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Linked multilevel hierarchical approach to the MINLP synthesis of mechanical structures

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Abstract

The paper present a Linked Multilevel Hierarchical (LMH) approach to the Mixed-Integer Nonlinear Programming (MINLP) synthesis of mechanical structures, by which the original MINLP problem is hierarchically decomposed into material, topology and standard dimension levels to be solved sequentially. We introduce a special case of a simplified MINLP model formulation for mechanical superstructures (MINLP-S), which are supposed to contain a number of (sub)groups of equal structural elements. The objective is to perform a continuous parameter optimization of the superstructure simultaneously with a discrete optimization of its material, topology and standard dimension alternatives. The Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm has been used for the optimization. In order to demonstrate this approach, we have included a practical example of a vacuum chamber structural synthesis for the high-frequency dryer for timber.

1 Introduction

The objective of the paper is to present simultaneous material, topology and parameter optimization of structures. The optimization is carried out by Mixed-Integer Nonlinear Programming (MINLP) approach. The MINLP performs a discrete optimization of materials, topology and standard dimensions while parameters are simultaneously calculated inside the continuous space. Since some structural elements are added or removed from the structure within the optimization process, so that they form various structural alternatives, the paper concentrates on the discussion of a structural synthesis.

The MINLP optimization approach to structural synthesis is performed through three steps [1]: i.e. the generation of a mechanical superstructure, the modeling of an MINLP model formulation and the solution of the defined MINLP problem.

The MINLP approach to structural synthesis requires the generation of a MINLP mechanical superstructure composed of various material, topology and standard dimension alternatives. The superstructure is described by means of structural elements and their interconnection nodes. Each structural element may have different material and standard dimension alternatives. Various selections of these elements give an extra topology alternatives. Therefore, the main goal of structural synthesis is to find a feasible structure within the given superstructure of alternatives, which is optimal with respect to material, topology, parameters and standard dimensions.

This give rise to a very complex nonlinear and nonconvex MINLP problem, which may not be solvable, or not solvable in a reasonable amount of time, if the number of discrete alternatives is too high. To overcome the problem, we are introducing a special MINLP strategy, called Linked Multilevel Hierarchical (LMH) approach. The Modified Outer-Approximation/ Equality-Relaxation (OA/ER) [2] algorithm has been used in order to solve such nonlinear and nonconvex problems efficiently.

2 The simplified MINLP model formulation for mechanical superstructures

2.1 The general MINLP model formulation (MINLP-G)

The general nonconvex and nonlinear discrete/continuous optimization problem can be formulated as an MINLP problem (MINLP-G) in the form:

$$\begin{aligned} \min \quad & z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{y} + f(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\ & \boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0} \\ & \boldsymbol{B}\boldsymbol{y} + \boldsymbol{C}\boldsymbol{x} \le \boldsymbol{b} \\ & \boldsymbol{x} \in X = \{\boldsymbol{x} \in R^n : \boldsymbol{x}^{\mathrm{LO}} \le \boldsymbol{x} \le \boldsymbol{x}^{\mathrm{UP}}\} \\ & \boldsymbol{y} \in Y = \{0,1\}^m \end{aligned} \qquad \text{(MINLP-G)}$$

where $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set X and $\boldsymbol{y}$ is a vector of discrete, mostly binary 0-1 variables. While continuous variables are linear and nonlinear, discrete variables appear only in the linear form. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are nonlinear functions involved in the objective function z, equality and inequality constraints, respectively. Finally, $\boldsymbol{By}+\boldsymbol{Cx}\le\boldsymbol{b}$ represents a subset of mixed linear equality/inequality constraints. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ must be continuous and differentiable.

It should also be noted that continuous variables define structural parameters, while binary variables correspond to different discrete decisions (e.g. the existence of units). Equality and inequality constraints and the bounds of the continuous variables represent a rigorous system of the analysis. Logical constraints that must be fulfilled for discrete decisions and structural configurations, which are selected from within the superstructure, are given by $\boldsymbol{By}+\boldsymbol{Cx}\leq\boldsymbol{b}$. The economical objective function z involves fixed costs charges in the term $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{y}$ for manufacturing while the dimension dependent costs are included in the function $f(\boldsymbol{x})$.

2.2 The simplified MINLP model formulation for mechanical superstructures (MINLP-S)

The general model formulation MINLP-G was adapted for the synthesis of mechanical superstructures (MINLP-SMS) that was already reported elsewhere, e.g. [1].

In this paper, we are introducing a special case of a simplified MINLP model formulation for mechanical superstructures (MINLP-S), which are supposed to contain a number of (sub)groups of equal structural elements. Some modifications have also been done in order to handle structural materials and topology more explicitly. For this purpose we introduce material and topological variables. The material variables represent different material alternatives subjected to individual structural elements, groups or to the entire superstructure while topological variables determine the number of structural elements in each group.

The economical objective function usually involves binary variables, which must appear linearly. Instead of binary variables we implement the continuous topological variables into the objective. This modification give some advantages, particularly in the case of complex and highly nonlinear objective function.

The resulted simplified model formulation for mechanical superstructures MINLP-S is given in the following form:

$$
\begin{aligned}
\min \quad & z=\sum_{i\in I} t_i\cdot\left\{c_i+f_i\left(m_i,d_i,p_i\right)\right\}+f\left(\boldsymbol{m},\boldsymbol{d},\boldsymbol{p}\right)\\
\text{s.t.} \quad & \boldsymbol{h}\left(\boldsymbol{m},\ \boldsymbol{t},\ \boldsymbol{d},\ \boldsymbol{p}\right)=\boldsymbol{0}\\
& \boldsymbol{g}\left(\boldsymbol{m},\ \boldsymbol{t},\ \boldsymbol{d},\ \boldsymbol{p}\right)\leq\boldsymbol{0}\\
& \boldsymbol{A}\left(\boldsymbol{m},\ \boldsymbol{t},\ \boldsymbol{d},\ \boldsymbol{p}\right)\leq\boldsymbol{a}\\
& \boldsymbol{Ey}\leq\boldsymbol{e}\\
& \boldsymbol{D}\,\boldsymbol{y}^{\mathrm{mat}}+\boldsymbol{M}(\boldsymbol{m})\leq\boldsymbol{r}\\
& \boldsymbol{K}\boldsymbol{y}^{\mathrm{e}}+\boldsymbol{T}(\boldsymbol{t})\leq\boldsymbol{k}\\
& \boldsymbol{P}\boldsymbol{y}^{\mathrm{st}}+\boldsymbol{S}(\boldsymbol{d}^{\mathrm{st}})\leq\boldsymbol{s}\\
& \boldsymbol{m}\in M,\quad \boldsymbol{t}\in T,\quad \boldsymbol{d}\in D,\quad \boldsymbol{p}\in P,\quad y\in Y=\{y\,|\,y\in\{0,1\}\}
\end{aligned}
\qquad \text{(MINLP-S)}
$$

The proposed MINLP-S model formulation consists of the superstructure's economical objective function, structural analysis constraints and logical constraints with continuous and binary variables. A detailed description of the MINLP-S model formulation is given in the following paragraphs.

2.2.1 Groups of equal structural elements

Let I denote the set of different groups of equal structural elements involved in the superstructure. In each i-th group, $i \in I$, all structural elements are equal. In general, different groups contain different number of structural elements and, in addition, structural elements from different groups are in principle distinguished one from another. There also may exist some structural elements, which belong to the entire superstructure and are not involved in any of the defined group. These elements are in general not equal to each other.

2.2.2 Variables

Variables for this simplified MINLP-S model formulation should include continuous variables $\boldsymbol{x}=\{\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{d}, \boldsymbol{p}\}$ as well as discrete binary variables $\boldsymbol{y}=\{\boldsymbol{y}^{\mathrm{mat}}, \boldsymbol{y}^{\mathrm{e}}, \boldsymbol{y}^{\mathrm{st}}\}$.

Continuous variables are partitioned into material variables $\boldsymbol{m}$, topological variables $\boldsymbol{t}$, design variables $\boldsymbol{d}=\{\boldsymbol{d}^{\mathrm{cn}}, \boldsymbol{d}^{\mathrm{st}}\}$ and into performance (state, nondesign) variables $\boldsymbol{p}$. Topological variables $\boldsymbol{t}=\{t_i\}$, $i \in I$, are defined to determine the number of structural elements for each i-th group. Design variables include dimensions/sizes of structural elements and a superstructure geometry. They are further partitioned into subsets $\boldsymbol{d}^{\mathrm{cn}}$ and $\boldsymbol{d}^{\mathrm{st}}$ of continuous design variables and standard dimensions, respectively. Performance variables $\boldsymbol{p}$ represent all other nondesign variables like cross-section characteristics of structural elements, load, loading, strains, deflections, coefficients for stability analysis, stresses, economical parameters, etc. As there may exist I different groups of elements in the superstructure, continuous variables may be further divided into i subvectors, subjected to these groups: $\boldsymbol{x}=\{\boldsymbol{x}_i\}=\{m_i, t_i, \boldsymbol{d}_i, \boldsymbol{p}_i\}$, $i \in I$.

Subvectors of binary variables $\boldsymbol{y}^{\mathrm{mat}}$, $\boldsymbol{y}^{\mathrm{e}}$ and $\boldsymbol{y}^{\mathrm{st}}$ denote potential choices of material alternatives, potential existence of structural elements and potential selection of standard dimension alternatives, respectively. By the same analogy as before, binary variables may be further portioned into i subsets: $\boldsymbol{y}^{\mathrm{mat}}=\{\boldsymbol{y}_i^{\mathrm{mat}}\}$, $\boldsymbol{y}^{\mathrm{e}}=\{\boldsymbol{y}_i^{\mathrm{e}}\}$ and $\boldsymbol{y}^{\mathrm{st}}=\{\boldsymbol{y}_i^{\mathrm{st}}\}$, $i \in I$.

2.2.3 The economical objective function

The most popular criteria used today are the minimization of mass, strain energy, stresses or costs as well as the maximization of stiffness, frequency of free vibration, selling price etc. In this paper, an economical objective function is proposed to minimize the superstructure's self manufacturing (material and labor) costs.

The economical objective function z involves fixed cost charges in the linear terms $t_i \cdot c_i$ and dimension dependent costs in the terms $t_i \cdot f_i(\boldsymbol{m}_i, \boldsymbol{d}_i, \boldsymbol{p}_i)$ for i different groups of equal elements, $i \in I$. Higher the numbers of selected structural elements involved in defined groups are, higher the values of topological variables t_i are and higher costs are therefore obtained. The objective function also involves dimension dependent costs of structural elements, which are subjected to none group $f(\boldsymbol{m}, \boldsymbol{d}, \boldsymbol{p})$.

2.2.4 Structural parameter constraints

Parameter nonlinear and linear constraints $\boldsymbol{h}(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{d}, \boldsymbol{p})=\boldsymbol{0}$, $\boldsymbol{g}(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{d}, \boldsymbol{p})\leq \boldsymbol{0}$ and $\boldsymbol{A}(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{d}, \boldsymbol{p})\leq \boldsymbol{a}$ represent the rigorous system of the design, loading, stress, deflection, stability, etc. constraints known from a structural analysis.

2.2.5 Pure integer logical constraints

Pure integer logical constraints $\boldsymbol{Ey}\leq \boldsymbol{e}$ are proposed to describe logical relations between binary variables to avoid equal topology solutions. They also define bounds of the topology.

2.2.6 Logical constraints for material variables

Mixed linear constraints $\boldsymbol{D}\,\boldsymbol{y}^{\text{mat}}+\boldsymbol{M}(\boldsymbol{m})\leq \boldsymbol{r}$ define material variables $\boldsymbol{m}$. Each material alternative is to be represented by material variable m, which is determined as a scalar product between a vector of j, $j \in J$, standard values of yield stresses alternatives $f_y=\{f_{y1}, f_{y2}, f_{y3},\dots f_{yj}\}$ and the vector of j associated binary variables $y^{\text{mat}}=\{y_1^{\text{mat}}, y_2^{\text{mat}}, y_3^{\text{mat}},\dots y_j^{\text{mat}}\}$, where the sum of j binary variables y_j has to be equal 1. Only one yield stress f_{y_j} is then assigned to the material variable:

$$m = \sum_{j\in J} f_{y_j} y_j^{\text{mat}} \tag{1}$$

$$\sum_{j\in J} y_j^{\text{mat}} = 1 \tag{2}$$

In the addition, to each i-th group an extra material variable m_i may be in principle defined, $i \in I$.

2.2.7 Logical constraints for topological variables

Mixed linear constraints $\boldsymbol{K}\boldsymbol{y}^{\text{e}}+\boldsymbol{T}(\boldsymbol{t})\leq \boldsymbol{k}$ are proposed to define topological variables $\boldsymbol{t}=\{t_i\}$, $i \in I$. Binary variables $\boldsymbol{y}^{\text{e}}$ are used to represent the potential existence of each structural element inside the superstructure: a structural element is selected if its assigned binary variable is 1, otherwise it is rejected. To each i-th group an extra subvector of n, $n \in N(i)$, binary variables

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$y_i^e = \{y_{i,1}^e, y_{i,1}^e, y_{i,1}^e, \ldots y_{i,n}^e\}$ is assigned. Topological variable t_i is then determined as a sum of these binary variables to represent the number of selected structural elements inside the i-th group:

$$t_i = \sum_{n \in N(i)} y_{i,n}^e \quad (3)$$

2.2.8 Logical constraints for standard design variables

Mixed linear constraints $\boldsymbol{P}\boldsymbol{y}^{st} + \boldsymbol{S}(\boldsymbol{d}^{st}) \leq \boldsymbol{s}$ define standard design variables $\boldsymbol{d}^{st}$. Each standard dimension d^{st} is determined as a scalar product between its vector of k, $k \in K$, standard dimension constants $\boldsymbol{q} = \{q_1, q_2, q_3, \ldots q_k\}$ and its vector of k binary variables $\boldsymbol{y}^{st} = \{y_1^{st}, y_2^{st}, y_3^{st}, \ldots y_k^{st}\}$. Only one discrete value can be selected for each standard dimension, since the sum of k binary variables y_k^{st} has to be equal 1:

$$d^{st} = \sum_{k \in K} q_k y_k^{st} \quad (4)$$

$$\sum_{k \in K} y_k^{st} = 1 \quad (5)$$

3 The Linked Multilevel Hierarchical Approach

The defined MINLP problem has to be solved by the use of a suitable MINLP algorithms and strategies.

For the solution of nonlinear and nonconvex problems we used the *Modified OA/ER algorithm* [2], in which many modifications like deactivation of linearizations, decomposition and deactivation of the objective function linearization, use of the penalty function, use of the upper bound on the objective function to be minimized as well as a global convexity test and a validation of the outer approximations have been applied for the master problem.

The OA/ER algorithm consists of solving an alternative sequence of Nonlinear Programming (NLP) and Mixed-Integer Linear Programming (MILP) master problem optimization subproblems. The former corresponds to continuous optimization of parameters for a mechanical structure with fixed material, topology and standard dimensions and yields an upper bound to the objective to be minimized. The latter involves a global approximation to the superstructure of alternatives in which new materials, topology and standard dimensions are identified so that its lower bound does not exceed the current best upper bound. The search is terminated when the predicted lower bound exceeds the upper bound.

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The optimal solution of complex MINLP problem with a high number of discrete decisions is in general very difficult to be obtained. Initialization scheme is weak since initial standard dimensions selected may be bad or infeasible. The convergence of the algorithm is usually poor.

To overcome the problem, a *Linked Multilevel Hierarchical (LMH) strategy* has been developed to accelerate the convergence of the Modified OA/ER algorithm. We hierarchically decompose the original MINLP problem into subproblems which are then easier to solve than the original one. The MINLP optimization of discrete decisions is sequentially performed at different decision levels, starting from the highest (the most important) one. Decision levels are hierarchically classified:

- from the first level of discrete material selection (the highest level),
- to the second level of discrete topology alternatives (the middle level), and
- to the third level of standard dimension decisions (the lower level).

When the dimensionality of standard dimensions is high, the problem may be further decomposed.

Higher levels give lower bounds to the original objective function to be minimized while lower levels give upper bounds. The MINLP subproblems are iterated around each level until there is no improvements in the NLP solution. Thus, we start with the discrete optimization of materials at the relaxed topology and standard dimensions. When the optimal material is reached, we proceed with simultaneous discrete material and topology optimization at the second level (standard dimensions are still relaxed). Finally, after the optimal material and topology are obtained, the MINLP is carried out once more for complete discrete decisions at the third level. Each higher lever accumulates a global linear approximation of the superstructure model representation to be used at its lower level, which can in this way be solved much more efficiently.

4 Structural synthesis of the vacuum chamber for the high-frequency dryer for timber

In order to demonstrate the synthesis of the proposed MINLP optimization approach, a practical example of structural synthesis of a vacuum chamber of the high-frequency dryer for timber is presented, see Figure 1.

First, the chamber superstructure has been generated in which three different material alternatives Fe 360, Fe 430 and Fe 510 are considered. All possible structures are defined by topology variation between 3 to 7 intermediate longitudinal stiffeners for each of the both horizontal skin-plates and, in addition, 3 to 7 intermediate longitudinal stiffeners for each of the both vertical skin-plates. All stiffeners of the both horizontal skin-plates are proposed to be equal, the same holds for stiffeners of the both vertical skin-plates, for stiffeners of the front skin-plate and for stiffeners for the four longitudinal edges. In this way, we defined four different groups of equal stiffeners. Three different standard thicknesses of sheet-iron plates 3.5 mm, 4.0 mm and 4.5 mm have been proposed for the skin-plate and three different standard cross-sections IPE 100, IPE 120 and IPE 140 for transversal frames. Finally, 32 different standard cross-sections of angles between 30x30x3 to 90x90x8 have been defined for longitudinal stiffeners, 8 for the each superstructure's group.

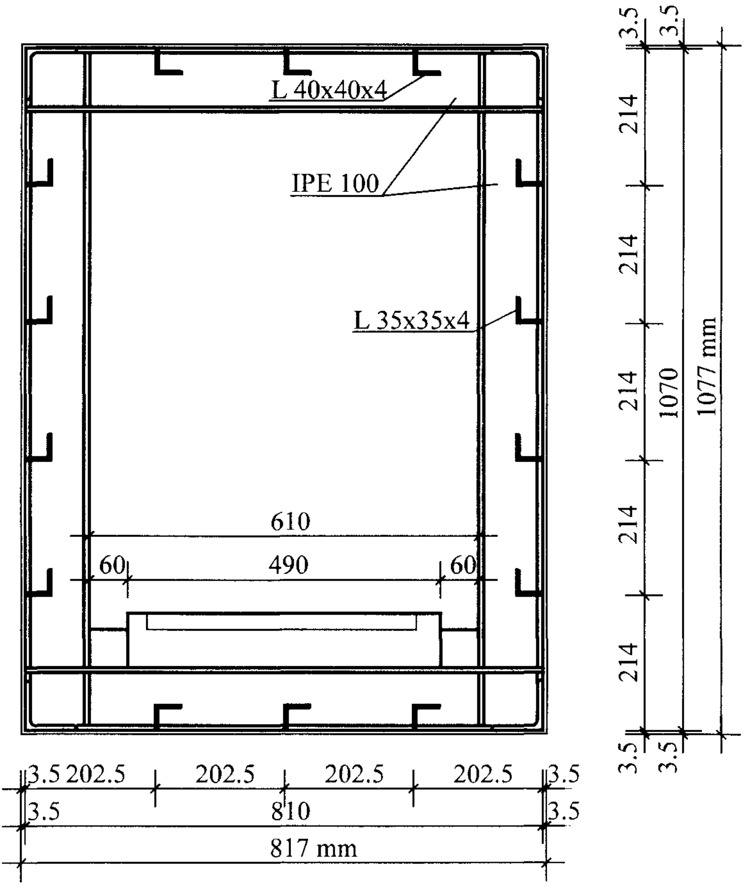

Figure 1: Vertical section of the high-frequency dryer for timber

For this purpose, an optimization model of the vacuum chamber structure has been developed according to the proposed simplified MINLP-S model formulation for mechanical superstructures. As an interface for mathematical modeling and data inputs/outputs GAMS (General Algebraic Modelling

System) by Brooke et al. [3], a high level language, has been used. The self manufacturing costs like material, sheet-iron cutting, welding and anti corrosion resistant painting have been accounted for in the economical type of the objective function, subjected to the given design, material, stress, deflection and stability constraints. Eurocode-3 [4] has been used for the design.

As the model is nonconvex and highly nonlinear, the Modified OA/ER algorithm has been used for the optimization. The LMH strategy has been proposed to accelerate the convergence of the mentioned algorithm. Structural synthesis of the vacuum chamber was carried out by our MINLP computer package TOP (Topology Optimization Program), [5]. MINOS [6] has been used to solve the NLP subproblems and OSL [7] to solve the MILP master problems.

The optimal solution yields the costs of DM 3870 at the optimal material Fe 510 and optimal topology 3-4 (3 intermediate longitudinal stiffeners for each of the both horizontal skin-plates and 4 intermediate longitudinal stiffeners for each of the both vertical skin-plates). The Modified OA/ER algorithm accompanied by the LMH approach converged very fast: 8 major MINLP iterations were needed (7 MILP and 8 NLP subproblems). 347 equations with 2325 nonzero elements, 181 continuous and 51 discrete variables are included in the NLP subproblem at which the optimal solution was found. Only 549 seconds of the CPU time on the computer VAX 4000-600 were spent.

5 Conclusions

For the solution of comprehensive nonlinear discrete/continuous structural design problems, the Linked Multilevel Hierarchical strategy has been developed in order to accelerate the convergence of the Modified OA/ER algorithm. We also introduced a simplified MINLP model formulation for mechanical superstructures, which are supposed to contain a number of (sub)groups of equal structural elements.

The synthesis of the proposed MINLP optimization approach has been demonstrated on a practical example of the vacuum chamber for the high-frequency dryer for timber. Alongside the optimal material and labor costs of the chamber and its optimal topology (the optimal number of longitudinal stiffeners), all the necessary standard thicknesses and cross-sections of structural elements as well as other continuous dimensions like the chamber's global geometry and the intermediate distances between structural elements were also simultaneously obtained. No feasible results were obtained without the implementation of the proposed LMH approach.

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