e.g. Bernardeau et al. 2002; Cooray & Sheth 2002; Pueblas & Scoccimarro 2008). Although including this effect is beyond the scope of this work, we note that accounting for it will lower the theory curves slightly, reducing the small discrepancy in the panels on the right-hand side of Fig. 2. Incorporating these refinements into our analysis is the subject of work in progress.

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APPENDIX A: LINEARLY BIASED TRACERS

Dark matter haloes are biased tracers of the underlying density field. If this bias is linear, but the halo velocities are unbiased, then pair-weighted (as opposed to volume-weighted) velocity statistics of haloes may be biased relative to those of the mass, with the effect increasing with spatial bias.

To see this, it is helpful to write the expressions in the main text explicitly as averages over pairs. In linear theory, the mean streaming motions are

$$\begin{aligned}\langle v_{||}(r)\rangle &\equiv \frac{\langle(1+b_1\delta_1)(1+b_2\delta_2)(v_{1||}-v_{2||})\rangle}{\langle(1+b_1\delta_1)(1+b_2\delta_2)\rangle}\\ &= \frac{-b_1\,\langle\delta_1 v_{2||}\rangle + b_2\,\langle\delta_2 v_{1||}\rangle}{1+b_1b_2\langle\delta_1\delta_2\rangle}\\ &= -\frac{b_1+b_2}{2}\,\frac{2\langle\delta_1 v_{2||}\rangle}{1+b_1b_2\,\xi(r)}\\ &= \frac{b_1+b_2}{2}\,v_{12}^{\rm dm}(r)\,\frac{1+\xi(r)}{1+b_1b_2\,\xi(r)}\\ &\to b\,v_{12}^{\rm dm}(r)\,\frac{1+\xi(r)}{1+b^2\,\xi(r)},\end{aligned} \tag{A1}$$

where

$$v_{1||}-v_{2||}\equiv(\boldsymbol{v}_1-\boldsymbol{v}_2)\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)}{|\boldsymbol{r}_1-\boldsymbol{r}_2|}, \tag{A2}$$

and it is understood that $r \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2|$. The factors of $1 + b_1\delta_1$ and $1 + b_2\delta_2$ represent the halo counts at positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, so their product is the pair-weight and the averages are over all pairs separated by r. The algebra above follows because $\langle \boldsymbol{v}_1 - \boldsymbol{v}_2\rangle = 0$, and because, in linear theory, $\langle\delta_i \boldsymbol{v}_i\rangle = 0$ and $\langle\delta_1\delta_2\boldsymbol{v}_1\rangle = 0$. On large scales, the difference in the pair counts is small ($\xi(r) \ll 1$), and $v_{12} \to bv_{12}^{\rm dm}(r)$ where $b = (b_1 + b_2)/2$. If the statistic is measured for a range of halo masses, then one simply sets both b_1 and b_2 equal to the average bias factor b in the expressions above; this is the form we have used for the final expression. Note that v_{12} is biased relative to the dark matter, even though the velocities themselves are explicitly unbiased.

Similarly, in linear theory, the mean correlation of halo line-of-separation velocities, averaged over halo pairs, is

$$\begin{aligned}\Psi_{||} &\equiv \frac{\langle(1+b_1\delta_1)(1+b_2\delta_2)\,v_{1||}\,v_{2||}\rangle}{\langle(1+b_1\delta_1)(1+b_2\delta_2)\rangle}\\ &= \frac{\langle v_{1||}\,v_{2||}\rangle + \langle b_1\delta_1\,b_2\delta_2\,v_{1||}\,v_{2||}\rangle}{1+b_1b_2\,\langle\delta_1\delta_2\rangle}\\ &= \frac{\langle v_{1||}\,v_{2||}\rangle + b_1b_2\langle\delta_1\delta_2\rangle\langle v_{1||}v_{2||}\rangle + b_1b_2\langle\delta_1 v_{2||}\rangle\langle\delta_2 v_{1||}\rangle}{1+b_1b_2\,\langle\delta_1\delta_2\rangle}\\ &= \langle v_{1||}\,v_{2||}\rangle + \frac{b_1b_2\langle\delta_1 v_{2||}\rangle\langle\delta_2 v_{1||}\rangle}{1+b_1b_2\,\langle\delta_1\delta_2\rangle}\\ &= \langle v_{1||}\,v_{2||}\rangle - \frac{b_1b_2}{4}\,\frac{4\langle\delta_1 v_{2||}\rangle^2}{1+b_1b_2\,\langle\delta_1\delta_2\rangle}\\ &\to \langle v_{1||}\,v_{2||}\rangle - \frac{\langle v_{||}(r)\rangle^2\,[1+b^2\,\xi(r)]}{4}.\end{aligned} \tag{A3}$$

Note that, in the second and third lines, we have assumed Gaussian statistics to neglect three-point averages and to write the four-point average as a product of two-point averages. The final expression is what one obtains if a range of halo masses, with average bias factor b, contributes to the statistic. On large scales, $v_{12}^{\rm dm}$ and $\xi(r)$ are both small, so $\Psi_{||} \approx \langle v_{1||}v_{2||}\rangle$. However, the second term in the expression may be non-negligible when $b \gg 1$; neglecting it will lead to unnecessary inaccuracies. A similar analysis of $\Psi_\perp$ does not yield an extra term because $\langle\delta_2 v_{1\perp}\rangle = 0$.

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