Link-level Vulnerability Indicators for Real-World Networks

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Abstract

It is computationally expensive to find out where vulnerable parts in a network are. In literature a variety of methods were introduced that use simple indicators (measured in real-life or calculated in a traffic simulator) to pre-determine the seriousness of the delays caused by the blocking of that link and thereafter perform a more detailed analysis. This article reviews the indicators proposed in the literature and assesses the quality of these indicators. Furthermore, a multi-linear fit of the indicators is made to find a better, combined, indicator to rank the links according to their vulnerability. The article shows that different indicators assess different links to be vulnerable. Also combined they cannot predict the vulnerability of a link. Therefore, it is concluded that to find vulnerable links, one has to look further than link-based indicators.

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1. Introduction

Numerous situations can be thought of in which large parts of a road network are blocked due to an event on one single location. For example, an incident in the peak hour in which a truck is involved could cause severe congestion on many roads in the surroundings of the accident location. Other less frequently occurring causes of disruptions are (terrorist) attacks, or disasters or calamities, or the thread thereof which causes the authorities to close the road. All disruptions cause delays, which is undesirable for road users. Road authorities want to know the most vulnerable links of their network because this enables them to protect or to improve those links or parts of their network.

The term "road network robustness" refers to these issues. In the literature different definitions of robustness can be found, but there is not yet a commonly accepted definition for robustness. The cause of disruptions is one of the most important differences. Sometimes only severe and non-recurrent disruptions are considered and sometimes daily variations are also taken into account. The terms robustness and vulnerability are often used as opposites and this is also done in this contribution. Vulnerability describes the weakness of a network and robustness describes the strength of a network. Here, robustness is defined as follows: "Robustness is the ability of the network to maintain its functionality under conditions that deviate from the normal conditions." In this definition, the normal conditions are conditions in which traffic operations are within the boundaries of the regular design specifications, i.e. without serious incidents or exceptional demands. In this paper we focus on (non-recurrent) incidents that block two lanes of a road. This choice was made because incidents on freeways with more than 2 lanes usually do not block the complete freeway. Incidents on roads with 1 or 2 lanes are assumed to block the road completely.

It is difficult to predict the robustness of a road network. Generally, there are two possibilities. Either one simulates all possible link blockings in a road network, which is computationally expensive. Alternatively, one pre-selects potentially vulnerable links based on an equilibrium assignment and certain criteria and performs an additional analysis for the selected links. The second approach raises the following questions. What is the quality of the selection criteria used in the second group? How large should the selection of possible vulnerable links be to be sure that the most vulnerable links are indeed included? And if the selection is good, is a detailed analysis really needed, or could the vulnerability and robustness of a network (or parts of the network) also be determined by applying only the selection criteria (without reducing the capacity for a selected link)? If this is possible, then it would make the modeling of the implications of protective measures much easier. A quick assessment of the vulnerability and the vulnerable parts of a network is also needed for the design of robust road networks with network design models. This problem of network design is very complex and computationally expensive even without the robustness aspect. A very long computation time for the robustness assessment would increase the computation time of the “robustness network design problem” to an unacceptable level. Rather than running one traffic simulation for each road network layout, one has to run many simulation runs in order to assess the consequences of incidents at all locations of the network. Therefore, it would be useful to have indicators showing the most vulnerable parts. The objective of this contribution is to assess the quality and validity of different selection criteria for measuring road network robustness.

This paper is restricted to assessing link-level criteria that could be calculated from one equilibrium run of the network, or, even better, can be measured from the everyday conditions in a road network. This choice was made because our aim is to limit the computation time and the required data.

The next section of the paper gives an overview of the state-of-the-art methodologies used to identify the vulnerable links. Then, we present a description of the method that is used for comparing the selection criteria and an overview of the networks on which this comparison is made. The results and conclusions about the quality of the selection criteria are presented in the sections thereafter.

2. Literature-overview of methodologies to find vulnerable links

The methods for finding vulnerable links can be divided into two groups. The first group contains the "full calculation methods" in which the capacity is reduced for each link separately. In order to find out which links in a network are the most vulnerable, a complete simulation could be made. That is, for each link the capacity could be reduced and a traffic assignment could be made. It would be best to take en-route route choice since people are not aware of the incident beforehand. The effects of the capacity reduction on for instance the total travel time could be regarded as an indicator for the vulnerability of a link. Jenelius (2007) uses the approach of blocking each of the links in a traffic simulation program without traffic jams. Knoop et al. (2008) use the same approach for calculating the consequences for a blocking at each link. However, they argue that the network effects including spillback are significant. Hence, they use a more accurate simulation that represents the dynamics of traffic jams, including spillback. The simulation consequently needs a time dependent OD-matrix. The advantage of the approach used by both, a full calculation, is that it gives a complete analysis. However, the computation time of this approach is very high which can be considered as a disadvantage; this brute force method is furthermore lacking a structure for searching weak links. Corthout et al. (2009) show a method which is based on a simulation of the network in equilibrium. They then only compute the changes due to an incident. Again, they compute the effects for each incident location. This method is much quicker, but still lacks any direction in the search for vulnerable links. Furthermore, the route choice is assumed constant, whereas travellers might change their route due to an incident. Furthermore, it is sensitive for the moment an incident occurs.

Several approaches have been introduced in order to overcome the disadvantage of computation time. This second group uses criteria of links to have a direction in the search for the most vulnerable links. These approaches first select links that are likely to be vulnerable based on certain criteria. For these links a more detailed analysis is made by reducing the capacity and by assessing the vulnerability of these links. Tamminga et al. (2005) were the first to introduce a method in which this approach was used. Also Tampère et al. (2007) introduced their own selection criteria. These methods are still computationally intensive because simulations for all the selected links are required.

There are also other approaches like the game-theoretical approach presented by Bell (2000). However, this has to the best of our knowledge never been applied in a dynamic simulation environment on a real-size network. These methods show the way people could avoid possible blockades and are more relevant for fully informed travellers.

3. Calculation of Vulnerability Indicators

This section describes the approach that is used for determining the quality of different selection criteria that are used in the second group of approaches that were described in the previous section. Section 3.1 shows the criteria we used to analyse. In section 3.2 it is described which traffic assignment method is chosen.

3.1. Selection Criteria

The different selection criteria are listed below. The selection criteria I^1-I^7 can be found in Tampère et al. (2007), Li (2008) adds I^8 and Tamminga et al. (2005) I^9. Despite the fact that most of these criteria were intended for identifying vulnerable links for different kinds of accidents compared to the kind we used (2 lane blockades), we still included them in order to get a wide range of criteria. As indicated in the introduction, we only evaluate the criteria that can be evaluated from one equilibrium assignment or that can be measured in a real-life situation without blocking.

Other criteria like the vulnerability index which was introduced by Murray-Tuite and Mahmassani (2004) and the more recently introduced criteria of Kurauchi et al. (2007) require more input because they also include the possible route choice if a link is blocked. Compared to the other criteria, these add extra computation time for the rerouting part. Furthermore, they need extra (calibrated) information about users' choices when they face an unexpected blocking, and this is not available. Other criteria mentioned by Tampère et al. (2007) and Li (2008) include the risk of a grid lock (cannot be calculated automatically), the quality on alternative routes (adds computational complexity) and the criterion that all off-ramps could be vulnerable (this is only one step in the selection process). The reasons for not including these criteria are mentioned between brackets. Finally, some criteria explicitly take the chances of an incident into account. This chapter discusses the possible consequences of an incident given that it happens and therefore also these criteria are excluded.

Below, a short description of each of the used criteria is given. Some of the criteria have been inverted, to get a better comparison. For each of the listed criteria, a higher value means that the predicted impact of the blocking of that link is bigger. The list shows the criteria and indicates the meaning. The symbols are explained in table 1.

1. $I^1 = q\,/(1 - q/C)$

   If the flow (q) increases with respect to the capacity (C) more travellers have to queue. I^1 expresses this influence of the flow.
2. $I^2 = 1/T_b$.

   T_b is the time it take before the tail of a queue reaches the upstream junction. The higher T_b is, the lower will be the impact of an blockage. T_b depends on the traffic inflow, the current density of the traffic and the length of the link. Tampère et al. (2007) shows the equation for T_b:

   $$T_b = L_i\,/q_i\,\left(l_i \cdot k_{j\,i} - q_i/v_{f\,i}\right) \tag{1}$$

3. $I^3 = I^1_i \cdot \vartheta\,(q - 2500)$ Criterion 1 will indicate the links where the queues will be the largests. However, network effects play an important role. Therefore, it is important to also include links with a low capacity. Therefore I^3 is the same as I^1, but limited to links with a capacity of 2500 pcu/hour. Mathematically, this expression uses the step function $\vartheta(x)$, which is 0 for $x < 0$ and 1 for $x > 0$. This criterion should capture the offramps.

4. $I^4 = I^1 \times q$. I^4 is a modification of I^1 which aims at expressing the effects of an incident. Tampère et al. (2007) argue that vulnerability needs an extra input, being the probability that an incident occurs. In the formulation for I^4 this probability is taken proportional with flow q.
5. $I_i^5 = I_i^2 \times q_i \times \sum_{j \in \mathcal{U}_i} I_j^1$. I^5 is equivalent to I^4, capturing both effects and incident probability. However, I^5 also takes the possible effet of blocking back into account. It does so by multiplying I^4 by the effect of a blockage on link j, (estimated as I_j^1).
6. $I_i^6 = I_i^3 \times q_i \times \sum_{j \in \mathcal{U}_i} I_j^1$. I^6 is the same as I^5, but restricted to lower-capacity links. This would capture for example risk-prone off ramps just downstream of a motorway junction.
7. $I^7 = \sum_{j \in \mathcal{U}_i} I_j^1$. I^7 is a sum of the effects (estimated by I^1) on all upstream links j of link i, which might be blocked due to spillback of congestion of a blocking on links i. This shows the links that cause large problems in blocking back effects: for example a link just downstream of a motorway junction.
8. $I^8 = \dfrac{q}{C}$. This captures the links that have a large volume compared to their capacity. This usually is an indication that the link is heavily used, and that if an blockade happens, the queue will grow quickly.
9. $I^9 = q_i - C_i^b$. This shows rate at which cars arrive in the queue when an incident occurs on a link and therefore shows the direct consequences; in this chapter, it is assumed that C_b equals 0.

3.2. Assignment

Assignments can be divided according to several criteria, like static or dynamic, user equilibrium or no equilibrium, stochastic or deterministic, path based or link based, single user class or multi user class, unimodal or multimodal and en-route route choice possibility or no en-route route choice possibility. For modelling robustness, especially the difference between static and dynamic assignments and the possibility for en-route assignment are important. It is generally accepted that dynamic assignments are required for correctly modelling robustness. Compared to static assignments, dynamic assignments are better at showing the exact location of congestion and at determining the development over time of congestion. This is important for correctly modelling the effects of variations in demand and capacity (e.g. incidents). The possibility of en-route route choice is important, because in practice a certain percentage of the travellers change their route when they are informed about congestion at a certain location. The importance of en-route route choice for the assessment of the impact of incidents is advocated in the thesis of Li (2008). Tampère et al. (2007) argue that en-route route choice can indeed be of added value, but that it is very difficult to correctly model the en-route route choice of travellers during incidents because of the uncertainty that is inherent to human behaviour (Bogers et al., 2005); recently, some experimental results about rerouting under incident conditions were reported (Knoop et al., 2010). Especially during incidents this uncertainty is important, because it is not known how many people have information about the incident and how they will respond to that information. Besides these two characteristics, Tampère et al. (2007) also claim that a correct modelling of the way in which congestion builds up (at least consistent with first order traffic flow theory) and a correct modelling of intersections is required for vulnerability analysis.

We used the traffic assignment model INDY (Bliemer, 2005, 2007) to calculate the equilibrium traffic situation. INDY is a dynamic path based multi-user class assignment model. The model finds an equilibrium route set for three driver types:

drivers which use a fixed path, drivers with deterministic route choice and drivers with stochastic route choice. In INDY congestion is modelled in line with the first order traffic flow theory. En-route route choice is not possible in INDY. However, since INDY was used to simulate non-incident situations only, the lack of en-route route choice is not relevant. The package gives a good representation of the network flows without incidents, and the criteria have to be calculated on the non-incident traffic patterns. Therefore, the assignment results can be used for the evaluation of the nine robustness criteria.

Obviously, when facing an incident, it is likely that drivers will deviate from their equilibrium paths. Therefore, for the full calculation a different, dynamic non-equilibrium traffic simulator was used, being the macroscopic simulator DS-MART, Zuurbier et al. (2006). It is an implementation of an LWR model (Lighthill and Whitham, 1955; Richards, 1956), and includes en-route route choice and blocking back, important features for incidents. Li (2008) shows the influence of subtle choices in modelling the traffic assignment. More details on the chosen DSMART simulator and the route-choice can be found in Knoop et al. (2008). The assessment of the vulnerability of each link was done by evaluating the impact of blocking single links using this simulator. In this case, blocking means that 2 lanes were blocked (or one if the link only contains one lane). The total travel time (including the delay at the origin) was used as performance indicator.

4. Analysis

Section 3 shows how different indicators can be calculated. This section shows how they are compared with each other (section 4.1). In section 4.2 it is shown how the vulnerability indicators are compared with a assessment of vulnerability by simulation (iteratively block a link and calculate the performance decrease).

4.1. *Redundancy of criteria*

Vulnerability indicators for all links are calculated. First of all, the mutual cross-correlations between all indicators were calculated. This indicates how good the mutual correlation between the criteria is. The statistical value R^2 is well known and indicates how two stochastic variables relate. The value of R^2 indicates which part of the variation in one variable (y) can be explained by a variation in the other, x. R^2 is the square of the "Pearson" R-statistic (see Chakravarti et al. (1967)). It is indicated with R, and is calculated as follows:

$$R = \frac{\sum (x - \bar{x})(y - \bar{y})}{s_x s_y}. \tag{2}$$

In this equation, $\bar{x}$ and $\bar{y}$ indicate the mean values, s_x and s_y indicate the standard deviations, and n is the size of the sample. The value of **R** lies between -1 and 1, and its absolute value shows the size of the correlation and its sign shows whether it is a positive (+1) or negative (-1) correlation.

The "Pearson" correlation is a linear correlation method of which the underlying assumption is that the numbers might be mutually linearly dependent. Any other relationship that would give the correct order of vulnerability, also non-linear relationships, could make a perfect prediction. Another correlation test, the Spearman Rank Correlation (Spearman (1904)) is a similar standard test to show how much the ranks are correlated. The Spearman **R** also has an outcome between -1 and 1. Here too, the absolute value shows the size of the correlation and its sign shows whether it is a positive (+1) or negative (-1) correlation.

The advantage of using this test is that it shows whether the ranking is correct. However, if the values of the criteria are similar, it can be more interesting to know whether there are similarities between the values than to see the differences within that group. In that case the rank correlation might be low, because the ranking within a group is changed, but the indicators might give an reasonable estimate for the vulnerability.

The correlation coefficients R between two indicators show the correlation coefficients of one criterion with all other criteria. Now, a sum of these variables, S, can be defined, which indicates whether that criterion shows the same trend as others.

$$S^k = \sum_{l \in \mathcal{S}; l \neq k} R(I^k, I^l) = -1 + \sum_{l \in \mathcal{S}} R(I^k, I^l) \tag{3}$$

In this formula, k and l are the numbers of the criteria. If criterion k matches positively linearly with criterion l, the value $R(I^k, I^l)$ equals one. If there is no correlation at all, $R(I^k, I^l)$ equals zero. Since I^k matches perfectly with itself, the value $R(I^k, I^k)$ equals one; this explains the second equal-sign in equation (3).

A high value of S^k now means that there is a high positive correlation between criterion k (I^k) and the other criteria. That means that its value can represent the average of the other criteria well, or the other way round, the average of the other criteria already tells something about the value of criterion I^k.

Note that statistics for sets with a low number of elements (links in a network in our cars) may be misleading. Typically, we would need at least 10-100 numbers in order to have useful outcomes.

4.2. Predictive value of criteria for simulation result

The goal of the indicators is to find the most vulnerable links. It is assumed that the complete dynamic simulation of all possible blockings gives an accurate result of the vulnerabilities (see e.g., Knoop et al. (2007)). A first step is assuming all indicators work equally well, and see how many links, indicated by each of the criteria, have to be selected to find the top-n most vulnarable links, calculated by a full dynamic simulator.

The above is expressed in this paragraph in words and mathematically, and a numerical example follows in the next paragraph. The goal is to have a selection of links that includes the n most vulnerable links according to the full calculation. This set to be found is indicated with T_F^n. We are going to pre-select the most vulnerable i links according to indicator j. This gives a set of links indicated by T_j^i. This process is repeated for all nine indicators. The resulting sets are now combined, and it is checked whether all links in T_F^n are in this combination. The larger the number of links one pre-selects for each of the indicators (i), the larger the combination of subsets of links, and from a certain number i this group is large enough to include all links of the top-n vulnerable links. The lowest number i for which this holds is indicated with z, and of course depends on n, the number of links one wants to find. In an equation, one would write as follows:

$$z(n) = \inf \left\{ i \in \mathbb{N} \leq \text{Nr of links} : \left(\bigcup_j T_j^i \right) \supset T_F^n \right\} \tag{4}$$

Note that $\inf$ stands for infimum operator which gives the minimum of the numbers in the set after the operator.

In an example it is now shown how this method works. If for instance link number 10 is the most vulnerable link according to the full analysis, then the position of link 10 is determined in the link ordering of the different criteria. Thereafter the minimum is determined. It could be that I^3 is the criterion that gives link 10 the highest rank: position 3. From this, it would be concluded that at least 3 links are to be selected by each criteria. Since it is likely that there is an overlap in the selected links by each criterion, the number of uniquely selected links is also presented.

If the indicators were perfectly aligned with the real world, $z(n)$ would be equal to n. Due to differences in indicators, one might even find values of $z(n)$ which are lower than n. For instance, think of the top-5 most vulnerable links, and these are all indicated as most vulnerable by at least one of the indicators. In that case $z(5) = 1$. However, if the indicators work badly, it is needed to pre-select a much larger group of links and $z(n) > n$.

Also the correlation coefficients and rank correlation coefficients for each indicator and the full calculation are determined for the simulation result. This shows how good each of the indicators approaches the result of the full dynamic simulation, which we assume here to be the correct result.

4.3. Multi-linear fit of criteria

For one network, the delay caused by the blocking of the link (D) is known as well as all indicators. We propose a linear model to predict the delay-values for each of the links based on the indicators, the predicted or estimated delay, indicated by $\widetilde{D_i}$. This way, we create a function which should be an approximation of the delay in case of a real blocking. Although we are acknowledge that the indicators do not aim at predicting the delays, they do give an indication of the delays. To test their combined capabilities, we create a function in which the indicators, together, predict the delay. It is acknowledged that this function possibly has a difficult functional form. However, there are no a-priori reasons to assume a particular functional form, we choose the simplest form, being a multi-linear form. Not necessarily all indicators are considered in this linear combination: the indicators which are considered are given by the set $\mathcal{K}$. The prediction, including the fit parameters β, is thus formulated as follows:

$$\widetilde{D_i} = \sum_{k \in \mathcal{K}} \beta^k I_i^k \tag{5}$$

Now, vector β is optimised in order to minimise the error, ε

$$\beta = \underset{b}{\operatorname{argmin}} (\varepsilon) = \underset{b}{\operatorname{argmin}} \sum_{i \in \mathcal{K}} \left(TDL_i - \widetilde{D_i} \right)^2 \tag{6}$$

The aim of the fit is that the predicted value for the delay ($\widetilde{D_i}$) is similar to the simulated total delay (D_i). For a set of validation links, we compute the residual error,

$$\varepsilon = \widetilde{D_i} - D \tag{7}$$

Ideally, this would be zero. It is most interesting to analyse the variations in the errors, rather than a constant offset. Therefore we assess the quality of the prediction model by the standard deviation of ε.

The vector β is estimated based on a calibration set of links which is a subset of all the links in the network. One third of the links will be kept out of the calibration. These links are used for the validation. In fact, it is calculated what is the predicted delay for a blocking on each of these links. These predicted delays are compared with the delays calculated in the dynamic traffic simulation program.

5. Networks

For the comparison of the selection criteria, we used three different sized networks. We used a simple test network to show clearly the characteristics of the different indicators. The second test network is a bit more detailed and shows the effects of on and off ramps. The simulation of traffic in a real-world, medium-sized network shows how the effects work out in practice (third network).

The first network studied is a test network with 11 directional links (figure 1b). It can be seen as a freeway that passes a city. There are 3 centroids (origins and/or destinations) 5 nodes and 11 links. There are connections to the city (links 7, 8, 9 and 10) and there is a local road that passes the city (link 11). All local connections have a speed limit of 50 km/h, whereas the freeway has a speed limit of 120 km/h. As congestion sets in, more drivers take the local road around the city.

The second network is a test network that is based on the network of Delft in the Netherlands (figure 1b). The freeways around the city are included as well as the largest two roads through the city. In total, 12 centroids are modelled, 90 nodes, and 150 links. All local roads are excluded. The on and off ramps are modelled in detail. Since the capacity and location of on and off ramps is likely to be of relevance for the robustness of a road network, this is an important addition compared to the first test network.

The network around the city of Rotterdam (about 600,000 inhabitants, shown in figure 1c) is the third network considered here. It has 44 centroids, 239 nodes and 454 links. The freeways around the city are modelled as well as the most important corridors through the city. The network is used for local traffic and for transit traffic. The period from 6.30 to 9.30 in the morning was simulated.

The assignment on the first two test networks was not calibrated, but equilibrium routing was assumed. Capacities have been put in based on speed limit and number of lanes. For the network of Rotterdam a calibration of capacities and speeds has been carried based on detector counts on the freeways (all major links, each 500 meter a detector).

6. Results

In the section, we present the interesting results for all three networks.

6.1. Simple Network

All indicators are formulated chosen in such a way that bigger values indicate a higher vulnerability for the network. It is therefore remarkable that some of the correlation indices are negative, meaning that a best fit is a negative relationship.

S is even negative for I^3 and I^6. For I^3, it can be explained by the exclusion of the freeway links. When the freeway links are vulnerable according to the other criteria and (by exclusion) they are not any more according to I^3, the correlation coefficient becomes negative. I^6 uses I^3 as input, so it was expected that it would follow the trend of I^3. As that counteracts the average, so will I^6. The cross correlation of I^3 and I^6 is relatively high (0.81). It is also the only combination with the same top-1, top-2, top-3 and top-5 of vulnerable links.

The correlation of the I^1 and the I^9 is the highest of all with an R of 0.99. It is, apart from I^3 and I^6, the only combination that produces the same top-5 (though not in the same order). Other related combinations are: I^2-I^4, I^1-I^5, I^1-I^9, I^2-I^8, and I^4-I^9.

6.2. Delft Network

The strong correlations are the same in the Delft network. The cross correlation values are in the same order of magnitude, but the accordance of the top-n values is lower. Due to the higher number of links, there is less chance of accidentally including the same links in the top-n (n is chosen as a percentage of the total number of links).

Here, we find strong correlations in the following combinations: I^1-I^9 and I^3-I^6. The value for S varies from 1.7 (I^2) to 4.4 (I^1). Note that S has a value between -8 (perfect negative linearity with all other indicators) and 8 (perfect positive linearity with all other indicators). This means that there is a relationship between the indicators, or that the fraction of linearity of them is small.

6.3. Rotterdam Network

6.3.1. Analysis of indicators

Since the statistics on this 454-link network have the least random error, for this network all results for the comparison with the full calculation are presented. Figure 2 shows the correlation between each of the indicators with the full calculation. They are very scattered, meaning that if one knows value of the indicator, one does not know much about the vulnerability (obtained by a full calculation). Numerically, one could conclude that the correlation in values and rank is low. The correlation results as proposed in section 4.1 are presented in table 2. None of the indicators can properly predict the consequences of a blocking. The highest R is 0.15, leading to an R^2 of 2%. This indicates that only 2% of the variation in one variable can be explained by the other variable. Figure 3 shows the ranks of the links (the lower the number the more vulnerable it is) for each of the indicators and for the full calculation, where there is no line at all. This means that if one knows the order of vulnerability predicted by one of the indicators, this does not imply anything on the order of vulnerability according to the full calculation.

In this real-world network, the same combinations of indicators are related as in the other networks. There is one relationship that correlates more than in the other networks, I^1-I^5, with a cross correlation value R of 0.85. This might be related to variation in different links, and a more spread of flows over the network.

The selection power of the links, as described by z in equation 4, is shown in figure 4a. The figure shows the number of unique links (z) that are to be selected by each criterion in order to get the complete top-n of the actual impact analysis. From the line “Number of links required to select per criteria” it can be concluded that more than 250 links (55% of all links) need to be selected in order to include what by full calculation shows is the the most vulnerable link. The figure also shows (blue line) the number of unique links that result from selecting z links by each criterion. In the example given above this corresponds to the union of the top-250 links from all criteria. Already for a very small search area (top-1 vulnerable link), a very large subset of links needs to be considered (almost all). This implies that, at least for this case, pre-selecting links has hardly any added-value. Finally, the overlap is shown between the top-n of links selected by the indicators and the top-n of links based on the actual impacts. If the high level of the blue line is caused by a few links that are not selected by the criteria, or in other words, if most of the vulnerable links are captured by the criteria, this would appear in the overlap. We selected the most vulnerable n links according to the full calculation and we analyzed which percentage of these links also appears in the top-n of any criteria. This is the overlap percentage. The line will go to 100% for all links: all links belong to the set of vulnerable links if there is no threshold. The “overlap line” shows that 10% of the top-10 of most vulnerable links are included in the selection of the criteria based top-10. For the top-150 this is 33%. This implies that it is not just 1 link that is missing in the indicators selection.

We looked for the most vulnerable links according to the simulation and tried to identify reasons why this was not included in the criteria. It showed that especially the freeway junctions, the links downstream of junctions and the main urban arterial are not well covered by the criteria. These are, in the full calculation, vulnerable due to spillback effects, which are not captured good enough in the criteria. Especially secondary spillback (spillback from one link to the next and to the next, like present on a motorway junction) are not properly included in the indicators. For each of the link, the vulnerability according to the full computation is taken as basis. This is compared with lowest ranking of this link in any of the indicators. If the ranking according to an incidator is low (very vulnerable), but according to the full computation, the link is not that vulnerable, this will give no problems in preselection. However, the other way around, a very vulnerable link which is ranked very low on all indicators will not be found and is difficult to find. Figure 4b shows the difference between the lowest rank for the indicator and the index according to the full compuation. Since a negative difference is no problem, combined with the readability of the figure, the values are minimized to 0. It shows that mainly connecting links and urban arterials are vulnerable according to the full calculation but are not found with the indicators.

6.3.2. *Multi-linear fit*

For the Rotterdam network, the full calculation results are known. We fit a multi-linear model on the indicators to approximate the full calculation results, as explained in section 4.3.

Note that there are $2^9 = 512$ possibilities to fit a multi-linear model if each of the 9 criteria could be included or not. Exactly half of them (256) includes I^1 and the other half (256) does not include it. Figure 5a shows the distribution of the error for the models which include each criteria. One bar indicates the spread of the fits of the 256 models which include the indicator mentioned on the horizontal axis. It shows that generally the performance is very poor because the spread in the error (equation 7) is more or less the same as the spread in the delay. So in fact, the model could not find an explanation for the deviations. In some cases, the standard deviation of the model becomes even worse, meaning the model overfitted the results. This could be due to the colinearity between the indicators, and the fit of a multi-linear model as proposed in equation 5. With errors on correlated variables, the model estimation becomes more sensitive for errors, thus leading to overfitting.

The boxes in figure 5a show the distribution of the model fits with different complexity. A square shows the performance of the linear fit including only one indicator (and none of the others). This is generally a slightly better fit than the fit which included more parameters.

This is also seen in figure 5b which shows the fitness for models with different complexity, i.e. the number of criteria that are included in the fit. The standard deviation of the residual error for models with one parameter lies slightly under the standard deviation of the the measurements. That shows that including 1 indicator is slightly better than not having any information at all. However, when more criteria are included in the model, the error increases, which means the results are overfitted. This means that in the calibration the model finds an explanation for the differences in model outcomes in the indicators. However, these are not “true explanantions” but a statistical fluctuation, which does not hold for another set. Thus, exploiting this effect in the model will lead to a *worse* prediction on another set. Obviously, if one would plot the fit results for the calibration set rather than the validation set, the error will go down.

7. Conclusions and Recommendations

This contribution compares different criteria that have been proposed in literature to indicate the most vulnerable links in a network. We found that the different criteria indicate different links as most vulnerable. They should therefore be seen as complementary. Excluding freeways gives a completely different list of vulnerable links. This implies that the freeways are usually (i.e., by the other indicators) indicated as vulnerable. The Incident Impact, $q/\left(1-q/C\right)$, gives the best correlation with the other criteria. When comparing it to the fully calculated results, though, it is not better than the others.

In fact, none of the indicators on their own give a good representation of the full consequences of the blocking of a link. It is also insufficient to take the top-level numbers and analyze them in depth, as there is no indication that the indicated top-level vulnerable links are indeed the most vulnerable. Apart from that, they differ among the indicators.

Furthermore, a combination of the indicators also did not result in a good prediction of the list of most vulnerable links. The combined selection power of the criteria in the network appeared to be minimal. Especially, the freeway junctions, the links after the junctions and the main urban arterial are not well covered by the criteria. This could imply that spillback effects are not properly included in the criteria. This is not surprising: link-based criteria, by the mere fact that they are link-based, are not able to properly capture the network dynamics.

From these results it can be concluded that the quality of these criteria is not good enough to properly identify the most vulnerable links in a network. Also a linear model combining the criteria cannot predict the right vulnerability of links. Link-based indicators are suitable to for indicating the vulnerability of the traffic flow on that specific link. However, to capture all network effects, these indicators are proven to be insufficient. The full calculation method, at the other hand, will capture all these effects, but is time-consuming.

Future research should analyse the quality of the selection criteria for identifying vulnerable links for other disruptions then 2-lane blockings. Furthermore, the conclusion that the existing criteria are insufficient, should lead to new research to find out whether new criteria can be introduced that enable us to identify vulnerable links without doing a full calculation. For instance, the indicators based on routes instead of links, for example mentioned by Murray-Tuite and Mahmassani (2004), can provide an interesting approach to this problem which is to be integrated in a future study.

8. Acknowledgements

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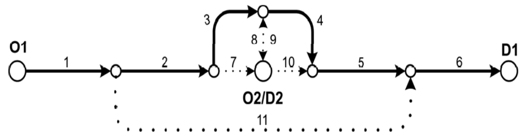
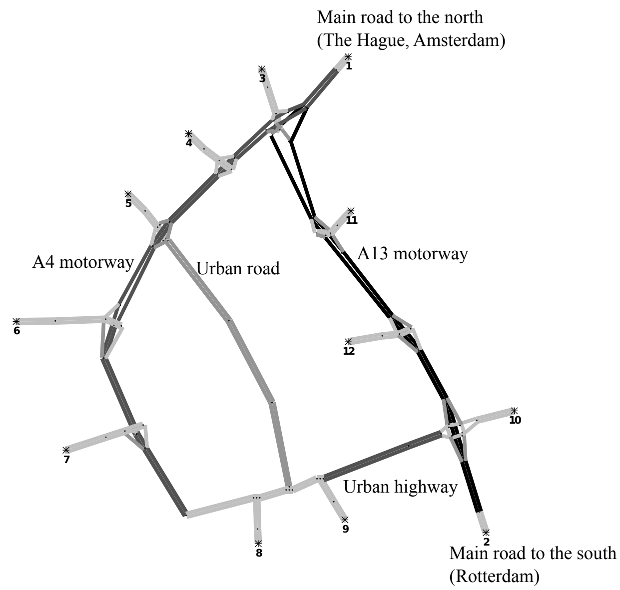


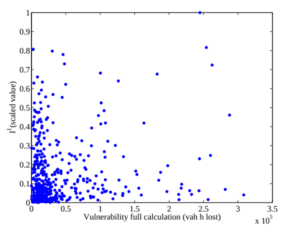
(a) Simple test network Source Li (2008)



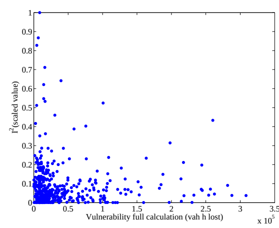
(b) Network of Delft, the Netherlands. The length of the urban highway in the south between the A4 and A13 motorway is approximately 5 km

(c) Network of Rotterdam, the Netherlands. The motorways are darker. Within the rectangular motorway layout is the urban area of Rotterdam; the major urban arterials are also modelled. The length of the northern east-west motorway between the two intersections with connections to motorways to the south is approximately 12.5 km

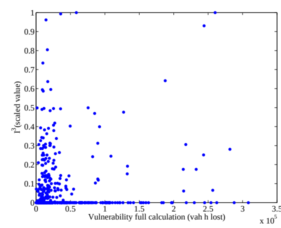
Figure 1: The used networks



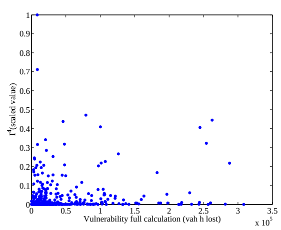
(a) I^1 and full calculation



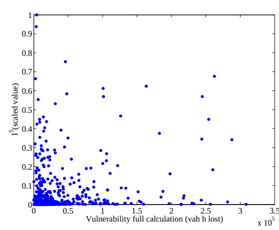
(b) I^2 and full calculation



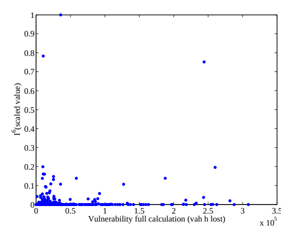
(c) I^3 and full calculation



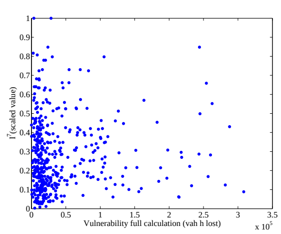
(d) I^4 and full calculation



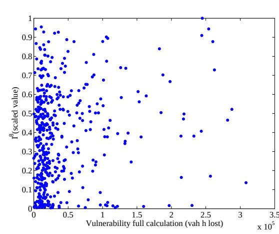
(e) I^5 and full calculation



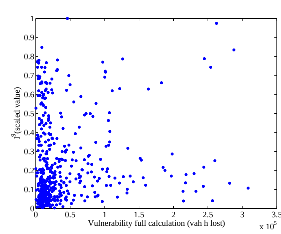
(f) I^6 and full calculation



(g) I^7 and full calculation

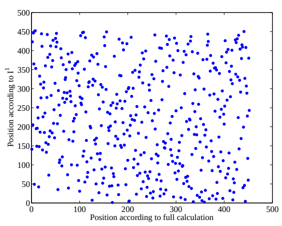


(h) I^8 and full calculation



(i) I^9 and full calculation

Figure 2: The scatterplot of the results of the full calculation compared with the calculated criteria



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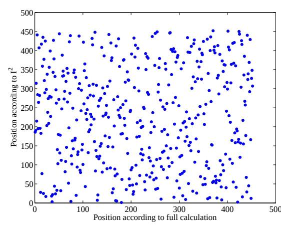
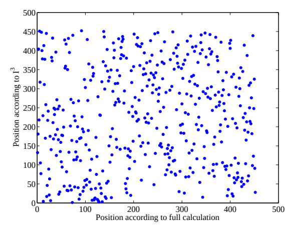


(b) I^2 and full calculation



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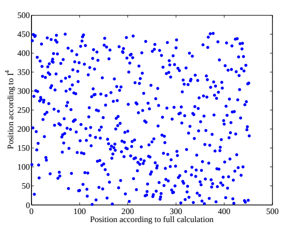

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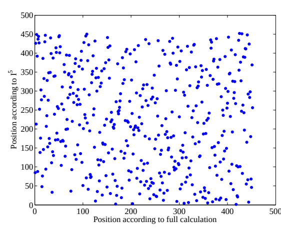
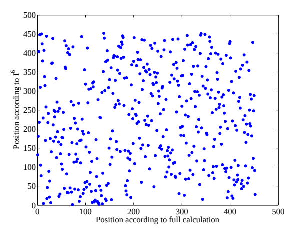


(e) I^5 and full calculation



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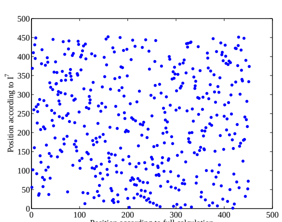
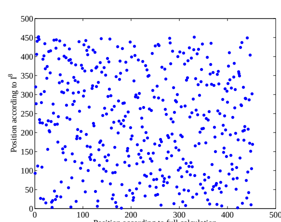
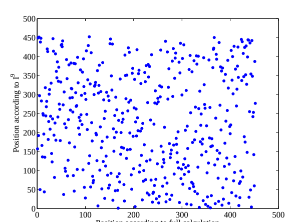


(g) I^7 and full calculation

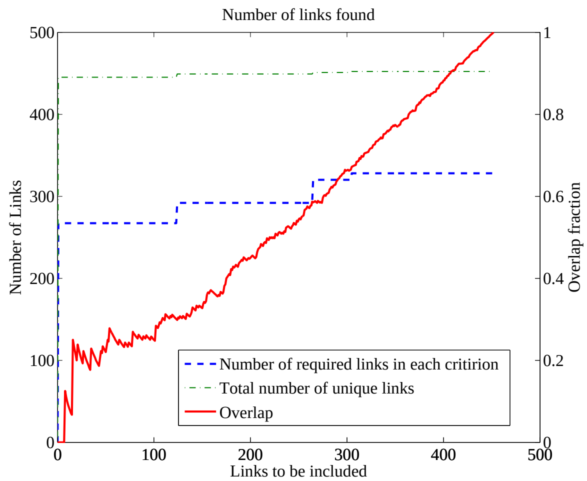


(h) I^8 and full calculation

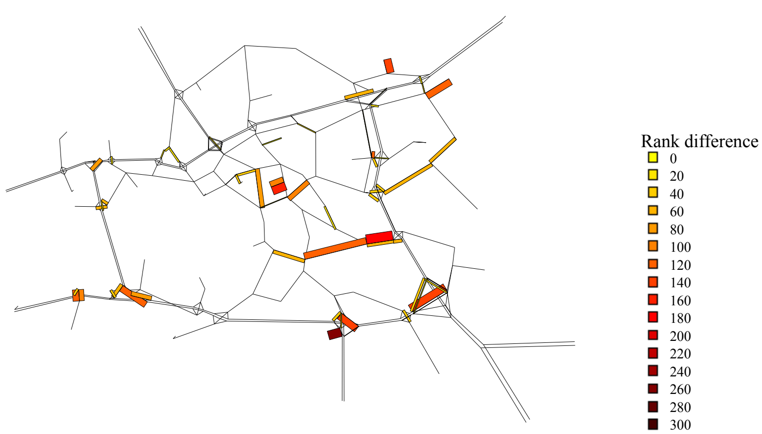


(i) I^9 and full calculation

Figure 3: Scatterplot of the ranks of the full calculation and the criteria

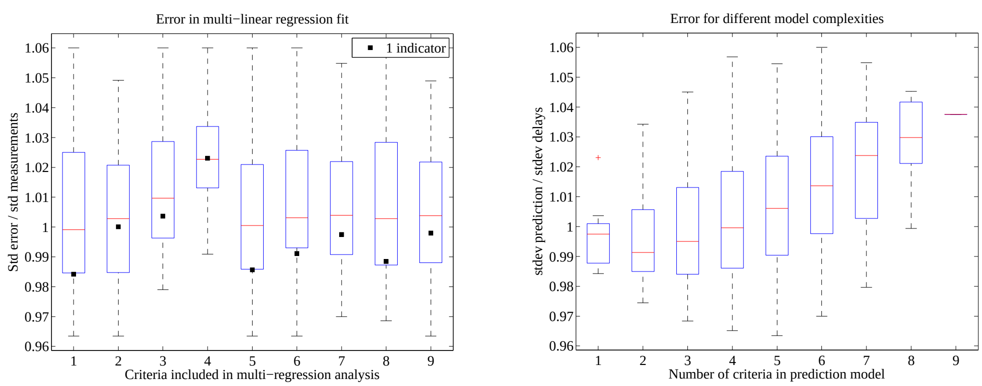


(a) The number of links that should be evaluated after pre-selecting using all criteria



(b) Links that are difficult to pre-select with the selection criteria shown by the difference between the minimum ranking of the indicators and the ranking according to the full calculation

Figure 4: Results if all criteria are used simultaneously



(a) The fit for different criteria included

(b) The fit for different model complexity

Figure 5: The goodness of fit of the multi-linear models

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Table 1: List of symbols used

Variable name	Description
Simulation level	
Δt	Time step
$\mathcal{S}$	The set of all criteria
R	The correlation coefficient
Per link i	
I_i^n	Criterion n for link i
q_i	Flow, also taken as incident probability
C_i	Capacity
C_i^b	Remaining capacity at blocking
$v_{f,i}$	Free flow speed
$k_{j,i}$	Jam density
L_i	Length
l_i	Number of lanes
$\mathcal{U}_i$	The set of links upstream of link i
S_k	The sum of the correlation of criterion I^k with the other criteria

Table 2: The correlation coefficients

Correlation	Linear	Rank
Full calculation and I^1	0.15	0.10
Full calculation and I^2	-0.01	-0.01
Full calculation and I^3	0.078	-0.05
Full calculation and I^4	0.13	0.15
Full calculation and I^5	0.15	0.11
Full calculation and I^6	0.09	-0.06
Full calculation and I^7	0.052	0.02
Full calculation and I^8	0.15	0.12
Full calculation and I^9	0.10	0.08