

## LINEAR WEINGARTEN HYPERSURFACES IN A UNIT SPHERE

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ABSTRACT. In this paper, we have considered linear Weingarten hypersurfaces in a sphere and obtained some rigidity theorems. The purpose of this paper is to give some extension of the results due to Cheng-Yau [3] and Li [7].

### 1. Introduction

Let  $M$  be a hypersurface in an  $(n + 1)$ -dimensional unit sphere  $S^{n+1}(1)$ . As is well known to us, there are many rigidity results for hypersurfaces in a unit sphere with constant mean curvature or with constant scalar curvature. Now we want to introduce an well-known theorem due to Cheng-Yau [3] as follows:

**Theorem 1.1.** *Let  $M$  be an  $n$ -dimensional compact hypersurface in a unit sphere  $S^{n+1}(1)$ . If*

- (1)  *$M$  has nonnegative sectional curvature,*
- (2) *the normalized scalar curvature  $r$  of  $M$  is constant and  $r \geq 1$ ,*

*then  $M$  is either totally umbilical, or  $M = S^{n-k}(a) \times S^k(\sqrt{1-a^2})$ ,  $1 \leq k \leq n - 1$ .*

On the other hand, Li [7] studied some hypersurfaces in a unit sphere with scalar curvature proportional to mean curvature and proved the following theorem.

**Theorem 1.2.** *Let  $M$  be an  $n$ -dimensional compact hypersurface in a unit sphere  $S^{n+1}(1)$ . If*

- (1)  *$M$  has nonnegative sectional curvature,*

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(2) the normalized scalar curvature  $r$  of  $M$  is proportional to mean curvature  $H$  of  $M$ , that is,

$$r = aH, \quad a^2 \geq \frac{4n}{n-1},$$

where  $a$  is a constant, then  $M$  is either totally umbilical, or  $M = S^{n-k}(a) \times S^k(\sqrt{1-a^2})$ ,  $1 \leq k \leq n-1$ .

Now let us introduce a notion for linear Weingarten hypersurfaces in an  $(n+1)$ -dimensional unit sphere  $S^{n+1}(1)$  as follows:

**Definition 1.1.** Let  $M$  be a hypersurface in an  $(n+1)$ -dimensional unit sphere  $S^{n+1}(1)$ . We call  $M$  a *linear Weingarten hypersurface* if  $cR+dH+e=0$ , where  $c$ ,  $d$  and  $e$  are constants such that  $c^2+d^2 \neq 0$ ,  $R$  and  $H$  respectively denote the scalar curvature and the mean curvature of  $M$ .

*Remark 1.1.* When the constant  $d=0$  in Definition 1.1, a linear Weingarten hypersurface  $M$  reduces to a hypersurface with constant scalar curvature. When the constant  $c=0$  in Definition 1.1, a linear Weingarten hypersurface  $M$  reduces to a hypersurface with constant mean curvature. In such a sense, the linear Weingarten hypersurfaces can be regarded as a natural generalization of hypersurfaces with constant scalar curvature or with constant mean curvature.

By investigating Cheng-Yau's operator  $\square$  given in [3] and using some new estimations, we want to study linear Weingarten hypersurfaces in a unit sphere as follows:

**Theorem 1.3.** Let  $M$  be an  $n$ -dimensional compact hypersurface in a unit sphere  $S^{n+1}(1)$ . If

- (1)  $M$  has nonnegative sectional curvature,
- (2) the normalized scalar curvature  $r$  and the mean curvature  $H$  of  $M$  satisfies the following conditions:  $r = aH + b$ ,  $(n-1)a^2 - 4n + 4nb \geq 0$ , then  $M$  is either totally umbilical, or  $M = S^{n-k}(a) \times S^k(\sqrt{1-a^2})$ ,  $1 \leq k \leq n-1$ .

*Remark 1.2.* Since  $R = n(n-1)r$ , a hypersurface  $M$  in Theorem 1.3 satisfying  $r = aH + b$  is just a linear Weingarten hypersurface in Definition 1.1.

*Remark 1.3.* When the constant  $a$  in above identically vanishes, our Theorem 1.3 reduces to Theorem 1.1. When the constant  $b$  vanishes, our Theorem 1.3 reduces to Theorem 1.2.

In all of theorems mentioned above, we have assumed that  $M$  has nonnegative sectional curvature. In the following theorem, we want to study linear Weingarten hypersurfaces in a unit sphere without the assumption of nonnegative sectional curvature. In fact, we prove the following:

**Theorem 1.4.** Let  $M$  be a hypersurface in  $S^{n+1}(1)$ . If

- (1)  $r = aH + b$ ,  $(n-1)a^2 - 4n + 4nb \geq 0$ ,
- (2)  $|B|^2 \leq 2\sqrt{n-1}$ ,

then either  $|B|^2 = 0$  and  $M$  is a totally umbilical hypersurface or  $|B|^2 = 2\sqrt{n-1}$  and  $M = S^1(c) \times S^{n-1}(\sqrt{1-c^2})$ .

**2. Preliminaries**

There are many studies on compact hypersurfaces in an  $(n + 1)$ -dimensional unit sphere  $S^{n+1}(1)$  (see [1], [2], [3], [5]-[12]). In this paper, let us also denote by  $M$  a compact hypersurfaces in  $S^{n+1}(1)$ . We choose a local orthonormal frame  $\{e_A\}_{1 \leq A \leq n+1}$  in  $S^{n+1}(1)$ , with dual coframe  $\{\omega_A\}_{1 \leq A \leq n+1}$ , such that, at each point of  $M$ ,  $e_1, \dots, e_n$  are tangent to  $M$  and  $e_{n+1}$  is the positively oriented unit normal vector. We shall make use of the following convention on the ranges of indices:

$$1 \leq A, B, C, \dots, \leq n + 1; \quad 1 \leq i, j, k, \dots, \leq n.$$

Then the structure equations of  $S^{n+1}(1)$  are given by

$$(2.1) \quad d\omega_A = \sum_{B=1}^{n+1} \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0,$$

$$(2.2) \quad d\omega_{AB} = \sum_{C=1}^{n+1} \omega_{AC} \wedge \omega_{CB} - \omega_A \wedge \omega_B.$$

When restricted to  $M$ , we have  $\omega_{n+1} = 0$  and

$$(2.3) \quad 0 = d\omega_{n+1} = \sum_{i=1}^n \omega_{n+1i} \wedge \omega_i.$$

By Cartan’s lemma, there exist functions  $h_{ij}$  such that

$$(2.4) \quad \omega_{n+1i} = \sum_{j=1}^n h_{ij} \omega_j, \quad h_{ij} = h_{ji}.$$

This gives the second fundamental form of  $M$ ,  $B = \sum_{i,j} h_{ij} \omega_i \omega_j e_{n+1}$ . The mean curvature  $H$  is defined by  $H = \frac{1}{n} \sum_i h_{ii}$ . From (2.1)-(2.4) we obtain the structure equations of  $M$

$$(2.5) \quad d\omega_i = \sum_{j=1}^n \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0,$$

$$(2.6) \quad d\omega_{ij} = \sum_{k=1}^n \omega_{ik} \wedge \omega_{kj} - \frac{1}{2} \sum_{k,l=1}^n R_{ijkl} \omega_k \wedge \omega_l$$

and the Gauss equation

$$(2.7) \quad R_{ijkl} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk} + h_{ik} h_{jl} - h_{il} h_{jk}.$$

Then it follows that

$$(2.8) \quad n(n-1)(r-1) = n^2 H^2 - |B|^2,$$

where  $R_{ijkl}$  denotes the components of the Riemannian curvature tensor of  $M$ ,  $n(n-1)r$  is the scalar curvature of  $M$  and  $|B|^2 = \sum_{i,j=1}^n h_{ij}^2$  is the square norm of the second fundamental form of  $M$ .

Let  $h_{ijk}$  denote the covariant derivative of  $h_{ij}$ . We then have

$$(2.9) \quad \sum_k h_{ijk} \omega_k = dh_{ij} + \sum_k h_{kj} \omega_{ki} + \sum_k h_{ik} \omega_{kj}.$$

Thus, by exterior differentiation of (2.4), we obtain the Codazzi equation

$$(2.10) \quad h_{ijk} = h_{ikj}.$$

The second covariant derivative of  $h_{ij}$  is defined by

$$(2.11) \quad \sum_i h_{ijkl} \omega_l = dh_{ijk} + \sum_m h_{mjk} \omega_{mi} + \sum_m h_{imk} \omega_{mj} + \sum_m h_{ijm} \omega_{mk}.$$

By exterior differentiation of (2.9), we can get that the following Ricci identities hold

$$(2.12) \quad h_{ijkl} - h_{ijlk} = \sum_m h_{mj} R_{mikl} + \sum_m h_{im} R_{mjkl}.$$

For a smooth function  $f$  defined on  $M$ , the gradient, the hessian ( $f_{ij}$ ) and the Laplacian  $\Delta f$  of  $f$  are defined by

$$(2.13) \quad df = \sum_i f_i \omega_i, \quad \sum_j f_{ij} \omega_j = df_i + \sum_j f_j \omega_{ji}, \quad \Delta f = \sum_i f_{ii}.$$

Let  $\phi = \sum_{i,j} \phi_{ij} \omega_i \otimes \omega_j$  be a symmetric tensor defined on  $M$ , where

$$(2.14) \quad \phi_{ij} = nH\delta_{ij} - h_{ij}.$$

Following Cheng-Yau [3], we introduce an operator  $\square$  associated to  $\phi$  acting on any smooth function  $f$  by

$$(2.15) \quad \square f = \sum_{i,j} \phi_{ij} f_{ij} = \sum_{i,j} (nH\delta_{ij} - h_{ij}) f_{ij}.$$

Since  $\phi_{ij}$  is divergence-free, it follows that the operator  $\square$  is self-adjoint relative to the  $L^2$  inner product of  $M$ , i.e.,

$$(2.16) \quad \int_M f \square g dv = \int_M g \square f dv.$$

We choose  $e_1, \dots, e_n$  such that

$$(2.17) \quad h_{ij} = \lambda_i \delta_{ij},$$

then we have

$$(2.18) \quad \begin{aligned} \square(nH) &= nH\Delta(nH) - \sum_i \lambda_i (nH)_{,ii} \\ &= \frac{1}{2} \Delta(nH)^2 - \sum_i (nH)_{,i}^2 - \sum_i \lambda_i (nH)_{,ii} \\ &= \frac{1}{2} n(n-1) \Delta r + \frac{1}{2} \Delta |B|^2 - n^2 |\nabla H|^2 - \sum_i \lambda_i (nH)_{,ii}. \end{aligned}$$

On the other hand, we can deduce from (2.10) and (2.12) that

$$(2.19) \quad \frac{1}{2}\Delta|B|^2 = \sum_{i,j,k} h_{ijk}^2 + \sum_i \lambda_i(nH)_{,ii} + \frac{1}{2} \sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2.$$

Putting (2.19) into (2.18), we obtain

$$(2.20) \quad \square(nH) = \frac{1}{2}n(n-1)\Delta r + |\nabla B|^2 - n^2|\nabla H|^2 + \frac{1}{2} \sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2.$$

From the Gauss equation, we have  $R_{ijij} = 1 + \lambda_i\lambda_j$ , putting this into (2.20), we get

$$(2.21) \quad \begin{aligned} \square(nH) &= \frac{1}{2}n(n-1)\Delta r + |\nabla B|^2 - n^2|\nabla H|^2 \\ &\quad + n|B|^2 - n^2H^2 - |B|^4 + nH \sum_i \lambda_i^3. \end{aligned}$$

Next we introduce the following lemma

**Lemma 2.1.** *Let  $M$  be a hypersurface in  $S^{n+1}(1)$ . If*

$$(2.22) \quad r = aH + b \quad \text{and} \quad (n-1)a^2 - 4n + 4nb \geq 0,$$

*then we have*

$$(2.23) \quad |\nabla B|^2 \geq n^2|\nabla H|^2,$$

*where  $n(n-1)r$  is scalar curvature of  $M$ ,  $H$  denotes the mean curvature of  $M$ .*

*Proof.* By the formula (2.7), we have

$$(2.24) \quad |B|^2 = n^2H^2 + n(n-1)(1-r) = n^2H^2 + n(n-1)(1-aH-b),$$

it follows that

$$(2.25) \quad 2h_{ij}h_{ijk} = 2n^2HH_{,k} - n(n-1)aH_{,k},$$

then we obtain

$$(2.26) \quad 4 \sum_k \left( \sum_{i,j} h_{ij}h_{ijk} \right)^2 = [2n^2H - n(n-1)a]^2|\nabla H|^2.$$

Hence we deduce that

$$(2.27) \quad \begin{aligned} 4 \left( \sum_{i,j} h_{ij}^2 \right) \left( \sum_{i,j,k} h_{ijk}^2 \right) &\geq 4 \sum_k \left( \sum_{i,j} h_{ij}h_{ijk} \right)^2 \\ &= [2n^2H - n(n-1)a]^2|\nabla H|^2, \end{aligned}$$

that is,

$$(2.28) \quad 4|B|^2|\nabla B|^2 \geq [2n^2H - n(n-1)a]^2|\nabla H|^2.$$

On the other hand,

$$\begin{aligned}
 & [2n^2H - n(n-1)a]^2 - 4n^2|B|^2 \\
 &= 4n^4H^2 + n^2(n-1)^2a^2 - 4n^3(n-1)Ha \\
 (2.29) \quad & - 4n^3[nH^2 + (n-1)(1-aH-b)] \\
 &= n^2(n-1)[(n-1)a^2 - 4n + 4nb] \\
 &\geq 0.
 \end{aligned}$$

From this, together with Lemma 2.1, we have

$$4|B|^2|\nabla B|^2 \geq [2n^2H - n(n-1)a]^2|\nabla H|^2 \geq 4n^2|B|^2|\nabla H|^2,$$

then we obtain either  $|B|^2 = 0$  and  $|\nabla B|^2 = n^2|\nabla H|^2 = 0$  or  $|\nabla B|^2 \geq n^2|\nabla H|^2$ . This completes the proof of Lemma 2.1.  $\square$

*Remark 2.1.* When the constant  $a$  vanishes, our Lemma 2.1 reduce to Lemma 3.2 in [6] due to Li.

In this paper, we also need the following lemma due to Okumura [9].

**Lemma 2.2.** *Let  $\mu_i$ ,  $i = 1, \dots, n$ , be real numbers such that  $\sum_i \mu_i = 0$  and  $\sum_i \mu_i^2 = \beta^2$ , where  $\beta = \text{constant} \geq 0$ . Then*

$$(2.30) \quad -\frac{n-2}{\sqrt{n(n-1)}}\beta^3 \leq \sum_i \mu_i^3 \leq \frac{n-2}{\sqrt{n(n-1)}}\beta^3$$

and the equality holds if and only if at least  $(n-1)$  of the  $\mu_i$  are equal.

### 3. Proofs of Theorem 1.3 and Theorem 1.4

First in this section, we want to prove Theorem 1.3 as follows:

By (2.20), we can obtain

$$(3.1) \quad 0 = \int_M (|\nabla B|^2 - n^2|\nabla H|^2)dv + \frac{1}{2} \int_M \sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2 dv.$$

By using of Lemma 2.1, we can deduce that

$$(3.2) \quad |\nabla B|^2 \geq n^2|\nabla H|^2.$$

Since  $M$  has nonnegative sectional curvature and (3.1), we know that

$$(3.3) \quad |\nabla B|^2 = n^2|\nabla H|^2, \quad \sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2 = 0,$$

that is,

$$(3.4) \quad |\nabla B|^2 = n^2|\nabla H|^2, \quad R_{ijij} = 0 \quad \text{if } \lambda_i \neq \lambda_j.$$

Then it follows from the Gauss equation  $R_{ijij} = 1 + \lambda_i \lambda_j$  and (3.4) that either  $M$  is totally umbilical or  $M$  has two distinct constant principal curvatures  $\lambda$ ,  $\mu$  and  $1 + \lambda\mu = 0$ . This completes the proof of Theorem 1.3.  $\square$

Next we want to give the proof of Theorem 1.4 as follows:

- Case 1:  $n = 2$ .

In this case, we obtain from the Gauss equation that

$$(3.5) \quad 2(r - 1) = 4H^2 - |B|^2 = 2\lambda_1\lambda_2.$$

Combining (2.21) and the Gauss equation (3.5), we obtain

$$\begin{aligned} \square(2H) &= \Delta r + |\nabla B|^2 - 4|\nabla H|^2 + 2|B|^2 - 4H^2 - |B|^4 + 2H \sum_{i=1}^2 \lambda_i^3 \\ &= \Delta r + |\nabla B|^2 - 4|\nabla H|^2 + 2|B|^2 - 4H^2 - |B|^4 \\ &\quad + 2H(\lambda_1 + \lambda_2)(\lambda_1^2 - \lambda_1\lambda_2 + \lambda_2^2) \\ (3.6) \quad &= \Delta r + |\nabla B|^2 - 4|\nabla H|^2 + 2|B|^2 - 4H^2 - |B|^4 \\ &\quad + 4H^2(|B|^2 - (r - 1)) \\ &= \Delta r + |\nabla B|^2 - 4|\nabla H|^2 + |B|^2 - 2(r - 1) - |B|^4 \\ &\quad + \{|B|^2 + 2(r - 1)\}\{|B|^2 - (r - 1)\} \\ &= \Delta r + |\nabla B|^2 - 4|\nabla H|^2 + r(|B|^2 + 2 - 2r) \\ &= \Delta r + |\nabla B|^2 - 4|\nabla H|^2 + (4H^2 + 2 - |B|^2)(|B|^2 - 2H^2), \end{aligned}$$

then we get the following integral equality

$$(3.7) \quad \int_M (|\nabla B|^2 - 4|\nabla H|^2)dv + \int_M (4H^2 + 2 - |B|^2)(|B|^2 - 2H^2)dv = 0.$$

Since  $|B|^2 \leq 2$ , we know that

$$(3.8) \quad (4H^2 + 2 - |B|^2)(|B|^2 - 2H^2) \geq 0.$$

By Lemma 2.1, we have

$$(3.9) \quad |\nabla B|^2 - 4|\nabla H|^2 \geq 0.$$

Combining (3.7), (3.8) and (3.9), we obtain  $|\nabla B|^2 - 4|\nabla H|^2 = 0$  and  $(4H^2 + 2 - |B|^2)(|B|^2 - 2H^2) = 0$ , that is, either  $H = 0$  and  $|B|^2 = 2$  or  $|B|^2 - 2H^2 = 0$ . If  $H = 0$  and  $|B|^2 = 2$ , we know that  $M = S^1(c) \times S^1(\sqrt{1 - c^2})$  from a result of [5]. If  $|B|^2 - 2H^2 = 0$ ,  $M$  is totally umbilical.

- Case 2:  $n \geq 3$ .

Let  $\mu_i = \lambda_i - H$  and  $|Z|^2 = \sum_i \mu_i^2$ , we get

$$(3.10) \quad \sum_i \mu_i = 0, \quad |Z|^2 = |B|^2 - nH^2,$$

$$(3.11) \quad \sum_i \lambda_i^3 = \sum_i \mu_i^3 + 3H|Z|^2 + nH^3.$$

By (3.10) and (3.11), we obtain

$$(3.12) \quad \begin{aligned} \square(nH) &= \frac{1}{2}n(n-1)\Delta r + |\nabla B|^2 - n^2|\nabla H|^2 \\ &\quad + |Z|^2(n + nH^2 - |Z|^2) + nH \sum_i \mu_i^3. \end{aligned}$$

Combining (3.12) and Lemma 2.2, we get

$$(3.13) \quad \begin{aligned} \square(nH) &\geq \frac{1}{2}n(n-1)\Delta r + |\nabla B|^2 - n^2|\nabla H|^2 \\ &\quad + |Z|^2 \left( n + nH^2 - |Z|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H||Z| \right). \end{aligned}$$

Hence we have the following:

**Lemma 3.1.** *Let  $M$  be a compact hypersurface in  $S^{n+1}(1)$ . Then we have*

$$(3.14) \quad \begin{aligned} 0 &\geq \int_M (|\nabla B|^2 - n^2|\nabla H|^2) dv \\ &\quad + \int_M |Z|^2 \left( n + nH^2 - |Z|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H||Z| \right) dv. \end{aligned}$$

Now, we assume  $r = aH + b$  and  $(n-1)a^2 - 4n + 4nb \geq 0$ . For a real number  $d = \frac{n+2\sqrt{n-1}}{n-2}\sqrt{n} > 0$ , we have

$$(3.15) \quad 2|H||Z| \leq dH^2 + \frac{1}{d}|Z|^2.$$

By Lemma 2.1, Lemma 3.1 and (3.15), we obtain

$$\begin{aligned} &\int_M |Z|^2 \left\{ n + nH^2 \left( 2 - \frac{(n-2)d}{2\sqrt{n(n-1)}} + \frac{n(n-2)}{2\sqrt{n(n-1)}d} \right) \right. \\ &\quad \left. - |B|^2 \left( 1 + \frac{n(n-2)}{2\sqrt{n(n-1)}d} \right) \right\} dv \leq 0, \end{aligned}$$

that is,

$$(3.16) \quad 0 \geq \int_M |Z|^2 \left\{ n - \frac{n}{2\sqrt{n-1}}|B|^2 \right\} dv.$$

Noting that  $|B|^2 \leq 2\sqrt{n-1}$ , we know that the right hand side of (3.16) is nonnegative, it follows that  $|\nabla B|^2 = n^2|\nabla H|^2$  and  $\int_M |Z|^2 \left\{ n - \frac{n}{2\sqrt{n-1}}|B|^2 \right\} dv = 0$ . Then we have either  $|Z|^2 = |B|^2 - nH^2 = 0$  and  $M$  is totally umbilical or  $|B|^2 = 2\sqrt{n-1}$ .

If  $M$  is not totally umbilical, we can see that

$$(3.17) \quad |B|^2 = 2\sqrt{n-1},$$

it follows from Lemma 2.2, Lemma 3.1 and (3.17) that

$$(3.18) \quad \lambda_1 = \cdots = \lambda_{n-1} \neq \lambda_n,$$



and  $M = S^1(c) \times S^{n-1}(\sqrt{1-c^2})$ . This completes the proof of Theorem 1.4.  $\square$

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