

# Liouville Cosmology at Zero and Finite Temperatures

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## Abstract

We discuss cosmology in the context of Liouville strings, characterized by a central-charge deficit  $Q^2$ , in which target time is identified with (the world-sheet zero mode of the) Liouville field: *Q-Cosmology*. We use a specific example of colliding brane worlds to illustrate the phase diagram of this cosmological framework. The collision provides the necessary initial cosmological instability, expressed as a departure from conformal invariance in the underlying string model. The brane motion provides a way of breaking target-space supersymmetry, and leads to various phases of the brane and bulk Universes. Specifically, we find a hot metastable phase for the bulk string Universe soon after the brane collision in which supersymmetry is broken, which we describe by means of a subcritical world-sheet  $\sigma$  model dressed by a space-like Liouville field, representing finite temperature (Euclidean time). This phase is followed by an inflationary phase for the brane Universe, in which the bulk string excitations are cold. This is described by a super-critical Liouville string with a time-like Liouville mode, whose zero mode is identified with the Minkowski target time. Finally, we speculate on possible ways of exiting the inflationary phase, either by means of subsequent collisions or by deceleration of the brane Universe due to closed-string radiation from the brane to the bulk. While phase transitions from hot to cold configurations occur in the bulk string universe, stringy excitations attached to the brane world remain thermalized throughout, at a temperature which can be relatively high. The late-time behaviour of the model results in dilaton-dominated dark energy and present-day acceleration of the expansion of the Universe, asymptoting eventually to zero.

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# 1 Introduction

Formal developments in string theory [1] over the past decade, in particular the discovery of a consistent way of studying quantum domain wall structures (D-branes) [2], have opened up novel ways of looking at both the microcosmos and the macrocosmos. In the *microcosmos*, there are novel ways of compactification, either via the observation [3] that extra dimensions that are large compared to the string scale may be consistent with the foundations of string theory, or by viewing our four-dimensional world as a brane embedded in a bulk space-time, allowing for large extra dimensions that might even be infinite in size [4], in a manner consistent with a large hierarchy between the Planck scale and the electroweak or supersymmetry breaking scale. In this modern approach, fields in the gravitational (super)multiplet of the (super)string, or more generally those neutral under the Standard Model (SM) group, are allowed to propagate in the bulk. This is not the case for non-Abelian gauge fields, nor fields charged under the SM group, which are attached to the brane world. In this approach, the weakness of gravity compared to the rest of the interactions is a result of the large compact dimensions, the compactification not necessarily being achieved through conventional means, i.e., closing up the extra dimensions in spatial compact manifolds, but perhaps also through the involvement of shadow brane worlds with special reflecting properties, such as orientifolds, which restrict the bulk dimension [5]. In such approaches the string scale  $M_s$  is not necessarily identical to the four-dimensional Planck mass scale  $M_P$ . Instead, the two scales are related through the large compactification volume  $V_6$ :

$$M_P^2 = \frac{8M_s^2 V_6}{g_s^2}. \quad (1)$$

As for the *macrocosmos*, this modern approach has offered new insights into the cosmic evolution of our Universe. Novel ways of discussing cosmology in brane worlds have been discovered over the past few years, which may revolutionize our way of approaching issues such as inflation [6, 7] and the present acceleration of the expansion of the Universe [8].

In parallel, mounting experimental evidence from diverse astrophysical sources presents some important puzzles that string theory must address if it is to provide a realistic description of Nature. Observations of distant Type-1a supernovae [9], as well as detailed studies of the cosmic microwave background fluctuations by the WMAP satellite [10], indicate that our Universe is currently in an accelerating epoch, and that 73% of its energy density consists of dark energy that does not cluster, but is present in ‘empty space’. These issues are highly significant for string theory, motivating a novel perspective on the treatment and understanding of string dynamics. If the dark energy turns out to be a true cosmological constant, leading to an asymptotic de Sitter horizon, then the entire concept of the scattering S-matrix, upon which perturbative string theory is built, breaks down<sup>1</sup>. This would cast doubt on the foundations of string theory, at least as they are conventionally formulated. On the other hand, even if models for relaxing the vacuum energy are invoked, leading asymptotically to a vanishing vacuum energy density at large cosmic times and consistency with an S-matrix, there is still the open issue of embedding such models in (perturbative) string theory. In particular, the formal question arises how to formulate consistently a world-sheet  $\sigma$ -model description of strings propagating in such time-dependent space-time backgrounds.

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<sup>1</sup>This is also true in more general scenarios for the vacuum energy with a de Sitter horizon.

The standard world-sheet conformal invariance conditions of critical string theory [1], which are equivalent to target-space equations of motion for the background fields on which the string propagates, are very restrictive, allowing only vacuum solutions of critical strings. The main problem may be illustrated as follows. Consider the graviton world-sheet  $\beta$  function, which is simply the Ricci tensor of the target space-time background to lowest order in  $\alpha'$ :

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu} , \quad (2)$$

in the absence of other fields. Conformal invariance would require that  $\beta_{\mu\nu} = 0$ , implying a Ricci-flat background, which is a solution to the *vacuum* Einstein equations. *A priori*, a cosmological-constant vacuum solution is inconsistent with this conformal invariance in strings, since it has a Ricci tensor  $R_{\mu\nu} = \Lambda g_{\mu\nu}$ , where  $g_{\mu\nu}$  is the metric tensor. The question then arises how to describe cosmological backgrounds for strings that are not vacuum solutions, but require the presence of a matter fluid yielding a non-flat Ricci tensor.

One proposal for obtaining a non-zero cosmological constant in string theory was made in [11], according to which dilaton tadpoles on higher-genus world-sheet surfaces produce additional modular infinities, whose regularization leads to extra world-sheet structures in the  $\sigma$  model. Since they do not appear at the world-sheet tree level, they lead to modifications of the  $\beta$  function such that the Ricci tensor of the space-time background is now that of an (anti) de Sitter Universe, with a cosmological constant fixed by the dilaton tadpole graph:  $J_D > 0$  ( $J_D < 0$ ). The problem with this approach is the above-mentioned existence of an asymptotic horizon in the de Sitter case, which prevents the proper definition of asymptotic states, and hence a scattering matrix. Since the perturbative world-sheet formalism is based on such an S-matrix, there is *a priori* an inconsistency in the approach.

It was proposed in [12] that one way out of this dilemma would be to assume a time-dependent dilaton background, with a linear dependence on time in the so-called  $\sigma$ -model frame. Such backgrounds, even when the  $\sigma$ -model metric is flat, lead to exact solutions (to all orders in  $\alpha'$ ) of the conformal invariance conditions of the pertinent stringy  $\sigma$ -model, and so are acceptable solutions from a perturbative viewpoint. It was argued in [12] that such backgrounds describe linearly-expanding Robertson-Walker Universes, which were shown to be exact conformal-invariant solutions, corresponding to Wess-Zumino models on appropriate group manifolds.

The pertinent  $\sigma$ -model action in a background with graviton  $G$ , antisymmetric tensor  $B$  and dilaton  $\Phi$  reads [1]:

$$S_\sigma = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\xi [\sqrt{-\gamma} G_{\mu\nu} \partial_\alpha X^\mu \partial^\alpha X^\nu + i\epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \alpha' \sqrt{-\gamma} R^{(2)} \Phi], \quad (3)$$

where  $\Sigma$  denotes the world-sheet, with metric  $\gamma$  and the topology of a sphere,  $\alpha$  are world-sheet indices, and  $\mu, \nu$  are target-space-time indices. The important point of [12] was the role of target time  $t$  as a specific dilaton background, linear in that coordinate, of the form

$$\Phi = \text{const} - \frac{1}{2} Q t, \quad (4)$$

where  $Q$  is a constant and  $Q^2 > 0$  is the  $\sigma$ -model central-charge deficit, allowing this *supercritical* string theory to be formulated in some number of dimensions different from the critical number.

Consistency of the underlying world-sheet conformal field theory, as well as modular invariance of the string scattering amplitudes, required *discrete* values of  $Q^2$ , when expressed in units of the string length  $M_s$  [12]. This was the first example of a non-critical string cosmology, with the spatial target-space coordinates  $X^i$ ,  $i = 1, \dots, D-1$ , playing the rôles of  $\sigma$ -model fields. This non-critical string was not conformally invariant, and hence required Liouville dressing [13]. The Liouville field had time-like signature in target space, since the central charge deficit  $Q^2 > 0$  in the model of [12], and its zero mode played the rôle of target time.

As a result of the non-trivial dilaton field, the Einstein term in the effective  $D$ -dimensional low-energy field theory action is conformally rescaled by  $e^{-2\Phi}$ . This requires a redefinition of the  $\sigma$ -model-frame space-time metric  $g_{\mu\nu}^\sigma$  to the ‘physical’ Einstein metric  $g_{\mu\nu}^E$ :

$$g_{\mu\nu}^E = e^{-\frac{4\Phi}{D-2}} G_{\mu\nu} . \quad (5)$$

Target time must also be rescaled, so that the metric acquires the standard Robertson-Walker (RW) form in the normalized Einstein frame for the effective action:

$$ds_E^2 = -dt_E^2 + a_E^2(t_E) (dr^2 + r^2 d\Omega^2) , \quad (6)$$

where we show the example of a spatially-flat RW metric for definiteness, and  $a_E(t_E)$  is an appropriate scale factor, which is a function of  $t_E$  alone in the homogeneous cosmological backgrounds we assume throughout.

The Einstein-frame time is related to the time in the  $\sigma$ -model frame [12] by:

$$dt_E = e^{-2\Phi/(D-2)} dt \quad \rightarrow \quad t_E = \int^t e^{-2\Phi(t')/(D-2)} dt' . \quad (7)$$

The linear dilaton background (4) yields the following relation between the Einstein and  $\sigma$ -model frame times:

$$t_E = c_1 + \frac{D-2}{Q} e^{\frac{Q}{D-2}t} , \quad (8)$$

where  $c_{1,0}$  are appropriate (positive) constants. Thus, a dilaton background (4) that is linear in the  $\sigma$ -model time scales logarithmically with the Einstein time (Robertson-Walker cosmic time)  $t_E$ :

$$\Phi(t_E) = (\text{const.}') - \frac{D-2}{2} \ln\left(\frac{Q}{D-2} t_E\right) . \quad (9)$$

In this regime, the string coupling [1]:

$$g_s = \exp(\Phi(t)) \quad (10)$$

varies with the cosmic time  $t_E$  as  $g_s^2(t_E) \equiv e^{2\Phi} \propto \frac{1}{t_E^{D-2}}$ , thereby implying a vanishing effective string coupling asymptotically in cosmic time. In the linear dilaton background of [12], the asymptotic space-time metric in the Einstein frame reads:

$$ds^2 = -dt_E^2 + a_0^2 t_E^2 (dr^2 + r^2 d\Omega^2) \quad (11)$$

where  $a_0$  a constant. Clearly, there is no acceleration in the expansion of the Universe (11).

The effective low-energy action on the four-dimensional brane world for the gravitational multiplet of the string in the Einstein frame reads [12]:

$$S_{\text{eff}}^{\text{brane}} = \int d^4x \sqrt{-g} \left\{ R - 2(\partial_\mu \Phi)^2 - \frac{1}{2} e^{4\Phi} (\partial_\mu b)^2 - \frac{2}{3} e^{2\Phi} \delta c \right\}, \quad (12)$$

where, as we discuss below,  $b$  is the four-dimensional axion field associated with a four-dimensional representation of the antisymmetric tensor, and  $\delta c = C_{\text{int}} - c^*$ , where  $C_{\text{int}}$  is the central charge of the conformal world-sheet theory corresponding to the transverse (internal) string dimensions, and  $c^* = 22(6)$  is the critical value of this internal central charge of the (super)string theory for flat four-dimensional space-times. The linear dilaton configuration (4) corresponds, in this language, to a background charge  $Q$  of the conformal theory, which contributes a term  $-3Q^2$  (in our normalization) to the total central charge. The latter includes the contributions from the four uncompactified dimensions of our world. In the case of a flat four-dimensional Minkowski space-time, one has  $C_{\text{total}} = 4 - 3Q^2 + C_{\text{int}} = 4 - 3Q^2 + c^* + \delta c$ , which should equal 26 (10). This implies that  $C_{\text{int}} = 22 + 3Q^2$  ( $6 + 3Q^2$ ) for bosonic (supersymmetric) strings.

An important result in [12] was the discovery of an exact conformal field theory corresponding to the dilaton background (9) and a constant-curvature (Milne) static metric in the  $\sigma$ -model frame (or, equivalently, a linearly-expanding Robertson-Walker Universe in the Einstein frame). The conformal field theory corresponds to a Wess-Zumino-Witten two-dimensional world-sheet model on a group manifold  $O(3)$  with appropriate constant curvature, whose coordinates correspond to the spatial components of the four-dimensional metric and antisymmetric tensor fields, together with a free world-sheet field corresponding to the target time coordinate. The total central charge in this more general case reads  $C_{\text{total}} = 4 - 3Q^2 - \frac{6}{k+2} + C_{\text{int}}$ , where  $k$  is a positive integer corresponding to the level of the Kac-Moody algebra associated with the WZW model on the group manifold. The value of  $Q$  is chosen in such a way that the overall central charge  $c = 26$  and the theory is conformally invariant. Since such unitary conformal field theories have *discrete* values of their central charges, which accumulate to integers or half-integers from *below*, it follows that the values of the central charge deficit  $\delta c$  are *discrete* and *finite* in number. From a physical point of view, this implies that the linear-dilaton Universe may either stay in such a state for ever, for a given  $\delta c$ , or tunnel between the various discrete levels before relaxing to a critical  $\delta c = 0$  theory. It was argued in [12] that, due to the above-mentioned finiteness of the set of allowed discrete values of the central charge deficit  $\delta c$ , the Universe could reach flat four-dimensional Minkowski space-time, and thus exit from the expanding phase, after a finite number of phase transitions.

The analysis in [12] also showed, as we discuss below, that there are tachyonic mass shifts of order  $-Q^2$  in the bosonic string excitations, but not in the fermionic ones. This implies the appearance of tachyonic instabilities and the breaking of target-space supersymmetry in such backgrounds, as far as the excitation spectrum is concerned. The instabilities could trigger the cosmological phase transitions, since they correspond to relevant renormalization-group world-sheet operators, and hence initiate the flow of the internal unitary conformal field theory towards minimization of its central charge, in accordance with the Zamolodchikov  $c$ -theorem [14]. As we discuss later on, in semi-realistic cosmological models [15] such tachyons decouple from the spectrum relatively quickly. On the other hand, as a result of the form of the dilaton in the Einstein frame (9), we observe that the dark-energy density for this (four-dimensional)

Universe,  $\Lambda \equiv e^{2\Phi} \delta c$ , is relaxing to zero with a  $1/t_E^{(D-2)}$  dependence on the Einstein-frame time for each of the equilibrium values of  $\delta c$ . Therefore, the breaking of supersymmetry induced by the linear dilaton is only an obstruction [16], rather than a spontaneous breaking, in the sense that it appears only temporarily in the boson-fermion mass splittings between the excitations, whilst the vacuum energy of the asymptotic equilibrium theory vanishes.

In [17] we went one step beyond the analysis in [12], and considered more complicated  $\sigma$ -model metric backgrounds that did not satisfy the  $\sigma$ -model conformal-invariance conditions, and therefore needed Liouville dressing [13] to restore conformal invariance. Such backgrounds could even be time-dependent, living in  $(d+1)$ -dimensional target space-times. Various mathematically-consistent forms of non-criticality can be considered, for instance cosmic catastrophes such as the collision of brane worlds [18, 19]. Such models lead to supercriticality of the associated  $\sigma$  models describing stringy excitations on the brane worlds. The Liouville dressing of such non-critical models results in  $(d+2)$ -dimensional target spaces with two time directions. An important point in [17] was the identification of the (world-sheet zero mode of the) Liouville field with the target time, thereby restricting the Liouville-dressed  $\sigma$  model to a  $(d+1)$ -dimensional hypersurface of the  $(d+2)$ -dimensional target space and maintaining the initial target space-time dimensionality. We stress that this identification is possible only in cases where the initial  $\sigma$  model is supercritical, so that the Liouville mode has time-like signature [12, 13]. In certain models [18, 19], such an identification was proven to be energetically preferable from a target-space viewpoint, since it minimized certain effective potentials in the low-energy field theory corresponding to the string theory at hand.

All such cosmologies require some initial physical reason for the initial departure from the conformal invariance of the underlying  $\sigma$  model that describes string excitations in such Universes. The reason could be an initial quantum fluctuation, or, in brane models, a catastrophic cosmic event such as the collision of two or more brane worlds. Such non-critical  $\sigma$  models relax asymptotically to conformal  $\sigma$  models, which may be viewed as equilibrium points in string theory space, as illustrated in Fig. 1. In some interesting cases of relevance to cosmology [15], which are particularly generic, the asymptotic conformal field theory is that of [12] with a linear dilaton and a flat Minkowski target-space metric in the  $\sigma$ -model frame. In others, the asymptotic theory is characterized by a constant dilaton and a Minkowskian space-time [18]. Since, as we discuss below, the evolution of the central-charge deficit of such a non-critical  $\sigma$  model,  $Q^2(t)$ , plays a crucial rôle in inducing the various phases of the Universe, including an inflationary phase, graceful exit from it, thermalization and a contemporary phase of accelerating expansion, we term such Liouville-string-based cosmologies *Q-Cosmologies*.

The use of Liouville strings to describe the evolution of our Universe has a broad motivation, since non-critical strings are associated with non-equilibrium situations, as are likely to have occurred in the early Universe. The space of non-critical string theories is much larger than that of critical strings. It is therefore remarkable that the departure from criticality may enhance the predictability of string theory to the extent that a purely stringy quantity such as the string coupling  $g_s$  may become accessible to experiment via its relation to the present-era cosmic acceleration parameter:  $g_s^2 = -q^0$  [8]. Another example arises in a non-critical string approach to inflation, if the Big Bang is identified with the collision [7] of two D-branes [19]. In such a scenario, astrophysical observations may place important bounds on the recoil velocity of the brane worlds after the collision, and lead to an estimate of the separation of the branes at the end of the inflationary period.

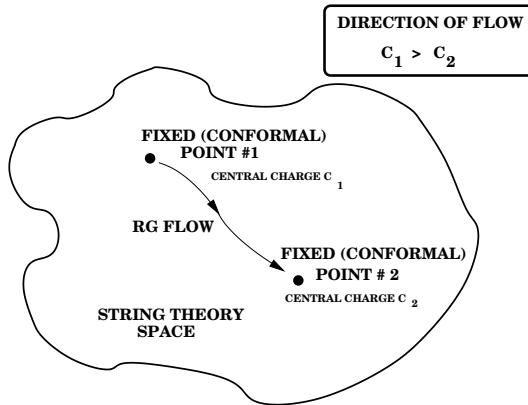


Figure 1: A schematic view of string theory space, which is an infinite-dimensional manifold endowed with a (Zamolodchikov) metric. The dots denote conformal string backgrounds. A non-conformal string flows (in a two-dimensional renormalization-group sense) from one fixed point to another, either of which could be a hypersurface in theory space. The direction of the flow is irreversible, and is directed towards the fixed point with a lesser value of the central charge, for unitary theories, or, for general theories, towards minimization of the degrees of freedom of the system.

In such a framework, the identification of target time with a world-sheet renormalization-group scale, the zero mode of the Liouville field [17], provides a novel way of selecting the ground state of the string theory. This is not necessarily associated with minimization of energy, but could simply be a result of cosmic chance. It may be a random global event that the initial state of our cosmos corresponds to a certain Gaussian fixed point in the space of string theories, which is then perturbed into a Big Bang by some relevant (in a world-sheet sense) deformation, which makes the theory non-critical, and hence out of equilibrium from a target space-time viewpoint. The theory then flows, as indicated in Fig. 1, along some specific renormalization-group trajectory, heading asymptotically to some ground state that is a local extremum corresponding to an infrared fixed point of this perturbed world-sheet  $\sigma$ -model theory. This approach allows for many ‘parallel universes’ to be implemented, and our world might be just one of these. Each Universe may flow between different fixed points, its trajectory following a perturbation by a different operator. It seems to us that this scenario is more attractive and specific than the landscape scenario [20], which has recently been advocated as an framework for parametrizing our ignorance of the true nature of string/M theory.

In this article we describe the main features of such non-critical string cosmological models. The structure of the article is as follows: in the next Section we review briefly the basic properties of Liouville strings at zero temperature, emphasizing the rôle of the Liouville mode as target time [17]. We start in Section 2.1 with a comprehensive description of the generic properties of Liouville dressing, and proceed in Section 2.2 to present the basic features of (compactified) cosmological models of non-critical strings upon which we rely later. We give physical reasons for the departure from conformal invariance, in Section 2.3 to discuss inflation in this framework, identifying the Hubble parameter with the central-charge deficit  $Q^2$  of the corresponding supercritical  $\sigma$  model describing string excitations in the pertinent non-conformal

cosmological backgrounds. In Section 2.4 we discuss the late stages of such universes, in particular the rôle of the (time-dependent) string coupling of the non-critical string in inducing the present-day acceleration of the Universe. In Section 2.5 we discuss the inclusion of matter in the late-stage evolution of the Universe, demonstrating the differences of our non-equilibrium Liouville formalism from standard Friedmann-Robertson-Walker (FRW) Cosmologies.

In Section 3 we present a concrete example of non-critical strings, that of colliding brane worlds [18, 21], where the departure from criticality results from the cosmic collision of the branes. Specific scenarios of this type are discussed in Sections 3.1 and 3.2, relevant aspects of Type-IIA supergravity are discussed in Section 3.3 and compactification issues in Section 3.4. Then, in Section 3.5 we present within this framework some scenarios for supersymmetry breaking at zero temperature, associated with either the presence of moving branes or the existence of magnetic fields in internal manifolds of the compactified space of brane worlds. The latter is compatible with the present value of the dark energy, as inferred from observations. In these models the dark energy is viewed as a relaxation energy of the brane world, which was excited after the collision.

In Section 4 we present a finite-temperature analysis of the early Universe in the context of Liouville strings. We commence our analysis in Section 4.1 with a review of the hot phase soon after the brane collision, and give estimates of the bulk and brane excitation energies. In Section 4.2 we review the Liouville approach to finite-temperature heterotic strings, whereby the temperature is associated with a space-like Liouville mode in a subcritical string describing the thermal vacuum of the heterotic string [22]. In Section 4.3 we discuss the finite-temperature properties of the Type-IIA vacuum which characterises the specific colliding-brane cosmological model mentioned above. In Section 4.4 we discuss metastability properties of the Type-IIA thermal supergravity vacuum, which, in contrast to the heterotic string case, is an unstable vacuum, leading to the exit of our Universe from the hot phase soon after the collision. In Section 4.5 we describe in some detail the various phases of the colliding brane scenario and the associated phase transitions, namely the transition from an initial hot phase to a cold inflationary phase, and its subsequent exit from it. We also pay attention to the fact that in these models the brane world appears thermalized throughout, leading to decelerating brane motion in the bulk, as a result of gravitational radiation leading towards thermal equilibrium between the brane and bulk worlds. This deceleration is the essential mechanism for the exit from the inflationary phase. We also discuss in Section 4.5 several open questions associated with this phase, in the context of our colliding-brane models, such as the possibility of a second collision and some delicate issues concerning nucleosynthesis in these models.

Finally, in Section 5 we present our conclusions and the outlook for future work in Liouville Q-cosmologies.

## 2 Non-Critical Liouville String Q-Cosmologies

### 2.1 Zero-Temperature Liouville Formalism

We consider a  $\sigma$ -model action deformed by a family of vertex operators  $V_i$ , corresponding to ‘couplings’  $g^i$ , which represent *non-conformal* background space-time fields from the massless string multiplet, such as gravitons,  $G_{\mu\nu}$ , antisymmetric tensors,  $B_{\mu\nu}$ , dilatons  $\Phi$ , their super-



symmetric partners, *etc.*:

$$S = S_0(X) + \sum_i g^i \int d^2z V_i(X) , \quad (13)$$

where  $S_0$  represents a conformal  $\sigma$  model describing an equilibrium situation. The non-conformality of the background means that the pertinent  $\beta^i$  function  $\beta^i \equiv dg^i/d\ln\mu \neq 0$ , where  $\mu$  is a world-sheet renormalization scale. Conformal invariance would imply restrictions on the background and couplings  $g^i$ , corresponding to the constraints  $\beta^i = 0$ , which are equivalent to equations of motion derived from a target-space effective action for the corresponding fields  $g^i$ . The entire low-energy phenomenology and model building of critical string theory is based on such restrictions [1].

In the non-conformal case  $\beta^i \neq 0$ , the theory is in need of dressing by the Liouville field  $\phi$  in order to restore conformal symmetry [13]. The field  $\phi$  acquires dynamics through the integration over world-sheet covariant metrics in the path integral, and may be viewed as a local dynamical scale on the world sheet [17]. If the central charge of the (supersymmetric) matter theory is  $c_m > 25(9)$  (i.e., supercritical), the signature of the kinetic term of the Liouville coordinate in the dressed  $\sigma$ -model is opposite to that of the  $\sigma$ -model fields corresponding to the other target-space coordinates. As mentioned previously, this opens the way to the important step of interpreting the Liouville field physically by identifying its world-sheet zero mode  $\phi_0$  with the target time in supercritical theories [17]. Such an identification emerges naturally from the dynamics of the target-space low-energy effective theory by minimizing the effective potential [18].

The action of the Liouville mode  $\phi$  reads [13]:

$$S_L = S_0(X) + \frac{1}{8\pi} \int_{\Sigma} d^2\xi \sqrt{\hat{\gamma}} [\pm(\partial\phi)^2 - QR^{(2)}\phi] + \int_{\Sigma} d^2\xi \sqrt{\hat{\gamma}} g^i(\phi) V_i(X) , \quad (14)$$

where  $\hat{\gamma}$  is a fiducial world-sheet metric, and the plus (minus) sign in front of the kinetic term of the Liouville mode pertains to subcritical (supercritical) strings. The dressed couplings  $g^i(\phi)$  are obtained by the following procedure:

$$\int d^2z g_i V_i(X) \rightarrow \int d^2z g_i(\phi) e^{\alpha_i\phi} V_i(X) , \quad (15)$$

where  $\alpha_i$  is the ‘‘gravitational’’ anomalous dimension. If the original non-conformal vertex operator has anomalous scaling dimension  $\Delta_i - 2$  (for closed strings, to which we restrict ourselves for definiteness), where  $\Delta_i$  is the conformal dimension, and the central charge surplus of the theory is  $Q^2 = \frac{c_m - c^*}{3} > 0$  (for bosonic strings  $c^* = 25$ , for superstrings  $c^* = 9$ ), then the condition that the dressed operator is marginal on the world sheet implies the relation:

$$\alpha_i(\alpha_i + Q) = 2 - \Delta_i . \quad (16)$$

Imposing appropriate boundary conditions in the limit  $\phi \rightarrow \infty$  [13], the acceptable solution is:

$$\alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + 2 - \Delta_i} . \quad (17)$$

The gravitational dressing is trivial for marginal couplings,  $\Delta_i = 2$ , as it should be. This dressing applies also to higher orders in the perturbative  $g^i$  expansion. For instance, at the next order, where the deviation from marginality in the deformations of the undressed  $\sigma$  model is due to the operator product expansion coefficients  $c_{jk}^i$  in the  $\beta^i$  function, the Liouville-dressing procedure implies the replacement [23]:

$$g^i \quad \rightarrow \quad g^i e^{\alpha_i \phi} + \frac{\pi \phi}{Q \pm 2\alpha_i} c_{jk}^i g^j g^k e^{\alpha_i \phi}, \quad (18)$$

in order for the dressed operator to become marginal to this order (the  $\pm$  sign originates in (14)).

In terms of the Liouville renormalization-group scale, one has the following equation relating Liouville-dressed couplings  $g^i$  and  $\beta$  functions in the non-critical string case:

$$\ddot{g}^i + Q\dot{g}^i = \mp \beta^i(g_j), \quad (19)$$

where the - (+) sign in front of the  $\beta$ -functions on the right-hand-side applies to super(sub)critical strings, the overdot denotes differentiation with respect to the Liouville zero mode,  $\beta^i$  is the world-sheet renormalization-group  $\beta$  function (but with the renormalized couplings replaced by the Liouville-dressed ones as defined by the procedure in (15), (18)), and the minus sign on the right-hand side (r.h.s.) of (19) is due to the time-like signature of the Liouville field. Formally, the  $\beta^i$  of the r.h.s. of (19) may be viewed as power series in the (weak) couplings  $g^i$ . The covariant (in theory space)  $\mathcal{G}_{ij}\beta^j$  function may be expanded as:

$$\mathcal{G}_{ij}\beta^j = \sum_{i_n} \langle V_i^L V_{i_1}^L \dots V_{i_n}^L \rangle_\phi g^{i_1} \dots g^{i_n}, \quad (20)$$

where  $V_i^L$  indicates Liouville dressing à la (15),  $\langle \dots \rangle_\phi = \int d\phi d\vec{r} \exp(-S(\phi, \vec{r}, g^j))$  denotes a functional average including Liouville integration, and  $S(\phi, \vec{r}, g^i)$  is the Liouville-dressed  $\sigma$ -model action, including the Liouville action [13].

In the case of stringy  $\sigma$  models, the diffeomorphism invariance of the target space results in the replacement of (19) by:

$$\ddot{g}^i + Q(t)\dot{g}^i = \mp \tilde{\beta}^i, \quad (21)$$

where the  $\tilde{\beta}^i$  are the Weyl anomaly coefficients of the stringy  $\sigma$  model in the background  $\{g^i\}$ , which differ from the ordinary world-sheet renormalization-group  $\beta^i$  functions by terms of the form:

$$\tilde{\beta}^i = \beta^i + \delta g^i \quad (22)$$

where  $\delta g^i$  denote transformations of the background field  $g^i$  under infinitesimal general coordinate transformations, e.g., for gravitons [1]  $\tilde{\beta}_{\mu\nu}^G = \beta_{\mu\nu}^G + \nabla_{(\mu} W_{\nu)}$ , with  $W_\mu = \nabla_\mu \Phi$ , and  $\beta_{\mu\nu}^G = R_{\mu\nu}$  to order  $\alpha'$  (one  $\sigma$ -model loop).

The set of equations (19),(21) defines the *generalized conformal invariance conditions*, expressing the restoration of conformal invariance by the Liouville mode. The solution of these equations, upon the identification of the Liouville zero mode with the original target time, leads to constraints in the space-time backgrounds [17, 18], in much the same way as the conformal invariance conditions  $\beta^i = 0$  define consistent space-time backgrounds for critical strings [1]. It is important to remark [24, 17] that the equations (21) can be derived from an action. This

follows from general properties of the Liouville renormalization group, which guarantee that the appropriate Helmholtz conditions in the string-theory space  $\{g^i\}$  for the Liouville-flow dynamics to be derivable from an action principle are satisfied.

To be specific, consider the case of the  $\sigma$  model (3), and the  $\mathcal{O}(\alpha')$  Weyl anomaly coefficients [12], assuming that the  $\sigma$ -model target space is a  $D$ -dimensional space-time  $(X^0, \vec{X})$ . The pertinent  $\beta$  functions are

$$\begin{aligned}\tilde{\beta}_{\mu\nu}^G &= \alpha' \left( R_{\mu\nu} + 2\nabla_\mu \partial_\nu \Phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu^{\rho\sigma} \right), \\ \tilde{\beta}_{\mu\nu}^B &= \alpha' \left( -\frac{1}{2} \nabla_\rho H_{\mu\nu}^\rho + H_{\mu\nu}^\rho \partial_\rho \Phi \right), \\ \tilde{\beta}^\Phi &= \beta^\Phi - \frac{1}{4} G^{\rho\sigma} \beta_{\rho\sigma}^G = \frac{1}{6} (C^{(D)} - 26),\end{aligned}\tag{23}$$

$$C^{(D)} = D - \frac{3}{2} \alpha' \left[ R - \frac{1}{12} H^2 - 4(\nabla\Phi)^2 + 4\Box\Phi \right],\tag{24}$$

where  $\alpha'$  is the Regge slope [1], the Greek indices are  $D$ -dimensional, and  $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$  is the field strength of the  $B$  field.

Dressing this  $\sigma$  model with a Liouville mode results in the appropriate equations (21), and it is straightforward to show that these can be *derived* from the following  $(D+1)$ -dimensional action in the  $\sigma$ -model (string) frame [23]:

$$\begin{aligned}I^{(D+1)} &= \int d\phi d^D X \sqrt{G} \sqrt{|G_{\phi\phi}|} e^{-2\Phi} \{ C^{(D)} - 25 + \\ &3G^{\phi\phi} [(\dot{\Phi} - \frac{1}{2} G^{\mu\nu} \dot{G}_{\mu\nu})^2 - \frac{1}{4} G^{\mu\nu} G^{\rho\lambda} (\dot{G}_{\mu\rho} \dot{G}_{\nu\lambda} + \dot{B}_{\mu\rho} \dot{B}_{\nu\lambda})] \},\end{aligned}\tag{25}$$

where  $G, B, \Phi$  are all  $D$ -dimensional fields, depending in general on  $X^0, \phi, \vec{X}$ , and the overdot denotes differentiation with respect to the Liouville zero mode  $\phi$ . In the Liouville dressing procedure [13] we employ in this work one has the normalization  $G_{\phi\phi} = -1$ . This justifies the presence of only spatial components of the metric and antisymmetric tensor fields in the terms inside the [...] in (25). This action may be schematically represented in the form [24]:

$$I^{(D+1)} = \int d\phi d^D X e^{-\varphi} \{ C^{(D)}(X) - 25 - 3[\dot{\varphi}^2 - \frac{1}{4} (\dot{\lambda}^I \mathcal{G}_{IJ} \dot{\lambda}^J)] \},\tag{26}$$

where  $\lambda_I = \{G, B\}$ ,  $\mathcal{G}_{IJ} = G^{\mu\rho} G^{\nu\lambda}$  is a Zamolodchikov metric in  $\lambda_I$  space<sup>2</sup>, and  $\varphi \equiv 2\Phi - \ln\sqrt{G}$  is a rescaled dilaton [23], which guarantees the diagonalization of the appropriate Zamolodchikov metric in the string theory space  $(\Phi, \lambda_I)$ , with  $\mathcal{G}_{\varphi\varphi} = 1$ .

Upon the identification of  $\phi$  with the target time  $X^0$ , the  $(D+1)$ -dimensional action is constrained onto a  $D$ -dimensional hypersurface  $(X^0 = \phi, \vec{X})$ . In that case the resulting  $D$ -dimensional target-space-time action reads:

$$\begin{aligned}I^D &= -\frac{3}{2} \alpha' \int d^D X \sqrt{G} e^{-2\Phi} \left( R + 4(\nabla\Phi)^2 - \frac{1}{12} H^2 - \frac{2}{3\alpha'} (D-25) \right) + \mathcal{I}_\phi, \\ \mathcal{I}_\phi &\equiv \int d^D X \sqrt{G} e^{-2\Phi} \left( -3[(\dot{\Phi} - \frac{1}{2} G^{\mu\nu} \dot{G}_{\mu\nu})^2 - \frac{1}{4} G^{\mu\nu} G^{\rho\lambda} (\dot{G}_{\mu\rho} \dot{G}_{\nu\lambda} + \dot{B}_{\mu\rho} \dot{B}_{\nu\lambda})] \right).\end{aligned}\tag{27}$$

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<sup>2</sup>Note that this is compatible with the definition of this metric in string space as a two-point correlation function of appropriate vertex operators, as explicit  $\mathcal{O}(\alpha')$  computations have demonstrated [25].

The extra piece,  $\mathcal{I}_\phi$ , as compared with a standard string theory target-space effective action, describes the non-equilibrium effects associated with the Liouville flow. In general, one may have critical target-space dimensionality  $D = 25$ , but deviations from conformal invariance, due for example to the recoil of brane worlds during collisions or other catastrophic cosmic events, will be the topic of interest to us here. The non-critical central-charge term proportional to  $\int d^D X \sqrt{G} e^{-2\Phi} (D - 25)$  in the action (27) may then be written in the form

$$\frac{3\alpha'}{2} \int d^D X \sqrt{G} e^{-2\Phi} \frac{2}{\alpha'} Q^2 \tag{28}$$

in the normalization of the Einstein term appearing in (27), where  $Q^2$  represents the central charge deficit (whatever its origin) of the appropriate  $\sigma$  model:  $Q^2 = (C - c^*)/3$ , describing closed-string excitations in an appropriate non-conformal background.

## 2.2 Cosmological Liouville Models: Generic Features

There are many cosmological models that fall in the above category of Liouville strings, notably models with catastrophic cosmic events such as the collision of two brane worlds [18, 21], which we concentrate upon later in this work. When one considers a brane moving in a bulk space in the presence of other brane worlds [21], there are in general non-trivial potentials between the moving branes: only the static configuration and certain other special configurations are supersymmetric in target space, with vanishing ground-state energy [21]. When one considers string theory excitations in such Universes, the corresponding  $\sigma$  model is *non-critical*. In the model of [21], as we discuss later, there are various phases for the bulk string Universe, which involve a passage from subcritical to supercritical strings, due to a change in sign of the pertinent supersymmetry-breaking potential of the moving brane world.

In such situations, the resulting  $\sigma$  model describing low-energy string excitations in the bulk lives in a  $(d + 1)$ -dimensional space-time,  $d$  denoting the number of spatial dimensions, and is not conformal. As already discussed, to restore the conformal symmetry required for consistency of the path-integral quantization, one needs Liouville dressing [13], resulting in equations of the form (21) for the background fields under consideration. Liouville dressing results in a critical string in  $(d + 2)$  dimensions, with restored conformal symmetry expressed by the vanishing of the  $(d + 2)$ -target-dimensional  $\sigma$ -model  $\beta$  functions. Eventually, dynamics which we review in due course results in the identification of the Liouville mode in supercritical situations with the target time, so the final target-space dimensionality of the dressed  $\sigma$  model remains  $(d + 1)$ .

One such model was considered in detail in [15]. The model is based on a specific string theory, namely ten-dimensional Type-0 [26], which leads to a non-supersymmetric target-space spectrum as a result of a special projection of the supersymmetric partners out of the spectrum. Nevertheless, the basic properties of its cosmology are sufficiently generic to be extended to the bosonic sector of any other effective low-energy string-inspired supersymmetric field theory. The model also involves flux compactification to four dimensions, which, as was pointed out in [15], plays an important rôle in ensuring the existence of large stable bulk dimensions.

The ten-dimensional metric configuration considered in [15] was:

$$G_{MN} = \begin{pmatrix} g_{\mu\nu}^{(4)} & 0 & 0 \\ 0 & e^{2\sigma_1} & 0 \\ 0 & 0 & e^{2\sigma_2} I_{5 \times 5} \end{pmatrix} \quad (29)$$

where lower-case Greek indices are four-dimensional space-time indices, and  $I_{5 \times 5}$  denotes the  $5 \times 5$  unit matrix. We have chosen two different scales for internal space. The field  $\sigma_1$  sets the scale of the fifth dimension, while the  $\sigma_2$  parametrize a flat five-dimensional space. In the context of the cosmological models we deal with here, the fields  $g_{\mu\nu}^{(4)}$ ,  $\sigma_i$ ,  $i = 1, 2$  are assumed to depend on the time  $t$  only. Type-0 string theory, as well as its supersymmetric versions appearing in brane models, contains appropriate form fields with non-trivial gauge fluxes (flux-form fields), which live in the higher-dimensional bulk space. In the specific model of [26], just one such field was allowed to be non-trivial. As was demonstrated in [15], a consistent background choice for the flux-form field has the flux parallel to the fifth dimension  $\sigma_1$ . This implies that the internal space is stabilized in such a way that this dimension is much larger than the remaining four  $\sigma_2$ . This demonstrates the physical importance of the flux fields for large radii of compactification.

Considering the fields to be time-dependent only, i.e., considering spherically-symmetric homogeneous backgrounds, restricting ourselves to the compactification (29), and assuming a Robertson-Walker form of the four-dimensional metric with scale factor  $a(t)$ , the generalized conformal-invariance conditions and the Curci-Pafutti  $\sigma$ -model renormalizability constraint [27] imply the set of differential equations (21), which were solved numerically in [15]. The set of  $\{g^i\}$  contains the graviton, dilaton, tachyon, flux and moduli fields  $\sigma_{1,2}$  whose vacuum expectation values control the sizes of the extra dimensions.

The detailed analysis of [15] indicated that the moduli fields  $\sigma_i$  freeze quickly to their equilibrium values, so, together with the tachyon field which also decays to a constant value rapidly, they decouple from the four-dimensional fields at very early stages in the evolution of this string Universe<sup>3</sup>. There is an inflationary phase in this scenario and dynamical exit from it. The important point to guarantee the exit is the fact that the central charge deficit  $Q^2$  is a time-dependent entity in this approach, which obeys specific relaxation laws determined by the underlying conformal field theory [15, 18, 19]. In fact, the central charge runs with the local world-sheet renormalization-group scale, the zero mode of the Liouville field, which is identified [17] with the target time in the  $\sigma$ -model frame. The supercriticality [12]  $Q^2 > 0$  of the underlying  $\sigma$  model is crucial, as already mentioned. Physically, the non-critical string provides a framework for *non-equilibrium* dynamics, which may be the result of some catastrophic cosmic event, such as a collision of two brane worlds [7, 18, 19], or an initial quantum fluctuation [24, 15]. It also provides, as we discuss below, a unified mathematical framework for analyzing various phases of string cosmology, from the early inflationary phase, graceful exit from it and reheating, until the current and future eras of accelerated cosmologies. Interestingly, one can constrain string parameters such as the separation of brany worlds at the

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<sup>3</sup>The presence of the tachyonic instability in the spectrum of the model of [15] is due to the fact that in Type-0 strings there is no target-space supersymmetry by construction. In other models with supersymmetry breaking [18, 21], due to either thermalization or other instabilities, e.g., brane motion, there are also tachyonic modes reflecting the broken supersymmetric spectrum. From a cosmological viewpoint such tachyon fields are not necessarily bad features, since they may provide the initial instability leading to cosmic expansion.

end of inflation, as well as the recoil velocity of the branes after the collision, by fits to current astrophysical data [19].

### 2.3 Liouville Inflation: the General Picture

As discussed in [24, 18, 19], a constant central-charge deficit  $Q^2$  in a stringy  $\sigma$  model may be associated with an initial inflationary phase with

$$Q^2 = 9H^2 > 0 , \quad (30)$$

where the Hubble parameter  $H$  can be fixed in terms of other parameters of the model. One may consider various scenarios for such a departure from criticality. For example, in the model of [18, 19] this was due to the collision of two brane worlds. In such a scenario, as we now review briefly, it is possible to obtain an initial *supercritical* central charge deficit, and hence a time-like Liouville mode in the theory. For instance, in the specific colliding-brane model of [18, 19],  $Q$  (and thus  $H$ ) is proportional to the square of the relative velocity of the colliding branes,  $Q \propto u^2$  during the inflationary era. As is evident from (30) and discussed in more detail below, in a phase of constant  $Q$  one obtains an inflationary de Sitter Universe.

However, catastrophic non-critical string scenarios for cosmology, such as that in [18], allow in general for a time-dependent deficit  $Q^2(t)$  that relaxes to zero. This may occur in such a way that, although during the inflationary era  $Q^2$  is (for all practical purposes) constant, as in (30), eventually  $Q^2$  decreases with time so that, at the present era, one obtains compatibility with the current accelerating expansion of the Universe. As already mentioned, such relaxing quintessential scenarios [18, 15, 19] have the advantage of asymptotic states that can be defined properly as  $t \rightarrow \infty$ , as well as a string scattering  $S$ -matrix <sup>4</sup>.

The specific normalization in (30) is due to the identification of the time  $t$  with the zero mode of the Liouville field  $-\varphi$  of the *supercritical*  $\sigma$  model. The minus sign may be understood both mathematically, as due to properties of the Liouville mode, and physically by the requirement of the relaxation of the deformation of the space-time following the distortion induced by the recoil. With this identification, the general equation of motion for the couplings  $\{g_i\}$  of the  $\sigma$ -model background modes is given by (19) [17]:

$$\ddot{g}^i + Q\dot{g}^i = -\tilde{\beta}^i(g) = -\mathcal{G}^{ij}\partial C[g]/\partial g^j , \quad (31)$$

where the dot denotes a derivative with respect to the Liouville world-sheet zero mode  $\varphi$ , i.e., target time, and  $\mathcal{G}^{ij}$  is an inverse Zamolodchikov metric in the space of string theory couplings  $\{g^i\}$  [14]. When applied to scalar, inflaton-like, string modes, (31) would yield standard field equations for scalar fields in de Sitter (inflationary) space-times, provided the normalization (30) is valid, implying a ‘Hubble’ expansion parameter  $H = -Q/3$  <sup>5</sup>. The minus sign in  $Q = -3H$  is due to the fact that, as we discuss below, one identifies the target time  $t$  with the world-sheet zero mode of  $-\varphi$  [17].

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<sup>4</sup>As mentioned in the Introduction, another string scenario for inducing a de Sitter Universe envisages generating the inflationary space-time from string loops (dilaton tadpoles) [11], but in such models a string  $S$ -matrix cannot be properly defined.

<sup>5</sup>The gradient-flow property of the  $\beta$  functions makes the analogy with the inflationary case even more profound, with the running central charge  $C[g]$  [14] playing the rôle of the inflaton potential in conventional inflationary field theory.

The relations (31) replace the conformal invariance conditions  $\beta^i = 0$  of the critical string theory, and express the conditions necessary for the restoration of conformal invariance by the Liouville mode [13]. Interpreting the latter as an extra target dimension, the conditions (31) may also be viewed as conformal invariance conditions of a *critical*  $\sigma$  model in (D+1) target space-time dimensions, where D is the target dimension of the non-critical  $\sigma$  model before Liouville dressing. In most Liouville approaches, one treats the Liouville mode  $\varphi$  and time  $t$  as independent coordinates. In our approach [17, 15, 18], however, we take a further step, basing ourselves on dynamical arguments which restrict this extended (D+1)-dimensional space-time to a hypersurface determined by the identification  $\varphi = -t$ . This means that, as time flows, one is restricted to this D-dimensional subspace of the full (D+1)-dimensional Liouville space-time.

In the work of [18, 19] which invoked a brane collision as a source of departure from criticality, this restriction arose because the potential between massive particles, in an effective field theory context, was found to be proportional to  $\cosh(t + \varphi)$ , which is minimized when  $\varphi = -t$ . However, the flow of the Liouville mode opposite to that of target time may be given a deeper mathematical interpretation. It may be viewed as a consequence of a specific treatment of the area constraint in non-critical (Liouville)  $\sigma$  models [17], which involves the evaluation of the Liouville-mode path integral via an appropriate steepest-descent contour. In this way, one obtains a ‘breathing’ world-sheet evolution, in which the world-sheet area starts from a very large value (serving as an infrared cutoff), shrinks to a very small one (serving as an ultraviolet cutoff), and then inflates again towards very large values (returning to an infrared cutoff). Such a situation may then be interpreted [17] as a world-sheet ‘bounce’ back to the infrared, implying that the physical flow of target time is opposite to that of the world-sheet scale (Liouville zero mode).

We now become more specific. We consider a non-critical  $\sigma$  model in metric ( $G_{\mu\nu}$ ), antisymmetric tensor ( $B_{\mu\nu}$ ), and dilaton ( $\Phi$ ) backgrounds. These have the following  $\mathcal{O}(\alpha')$   $\beta$  functions (24), where  $\alpha'$  is the Regge slope [1]:

$$\begin{aligned}\beta_{\mu\nu}^G &= \alpha' \left( R_{\mu\nu} + 2\nabla_\mu \partial_\nu \Phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu^{\rho\sigma} \right) , \\ \beta_{\mu\nu}^B &= \alpha' \left( -\frac{1}{2} \nabla_\rho H_{\mu\nu}^\rho + H_{\mu\nu}^\rho \partial_\rho \Phi \right) , \\ \tilde{\beta}^\Phi &= \beta^\Phi - \frac{1}{4} G^{\rho\sigma} \beta_{\rho\sigma}^G = \frac{1}{6} (C - 26) .\end{aligned}\tag{32}$$

The Greek indices are four-dimensional, including target-space-time components  $\mu, \nu, \dots = 0, 1, 2, 3$  on the D3-branes of [18], and  $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$  is the field strength. We consider the following representation of the four-dimensional field strength in terms of a pseudoscalar (axion-like) field  $b$ :

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b ,\tag{33}$$

where  $\epsilon_{\mu\nu\rho\sigma}$  is the four-dimensional antisymmetric symbol. Next, we choose an axion background that is linear in the time  $t$  [12]:

$$b = b(t) = \beta t , \quad \beta = \text{constant},\tag{34}$$

which yields a constant field strength with spatial indices only:  $H_{ijk} = \epsilon_{ijk} \beta$ ,  $H_{0jk} = 0$ . This implies that such a background is a conformal solution of the full  $\mathcal{O}(\alpha')$   $\beta$  function for the

four-dimensional antisymmetric tensor. We also consider a dilaton background that is linear in the time  $t$  [12]:

$$\Phi(t, X) = \text{const} + (\text{const})'t. \quad (35)$$

This background does not contribute to the  $\beta$  functions for the antisymmetric tensor and metric.

Suppose now that only the metric is a non-conformal background, due to some initial quantum fluctuation or catastrophic event, such as the collision of two branes discussed above, which results in an initial central charge deficit  $Q^2$  (30) that is constant at early stages after the collision. Let

$$G_{ij} = e^{\kappa\varphi + Hct} \eta_{ij}, \quad G_{00} = e^{\kappa'\varphi + Hct} \eta_{00}, \quad (36)$$

where  $t$  is the target time,  $\varphi$  is the Liouville mode,  $\eta_{\mu\nu}$  is the four-dimensional Minkowski metric, and  $\kappa, \kappa'$  and  $c$  are constants to be determined. As already discussed, the standard inflationary scenario in four-dimensional physics requires  $Q = -3H$ , which partially stems from the identification of the Liouville mode with time [17]

$$\varphi = -t. \quad (37)$$

The restriction (37) is imposed dynamically [18] at the end of our computations. Initially, one should treat  $\varphi, t$  as independent target-space-time components.

The Liouville dressing induces [13]  $\sigma$ -model terms of the form  $\int_{\Sigma} R^{(2)} Q \varphi$ , where  $R^{(2)}$  is the world-sheet curvature. Such terms provide non-trivial contributions to the dilaton background in the (D+1)-dimensional space-time  $(\varphi, t, X^i)$ :

$$\Phi(\varphi, t, X^i) = \frac{1}{2} Q \varphi + (\text{const})'t + \text{const}. \quad (38)$$

If we choose

$$(\text{const})' = \frac{1}{2} Q, \quad (39)$$

then (38) implies a *constant* dilaton background during the inflationary era, in which the central charge deficit  $Q^2$  is constant. We justify physically the choices (38) and (39) later in the article, when we discuss a specific example of non-criticality induced by the collision of brane worlds.

We now consider the Liouville-dressing equations [13] (31) for the  $\beta$  functions of the metric and antisymmetric tensor fields (32). For a constant dilaton field, the dilaton equation yields no independent information, apart from expressing the dilaton  $\beta$  function in terms of the central charge deficit, as usual. For the axion background (34), only the metric yields a non-trivial constraint (we work in units with  $\alpha' = 1$  for convenience):

$$\ddot{G}_{ij} + Q\dot{G}_{ij} = -R_{ij} + \frac{1}{2}\beta^2 G_{ij}, \quad (40)$$

where the dot indicates differentiation with respect to the (world-sheet zero mode of the) Liouville mode  $\varphi$ , and  $R_{ij}$  is the (non-vanishing) Ricci tensor of the (non-critical)  $\sigma$  model with coordinates  $(t, \vec{x})$ :  $R_{00} = 0$ ,  $R_{ij} = \frac{c^2 H^2}{2} e^{(\kappa - \kappa')\varphi} \eta_{ij}$ . One should also take into account the temporal ( $t$ ) equation for the metric tensor (which is identically zero for antisymmetric backgrounds):

$$\ddot{G}_{00} + Q\dot{G}_{00} = -R_{00} = 0, \quad (41)$$



where the vanishing of the Ricci tensor stems from the specific form of the background (36). We seek metric backgrounds of Robertson-Walker inflationary (de Sitter) form:

$$G_{00} = -1, \quad G_{ij} = e^{2Ht} \eta_{ij}. \quad (42)$$

Then, using (42), (36), (35) and (34), and imposing (37) at the end, we observe that there is indeed a consistent solution with:

$$Q = -3H = -\kappa', \quad c = 3, \quad \kappa = H, \quad \beta^2 = 5H^2, \quad (43)$$

corresponding to the conventional form of inflationary equations for scalar fields.

## 2.4 Current Stages of Cosmic Liouville Evolution: Acceleration, Dark Energy and the String Coupling

In the generic class of non-critical string models of interest in this work, the  $\sigma$  model always asymptotes, for long enough cosmic times, to the linear dilaton conformal  $\sigma$ -model field theory of [12]. But it is important to stress that this is only an asymptotic limit. In this respect, the current era of our Universe may be viewed as being close to, but still not quite at, the relaxation (equilibrium) point, in the sense that the dilaton is almost linear in the  $\sigma$ -model-frame time, and hence varies logarithmically with the Einstein-frame time (9). It is expected that this slight non-equilibrium will lead to a time-dependence of the unification gauge coupling and other constants (e.g., the four-dimensional Planck length (1)), that characterize the low-energy effective field theory, mainly through the time-dependence of the string coupling (10) as a result of the time-dependent linear dilaton (4).

The asymptotic time regime of the Type-0 cosmological string model of [15] was obtained analytically, by solving the pertinent equations (19) for the various fields. As already mentioned, at late times the theory becomes four-dimensional, and the only non-trivial information is contained in the scale factor and the dilaton, given that the topological flux field remains conformal in this approach, and the moduli and initial tachyon fields decouple very fast in the initial stages after inflation in this model. For times long after the initial fluctuations, such as the present epoch, when the linear approximation is valid, the solution for the dilaton in the  $\sigma$ -model frame follows from the equations (19) and takes the form:

$$\Phi(t) = -\ln \left[ \frac{\alpha A}{F_1} \cosh(F_1 t) \right], \quad (44)$$

with  $F_1$  a positive constant,  $\alpha$  a numerical constant of order one, and

$$A = \frac{C_5 e^{s_{01}}}{\sqrt{2} V_6}, \quad (45)$$

where  $s_{01}$  is the equilibrium value of the moduli field  $\sigma_1$ , associated with the large bulk dimension, and  $C_5$  the corresponding flux of the five-form flux field. Notice that  $A$  is independent of this large bulk dimension.

For very large times  $F_1 t \gg 1$  (in string units) one therefore approaches a linear solution for the dilaton  $\Phi \sim \text{const} - F_1 t$ . From (44), (10) and (1), we thus observe that the asymptotic

weakness of gravity in this Universe [15] is due to the smallness of the internal space  $V_6$  as compared with the flux  $C_5$  of the five-form field. The constant  $F_1$  is related to the central charge deficit of the underlying the non-conformal  $\sigma$  model [15]:

$$Q = Q_0 + \frac{Q_0}{F_1} \left( F_1 + \frac{d\Phi}{dt} \right), \quad (46)$$

where  $Q_0$  is a constant, and the numerical solution of (19), studied in [15], requires that

$$Q_0/F_1 = (1 + \sqrt{17})/2 \simeq 2.56, \quad (47)$$

which follows from the dilaton equation of motion. This connection of  $F_1$  to  $Q_0$  supports the above-described asymptotic conformal theory considerations of [12], where the model relaxes to for large times. In this spirit, we require that the value of  $Q_0$  to which the central charge deficit (46) asymptotes must be, for consistency of the underlying string theory, *one of the discrete values* obtained in [12], for which factorization (unitarity) of the string scattering amplitudes occurs. Notice that this asymptotic string theory, with a constant (time-independent) central-charge deficit,  $Q_0^2 \propto c^* - 25$  (or  $c^* - 9$  for superstring) is considered an *equilibrium* situation, where an  $S$ -matrix can be defined for specific (discrete) values of the central charge  $c^*$  [13, 12].

Defining the Einstein frame time  $t_E$  through (7), we obtain in this case

$$t_E = \frac{\alpha A}{F_1^2} \sinh(F_1 t), \quad F_1 t = \ln \left( \sqrt{1 + \gamma^2 t_E^2} + \gamma t_E \right), \quad (48)$$

where

$$\gamma \equiv \frac{F_1^2}{\alpha A}. \quad (49)$$

In terms of the Einstein-frame time, one obtains a logarithmic time dependence [12] for the dilaton (44) <sup>6</sup>:

$$\Phi_E = \text{const} - \ln(\gamma t_E), \quad (50)$$

For large  $t_E$ , e.g., present or later cosmological time values, one has [15, 8]

$$a_E(t_E) \simeq \frac{F_1}{\gamma} \sqrt{1 + \gamma^2 t_E^2}. \quad (51)$$

At very large (future) times  $a(t_E)$  scales linearly with the Einstein-frame cosmological time  $t_E$  [15], and hence there is no cosmic horizon. From a field theory viewpoint, this would allow for a proper definition of asymptotic states and thus a scattering matrix. As we mentioned briefly above, however, from a stringy point of view, there are restrictions in the asymptotic values of the central charge deficit  $Q_0$ , and only a discrete spectrum of values of  $Q_0$  allow for a full stringy  $S$ -matrix to be defined, respecting modular invariance [12]. The Universe relaxes asymptotically to its ground-state equilibrium situation, and the non-criticality of the string caused by the initial fluctuation disappears, yielding a critical (equilibrium) string Universe with Minkowski metric and a linear-dilaton background. This is the generic feature of the models we consider here and in [8], allowing the conclusions to be extended beyond the Type-0

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<sup>6</sup>Notice that in this subsection we work in  $D = 4$  space-time dimensions. For higher-dimensional models, the normalisations given in the Introduction, see (9), should be used.

string theory to incorporate also target-space supersymmetric strings/brane models, such as those in [21, 19].

An important comment is in order at this point, regarding the form of the Einstein metric corresponding to (51):

$$g_{00}^E = -1, \quad g_{ii} = a_E^2(t_E) = \frac{F_1^2}{\gamma^2} + F_1^2 t_E^2. \quad (52)$$

Although asymptotically, for  $t_E \rightarrow \infty$ , the above metric asymptotes to the linearly-expanding Universe (11), the presence of a constant  $F_1^2/\gamma^2$  contribution implies that the solution for large but finite  $t_E$ , such as the current era of the Universe, is different from that of [12]. Indeed, the corresponding  $\sigma$ -model-frame metric (5) is not Minkowski flat, and in fact the pertinent  $\sigma$  model does not correspond to a conformal field theory. This should come as no surprise since, for finite  $t_E$  no matter how large, the  $\sigma$ -model theory requires Liouville dressing. *It is only at the end-point of time/flow  $t_E \rightarrow \infty$  that the underlying string theory becomes conformal, and the system reaches equilibrium.*

The Hubble parameter of such a Universe for large  $t_E$  is

$$H(t_E) \simeq \frac{\gamma^2 t_E}{1 + \gamma^2 t_E^2} = \frac{F_1^2 t_E}{a^2(t_E)}. \quad (53)$$

On the other hand, the Einstein-frame effective four-dimensional ‘vacuum energy density’, defined through the running central-charge deficit  $Q^2$ , upon compactification to four dimensions of the ten-dimensional expression  $2 \int d^{10}x \sqrt{-g} e^{2\Phi} Q^2(t_E)$  in the Einstein frame, is [15]:

$$\Lambda_E(t_E) = 2e^{2\Phi - \sigma_1 - 5\sigma_2} Q^2(t_E) \simeq \frac{2Q_0^2 \gamma^2}{F_1^2 (1 + \gamma^2 t_E^2)} \sim \frac{13.11 \gamma^2}{1 + \gamma^2 t_E^2} \quad (54)$$

in the normalization of (12). Here we used (46) for  $Q$  at large  $t_E$ , approaching its equilibrium value  $Q_0$ , and we have also used (47). Thus, the dark energy density relaxes to zero for  $t_E \rightarrow \infty$ . Notice an important feature of the relaxation form (54), namely that the proportionality constants in front are such that, for asymptotically large  $t_E \rightarrow \infty$ ,  $\Lambda(t_E \rightarrow \infty)$  is independent of the equilibrium conformal field theory central charge  $Q_0$ .

Finally, and most importantly for our purposes here, the deceleration parameter in the same regime of  $t_E$  becomes:

$$q(t_E) = -\frac{(d^2 a_E / dt_E^2) a_E}{(da_E / dt_E)^2} \simeq -\frac{1}{\gamma^2 t_E^2}. \quad (55)$$

*As is clear from (50), (10), this expression can be identified, up to irrelevant constant factors which by normalization are set to one, with the square of the string coupling (10) [8]:*

$$|q(t_E)| = g_s^2. \quad (56)$$

To guarantee the consistency of perturbation theory, one must have  $g_s < 1$ , which can be achieved in our approach if one defines the *present era* by the time regime

$$\gamma^2 \sim \beta^2 t_E^{-2} \quad (57)$$

in the Einstein frame. In view of its relation with the deceleration parameter at late epochs (55),  $q = -1/\beta^2$ , the numerical value of  $\beta^2$  is determined by requiring agreement with the data [10]. As we discuss below, phenomenologically  $\beta^2 = \mathcal{O}(1)$ .

This is compatible with the time  $t_E$  being large enough (in string units) for

$$|C_5|e^{-5s_{02}}/F_1^2 \sim |C_5|e^{-5s_{02}}/Q_0^2 \gg 1, \quad (58)$$

as becomes clear from (45),(49), (47). This condition can be guaranteed *either* for small radii of five of the extra dimensions, *or* for a large value of the flux  $|C_5|$  of the five-form of the Type-0 string, compared with  $Q_0$ . We discuss in the next subsection concrete examples of non-critical string cosmologies, in which the asymptotic value of the central charge  $Q_0 \ll 1$  in string units. Recalling that the relatively large extra dimension in the direction of the flux  $s_{01}$  decouples from this condition, we thus observe that there is the possibility of constructing effective five-dimensional models with a large uncompactified fifth dimension that are consistent with the condition (57). Notice that, in the regime (57) of Einstein-frame times, the Hubble parameter and the cosmological constant continue to be compatible with the current observations, and in fact to depend on  $\gamma \sim t_E^{-1}$  in the same way as in their large- $\gamma t_E$  regime given above (53),(54), but now the string coupling (56) is kept smaller than one and finite, of order 1/2, as also suggested by grand unification phenomenology [1].

We next turn to the equation of state of our Universe. As discussed in [15], it resembles a quintessence model with the dilaton playing the rôle of the quintessence field. Hence the equation of state for our Type-0 string Universe reads [28]:

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{\frac{1}{2}(\dot{\Phi})^2 - V(\Phi)}{\frac{1}{2}(\dot{\Phi})^2 + V(\Phi)}, \quad (59)$$

where  $p_\Phi$  is the pressure and  $\rho_\Phi$  is the energy density, and  $V(\Phi)$  is the effective potential for the dilaton, which in our case is provided by the central-charge deficit term. Here the dot denotes Einstein-frame differentiation. In the Einstein frame, in the normalization of (12), the potential  $V(\Phi)$  is given by

$$V(\Phi) = \frac{\Lambda_E}{4} \sim \frac{6.56\gamma^2}{2(1 + \gamma^2 t_E^2)}, \quad (60)$$

where  $\Lambda_E$  is given in (54) and we have used (47). Defining the present era by the condition (57), we obtain from (44),(48):

$$\frac{1}{2} \left( \frac{d\Phi}{dt_E} \right)^2 = \frac{1}{2}\beta^2 \cdot \tanh^2 \ln \left( \sqrt{1 + \beta^2} + \beta \right) \cdot \frac{1}{t_E^2(1 + \beta^2)}, \quad V(\Phi) \sim \frac{6.56\beta^2}{2(1 + \beta^2)t_E^2}. \quad (61)$$

This implies a *constant* equation of state (59) in the current era:

$$w_\Phi(t_E \gg 1) = \frac{\tanh^2 \ln \left( \sqrt{1 + \beta^2} + \beta \right) - 6.56}{\tanh^2 \ln \left( \sqrt{1 + \beta^2} + \beta \right) + 6.56}. \quad (62)$$

We now remark that, if we use as the value of  $q$  today the one inferred by best fits of FRW cosmology to the data on high-redshift supernovae and the CMB [9, 10]:

$$q_{FRW,data} = -\frac{1}{\beta^2} \simeq -0.57 \quad (\text{today}), \quad (63)$$

this corresponds by (62) to an equation of state with

$$w_\Phi = -0.82, \quad (64)$$

which is in the region allowed by the data [10, 9, 29]<sup>7</sup>. On the other hand, an equation of state  $w_\Phi = -0.78$ , which is the upper bound given by the WMAP data [10, 29], yields a current-era deceleration (55),(57)  $q \sim -0.25$ . All such values of  $|q| < 1$  imply via (56) perturbative values for the string coupling, close to the value used frequently in string-inspired particle phenomenology:  $|q| = g_s^2 \sim 1/2$ .

The inclusion of matter modifies the situation, and allows for a more complete expression of the equation of state in terms of the current acceleration of the Universe for our model. As we discuss in the next subsection, this is different from the analogous relation in conventional FRW cosmologies. However, the function  $q(z)$  is not yet measurable with sufficient accuracy using supernovae alone: in order to infer a precise form of  $q(z)$  from such measurements, one has to make certain assumptions about the underlying dynamics. In conventional FRW cosmologies,  $q(z)$  is expressed in terms of the various energy-density components  $\Omega_i(z)$  using the underlying Einstein cosmology. In a similar vein, in our model  $q(z)$  can be expressed in terms of the various energy-density components, in units of the critical density, using the dynamics encoded in (78). However, due to the off-shell Liouville modifications, the critical density for a spatially flat Universe and the relation of  $q(z)$  to the various energy-density components are different from those in conventional FRW models. Thus the above-used values of  $\beta^2$  should not be taken for granted but only as indicative. For us,  $\beta^2 = 1/|q|$  can only be determined properly after a detailed direct fit of our model to the data. We discuss these issues in the next Section, where we show that the above considerations can be made compatible with the observations that suggest there was a past epoch of deceleration at redshifts larger than 0.5 [30].

These considerations concern the specific model of [15]. One can be more generic when considering equations of state for dilatonic dark energy Liouville models, by simply requiring that the present era is described by a linear dilaton solution (9), asymptotic to a conformal field theory with central charge deficit  $Q^2$ . In this case, the dilaton potential and kinetic energy are given by

$$\begin{aligned} V(\Phi) &= \frac{Q^2}{2} e^{2\Phi} = \frac{2}{t_E^2} e^{2\Phi_0}, \quad \Phi = \Phi_0 - \ln \frac{Q t_E}{2}, \\ \frac{d\Phi}{dt_E} &= \frac{1}{t_E}, \end{aligned} \quad (65)$$

where  $\Phi_0$  is a constant, denoting the initial value of the dilaton field in a generic situation. As we have seen previously (44), this constant is determined in the model of [15] by the values of the flux field and the frozen moduli. In the general situation, where no microscopic model is specified, this constant is free to be determined by phenomenology, as we see below. In such a generic situation, the dilatonic dark-energy equation of state (in the same normalization of (12), reads:

$$w_\Phi(t_E \gg 1) \simeq \frac{1 - 4e^{2\Phi_0}}{1 + 4e^{2\Phi_0}}. \quad (66)$$

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<sup>7</sup>Although the data at present are not sufficient for an accurate measurement of  $w(z)$ , they seem to indicate [29] negative values smaller than  $-0.6$  for  $z \simeq 1$  and  $w(z \rightarrow 0) \rightarrow -1$ .

One can easily obtain phenomenologically acceptable values of  $w_\Phi$  [10] by adjusting the value of the constant  $\Phi_0$ . For instance, for  $e^{2\Phi_0} > \frac{7}{4}$  one obtains:

$$w_\Phi \leq -3/4, \tag{67}$$

in agreement with the WMAP value [10, 29].

Such linear dilaton models can be made compatible with perturbative string couplings  $g_s < 1$ , as required by string-inspired particle-physics phenomenology, provided one chooses the asymptotic central charge  $Q$  in the region

$$Qt_E > \sqrt{7}. \tag{68}$$

Notice that the dark energy (65) is independent of the value of the central charge, and its magnitude today in such models depends only on the age of the Universe. The important question in such models is the precise form of the scale factor, which should be obtained as a specific solution of the appropriate dynamical equations (31), whose form depends on the details of the underlying string theory. To be specific, in what follows we adopt [8] the class of string models that yield predictions similar to those in [15] as best describing the current era of our string Universe.

## 2.5 Inclusion of Matter and Radiation

So far, our model has not included ordinary matter or radiation, as only fields from the gravitational string multiplet have been included. The inclusion of ordinary matter is not expected to change the results significantly, and we conjecture that the fundamental relation (56) will continue to hold, the only difference being that probably the inclusion of ordinary matter will tend to reduce the string acceleration, due to the fact that matter, being subjective to attractive gravity, resists the acceleration of the Universe.

We now discuss in some detail the formalism that allows the inclusion of matter in the Liouville framework. The important thing to notice is that, in the absence of matter, the Liouville-dressing approach of [17], together with the eventual (dynamical) identification of the Liouville zero mode with the target time, as explained above, leads to the generalized conformal invariance conditions (31) for the fields of the gravitational multiplet of the string propagating in a four-dimensional background<sup>8</sup>. These *are not* the ordinary equations of motion corresponding to a four-dimensional gravitational effective action (12), but describe the dynamics of an *off-shell relaxation process*.

Matter coupling to on-shell dilatonic gravity theory has been considered in the past, see, e.g., [31, 32], but in an *on-shell* formalism of critical strings, where the various target-space fields satisfied classical equations of motion derived from a four-dimensional action. As explained above, this is distinct from our Liouville cosmology approach. Moreover, the analysis of [32], although dealing with the possibility of a dilaton playing the rôle of a quintessence field responsible for the current acceleration of the Universe, nevertheless considers models in which the dilaton as well as its potential increases to positive infinity, as the cosmic time elapses. This is exactly opposite to our situation here and in [15, 8], where the dilaton  $\Phi \rightarrow -\infty$  asymptotically. In our situation, for large cosmic times, the string coupling  $e^\Phi \rightarrow 0$ , and this is the reason

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<sup>8</sup>Or five-dimensional, in the case of compactified brane models in a single large bulk dimension.

why in present and future eras of the Universe the string-tree-level approximation is sufficient. This is to be contrasted with the case of [32], where one should include asymptotically (all) higher loop corrections to the string effective action, which are not known at present.

We now discuss in some detail the proper inclusion of matter in our Liouville framework. The essential formalism is that of [12], in which all physically relevant quantities should be reduced to the Einstein frame (5) and Einstein cosmic time (7) framework. Examining the four-dimensional matter action (including radiation fields), we observe that in string theory this action couples to the dilaton field non-trivially, in a way that is specific to the various matter species, as a result of purely stringy properties of the effective action [1]. A generic  $\sigma$ -model-frame effective four-dimensional action with dilaton potential  $V(\Phi)$ , which could even include higher-string-loop corrections, has the form:

$$S^{(4)} = \frac{1}{2\alpha'} \int d^4x \sqrt{-G} [e^{-\Psi(\Phi)} R(G) + Z(\Phi)(\nabla\Phi)^2 + 2\alpha' V(\Phi) \dots] - \frac{1}{16\pi} \int d^4x \sqrt{G} \frac{1}{\alpha(\Phi)} F_{\mu\nu}^2 - I_m(\Phi, G, \text{matter}) , \quad (69)$$

in the notation of [32], with the various factors  $\Psi, Z, \alpha$  encoding information about higher string loop corrections. Also,  $F_{\mu\nu}$  denotes the radiation field strength and  $I_m(\Phi, G, \text{matter})$  represents matter contributions, which couple to the dilaton  $\Phi$  in a manner dictated by string theory scaling laws [1] with shifts of the dilaton field  $\Phi \rightarrow \Phi + \text{const}$ . In our situation, where only the string tree level plays a rôle at late times, the various form factors simplify, e.g.,  $\Psi(\Phi) = 2\Phi, Z(\Phi) = 4e^{-2\Phi}$ , *etc.* However, for purposes of generality, in this section we keep the form (69). When higher loop corrections are important, these factors have a complicated form, for instance one has  $e^{\Psi(\Phi)} = c_0 e^{-2\Phi} + c_1 + c_2 e^{2\Phi} + \dots$ , with  $c_i$  constants, and the powers of the square of the string coupling  $g_s^2 = e^{2\Phi}$  count the numbers of closed string loops, as appropriate for the gravitational multiplet. For simplicity in this subsection we ignore the four-dimensional antisymmetric tensor field, which, as discussed in [12] and mentioned above, corresponds to an axion field.

According to our discussion in Section 2.1, the action  $S^{(4)}$  coincides with the  $I^{(D)} - \mathcal{I}_\phi$  part of the  $D$ -dimensional action (27), obtained from (26) when  $D = 4$ , upon the identification of the Liouville mode  $\phi$  with the target time  $X^0$ :

$$S^{(4)} = I^{(4)} - \mathcal{I}_\phi = \int d^4X e^{-\varphi} \{C^{(4)}(X) - 25\}. \quad (70)$$

In contrast to the critical-string case considered in [31, 32], the field variations of (70) do not yield zero, but are such that they compensate the variations of the remaining (Liouville) part of (26), in order to yield the generalized conformal invariance conditions (21), augmented by the inclusion of matter fields. In particular, the set of couplings  $\lambda^I$  in (26), as well as the action  $C^{(4)}(X)$  (c.f. (69)), should now include matter fields in addition to the fields of the gravitational multiplet of the string. For simplicity, however, we may assume that, at least at the late epochs of the Universe which are of interest to us here, the matter couplings are almost conformal, and the dominant reason for departure from criticality lies in the fields of the gravitational multiplet. This is the case, for instance, in the colliding-brane scenario discussed in the next Section. This leaves the off-shell Liouville part of (26) in the form discussed in the previous Section. This will be understood in what follows.

We have:

$$\frac{\delta S^{(4)}}{\delta g^i} = -\frac{\delta \mathcal{I}_\phi}{\delta g^i}, \quad (71)$$

where  $g^i = (\Phi, G, \dots) \equiv (\Phi, \lambda^I)$  and we took into account the fact that the action  $\mathcal{I}^{(4)} \equiv \mathcal{I}^{(4+1)}|_{\phi=X^0}$  is critical (the identification of the Liouville mode with the target time  $X^0$  is done after the respective variation is taken). Near a fixed point one has  $\frac{\delta S^{(4)}}{\delta \lambda^I} = \ddot{g}_I + Q\dot{g}_I$ , with the overdot denoting differentiation with respect to the Liouville zero mode. When passing to the Einstein frame (5), and expressing the time in terms of the cosmic time  $t_E$  (7), the left-hand side of (21) in the supercritical string case for the graviton fields  $G_{\mu\nu}$  yields:

$$\begin{aligned} \dot{G}_{\mu\nu} &= e^\Phi \left( 2 \frac{d\Phi}{dt_E} g_{\mu\nu}^E + \frac{dg_{\mu\nu}^E}{dt_E} \right), \\ \ddot{G}_{\mu\nu} &= 2 \left( \frac{d\Phi}{dt_E} \right)^2 g_{\mu\nu}^E + 2 \frac{d^2\Phi}{dt_E^2} g_{\mu\nu}^E + 3 \frac{d\Phi}{dt_E} \frac{dg_{\mu\nu}^E}{dt_E} + \frac{d^2 g_{\mu\nu}^E}{dt_E^2}, \\ (0,0) - \text{component} : \quad \tilde{\mathcal{G}}_{00} &\equiv \ddot{G}_{00} + Q\dot{G}_{00} = -2Q \frac{d}{dt_E} (e^\Phi) - 2 \left( \frac{d\Phi}{dt_E} \right)^2 - 2 \frac{d^2\Phi}{dt_E^2}, \quad (g_{00}^E = -1), \\ (i,i) - \text{component} : \quad \tilde{\mathcal{G}}_{ii} &\equiv \ddot{G}_{ii} + Q\dot{G}_{ii} = \\ 2a^2(t_E) &\left( \left( \frac{d\Phi}{dt_E} \right)^2 + 3 \frac{d\Phi}{dt_E} H + \frac{d^2\Phi}{dt_E^2} + (1-q)H^2 \right) + 2Qa^2(t_E)e^\Phi \left( \frac{d\Phi}{dt_E} + H \right), \\ H &\equiv a^{-1}(t_E) \frac{da(t_E)}{dt_E}, \quad q \equiv -\frac{a(t_E) \frac{d^2}{dt_E^2} a(t_E)}{\left( \frac{da(t_E)}{dt_E} \right)^2}, \end{aligned} \quad (72)$$

with  $q$  the deceleration parameter (55). The dilaton variation of the function  $\mathcal{I}_\phi$  of (27), on the other hand, reads:

$$\begin{aligned} \frac{\delta \mathcal{I}_\phi}{\delta \Phi} &\equiv \mathcal{I}'_\phi = 6 \int d^{D-1}X e^{-\varphi} \left( \dot{\varphi}^2 - \ddot{\varphi} + \frac{1}{8}(\dot{\lambda}^I)^2 \right) = \\ 6V^{(3)} &\left\{ \left( 2 \frac{d\Phi}{dt_E} + H \right)^2 - \frac{d\Phi}{dt_E} \left( 2 \frac{d\Phi}{dt_E} + H \right) + \frac{dH}{dt_E} + 2 \frac{d^2\Phi}{dt_E^2} + 2 \left( \frac{d\Phi}{dt_E} \right)^2 + 3H \frac{d\Phi}{dt_E} + \frac{3}{2}H^2 \right\}, \end{aligned} \quad (73)$$

in the notation of (26), where  $V^{(3)}$  denotes the three-dimensional spatial volume. A complete analysis of matter effects requires solving the equations emerging from considering the variations (71),(72) and (73) with respect to the metric field in the Einstein and cosmic time frames [12]. This depends on the specific form of matter action considered.

At this stage we remind the reader of a few crucial technical details on the equivalence of the generalized conformal invariance conditions (31) to target-space dynamical equations. The Zamolodchikov metric in theory space,  $\mathcal{G}_{ij} = z^2 \bar{z}^2 \langle V_i(z) V_j(0) \rangle$ , where  $\langle \dots \rangle$  denotes a  $\sigma$ -model average including Liouville contributions, acts as a link between the  $\sigma$ -model  $\beta$  functions and field variations of the target-space effective actions  $S[g]$ :

$$\mathcal{G}_{ij} \beta^j = \frac{\delta S[g]}{\delta g^i}, \quad (74)$$



where, in what follows, the  $g^i$  denote various background target-space fields other than the dilaton, which is treated separately. To order  $\alpha'$ , standard analysis [33, 17] shows that one can find a renormalization scheme on the world sheet in which the Zamolodchikov metric, in the case of the graviton and dilaton backgrounds we are restricting ourselves here, becomes near a fixed point

$$\mathcal{G}_{ij} = e^{-2\Phi} (\delta_{ij} + \mathcal{O}(g^2)) , \quad \delta_{ij} \rightarrow \frac{1}{2} (-G^{\mu\nu} G^{\alpha\beta} + G^{\mu\alpha} G^{\nu\beta} + G^{\mu\beta} G^{\nu\alpha}) , \quad (75)$$

where  $G_{\mu\nu}$  is a  $\sigma$ -model-frame target-space metric (which, in our case, is four-dimensional after appropriate compactification or restriction on a three-brane). The exponential dilaton term arises from world-sheet zero-mode contributions to the  $\sigma$ -model average at tree level, and includes (linear) Liouville-zero-mode- ( $\phi_0$ -) dependent terms in the non-critical string case. This Liouville dependence is crucial [17] in ensuring the following property for the Zamolodchikov metric:  $d\mathcal{G}_{ij}/d\phi_0 = Q\mathcal{G}_{ij}$ , with  $Q^2$  the central-charge deficit, which guarantees the derivation of the Liouville terms on the left-hand side of (31) from a target-space action, as seen above, and therefore the canonical quantization of the Liouville-dressed couplings/fields  $g^i$  in string-theory space (upon summation over world-sheet topologies).

The form (75) implies that a contraction with a Ricci (or any other symmetric) tensor in target space, which is contained in the graviton  $\beta$  function to  $\mathcal{O}(\alpha')$ , results in an Einstein tensor on the right-hand side of (74), as appropriate for proper target-space dynamics. In a similar manner, upon contraction with (75), one obtains appropriate Einstein-like tensor structures for the Liouville modifications appearing in the left-hand side of (31) for the graviton case. On the other hand, considering the variation with respect to the four-dimensional graviton  $g^i \equiv G_{\mu\nu}$ , and passing to the Einstein-frame (5), we obtain:

$$\frac{\delta S[g]}{\delta G_{\mu\nu}} = e^{-2\Phi} \frac{\delta S[g]}{\delta g_{\mu\nu}^E} , \quad (76)$$

where the precise form of the exponential factor is exclusive to the four target space-time dimensions we consider here. As a result of (75), (76), the exponential factors cancel out in (74). The above results and properties will be understood in what follows.

Defining the Einstein-like tensor

$$\mathcal{J}_{\mu\nu} \equiv \tilde{\mathcal{G}}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}^E (g_E^{\nu\lambda} \tilde{\mathcal{G}}_{\nu\lambda}) , \quad (77)$$

and assuming a normal fluid form for matter or radiation, with stress tensor  $T_\mu^{\nu E} = \text{diag}(-\rho, p\delta_i^j)$  in the Einstein frame (5),(7), we then obtain the following gravitational and dilaton equations

of motion (in units  $M_P^2 = 1/8\pi G_N = 2$ , where  $M_P$  is the four-dimensional Planck constant):

$$\begin{aligned}
6H^2 &= \rho + \rho_\Phi + \mathcal{J}_{00} , \\
4\frac{d}{dt_E}H + 6H^2 &= -p - p_\Phi - a^{-2}(t_E)\mathcal{J}_{ii} , \quad i = 1, 2, 3, \\
\frac{d^2\Phi}{dt_E^2} + 3H\frac{d\Phi}{dt_E} + V'(\phi) + \frac{1}{2}[\Psi'(\Phi)(\rho - 3p) + \sigma + \sigma_\phi] &= 0 , \\
\sigma &\equiv -2\frac{1}{V^{(3)}\sqrt{-g_E}}\frac{\delta(I_m + \int(16\pi\alpha(\Phi))^{-1}F^2)}{\delta\Phi} , \\
\sigma_\phi &\equiv -2\frac{1}{V^{(3)}\sqrt{-g_E}}\mathcal{I}'(\Phi) = -12\left\{4\left(\frac{d\Phi}{dt_E}\right)^2 + 6H\frac{d\Phi}{dt_E} + 2\frac{d^2\Phi}{dt_E^2} + \frac{dH}{dt_E} + \frac{5}{2}H^2\right\} , \\
\rho_\Phi &\equiv \frac{1}{2}\left(\frac{d\Phi}{dt_E}\right)^2 + V(\Phi) , \quad p_\Phi \equiv \frac{1}{2}\left(\frac{d\Phi}{dt_E}\right)^2 - V(\Phi) ,
\end{aligned} \tag{78}$$

where the prime denotes differentiation with respect to  $\Phi$ ,  $\mathcal{I}'_\phi(\Phi)$  is defined in (73), and we use canonically-normalized dilaton fields. Notice that (78) differ from the corresponding on-shell equations in [32] by the Liouville *out-of-equilibrium* contributions  $\mathcal{J}$  and  $\sigma_\phi$ , which are exclusive to our treatment [17, 24, 15].

The equations (78) lead, after standard manipulations, to the coupled (non-) conservation equations of matter and dilaton energy density, in the presence of the non-equilibrium contributions:

$$\begin{aligned}
\frac{d\rho_\Phi}{dt_E} + 3H(\rho_\Phi + p_\Phi) + \frac{1}{2}\frac{d\Phi}{dt_E}[\Psi'(\Phi)(\rho - 3p) + \sigma + \sigma_\phi] &= 0 , \\
\frac{d\rho}{dt_E} + 3H(\rho + p) - \frac{1}{2}\frac{d\Phi}{dt_E}[\Psi'(\Phi)(\rho - 3p) + \sigma + \sigma_\phi] + \left(\frac{d}{dt_E} + 3H\right)\mathcal{J}_{00} + 3Ha^{-2}(t_E)\mathcal{J}_{ii} &= 0 ,
\end{aligned} \tag{79}$$

with the values of  $\mathcal{J}_{00}$  and  $\mathcal{J}_{ii}$  (common for all  $i = 1, 2, 3$  in our case) given by (77),(72). As we see from (79) the covariant conservation of the matter stress tensor (the first three terms in the second of eqs. (79)) breaks down, due not only to the presence of a dilaton field, but also to the off-shell Liouville contributions given by the  $\mathcal{J}$ -dependent terms, which express the non-equilibrium nature of the Liouville cosmology.

To solve (79) in the various epochs of the Universe, it is convenient first to split the energy density of matter as well as the function  $\sigma$  into radiation  $\rho_r$ , baryonic  $\rho_b$  and dark-matter  $\rho_d$  components, and to use the simple equation of state (59) for the dilaton fluid:

$$\begin{aligned}
\rho &= \rho_r + \rho_b + \rho_d \equiv \rho_r + \rho_m , \\
\sigma &= \sigma_r + \sigma_b + \sigma_d \equiv \sigma_r + \sigma_m , \\
p_b = p_d = 0, \quad p_r &= \frac{1}{3}\rho_r, \quad p_\Phi = w_\Phi\rho_\Phi .
\end{aligned} \tag{80}$$

Using (80), one can split the matter evolution equation (second of eqs. (79)) into various

components, which for an expanding Universe can be cast in the form:

$$\begin{aligned}
\frac{d\rho_r}{d\chi} + 4\rho_r - \frac{1}{2} \frac{d\Phi}{d\chi} [\sigma_r + \sigma_\phi] + \frac{d}{d\chi} \mathcal{J}_{00} + 3(\mathcal{J}_{00} + a^{-2}(t_E) \mathcal{J}_{ii}) &= 0 , \\
\frac{d\rho_A}{d\chi} + 3\rho_A - \frac{1}{2} \frac{d\Phi}{d\chi} [\Psi'(\phi) \rho_A + \sigma_A + \sigma_\phi] + \frac{d}{d\chi} \mathcal{J}_{00} + 3(\mathcal{J}_{00} + a^{-2}(t_E) \mathcal{J}_{ii}) &= 0 , \quad A = b, d , \\
\frac{d\rho_\Phi}{d\chi} + 3(1 + w_\Phi) \rho_\Phi + \frac{1}{2} \frac{d\Phi}{d\chi} [\Psi'(\phi) \rho_m + \sigma + \sigma_\phi] &= 0 ,
\end{aligned} \tag{81}$$

with

$$\chi = \ln(a/a_{\text{init}}) = -\ln(1+z) + \ln(a(0)/a_{\text{init}}), \tag{82}$$

where  $z$  is the redshift,  $a_{\text{init}}$  is an initial scale, and  $a(0)$  is the present value of the scale factor, evaluated at redshift zero.

Solving the above equations rigorously is a complicated task, and depends on the details of the matter theory. In general, one may relate [32] the various  $\sigma_i$ ,  $i = r, b, d$  with the corresponding energy densities  $\rho_i$ , through proportionality factors that depend on the dilaton field  $\Phi$ . However, for our purposes we may assume that at the current era of the Universe's evolution the dark matter component dominates over ordinary matter and that the dilaton is approximated by its logarithmic evolution (9) in cosmic Einstein-frame time. We also assume that the current scale factor is also approximately given by the expression (51). These assumptions guarantee that the relation (56) between the (square of) the string coupling and the acceleration of the Universe is valid today.

Using the familiar (model-independent) relation of the scale factor with the redshift  $z$ :  $a(z) = a(0)(1+z)^{-1}$ , we may then determine the region of  $z$  for which the approximation (51) is consistent. Recalling that in our model the current era of the Universe is defined by the relations (57),(55), with  $\beta^2 = -1/q(z=0)$  where  $q(z=0)$  is the acceleration of the Universe today at  $z=0$ , it is straightforward to arrive at:

$$a(0) = (F_1/\gamma)(1 + \beta^2)^{1/2}. \tag{83}$$

Making the (wrong) hypothesis that the formula for (51) is valid all the way down to  $t_E = 0$ , we would then find that  $z$  should lie in the region:

$$0 < z < z_{\text{init}}; \quad z_{\text{init}} = \sqrt{1 + \beta^2} - 1, \tag{84}$$

in order that the form (51) of the space-time metric be valid. For  $q(z=0) = \beta^{-2} = -0.57$  (c.f., (63)), we would then have  $z_{\text{init}} \simeq 0.66$ .

We assume  $\sigma_d \simeq \eta \rho_d$  for the dark matter, with  $\eta$  an approximately constant proportionality factor  $\eta = \mathcal{O}(1)$  for the present and future eras. These assumptions lead to simplifications of some of the equations. With  $\Psi \simeq 2\Phi$  (the string tree-level approximation), one obtains after some elementary algebraic manipulations:

$$\begin{aligned}
\mathcal{J}_{00} &\simeq -\frac{7H}{t_E} + H^2(1-q) \simeq -\frac{6F_1^2}{a^2(t_E)}, \quad \mathcal{J}_{ii} \simeq a^2(t_E) \left( \frac{H}{t_E} - H^2(1-q) \right) \simeq 0, \quad i = 1, 2, 3, \\
\frac{d}{d\chi} \mathcal{J}_{00} + 3(\mathcal{J}_{00} + a^{-2}(t_E) \mathcal{J}_{ii}) &\simeq -\frac{6F_1^2}{a^2(t_E)} = \mathcal{J}_{00} \rightarrow |\mathcal{J}_{00}| \simeq \left( \frac{a_{\text{init}}}{a(t_E)} \right)^2 = \left( \frac{a_{\text{init}}}{a(0)} \right)^2 (1+z)^2,
\end{aligned} \tag{85}$$

for the configuration (9), (51), (53) and (55), assumed to characterize the current era of the Universe.

Equation (81) yields, then, for the dark matter energy density, (which is assumed to dominate the matter sector:  $\rho_m \simeq \rho_d$ ,  $\sigma \simeq \sigma_d$ ):

$$\frac{d\rho_d}{d\chi} + 3\rho_d + \frac{d\rho_\Phi}{d\chi} + 3(1 + w_\Phi)\rho_\Phi + \mathcal{J}_{00} = 0 . \quad (86)$$

This is a rather complicated equation to solve in general <sup>9</sup>. Its solution, in conjunction with the rest of eqs. (78), will provide a scaling for the energy densities of matter and dark energy with the scale factor, which is modified in general.

A simplification can occur, however, if one concentrates on the epoch of large cosmic time (present era), and uses the asymptotic behaviour of the dilaton dark energy density  $\rho_\Phi$ , dictated by (60), (61) <sup>10</sup>, i.e.,  $\rho_\Phi \sim \rho_\Phi^0 a^{-2}(t_E)$ . Assuming then mixed scaling behaviours

$$\rho_d \sim \rho_{\text{dust}}^0 a^{-3} + \rho_{\text{exotic}}^0 a^{-2} , \quad (87)$$

where the first term is compatible with dust properties, and the second expresses the entanglement with the off-shell Liouville environment, and the value (64) for the equation of state, in agreement with recent WMAP data [10, 29], we observe that (86) is satisfied, provided

$$\rho_{\text{exotic}}^0 = 6F_1^2 + 1.46\rho_\Phi^0 . \quad (88)$$

Note that, for the model of [15]),  $\rho_\Phi^0$  is in most cases also of order  $F_1^2$  (c.f., (61), which in turn is of the order of the square of the asymptotic central charge (47)). The latter can be very small, e.g., of order  $10^{-60}$  in string units, in models [18] involving compactification on magnetized tori (c.f. (107), (109) below), which guarantees compatibility of the order of the magnetically-induced target-space supersymmetry breaking with realistic phenomenological considerations. In this way,  $\rho_{\text{exotic}}^0$  can be very small, and hence both terms in  $\rho_d$  may be of comparable magnitude today. Specifically, from (61) it follows that  $\rho_\Phi \simeq 3.78 \frac{F_1^2}{a^2}$ , which implies that  $\rho_\Phi^0 \simeq 3.78 F_1^2$ . Thus, (88) would yield in that case:

$$\rho_{\text{exotic}}^0 \simeq 3.25\rho_\Phi^0 . \quad (89)$$

Thus, we may write for the (dark) matter energy density today

$$\rho_d^0 \simeq \rho_{\text{dust}}^0 + \lambda\rho_\Phi^0 , \quad 0 < \lambda = \mathcal{O}(1 - 10) . \quad (90)$$

We stress once more that this mixed scaling in the matter energy density is due not only to the entanglement with the dilaton, but also to the non-trivial rôle of the off-shell Liouville

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<sup>9</sup>Notice that most of the complications arise from the presence of the off-shell Liouville modifications  $\mathcal{J}_{00}$ . In their absence, i.e., in ‘conventional’ dilaton cosmologies, one can solve this equation straightforwardly and obtain the standard scaling for the various energy-density components  $\rho_d \sim \rho_d^0 a^{-3}$ ,  $\rho_\Phi \sim \rho_\Phi^0 a^{-3(1+w_\Phi)}$ , with  $\rho_d^0 + \rho_\Phi^0 \simeq 1$  in the case of dominant dark matter. This is no longer true when  $\mathcal{J}_{00} \neq 0$ , and, as we shall see below, one obtains in that case a mixed scaling for the matter energy density.

<sup>10</sup>Notice that, despite the  $a^{-2}$  scaling of  $\rho_\Phi$  and the off-shell Liouville term  $\mathcal{J}_{00}$ , none of these contributions is equivalent to a (negative) curvature contribution. This is due to the fact that the dilaton dark energy and the off-shell Liouville modifications enter the relevant dynamical equations (78) in a different manner than the curvature term. This is consistent with the fact that our brane/string Universe is spatially flat by construction.

$\mathcal{J}$ -dependent contributions. This is an exclusive feature of our non-equilibrium Liouville-string approach to cosmology [17, 8, 15], which does not apply in conventional on-shell treatments [31, 32]. It has its roots in viewing target time as a world-sheet renormalization-group dynamical scale in non-critical string theory, which is a cornerstone of our approach.

As for the dilaton, a form linear in the  $\sigma$ -model frame is assumed throughout, which in turn determines the time dependences of the quantities  $\sigma, \sigma_\phi$  via the respective dilaton equation (78). Since the form of matter action is not in general fully known in our generic low-energy considerations, and depends on the details of the underlying microscopic string/brane model, we do not analyse this equation further here. The existence of self-consistent solutions to the graviton equations (78) including matter and radiation, inferred by the above analysis, justifies *a posteriori* our assumption that the important relation (56), which is based on the solution [8, 15] (52), (50) for the space-time metric and the dilaton fields, survives the inclusion of matter.

We now present the formalism for fitting our Liouville Cosmology to cosmological data in a rather model-independent way. Consider the first of the Einstein equations (78) for our Liouville cosmology. In the present era, we may assume the following asymptotic behaviour for the Liouville part  $\mathcal{J}$  (c.f. (85)):  $\mathcal{J}_{00} \simeq -6H^2(1 - q)$ . From the first of eqs. (78), we may then conclude that, as a result of the Liouville out-of-equilibrium contributions, the *critical* total mass (energy) density of the fluid,  $\rho_c$  required to have a spatially flat Universe, is no longer  $6H^2$ , as in the conventional on-shell Einstein cosmologies, but

$$\rho_c = 6H^2(2 - q). \quad (91)$$

One may then define modified  $\Omega'_i$  fractions:

$$\Omega' \equiv \frac{\rho_i}{\rho_c} = \frac{\rho_i}{6H^2(2 - q)}, \quad i = \text{matter, dilaton } \Phi \text{ dark energy } \textit{etc.} \quad (92)$$

With this definition the first of equations (78) would imply the standard relation for a spatially flat Universe today:

$$\Omega'_{\text{Matter}} + \Omega'_\Phi = 1. \quad (93)$$

Notice that the critical density (91) scales with  $a = a(0)(1 + z)^{-1}$  as:

$$\rho_c(z) = \frac{6F_1^2}{a^2} (2 - \xi(1 + z)^2), \quad \xi \equiv \frac{F_1^2}{\gamma^2 a^2(0)} = \frac{1}{1 + \beta^2}, \quad \beta^2 = -1/q(z = 0), \quad (94)$$

for the (rather generic) string model of [15] used here, where we took into account (83). The reader should also recall that  $0 < z < 0.66$  for the validity of the approximations leading to the above analysis. Thus, we have the following scaling with the redshift:

$$\begin{aligned} \Omega'_\Phi(z) &= \frac{\rho_\Phi^0}{6F_1^2(2 - \xi(1 + z)^2)} \equiv \Omega'_\Phi{}^0 \frac{2 - \xi}{2 - \xi(1 + z)^2}, \\ \Omega'_{\text{Matter}} &= \frac{1}{6F_1^2(2 - \xi(1 + z)^2)} \left( \frac{\rho_{\text{dust}}^0}{1 + z} + \lambda \rho_\Phi^0 \right), \end{aligned} \quad (95)$$

where  $\lambda = \mathcal{O}(1 - 10)$  depending on model details, and  $\Omega'_\Phi{}^0$  denotes the corresponding quantity today, i.e. at  $z = 0$ .

Alternatively, one may use the first of eqs. (78) to express the critical density in terms of the various energy-density components instead of the parameter  $\xi$ . This allows a more convenient model-independent formalism to be used for comparison with data. In this way, using the mixed scaling (87) for the matter sector (including the dominant dark matter) of our spatially flat Universe,  $\rho_{\text{Matter}} = \rho_{\text{dust}}^0(1+z)^3 + \rho_{\text{exotic}}^0(1+z)^2$  with  $\rho_{\text{dust}}^0 + \rho_{\text{exotic}}^0 + \rho_{\Phi}^0 = \rho_c^0$ , one obtains the following scaling of  $\Omega'_i$  with the redshift  $z$ :

$$\begin{aligned}\Omega'_{\Phi}(z) &= \frac{1}{1 + \rho_{\text{Matter}}/\rho_{\Phi}} = \frac{1}{1 + \frac{\Omega'_{\text{dust}}{}^0}{\Omega'_{\Phi}{}^0}(1+z) + \frac{\Omega'_{\text{exotic}}{}^0}{\Omega'_{\Phi}{}^0}}, \\ \Omega'_{\text{Matter}}(z) &= 1 - \Omega'_{\Phi}(z) = \frac{1}{1 + \frac{\Omega'_{\text{dust}}{}^0}{\Omega'_{\Phi}{}^0}(1+z) + \frac{\Omega'_{\text{exotic}}{}^0}{\Omega'_{\Phi}{}^0}}.\end{aligned}\tag{96}$$

These expressions can be used to fit the astrophysical data and derive values for the cosmological parameters of our Q-cosmology model.

Since the dilaton, matter and radiation energy densities scale differently, there is a (past) era of this Liouville Universe, corresponding to redshifts larger than a critical value,  $z > z^*$ , in which matter effects dominate over the dilaton dark energy, leading to a decelerating phase of the Universe. In fact, such a past early era when there was deceleration of the Universe was present also in the model of [15], even in the purely gravitational and moduli sector. Such a feature is simply pronounced by the inclusion of matter, since the latter feels the attractive feature of gravity. The critical  $z^*$  is shifted from an early era in the purely gravitational case of [15]) towards the current epoch:  $z^* \rightarrow \sim \mathcal{O}(1)$ , as a result of the inclusion of matter in the model. The past deceleration in our Universe is a feature confirmed by astrophysical data [10, 30], which indicate a value  $z^* \sim 1/2$ . However, our model is too generic, at this stage, to claim a specific prediction for  $z^*$ .

### 3 Concrete Non-critical String Examples: Colliding Branes

The above considerations are rather generic for models which relax asymptotically to the linear-dilaton conformal field theory solutions of [12], and from this point of view are physically interesting. We have not yet specified the microscopic theory underlying the deviation from criticality. For this purpose, one needs specific examples of such deviations from the conformal invariant points in string theory space. One such example with physically interesting consequences is provided by a colliding-brane-world scenario, in which the Liouville string  $\sigma$  model describes stringy excitations on the brane worlds for relatively long times after the collision, so that string perturbation theory is valid. This Section is devoted to a detailed discussion of such a scenario [18, 21].

#### 3.1 Example I: Colliding Type-IIB Five-Branes

We now concentrate on particular examples of the previous general scenario [24], in which the non-criticality is induced by the collision of two branes, as seen in Fig. 2. We first discuss the basic features of this scenario. For our purposes below we assume that the string scale is of the same order as the four-dimensional Planck scale. However, this is an assumption which can be

relaxed in view of recent developments in strings with large compactification directions, as was mentioned in the Introduction.

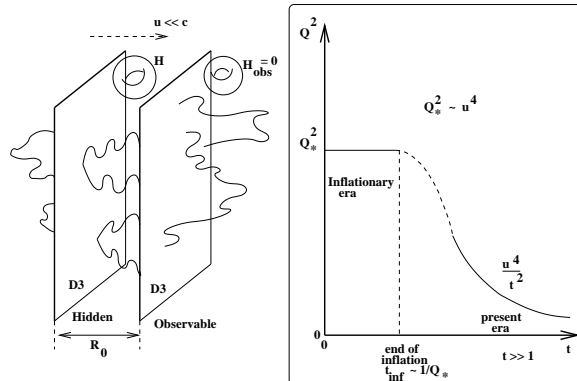


Figure 2: *A scenario in which the collision of two Type-II five-branes provides inflation and a relaxation model for cosmological vacuum energy.*

Following [18], we consider two five-branes of Type-II string theory, in which the extra two dimensions have been compactified on tori. On one of the branes (assumed to be the hidden world), the torus is magnetized with a field intensity  $\mathcal{H}$ . Initially our world is compactified on a normal torus, without a magnetic field, and the two branes are assumed to be on a collision course with a small relative velocity  $v \ll 1$  in the bulk, as illustrated in Fig. 2. The collision produces a non-equilibrium situation, which results in electric current transfer from the hidden brane to the visible one. This causes the (adiabatic) emergence of a magnetic field in our world.

The instabilities associated with such magnetized-tori compactifications are not a problem in the context of the cosmological scenario discussed here. In fact, as discussed in [18], the collision may also produce decompactification of the extra toroidal dimensions at a rate much slower than any other rate in the problem. As discussed in [18], this guarantees asymptotic equilibrium and a proper definition of an  $S$ -matrix for the stringy excitations on the observable world. We come back at this issue at the end of this Section.

The collision of the two branes implies, for a short period afterwards while the branes are at most a few string scales apart, the exchange of open-string excitations stretching between the branes, where their ends are attached. As argued in [18], the exchanges of such pairs of open strings in Type-II string theory result in an excitation energy in the visible world. The latter may be estimated by computing the corresponding scattering amplitude of the two branes, using string-theory world-sheet methods [34]: the time integral for the relevant potential yields the scattering amplitude. Such estimates involve the computation of appropriate world-sheet annulus diagrams, due to the existence of open string pairs in Type-II string theory. This implies the presence of ‘spin factors’ as proportionality constants in the scattering amplitudes, which are expressed in terms of Jacobi  $\Theta$  functions. For the small brane velocities  $v \ll 1$  we are considering here, the appropriate spin structures start at *quartic order* in  $v$ , for the case of identical branes, as a result of the mathematical properties of the Jacobi functions [34]. This in turn implies [18, 21] that the resulting excitation energy on the brane world is of

order  $V = \mathcal{O}(v^4)$ , which may be thought of as an initial (approximately constant) value of a *supercritical* central-charge deficit for the non-critical  $\sigma$  model that describes stringy excitations in the observable world after the collision:

$$Q^2 = \left( \sqrt{\beta} v^2 + \mathcal{H}^2 \right)^2 > 0, \quad (97)$$

where, in the model of [21, 19], the proportionality factor  $\beta$ , computed using string amplitude computations, is of order

$$\beta \sim 2\sqrt{3} \cdot 10^{-8} \cdot g_s, \quad (98)$$

with  $g_s$  the string coupling, which is of order  $g_s^2 \sim 0.5$  for interesting phenomenological models [1, 5]. The supercriticality, i.e., the positive definiteness of the central charge deficit (97) of the model, is essential [12] for a time-like signature of the Liouville mode and hence its interpretation as target time.

At times long after the collision, the branes slow down and the central charge deficit is no longer constant but relaxes with time  $t$ . In the approach of [18], this relaxation has been computed by using world-sheet logarithmic conformal field theory methods [35], taking into account recoil (in the bulk) of the observable-world brane and the identification of target time with the (zero mode of the) Liouville field. In that work it was assumed that the final equilibrium value of the central-charge deficit was zero, i.e., the theory approached a critical string. This late-time varying deficit  $Q^2(t)$  scales with the target time (Liouville mode) as follows (in units of the string scale  $M_s$ ):

$$Q^2(t) \sim \frac{(\mathcal{H}^2 + v^2)^2}{t^2}. \quad (99)$$

Some explanations are necessary at this point. In arriving at (99), one identifies the world-sheet renormalization group scale  $\mathcal{T} = \ln(L/a)^2$ , where  $(L/a)^2$  is the world-sheet area, which appears in the Zamolodchikov  $c$ -theorem used to determine the rate of change of  $Q$  with  $\mathcal{T}$ , with the zero mode of a normalized Liouville field  $\phi_0$ , such that  $\phi_0 = Q\mathcal{T}$ . This normalization guarantees a canonical kinetic term for the Liouville field in the world-sheet action [13]. Thus,  $\phi_0$  is identified with  $-t$ , where  $t$  is the target time. This will always be understood in what follows.

On the other hand, in other models [15] that we discuss below, the asymptotic value of the central-charge deficit may not be zero, in the sense that the asymptotic theory is that of a dilaton field that is linear in time, with a Minkowski metric in the  $\sigma$ -model frame [12]. This theory is still a conformal model, but the central charge is a constant  $Q_0$ , and in fact the dilaton is of the form  $\Phi = Q_0 t + \text{const}$ , where  $t$  is the target time in the  $\sigma$ -model frame. Conformal invariance, as already mentioned previously, suggests [12] that  $Q_0$  takes on one of a *discrete* set of values, in the way explained in [12]. In such a case, following the same method as in the  $Q_0 = 0$  case of [18], one arrives at the asymptotic form

$$Q^2(t) \sim Q_0^2 + \mathcal{O}\left(\frac{\mathcal{H}^2 + v^2}{t} Q_0\right) \quad (100)$$

for large times  $t$ .



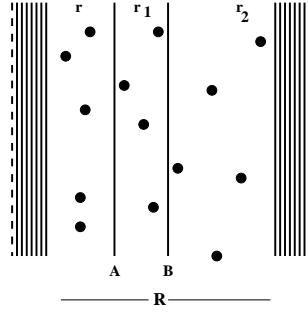


Figure 3: A model for supersymmetric  $D$ -particle foam consisting of two stacks each of sixteen parallel coincident  $D8$ -branes, with orientifold planes (thick dashed lines) attached to them [21]. The space does not extend beyond the orientifold planes. The bulk region of ten-dimensional space in which the  $D8$ -branes are embedded is punctured by  $D0$ -branes ( $D$ -particles, dark blobs). The two parallel stacks are sufficiently far from each other that any Casimir contribution to the vacuum energy is negligible. If the branes are stationary, there is zero vacuum energy, and the configuration is a consistent supersymmetric string vacuum. To obtain excitations corresponding to interesting cosmologies, one should move one (or more) of the branes from each stack, let them collide (Big Bang), bounce back (inflation), and then relax to their original position, where they collide again with the remaining branes in each stack (exit from inflation, reheating).

### 3.2 Example II: Orientifold/Eight-Brane/ $D$ -particles Colliding-Brane Model

The colliding-brane model of [18] can be extended to incorporate proper supersymmetric vacuum configurations of string theory [21]. As illustrated in Fig. 3, this model consists of two stacks of  $D8$ -branes with the same tension, separated by a distance  $R$ . The transverse bulk space is restricted to lie between two orientifold planes, and is populated by  $D$ -particles. It was shown in [21] that, in the limit of static branes and  $D$ -particles, this configuration constitutes a zero vacuum-energy supersymmetric ground state of this brane theory.

The bulk low-energy effective theory in such configurations is known to be the ten-dimensional Type-IIA supergravity, whose bosonic part is given in the string frame by [2]:

$$S = S_{NS} + S_R + S_{CS}, \quad (101)$$

where

$$S_{NS} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left( R + 4|\nabla_\mu \Phi|^2 - \frac{1}{2}|H_3|^2 \right), \quad (102)$$

$$S_R = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left( |F_2|^2 + |\tilde{F}_4|^2 \right), \quad (103)$$

$$S_{CS} = -\frac{1}{4\kappa_{10}^2} \int B_2 \wedge dC_3 \wedge dC_3, \quad (104)$$

in standard notation. However, in order to incorporate eight-dimensional branes, one needs actually a modified version of the Type-IIA supergravity, which we now proceed to describe

briefly, along with its possible compactifications, since we are eventually interested in four space-time dimensional theories of phenomenological interest.

### 3.3 Dual Formulation of Type-IIA Supergravity

Type-IIA string theory contains all even-dimension D-branes from zero to eight dimensions  $p$  [2].  $Dp$ -branes couple to R-R  $p + 1$ -forms, but the action for Type-IIA supergravity only contains 1-form (D0-brane) and 3-form (D2-brane) gauge potentials - how are the other D-branes incorporated into the action? In [36], a *dual* formulation of Type-IIA supergravity was constructed, which contains higher-dimensional R-R potentials and hence allows objects like the D8-brane to be incorporated. The dual Type-IIA supergravity allows for the construction of Type-IA supergravity, and has the action:

$$\begin{aligned}
S_{\text{bulk}} = & -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\Phi} [R(\omega(e)) + 4(\partial\Phi)^2 + \frac{1}{2}H \cdot H - 2\partial^\mu\Phi\chi_\mu^{(1)} + H \cdot \chi^{(3)} + \right. \\
& + 2\bar{\psi}_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar{\lambda}\Gamma^\mu\nabla_\mu\lambda + 4\bar{\lambda}\Gamma^{\mu\nu}\nabla_\mu\psi_\nu] + \sum_{n=0,1,2} \frac{1}{2}G^{(2n)} \cdot G^{(2n)} + G^{(2n)} \cdot \Psi^{(2n)} + \\
& - \star \left[ \frac{1}{2}G^{(4)}G^{(4)}B - \frac{1}{2}G^{(2)}G^{(4)}B^2 + \frac{1}{6}G^{(2)^2}B^3 + \frac{1}{6}G^{(0)}G^{(4)}B^3 - \frac{1}{8}G^{(0)}G^{(2)}B^4 + \right. \\
& \left. \left. + \frac{1}{40}G^{(0)^2}B^5 + e^{-B}\mathbf{G}d(A^{(5)} - A^{(7)} + A^{(9)}) \right] \right\} + \text{quartic fermionic terms}, \quad (105)
\end{aligned}$$

in conventional notation [36]. However, this dual formulation only describes branes of dimension 4, 6 and 8 because of the problem of consistently introducing all of the available R-R forms. A *democratic* formulation was also constructed, which contained all potentials, but this version has no proper action [36]. Since the D-foam model of [21] contains D8-branes, O8-planes and D0-branes, along with fundamental strings when the D0-branes are between an odd number of D8-branes, neither the dual nor the democratic action is appropriate. One needs a combined action, which has been constructed in [37]:

$$\begin{aligned}
S = & -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\Phi} [R + 4(\partial\Phi)^2 + \frac{1}{12}(H^{(3)})^2] + \frac{1}{2}(G^{(0)})^2 + \frac{1}{2}(G^{(2)})^2 - \star[G^{(0)}dA^{(9)}] \right\} \\
& - T_8(n-8) \int d^{10}x \left( e^{-\Phi} \sqrt{|G_{(9)}|} + \alpha \frac{1}{9!} \epsilon^{(9)} A^{(9)} \right) [\delta(x^9) - \delta(x^9 - \pi R)] \\
& - T_2 \int d^{10}x \left[ \left( e^{-\Phi} \sqrt{-G_{tt}} - bA_t \right) + \frac{e^{-\phi}}{\sqrt{G_{zz}}} \left( \sqrt{-G_{tt}G_{zz}} - \frac{a}{2!} \epsilon^{\nu\lambda} B_{\nu\lambda} \right) \right] \left[ \sum_k N_k \delta^8(\vec{x} - \vec{x}_k) \right], \quad (106)
\end{aligned}$$

where  $\vec{x}$  denotes an eight-dimensional vector,  $G^{(n)}, A^{(n)}$  are appropriate gauge flux fields,  $G$  denotes the ten-dimensional  $\sigma$ -model-frame target-space metric,  $G_{00}, G_{zz}$  are the temporal and bulk components of this metric respectively, and  $G_{(9)}$  is a nine-dimensional metric. The second line describes the D8-branes and orientifold planes and the third the combined action for the D0-brane and fundamental strings. These brane-bulk actions describe all of the dynamics relevant to the branes of interest.

### 3.4 Towards Realistic Compactifications

To compactify such an action, e.g., on  $T^4/\mathbb{Z}_2$ , the fields which survive the orbifold projection must be determined. An example of this for D6-O6 branes is given in [38]. Once the remaining field content is found, calculation of the dimensionally-reduced Bianchi identities then leads to the lower-dimensional effective potential and corresponding superpotential. Instead of a normal Kaluza-Klein dimensional reduction, Scherk-Schwarz fluxes [39] can be added [40], which have the advantage of allowing a greater range of vacua: see [41, 38, 42] and references therein <sup>11</sup>.

The overall structure of Type IA string theory/supergravity is  $\mathcal{M}^9 \times S^1/\mathbb{Z}_2 I_9 \Omega$ , corresponding to an orbifold of the Type-IIA theory with an orientifold projection in the ninth dimension. A realistic compactification would result in either a Randall-Sundrum type scenario [4], i.e., a 3-brane embedded in five dimensions, or a conventional intersecting D-brane model [43], with the unusual feature of using D8-branes instead of D6-branes [44]. For Randall-Sundrum-II (RS-II) scenarios, it has been suggested [36] that a metric product of  $AdS_5$  and some Euclidean 5-manifold would give 3-branes in 5-dimensional Minkowski space, with the bulk solution being uplifted from [45] <sup>12</sup>.

The range of compactification choices can be summarized as follows:

- Compactify D8-O8 on  $AdS_5 \times \mathcal{M}^5$  to get an RS-II scenario [36],
- Compactify on appropriate torii to get an intersecting brane model [44],
- Compactify on  $K3 \times S^d$ , giving another intersecting brane model based upon Calabi-Yau manifolds [43].

The important requirement is to obtain  $D = 4$ ,  $\mathcal{N} = 1$  supersymmetry on the brane, which in the (compactified) model of [21] would correspond to the static D-brane/D-particle configuration. We recall that Type-IIA supergravity has 32 supersymmetries [2]. In the the model of [21], the bulk space has  $\mathcal{N} = 2$ , and on the brane there is [36]  $\mathcal{N} = 1$ . It should be noted that, for simplicity in this case, we are assuming that all of the branes are located on the orientifolds, and not in the bulk. Toroidal compactification of supergravity does not break any supersymmetries, so compactification on  $T^5$  would give  $D = 5$ ,  $\mathcal{N} = 4$  supersymmetry in the bulk, with  $\mathcal{N} = 2$  on the brane. Changing this to  $T^5/\mathbb{Z}_2$  breaks half of the supersymmetries resulting in  $\mathcal{N} = 2$  in the bulk and  $\mathcal{N} = 1$  on the brane. More complicated compactifications would change the precise way in which the supersymmetries are broken, as in the example suggested by [36], where the metric is the product of  $AdS_5$  and a Euclidean 5-manifold. This would also result in  $D = 4$ ,  $\mathcal{N} = 1$  on the brane. Inspired by the analysis on the colliding five-brane model of [18], in which the five-branes were compactified on magnetized tori to yield three-brane worlds with broken supersymmetry, as a result of internal magnetic fields, it would be desirable to discuss similar magnetized compactifications for the eight-branes of [21]. This may be subtle due to the presence of the orientifolds, but some progress has already been made in this direction [49].

In this work we deal no further with the important issue of compactification, but postpone a detailed analysis to a future publication.

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<sup>11</sup>As we saw in the generic analysis of [15], reviewed above, such flux fields play an important rôle in ensuring the stabilisation of large bulk dimensions. A similar scenario is envisaged for the compactified version of the eight-brane model of [21].

<sup>12</sup>We recall that the strong-coupling limit of Type-IA string theory is equivalent to the solution of Horava and Witten (HW) [46, 47, 48].

## 3.5 Supersymmetry Breaking and Vacuum Energy in the Post-Inflationary Era

We now discuss briefly issues related to the supersymmetry breaking that would result from brane motion in the model.

### 3.5.1 Supersymmetry Breaking via Internal Magnetic Fields

In the colliding-brane scenario of [19], which uses the orientifold configuration of [21] shown in Fig. 3), one may imagine that the exit from inflation and the reheating phase corresponds to a second collision, when the moving brane world returns to its initial position and hits the original stack of branes again. In such a case the recoil velocity of the brane world vanishes, but one may still have a magnetic field  $\mathcal{H}$  on the brane world, corresponding to a contribution to the four-dimensional energy density on the brane of order  $\mathcal{H}^2$ , for compactification radii of the extra dimensions of order  $M_s$ . One may identify therefore

$$Q_0^2 \sim \mathcal{H}^2 \quad (107)$$

in the  $\sigma$ -model frame, leading to a dilaton of the form  $\Phi \sim \mathcal{H}t + \text{const}$ . For consistency with the results of [12], one would then discretize  $\mathcal{H}$  with one of the values dictated by conformal invariance in this asymptotic  $\sigma$  model.

An important issue that we have already mentioned, but would like to stress again, is that in the Einstein frame the constants in the expression for the dilaton are such that the dark-energy density (c.f., (54) below) relaxes to zero with the cosmic time  $t_E$  in the Einstein frame (c.f. (48) below) as  $1/t_E^2$ , in a manner independent of the magnitude of  $Q_0$ . In this way, the magnitude of the supersymmetry breaking in target space induced by the presence of  $\mathcal{H}$  [34, 18] may be large enough to be of phenomenological interest, whilst the observed value of the vacuum energy may be acceptably small, as we now explain.

The reason why the magnetic field  $\mathcal{H}$  in the extra dimensions [18] breaks target-space supersymmetry [34] is that bosons and fermions on the brane worlds couple differently to  $\mathcal{H}$ . This is nothing other than a Zeeman-type energy-splitting effect. In our problem, where the magnetic field is turned on adiabatically, the resulting mass difference between bosonic and fermionic string excitations is found to be [18]:

$$\Delta m_{\text{string}}^2 \sim 2q_e |\mathcal{H}| \cosh(\epsilon\varphi + \epsilon t) \Sigma_{45}, \quad (108)$$

where  $q_e$  is the electric charge,  $\Sigma_{45}$  is a standard spin operator in the plane of the torus, and  $\epsilon \rightarrow 0^+$  is the regulating parameter of the Heaviside operator  $\Theta_\epsilon(t) = -i \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i\epsilon} e^{i\omega t}$  appearing in the D-brane recoil formalism [35]. The dependence in (108) implies that the formalism selects dynamically a Liouville mode which flows opposite to the target time  $\varphi = -t$ , as mentioned earlier, as a result of minimization of the effective field-theoretic potential of the various stringy excitations.

In the scenario of [18], where the dilaton remains constant asymptotically in time, the mass splitting (108) with  $\varphi = -t$  is the only contribution to supersymmetry breaking as far as excitations are concerned. Since in that scenario the dark energy in target space relaxes to zero asymptotically (99) while the mass splittings remain finite, provided  $\mathcal{H}$  remains constant one has a supersymmetry *obstruction* [16], rather than breaking, on the brane world, since the

cosmological constant of the vacuum state is still zero, as required by a supersymmetric theory, but the excitation spectrum is not supersymmetric. By choosing appropriately

$$q_e|\mathcal{H}| \sim 10^{-30} \quad (\text{in string - scale units}), \quad (109)$$

we may arrange for the supersymmetry-breaking/obstruction scale to be of the order of a few TeV. Such contributions would therefore be significantly subdominant, compared with the velocity contribution, in the expressions (97), (99) during the inflationary era. (We recall that phenomenological analyses such as those in [19, 21], yield recoil velocities as large as  $v \sim \mathcal{O}(10^{-1})$  towards the end of inflation.) However, we note here that if the string scale is itself of the order of a few TeV, then  $q_e|\mathcal{H}|$  in (108) may be chosen of order one in string units in order to reproduce supersymmetry breaking at TeV scales.

There is an issue with the scenario of [18] concerning the recoil velocity of the brane worlds after inflation. In [18] it was assumed that the brane worlds eventually stop moving in the bulk, as a result of gravitational radiation, i.e., emission of closed strings from the brane towards the bulk. This would imply that there were no velocity-dependent contributions to the supersymmetry breaking in the bulk asymptotically. We stress that, if the branes are moving relative to each other, the asymptotic vacuum energies on the brane world are non-zero, but depend on some power of the recoil velocity (in the case of identical recoiling branes this is  $v^4$  [21]), breaking the (bulk) supersymmetry.

On the other hand, in scenarios with an asymptotically linear dilaton one has [12], as a result of the presence of the background charge  $Q$ , *tachyonic shifts*  $-Q^2$  in the masses of *bosons*, while the fermion masses remain *unaffected*. Such shifts induce *additional* contributions to supersymmetry-breaking mass splittings (108) asymptotically<sup>13</sup>:

$$\Delta m_{\text{susy-br}}^2 \sim q_e|\mathcal{H}| + O(\mathcal{H}^2), \quad (110)$$

and in this type of breaking one has [34]  $\text{Str}m^2 = 0$ , where  $\text{Str}$  denotes the supertrace.

In the scenario of [19], one uses the supersymmetric vacuum configuration of Fig. 3, where one or more of the branes of one stack collide with branes of the other stack before returning to their original position, where they collide for a second time, and eventually stop. The end of the inflationary era in this framework corresponds to this *second collision*. We assume for simplicity that there is only one collision between the Big Bang and the exit from inflation, where our brane world collides with its original stack of branes and stops. This second collision results in a phase transition and *reheats* the Universe, as a result of entropy production due to the collision. This second collision is much milder than the initial one, because the recoiling brane world may lose energy not only via its collisions with D-particles in the foam, but also due to gravitational radiation, i.e., closed string emission in the bulk. The precise mechanism for reheating is still open: one possible contribution is the gravitational collapse of the bulk D-particles in the model of [21] into black holes, due to distortions of their populations following the second collision. Evaporation of such bulky black holes on the brane worlds would result in Hawking radiation, represented by open string excitations attached to the brane worlds, thereby contributing to reheating.

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<sup>13</sup>Note that, since during inflation the dilaton remains constant, there are no extra shifts in the boson masses due to the central charge deficit in that era.

In such scenarios, the asymptotic value of  $Q^2 \sim \mathcal{H}^2$ , since there is no recoil velocity of the brane after the second collision. For the order of magnitude of the magnetic field chosen above (109), such contributions are negligible compared with the Zeeman mass splittings (108). Moreover, for such values of the magnetic field, the equilibrium central charge (107) is of order  $Q_0^2 \sim 10^{-60}$  (in string units), and the value of  $\gamma$  in such a Universe is compatible with the current-era condition (57), provided (c.f., (49),(45), (58)):  $|C_5|e^{-5s_{02}} \sim 1$  (in string units), which is a natural value for the flux field in the model of [15]. This guarantees a present-era vacuum energy (99) of the observed order, compatible with a phenomenologically-viable scenario for supersymmetry-breaking mass splittings (110). On the other hand, if the string scale is of the order of a few TeV, then the age of the Universe today is  $t_E \sim 10^{44}$  in string units, and since  $Q_0^2 \sim \mathcal{H}^2 \sim 1$  in these units, one needs very large five-dimensional fluxes  $|C_5|e^{-5s_{02}} \sim 10^{44}$  to ensure the condition (57).

The basic features of the low-energy limit of the non-supersymmetric Type-0 string theory that we used in [19] and in [15], can be extended appropriately to the supersymmetric brane/orientifold compactification model of [21], without affecting the basic characteristics of the model, such as the existence of one large extra dimension, the presence of flux bulk fields, tachyons and extra moduli fields which freeze out quickly, and play no rôle in the phenomenology of the Universe in the present era. As far as tachyons are concerned, these fields existed in Type-0 string theory as a result of the explicit breaking of supersymmetry due to projecting the partners out of the string spectrum. In the supersymmetric models of colliding-brane worlds [21, 19], the motion and collision of the brane worlds breaks supersymmetry explicitly, both on the branes and in the bulk, as a result of the non-zero relative velocities. This also results in tachyonic excitations in the string spectrum, reflecting the instability of the configuration. This instability is essential in cosmological situations, such as the one we encounter here. The same analysis as in [15] can then be performed for the bosonic sector of the low-energy field theory in this case, to demonstrate the existence of solutions of cosmological relevance, in which the tachyon fields decouple quickly, leading to a similar late-stage analysis and results like those in [15, 8], as reviewed in the previous Section.

We would like to call the reader's attention to one final point. As mentioned above, magnetized toroidal compactifications are known to have Nielsen-Olesen instabilities [34]. It may well happen [18], therefore, that as a result of the collision(s) a *decompactification* process takes place at a rate slower than any other time scale in our physical Universe, which implies, however, that the compactification radius  $R \rightarrow \infty$  asymptotically in cosmic time, whilst the magnetic field energy  $\mathcal{H}^2 R^p$ , for  $p$  compact dimensions on the brane worlds, remains *finite*. This would imply vanishing magnetic fields asymptotically, and hence restoration of supersymmetry.

However, the compactification on magnetized internal manifolds is not the only way for supersymmetry to be broken in such cosmologies. As already mentioned, in the models of [21] the motion of the brane world constitutes another source of breaking of supersymmetry. Moreover, as we also discuss below, the thermalization of the bulk and brane worlds soon after the collision could in principle result in yet another (independent) contribution to supersymmetry breaking. However, the finite recoil velocity of the colliding brane world and the temperature will be related, and hence there will be only one independent type of supersymmetry breaking in the scenario of [21]<sup>14</sup>.

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<sup>14</sup>Notice that the model of [21] does not involve compactification, and hence the considerations on phases with

### 3.5.2 Moving Branes and Supersymmetry Breaking

The colliding-brane scenario can be realized [19] in this framework by allowing (at least one of) the D-branes to move, keeping the orientifold planes static. One may envisage a situation in which the two branes collide, at a certain moment in time corresponding to the Big Bang - a catastrophic cosmological event setting the beginning of observable time - and then bounce back. The width of the bulk region is assumed to be long enough that, after a sufficiently long time following the collision, the excitation energy on the observable brane world - which corresponds to the conformal charge deficit in a  $\sigma$ -model framework [18, 21] - relaxes to tiny values.

It is expected that a ground state-configuration will be achieved when the branes reach the orientifold planes again (within stringy length uncertainties of order  $\ell_s = 1/M_s$ , the string scale). In this picture, since observable time starts ticking after the collision, the question how the brane worlds started to move is merely philosophical or metaphysical. The collision results in a kind of phase transition, during which the system passes through a non-equilibrium phase, in which one loses the conformal symmetry of the stringy  $\sigma$  model that describes perturbatively string excitations on the branes. At long times after the collision, the central charge deficit relaxes to zero [18], indicating that the system approaches equilibrium again. The dark energy observed today may be the result of the fact that our world has not yet relaxed to this equilibrium value. Since the asymptotic ground state configuration has static D-branes and D-particles, and hence has zero vacuum energy as guaranteed by the exact conformal field theory construction of [21, 19], it avoids the fine-tuning problems in the model of [18].

Thus, the bulk motion of either the D-branes or the D-particles <sup>15</sup> results in non-zero ‘vacuum’ (or, rather, ‘excitation’) energy [21], and hence the *breaking of target-space supersymmetry*, proportional to some power of the average (recoil) velocity squared, which depends on the precise string model used to describe the (open) stringy matter excitations on the branes. Sub-asymptotically, there are several contributions to the excitation energy of our brane world in this picture. One comes from the interaction of the brane world with nearby D-particles, i.e., those within distances at most of order  $\mathcal{O}(\ell_s)$ , as a result of open strings stretched between them. The other contribution comes from the collision of the identical D-branes.

A detailed analysis, using world-sheet methods for the computation of the various potentials felt by the D-branes/D-particles in the colliding-brane model of [21], yields two types of effective potentials. One is a potential in the bulk space, felt by closed-string excitations from the gravitational multiplet that are allowed to propagate in the bulk. The bulk potential is given by:

$$\mathcal{V}_{\text{sym}} \simeq V_8 \frac{(30R - 64r)v^4}{2^{13}\pi^9\alpha'^5} - N \left(\frac{v}{\alpha'}\right)^{1/2}, \quad (111)$$

where the distances  $R, r$  are defined in Fig. 3,  $v$  is the recoil velocity of our brane world, and  $N$  is the number of D-particles near the moving brane world, which are the only type of D-particles that contribute significantly to the potential [21]. A symmetric configuration of branes has been considered in Fig. 3 for concreteness and simplicity. For a sufficiently dilute gas of

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broken supersymmetry pertain to eight-dimensional brane worlds, moving in the ninth bulk dimension. Upon subsequent compactification it is possible to have additional sources of supersymmetry breaking, including the ones associated with possible internal magnetic fields, as discussed elsewhere.

<sup>15</sup>The latter could arise from recoil effects following scattering with closed-string states propagating in the bulk.

nearby D-particles, one may assume that this latter contribution is the dominant one. In this case, one may ignore the D-particle/D-brane contributions to the vacuum energy, and hence apply the previous considerations on inflation, based on the  $\mathcal{O}(v^4)$  central charge deficit, with  $v$  the velocity of the brane world in the bulk.

The other type of potential, generated in the moving-brane scenario of [21], is an effective potential felt by the brane world itself, as a result of its interactions with the other branes and D-particles. This second type of potential is felt by the open-string excitations whose ends are attached to the brane, which constitute the Standard Model matter and radiation, living on the brane world. The brane potential is [21]:

$$V_{\text{brane}} = -V_8 \frac{31(R - 2r)v^4}{2^{13}\pi^9\alpha'^5}, \quad (112)$$

where  $r_1 = R - 2r$  denotes the relative separation of the branes in the symmetric situation of Fig. 3 with  $r_2 = r$ . Notice that the potential is negative, which expresses the fact that the brane world feels an attractive force towards its original stack, and the configuration is stabilized when  $v \rightarrow 0$ . In Section 4 we return to a physical interpretation of the above potentials, which determine the various phases of our early Q-cosmology.

From the point of view of the low-energy bulk action (102) and (106), the bulk potential (111) would correspond to a non-zero contribution to the scalar potential of the Type-IIA supergravity theory, proportional to a central charge deficit:  $e^{-2\Phi}Q^2$ , expressing the non-criticality of the associated  $\sigma$  model describing bulk string excitations. The fact that the potential (111) changes sign, depending on the value of  $r$ , will lead to a rich phase structure, as we discuss in Section 4. However, due to the fact that we consider here a brane excitation, the system does not sit at a global minimum of the potential, but rather in a local (metastable) extremum. We return to this important point in Section 4, when we discuss the various phases of the bulk theory. As we show there, the compactified Type-IIA theory may not be characterized by such a global minimum as a result of purely stringy properties (lack of certain T-duality symmetries [22]).

For the effective low-energy of the open-string excitations on the brane, a similar excitation ‘vacuum energy’ is provided by the potential (112), but with subtleties because its value is always negative. As we discuss in Section 4, this may be interpreted as thermalization of the brane world, throughout the inflationary period and its exit phase (this mechanism is an alternative to the usual description of reheating). Moreover, the presence of matter on the brane world causes back-reaction onto the space-time, along the lines discussed earlier. Matter is assumed to satisfy classical equations in an effective four-dimensional supergravity field theory on the brane world. There are of course subtleties associated with specific compactification scenarios, which we do not discuss here.

A final comment concerns the rôle of the D-particles in the above models. The presence of these space-time defects, which inevitably cross the D-branes as the latter move in the bulk, even if the D-particle defects are static initially, distorts slightly [50] the inflationary metric on the observable brane world at early times after the collision, during an era of approximately constant central charge deficit. However, this effect does not lead to significant qualitative changes. Moreover, the existence of D-particles on the branes affects the propagation of string matter on the branes, in the sense of modifying their dispersion relations by inducing local curvature in space-time, as a result of recoil following collisions with string matter. However, it



was argued in [51] that only photons are susceptible to such effects in this scenario, due to the specific gauge properties of the membrane theory at hand. The dispersion relations for chiral matter particles, or in general fields on the D-branes that transform non-trivially under the Standard Model gauge group, are protected by special gauge symmetries in string theory, and as such are not modified.

## 4 Finite Temperature in the Liouville Framework

We now discuss thermalized strings in the context of our Liouville formalism, and describe the thermal phase diagram of our early Universe.

### 4.1 Brane Collisions and Hot Universes

In our colliding-brane scenario, each brane collision thermalizes the string excitation spectrum on the brane worlds and in the bulk, as a result of the conversion of the kinetic energy of the moving branes into thermal energy. In the scenario with two moving colliding branes of [21] (c.f., Fig. 3), the string excitations may be thermalized immediately after the collision. Indeed, as the detailed computations of [21] have shown, the effective potential of the configuration when the two branes lie a distance  $r_1 = R - 2r$  apart (with the symbols as in Fig. 3, restricting ourselves to the symmetric case  $r_2 = r$  for simplicity) is given by (111).

We observe from (111) that for a *sufficiently dilute gas* of D-particles, the potential is positive for  $r \ll R/2$ . This implies that, for relatively long times after the collision when the distance of our brane world from its original stack satisfies the above constraint  $r \ll R/2$ , closed-string excitations in the bulk feel this positive vacuum energy, which means that the corresponding  $\sigma$  model is *supercritical*. It must therefore be dressed by a time-like Liouville field, which is eventually identified with the target time. In fact, in the analysis of [18], for reasons associated with the convergence of the world-sheet path integral, we considered the initial time coordinate  $X^0$  (before Liouville dressing) as *space-like (Euclidean time)*. This was important, because dressing with a time-like Liouville field implied a  $(D + 1)$ -dimensional target-space metric (in our normalization here) [18]:

$$ds_{D+1}^2 = -2(d\varphi)^2 + (dX^0)^2 + d\vec{x}^2. \quad (113)$$

Upon the identification (37)  $\varphi = -t$ , where now  $t = X^0$  is a Euclidean target time, one obtains a *Minkowski-signature*  $D$ -dimensional space-time in a dynamical way. Although in [18] we viewed the use of Euclidean time merely as a mathematical peculiarity of the world-sheet path integral, it may be given a physical meaning in the context of the colliding-brane scenarios, as follows.

Assuming that the adiabatic analysis of [21] is valid soon after the initial collision, and ignoring again the contributions from D-particles, assuming them sufficiently dilute, we observe that in that early epoch of the Universe the potential (??) is negative, since in that era  $30R/64 < r < R/2$  (c.f., Fig. 3). The closed-string excitations find themselves described by a *subcritical*  $\sigma$  model, which can become critical upon Liouville dressing by a *space-like* Liouville mode. This correspond to *thermalization* as a result of the collision, during which the initial kinetic energy of the D-branes is transformed into thermal energy. In fact, if we assume that the initial relative

velocity of the D-branes is of the same order as the recoil velocity, which in [21, 19] was estimated to be of order  $10^{-3} < v < 10^{-1}$  in units of  $c = 1$ , then we observe that the induced temperature  $\frac{1}{2}\mathcal{M}v^2 \sim k_B T$ , where  $\mathcal{M}$  is the D-brane mass, could be close to the Hagedorn temperature of the corresponding string theory,  $T \simeq T_H \sim \frac{1}{2\pi\sqrt{2\alpha'}}$  in order of magnitude<sup>16</sup>. Thus, during the collision phase, the branes and the bulk (closed) string excitations find themselves at a finite (high) temperature.

It is interesting to describe the stringy excitations under such conditions. In what follows we review the rôle of the Liouville formalism in describing generic strings at finite temperature. We commence our analysis with the heterotic string case, which is the simplest and among the most interesting cases for phenomenology.

## 4.2 Liouville Approach to Finite-Temperature Strings: the Case of Heterotic Strings

Historically [22], there has been interest in obtaining a description of a hot, stable phase of strings at temperatures beyond the Hagedorn phase transition at  $T_H \sim 1/2\pi\sqrt{\alpha'}$ . In our case, we are interested in the description of strings much below such high temperatures. However, it is instructive for our purposes to review first the Hagedorn phase, as studied for heterotic strings in [22]. We then return to the brane model of [21], characterized by a bulk low-energy Type-IIA effective supergravity theory in the next Section.

The easiest approach to discussing strings at finite temperature  $T$  is to compactify the time direction on a circle of radius  $R = 1/\pi T$  and discuss the mass spectrum of the winding modes of the string. Using appropriate T-dualities the authors of [22] have discussed the instabilities arising from the fact that some of these T-winding modes of the string become tachyonic above the Hagedorn temperature of a gas of strings. This defines the high-temperature phase of strings, and in our case we could identify it with the epoch soon after the initial collision, where the separation between the branes is small.

The presence of a non-zero temperature leads in general to additional contributions to supersymmetry breaking beyond the ones discussed so far. An important result [52] in the context of strings is that  $D$ -dimensional superstrings at finite temperature look like  $(D - 1)$ -dimensional superstrings with *spontaneously* broken supersymmetry. Restricting our attention to the (bulk) closed string winding sector, this observation implies [22] that the corresponding low-energy effective supergravity field theory is characterized by a non-zero (negative) value of the (global) minimum of its corresponding scalar potential, proportional to the square of the gravitino mass:

$$V_{\min} = -2m_{3/2}^2\kappa^{-2} = -1/2S\kappa^{-2}, \quad (114)$$

where  $S = e^{-\Phi}$  denotes the dilaton field in the supergravity multiplet, and  $\kappa$  is the (ten-dimensional) gravitational constant. From a stringy  $\sigma$ -model viewpoint, such a minimum corresponds to the propagation of strings in a space-time with a tree-level non-constant cosmological term, providing a runaway-dilaton potential. A detailed analysis of heterotic superstrings in high-temperature phases has been performed in [22], where it was shown that there exists a conformal field theory description of this high-temperature phase, corresponding to a  $\sigma$  model where the central charge has been lowered by four units.

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<sup>16</sup>Slight differences in the proportionality factors occur between the various string theories.

Indeed, the conformal field theory is nothing other than the strongly-coupled Liouville theory [13], which is not yet very well understood. This Liouville conformal theory corresponds in target space to a *subcritical* superstring whose  $\sigma$ -model-frame metric is the flat Minkowski one:  $G_{\mu\nu}^{\sigma} = \eta_{\mu\nu}$ , with a dilaton linear in a *space-like* coordinate, playing the rôle of the Euclidean compactified time  $\Phi = QX^0$ , where  $Q$  is a background charge. The space-like nature of the Liouville mode (temperature) is due to the fact that there is a central-charge *deficit* and not a *surplus* as in the time-like Liouville case of [12, 17] discussed in previous Sections. The physical metric, corresponding to a canonically normalized Einstein term in the effective action, is again given by (5), and the corresponding target-space effective action by (12). From the cosmological term of this action, and its identification with the the minimum of the supergravity scalar potential (114), one can compute the central-charge deficit  $\delta c \propto Q^2 < 0$  of the conformal theory (paying particular attention to the appropriate normalization factors [22]), essentially by identifying it with the numerical coefficient of  $1/S$  in the expression (114):

$$Q^2 = \frac{\delta c}{8\alpha'} = -\frac{1}{2\alpha'} < 0, \quad (115)$$

implying a central charge deficit:  $\delta c = -4$  for the superstring. Taking into account the fact that for flat  $\sigma$ -model-frame metric backgrounds  $\delta c = D - 10$ , where  $D$  is the space-time dimensionality of the free superstring, we therefore observe that in the high-temperature phase the thermalized closed-string system corresponds to a non-critical superstring living in 5+1 dimensions. In such a phase it was remarked in [22] that five-branes condense. Such features may turn out to be quite important for cosmological model building. For instance, this would imply that in the original model of colliding five-branes of [18], immediately after the collision one would have condensation of the five-branes, which does not happen in the model of [21].

An important feature of such finite-temperature superstrings is the existence of a space-like supersymmetry at a perturbative level which characterizes the hot phase [22]. Indeed, before Liouville dressing, the finite temperature contributes to supersymmetry breaking mass shifts between the bosonic ( $M_B$ ) and fermionic ( $M_F$ ) excitations of the corresponding supergravity theory:

$$(M_B)_{i\bar{j}}^2 = (M_F)_{i\bar{j}}^2 - m_{3/2}^2 \delta_{i\bar{j}} \quad (116)$$

in the mass-matrix notation of [22]. After the Liouville dressing by the space-like linear dilaton, the fermion masses remain unaffected, but the boson masses undergo mass shifts [12]. However, this time, due to the subcriticality of the string, which is to be contrasted with the case of [12], the mass shifts are not tachyonic, but real:

$$\delta(M_B)^2 = Q^2 = m_{3/2}^2, \quad \delta(M_F)^2 = 0 \quad (117)$$

in our normalization. These mass shifts are additive to (116), which implies that *at a perturbative level* supersymmetry is restored in this hot phase of strings. In our case, therefore, this means that, at a perturbative level, supersymmetry breaking is still be given by the magnetic terms as described above.

The supersymmetry is however broken, or rather *obstructed* [16] at a *non-perturbative* level, due to the fact that masses in three space dimensions produce conical singularities, and as such they break supersymmetry at the level of the excitation spectrum, although the vacuum may still be supersymmetric. Such non-perturbative breaking has been discussed explicitly

in [22], and we do not discuss it further here. We mention, though, that this corresponds to an instability of the high-temperature phase, because this breaking of supersymmetry produces tachyonic states.

The string must leave this unstable phase and re-enter a phase where such instabilities eventually disappear, and the string system relaxes to an equilibrium situation (with unbroken supersymmetry, modulo the effects of the magnetic field, if compactification on magnetized manifolds is considered). We now discuss how this may be understood from a world-sheet view point, in the context of our cosmological model of colliding branes presented in [21]. Since the effective low-energy theory in the bulk in this model is Type-IIA supergravity, we first review a finite-temperature analysis of this special theory, with the aim of repeating the analysis of [22] for this case. There are important physical differences, however, associated with the lack of global minima in Type-IIA theories, which we outline in due course. We commence our analysis with a finite-temperature study of the effective Type-IIA supergravity theory, which characterizes the low-energy bulk dynamics in the model of [21].

### 4.3 Type-IIA Supergravity at Finite Temperature

As already mentioned, in [21] we have a system of D8-branes and orientifolds, in the configuration known as Type-IA string theory. Between the two stacks of D8-branes, the bulk space corresponds to Type-IIA supergravity. When some of the D8-branes move into the bulk, the overall bulk potential induced by this motion can become negative [21], which can be interpreted as the system moving into a finite-temperature phase. Finite-temperature field theory is realized by the Euclidean compactification of the time dimension, and its effects can be calculated using the Scherk-Schwarz mechanism [39].

When one of the D8-branes from each stack moves into the bulk, there are two potentials which must be taken into account. First there is the bulk potential (??), describing the overall energy of the system, where the potential is positive as long as the distance between the brane and its originating stack,  $r$ , is less than  $15R/32$ . Secondly there is the potential on the moving brane itself (as we are dealing with a symmetric case, we consider the left-hand brane). This case is more complex and will be discussed later on.

As stressed above, the important result when considering-finite temperature supergravity is that  $D$ -dimensional superstrings at finite temperature look like  $(D - 1)$ -dimensional superstrings with spontaneously broken supersymmetry [52, 22]. Thus, spontaneous supersymmetry breaking via the Scherk-Schwarz mechanism is equivalent to considering the system at finite temperature. The Scherk-Schwarz mechanism [39] works by generalizing the standard dimensional reduction procedure [53], in which all of the fields are taken to be independent of the compact coordinates. Instead, the fields are given a specific dependence on the internal coordinates of the compact manifold, namely twisting the boundary conditions of the compact dimensions by a global symmetry of the action. This twist induces a shift in the mass terms of the lower-dimensional fields.

In finite-temperature QFT [54], bosons are periodic and fermions anti-periodic in the compact Euclidean time dimension [22]:

$$\Phi(t + 2L\pi R) = (-)^{La}\Phi(t), \tag{118}$$

where for a  $2\pi$  rotation  $L = 1$ , and  $a = 0, 1$  for bosons and fermions respectively. The modular

invariance of Type II string theory requires further constraints to be placed on the periodicity conditions [52, 22, 54]:

$$(-)^{aL+bn} \tag{119}$$

where  $m, n$  are winding numbers and  $a$  and  $b$  are the fermionic spin structures along the world-sheet torus. The resulting shift in the lattice momenta along the compact coordinate is <sup>17</sup>:

$$p_{L,R} = \frac{m + a/2}{R} \pm \frac{nR}{2}, \tag{120}$$

with an additional sign factor of  $(-)^{ab}$  which reverses the GSO projection in the odd winding-number sectors. By redefining  $m$ ,  $a$  can be identified with the  $D$ -dimensional helicity operator  $\mathcal{Q} = \mathbb{Z} + a/2$ , so that  $(D - 1)$ -dimensional thermal states are mapped to a supersymmetric theory on  $S^1$  without the momentum shift (120). The helicity vector  $\vec{\mathcal{Q}} = (\mathcal{Q}_L, \mathcal{Q}_R)$  is defined in terms of the left- and right-moving string helicities, and the vector  $\vec{e} = (1, 1)$  for the Type II string. The inner product is Lorentzian,  $\vec{A} \cdot \vec{B} = A_L B_L - A_R B_R$ .

Thus there is a mapping between the  $(D - 1)$ -dimensional supersymmetric theory with quantum numbers  $(n, m, \mathcal{Q})$  and the  $(D - 1)$ -dimensional thermal theory, which results in the quantum numbers of the supersymmetric theory being shifted:

$$\begin{pmatrix} n \\ m \\ \mathcal{Q} \end{pmatrix} \longrightarrow \begin{pmatrix} n \\ m + \mathcal{Q} \cdot e + \frac{1}{2} n e \cdot e \\ \mathcal{Q} - n e \end{pmatrix}. \tag{121}$$

Clearly, all of the previously massless fermionic states with  $n = m = 0$  have their masses shifted to non-zero values, which means that supersymmetry is broken, with a supersymmetry-breaking mass

$$m_{3/2} = \frac{\mathcal{Q} \cdot e}{R}. \tag{122}$$

In the case of Type-IIA string theory, the vector product  $\mathcal{Q} \cdot e = 1/2$  [22], giving a supersymmetry-breaking mass of  $m_{3/2} = 1/(2R)$ . As already noted, a  $D$ -dimensional theory at finite temperature is equivalent to a  $(D - 1)$ -dimensional theory with broken supersymmetry, so the radius  $R$  can be identified with the temperature of the system,  $2\pi R = T^{-1}$ , giving

$$m_{3/2} = \pi T, \tag{123}$$

from which it is clear that supersymmetry is restored when  $R \rightarrow \infty$ , i.e., at zero temperature.

#### 4.4 Effective Potentials in Type-IIA Supergravity

The spontaneously broken  $(D - 1)$ -dimensional theory can be used in certain cases to determine the value of the gravitino mass, via minimization of the scalar potential. As discussed in [22], for the case they considered of  $D = 5$  heterotic theory at temperature, a global minimum was found which could be used to calculate  $m_{3/2}$  in terms of the dilaton field. For the case of Type-IIA strings, a similar analysis was performed, but the form of the scalar potential was such that there was no global minimum.

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<sup>17</sup>The following discussion is taken from [22].

This is understandable in view of the duality symmetries which occur at finite temperature. At finite temperature the heterotic string possesses a duality which relates the original Hagedorn temperature to an upper Hagedorn temperature, above which the tachyon disappears [54, 55]:

$$R \rightarrow \alpha'/R, \quad T \rightarrow (4\pi^2 \alpha' T)^{-1}. \quad (124)$$

This temperature duality of the heterotic string is directly related to a duality in the scalar potential found by [22], the existence of which appears to determine the existence of the global minimum. For the five-dimensional Type-IIA string theory considered in [22], there is no such duality, thus no global minimum. These considerations, however, concern the compactified theories<sup>18</sup>. In the compactified Type-IIA case, the scalar potential assumes the form:

$$V = -\frac{1}{S} \cdot a(Z, \Omega, \dots) \propto -m_{3/2}^2, \quad (125)$$

where  $S = e^{-\Phi}$  is the dilaton, and the (positive) function  $a(Z, \Omega, \dots)$ , with  $Z, \Omega, \dots$  appropriate moduli fields in the supergravity multiplet, is given in [22] for the D=5 case. As discussed in [22], minimization with respect to the  $\Omega$  field leads to a runaway potential in the  $Z$  direction, thereby leading to the absence of a global minimum, in accordance with the above-mentioned duality argument.

The absence of a global minimum is not an unwelcome situation for the cosmological model of [21], where the collision of branes causes an excitation of the brane world, which no longer sits at its stable minimum and becomes metastable. The excitation energy is determined in this case by the bulk potential (??), which in turn is identified with the central charge deficit of an appropriate non-critical  $\sigma$  model, describing (perturbative) string (bulk) excitations.

## 4.5 Colliding-Brane Scenario, Non-Critical Strings and Effective Potentials in Type-IIA Theories

We now examine the previous case in some detail, with the aim of understanding from a world-sheet framework the various hot and cold phases of the theory.

### 4.5.1 Thermal Type-IIA Phase following the Collision

We return to the colliding brane scenario described in [21], in particular in the phase shortly after the first collision in the configuration of Fig. 3, when the relative separation  $r_1$  of the colliding branes is

$$r_1 \leq \frac{R}{16}. \quad (126)$$

In this region the bulk effective potential (??) is negative.

In the colliding-brane scenario [21], we are dealing essentially with a *non-equilibrium* situation. The bulk potential (??), therefore, should *not* be viewed as indicating a minimum value of a superpotential of the low-energy supergravity theory in the bulk. Indeed, as we discussed

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<sup>18</sup>The spontaneously broken supersymmetric 9-dimensional effective theory, representing the ten-dimensional supergravity at finite temperature, has a scalar superpotential proportional, as usual, to the square of the gravitino mass.

in the previous Section, the effective potential of Type-IIA supergravity does not have a global minimum. Instead, we view the potential (??) as a *non-equilibrium* excitation energy of the vacuum due to the collision of the brane worlds. From the point of view of the low-energy effective theory this is a *metastable vacuum* (local minimum), which is potentially interesting in the cosmological context considered here.

Following the analysis in [21], we may associate the negative potential (??) with a central charge deficit  $Q^2 = C - c^* < 0$  of a *subcritical*  $\sigma$  model describing (perturbative) string bulk excitations in this hot phase. The analysis of [21] assumed configurations of the bulk D-particles that were sufficiently dilute that the dominant contribution to the central-charge deficit, identified as the ten-dimensional energy density corresponding to the potential (??), is:

$$|Q^2| \simeq 1.2 \cdot 10^{-8} v^4 g_s^2, \quad (127)$$

where  $g_s$  is the string coupling, and  $v$  is the brane-world recoil velocity, which is constrained by WMAP [10] data to be at most of order [21, 19]:  $v \leq 0.8$  for the symmetric model of colliding branes of [21], as depicted in Fig. 3. This last relation is obtained upon compactifying (formally) the model into one large dimension along the ninth (bulk) direction, and five small directions of order  $\sqrt{\alpha'}$  <sup>19</sup>.

Since, according to [52],  $D$ -dimensional strings at finite temperature are equivalent to  $(D - 1)$ -dimensional strings with spontaneously broken supersymmetry, we may view the effective target-space supergravity theory corresponding to the hot phase of the colliding branes of [21] as living in 9 target dimensions, and corresponding to the effective action of a non-critical string with an anti-de-Sitter (negative) cosmological constant whose magnitude is given by (46). The pertinent nine-target-dimensional  $\sigma$ -model theory needs Liouville dressing [13] to restore conformal symmetry, but with a *space-like* Liouville mode. The pertinent dressed  $\sigma$  model is characterized by a flat Minkowski target-space metric  $G_{\mu\nu} = \eta_{\mu\nu}$  and a background dilaton linear in the Liouville coordinate [12], which is viewed as a Euclidean time  $X_E^0$ :

$$\Phi = -\frac{1}{2} Q X_E^0. \quad (128)$$

The corresponding target space of the dressed theory is again ten-dimensional:  $(9, X_E^0)$ , and the corresponding effective action in the  $\sigma$ -model frame is given by

$$S_{\sigma\text{-frame}} = \int_0^\beta dX_E^0 d^9 X \sqrt{G} e^{-2\Phi} (R - Q^2 + 4(\nabla_\mu \Phi)^2 + \dots), \quad (129)$$

where  $\beta = 1/2\pi T$  is the inverse temperature, which should be compared with the appropriate parts of (102). We see that the difference from (102) is the presence of a dark energy term proportional to  $e^{-2\Phi} Q^2$ , which plays the rôle of a non-zero contribution to the appropriate scalar potential, and is responsible for supersymmetry breaking. Additional contributions/modifications will result from compactification, but for the purposes of this Section we restrict ourselves to the uncompactified thermal case.

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<sup>19</sup>The compactification issue is a non-trivial one in our case, and the resulting four-dimensional supergravity may present complications. For our purposes here we only present generic qualitative arguments, postponing a detailed compactification analysis for a future publication.

The equations of motion obtainable from this action are equivalent to the conformal invariance conditions of the Liouville-dressed ten-target-dimensional stringy  $\sigma$  model. The dilaton equation (equivalently the vanishing of the ten-euclidean-dimensional dilaton  $\beta$  function) reads [1]:

$$R + 4(\partial_\mu \Phi)(\partial^\mu \Phi) - 4\Box\Phi = Q^2, \quad (130)$$

from which we see that the linear-dilaton background [12] (128) in a flat  $\sigma$ -model-frame target metric satisfies this equation, as expected from the fact that the Liouville dressing restores the conformal symmetry. This implies that this background is at least a *local minimum* of the action. Upon compactification of the type IIA theory, we know from the work of [22] that there may be no global minimum, thereby making the above-mentioned extremum of the action a metastable vacuum. This is a welcome fact, because this will lead to the cosmological evolution of our brane world, and its eventual exit from this hot phase.

One may go one step further, and derive a relation between the temperature of the hot phase and the recoil velocity by requiring a perturbative space-like supersymmetry between bosonic and fermionic degrees of freedom, as in the heterotic string case (116),(117). Indeed, since we *have postulated* that the string theory describing the excitations in the bulk of this situation is a *subcritical Liouville* string [17, 12], we know from the generic analysis of [12] that in such a non-critical string the bosonic masses will acquire a shift by  $Q^2$ , as compared with the  $Q = 0$  case, while the fermion masses remain unshifted (c.f., (117)). We now *require* that there should be no supersymmetry breaking at the perturbative level in the bulk theory, exactly as happens in the heterotic string case. We base this postulate on duality symmetries between the heterotic and Type-IIA theories. It means that the finite-temperature mass shift of the gravitino (116) should compensate the Liouville shift (117). This would result to a restoration of a bulk space-like supersymmetry, at the perturbative level. From the point of view of the original model of [21], this supersymmetry restoration would be compatible with the anti-de-Sitter nature of the bulk geometry in the regime where the effective potential (??) is negative.

From (123) and (117), then, we may determine a relationship between the central charge deficit  $Q^2$  of the Liouville  $\sigma$  model, describing bulk string excitations, and the temperature  $T$ . Furthermore, as we mentioned above, the analysis of [21] relates the central charge deficit to the brane recoil velocity  $v$  (127). The result of such an analysis is therefore:

$$m_{3/2}^2 = \pi^2 T^2 = Q^2 \simeq 1.2 \cdot 10^{-8} v^4 g_s^2. \quad (131)$$

From the point of view of the spontaneously-broken nine-dimensional target-space theory, this gravitino mass is proportional to the scalar potential at a local minimum. Upon compactification of the theory, this minimum is not a global one, as can be seen by an analysis similar to that of [22], mentioned previously. The metastable vacuum state of the thermal vacuum of the compactified Type IIA theory can then be found by solving the appropriate dilaton equation. The cleanest method is to use the equation of motion in the Einstein frame (5), where the gravitational curvature term in the effective action has a canonical normalization. The pertinent dilaton equation in a conformally flat target-space background reads:

$$4\Box\Phi = -Q^2 e^{2\Phi}. \quad (132)$$

The solution of such equations (of the compactified theory), together with the Einstein equations, determines the metastable thermal Type-IIA vacuum corresponding to our case, with



an excitation energy proportional to  $Q^2 < 0$ , given in (in magnitude) by (127). Notice that a dilaton of the form (9) in the Euclidean-time Einstein frame satisfies the above equation for  $D = 4$  uncompactified dimensions. The non-trivial issue in the higher-dimensional case of [21] is to find, upon compactification, the dependence of a dilaton satisfying (132) on the radii of the compact dimensions/moduli [22]. We do not consider this issue further here.

We now remark that the equality (131) allows us to determine a recoil-velocity dependence of the temperature of the early phase of the brane Universe after the collision, in the adiabatic situation considered here:

$$T \sim 10^{-4} 2\sqrt{2} g_s v^2 / (2\pi\sqrt{2\alpha'}) \leq 1.28 \cdot 10^{-4} T_H, \quad (133)$$

where  $T_H = 1/2\pi\sqrt{2\alpha'}$  is the respective Hagedorn temperature, and we assumed standard weakly-coupled strings with  $g_s^2 \sim 1/2$ . The fact that the temperature turns out to be proportional to  $v^2$  is in agreement with the arguments given above on the transformation of most of the (non-relativistic) kinetic energy of the colliding branes into thermal energy, in the adiabatic approximation we use here.

We see from (133) that this early phase of the brane Universe, soon after the collision, could be characterized by quite a high temperature, up to  $10^{13}$  GeV, if we accept that a typical string scale corresponds to an energy of  $10^{18}$  GeV [1]. Of course, the above estimate has been obtained by saturating the upper bounds for the recoil velocity that fit the WMAP data [21, 19], and in practice one may have somewhat lower temperatures. In fact, lower temperatures may be required in order to avoid massive gravitino overproduction. Such constraints would restrict further the upper bound on the recoil velocities in the (compactified version of the) model of [21].

However, despite the perturbative supersymmetry restoration, one would have non-perturbative thermal instabilities, for the same reason as in the heterotic case examined above [22], associated with supersymmetry obstruction. Such non-perturbative instabilities would result in the presence of tachyonic states in the string spectrum, which could provide the initial cosmological instability. As discussed in [15], it seems to be a generic feature of such tachyonic states to decouple quickly in the cosmological Liouville evolution. In addition to these non-perturbative instabilities, compactification of Type-IIA theories leads to extra instabilities, due to the above-mentioned lack of a global minimum in the low-energy effective scalar potentials arising from thermal supersymmetry breaking. The metastable nature of the hot phase of the Type-IIA vacuum leads to an exit from this phase, which is succeeded by a cold inflationary phase that we now proceed to discuss.

#### 4.5.2 Inflationary Phase

Some time after the initial collision, the recoiling D-brane world's bulk potential (??) becomes *positive*. From a conformal field theory point of view, and in the adiabatic approximation we assumed in [21], this phase might be described by *an analytic continuation* of the above linear-space-like dilaton solution:

$$Q \rightarrow iQ, \quad X_E^0 \rightarrow i t. \quad (134)$$

The corresponding  $\sigma$ -model-frame metric, which in the hot phase was a flat Minkowski metric, becomes now conformally flat:

$$G_{\mu\nu} = e^{-2\Phi} \eta_{\mu\nu} , \quad \Phi = \frac{1}{2} Q t. \quad (135)$$

In the model of [21] this could be the full ten-dimensional metric, although appropriate compactifications can restrict the indices to the four dimensions relevant for a three-brane, as seen in Fig. 2, which represents the inflationary model [19] reviewed in Section 2.3 above. The normalizations in (135) pertain to the four-dimensional case of the three-branes, and would change for higher-dimensional branes [21].

It is a curiosity that, setting  $Q = -3H$ , the physical (Einstein-frame) dilaton and metric fields (5) of the hot phase, which remain *real* under (134), become equivalent to the corresponding Liouville-undressed fields (135) (c.f., (36),(38)) when one sets the Liouville field  $\varphi = 0$ . However, this is only a coincidence, since in the inflationary phase it is the  $\sigma$ -model-frame metric that acquires the conformally-flat form. As a  $\sigma$ -model-frame metric, (135) is *not* conformal invariant, since its one-loop  $\beta$  function is non-vanishing:  $\beta_{\mu\nu}^G = R_{\mu\nu} = Q^2 G_{\mu\nu} \neq 0$ . The central charge deficit  $Q^2$  in this case is given by the potential (??) in the regime in which is positive, which is treated as a constant in the adiabatic approximation.

Because of the positive central charge deficit, the string system requires Liouville dressing by a *time-like* field,  $\varphi$ , which is an extra time-like coordinate, in addition to  $t$ . The eventual identification (37), which in this scenario is dictated by dynamical reasons [18], ensures that there is only one time variable in the formalism, and leads to an eventual constant dilaton during the inflationary phase. This phase in which the Universe cools down is nothing other than the inflationary phase, described in Section 2.3 above. The analytic continuation procedure (134) describes simply a phase transition of the bulk superstrings from the hot phase to a cold inflationary one, within the colliding-brane system of Fig. 3.

We recall that, as a result of the non-perturbative breaking of target-space supersymmetry in the hot phase, there are tachyonic states in the spectrum, which trigger the initial cosmological instability. However, as discussed in [15], such states decouple relatively quickly in the cosmic evolution.

### 4.5.3 Exit from the Inflationary Phase: Reheating and Possible Subsequent Collision(s)

In a similar vein, one may discuss the phase transition associated with the second collision, and the subsequent reheating of the Universe. However, the physics of reheating is not understood at a satisfactory level in this framework. In the context of the model of [21], one has to understand technical details associated with internal magnetic fields in the respective orientifold compactification [49], as well as issues with the potential felt by open-string excitations on a brane world. These issues still raise many open questions, but, for completeness, we now present some relevant speculations.

The potential  $V_{\text{brane}}$  felt by our brane world in the model of [21], in the configuration of Fig. 3, is by itself negative even during the inflationary phase [17],

$$V_{\text{brane}} = -V_8 \frac{31(R-2r)v^4}{2^{13}\pi^9\alpha'^5}, \quad (136)$$

where  $r_1 = R - 2r$  denotes the relative separation of the branes in the symmetric situation of Fig. 3 with  $r_2 = r$ . This negative value can be understood by recalling that the brane world feels an attractive force towards its original stack, and the configuration is stabilized when  $v \rightarrow 0$ .

An issue arises at this point, concerning the boundary conformal field theory of open-string excitations, with their ends attached to the brane. In view of this negative potential, one may think of dressing the open-string  $\sigma$  model with a *space-like* Liouville mode to restore conformal symmetry, already during the inflationary phase. In view of the corresponding situation in the closed string sector [22], discussed above, one is tempted to take the view that such a negative brane potential represents some sort of thermalization of open string excitations on the brane world during the inflationary era.

Indeed, for  $r < R/2$ , i.e., very soon after the initial brane collision *both* the brane and bulk Universes are *hot* and in thermal equilibrium. As the two bouncing brane worlds of Fig. 3 move further apart, the closed-string excitations in the bulk cool down, since the available space becomes larger and their collisions rarer, whilst the brane Universe remains initially hot, because the inflationary expansion is only a ‘mirage’ due to the brane motion.

The order of magnitude of the brane potential (136) is the same as the bulk one (??), which implies that, after the exit from the inflationary phase, our brane Universe remains thermalized with a temperature at intermediate energy scales  $\sim 10^{13}$  GeV, according to the calculation above<sup>20</sup>. This may provide an alternative to conventional reheating scenarios in the following sense: although the bulk cools down significantly, and the Universe exits from inflation, undergoing an appropriate phase transition, expressed by the change in sign of the bulk potential (??), the brane world remains thermalized after the exit from inflation at temperatures of order  $10^{13}$  GeV. This is simply a result of the initial collision, without the need for other reheating mechanisms.

The usual constraints on gravitino overproduction in spontaneously-broken supergravity models, such as those pertaining to the brane Q-cosmologies of [21], restrict the allowed temperature to values much smaller than  $10^{13}$  GeV. This in turn implies an upper bound on the brane recoil velocities, according to the discussion following (133). However, one should bear in mind that in brane models the produced gravitino will escape in the bulk, since it is an excitation of the closed superstring multiplet, and therefore these constraints may not be so strict as in conventional supergravity cosmologies.

This approach may provide an explicit realization of the ideas in the ekpyrotic scenario for inflation and reheating of the Universe [7]. Immediately after inflation there is a difference in temperature between the bulk and the brane worlds. As time passes, this difference in temperature will cause significant closed string (gravitational) emission from the brane to the bulk, in order to equilibrate the situation with a common (low) temperature in both brane and bulk worlds.

Due to energy conservation, this causes non-adiabatic motion, with the brane world decelerating towards an eventually zero velocity. This would correspond to the exit phase from the inflationary epoch, given that the central charge deficit of the pertinent stringy  $\sigma$  model would vanish asymptotically and, according to the discussion in Section 2.3, the space-time metric would tend to that of a static flat Minkowski space-time. In some models, however, e.g., those

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<sup>20</sup>Such temperatures may characterize no-scale supergravity models [56].

with internal magnetic flux contributions, such as the Type-0 models discussed previously, the asymptotic state may be that of a linear dilaton, leading to a linearly expanding Einstein-frame Universe. In addition, as a result of brane recoil effects also discussed above, one would have contributions to the dark energy, relaxing asymptotically either to zero or to a constant contribution (set, for instance, by the internal magnetic field contributions in magnetized compactifications [34]), as in (99) or (100), (107). In all such models, current-era cosmology can be made compatible with observations by fixing the various stringy parameters [15, 19, 8].

#### 4.5.4 Open Issues: a Second Collision? Nucleosynthesis? ...

There are several open issues regarding the fate of the inflationary Universe. It is an open, and certainly model-dependent question whether the gravitational radiation from the brane to the bulk makes the brane world stop before the second collision takes place (in which case the latter will never occur). Indeed, if the brane gravitational radiation causes a significant reduction of the brane velocity  $v$  before the second collision with the stack of D-branes in Fig. 3, it is possible that the velocity contribution to the bulk potential (??) diminishes significantly, in such a way that the D-particle term overcomes the positive  $v^4$  term. In such a case the bulk potential becomes unstable (negative) and the bulk string system may thermalize in the way described earlier. The thermalization may also have a conformal field theory description, with the bulk background central charge deficit being given by  $-Nv^{1/2}$ . If the second collision takes place before the brane stops due to radiation, there may be a local disturbance in the population of the D-particles near the brane worlds during the (second) collision, such that  $N$  is significantly higher than before, when the brane was moving in the bulk. Such local disturbances may even cause the massive D-particles to collapse forming black holes, whose Hawking evaporation leads to additional thermal contributions to the brane world after the second collision.

It is an open issue whether the potential energy of these re-thermalized bulk closed strings becomes of similar order as the brane potential (136). If such were the case, one would reach thermal equilibrium between brane and bulk worlds, the gravitational radiation towards the bulk could counterbalance the breaking up of bulk closed strings on the brane, and the brane world would stop decelerating before the second collision take place. The brane world could then either move again adiabatically in the bulk with a very small velocity, in which case there could still be a long time before it starts accelerating again due to the influence of the other branes, either until the second collision takes place, or until it stops. The low (equilibrium) temperature in either case could be identified with the CMB temperature of the present era of the Universe.

Another open issue concerns the mechanism for *nucleosynthesis* in such a scenario. Nucleosynthesis requires a delicate balance between the expansion of the Universe and the rate of nuclear reactions for the formation of the light elements, which appears to work very well in scenarios with a negligible cosmological constant. It may therefore be desirable that the reduction in brane velocity due to radiation occurs around the nucleosynthesis era, so that in such a case the brane Universe has only a very small vacuum energy. For instance, in the class of models with compactified branes in magnetized internal manifolds [18] it could be that only the magnetic field supersymmetry-breaking contributions to the vacuum energy are present during the nucleosynthesis era. At the end of nucleosynthesis a second collision of the brane world with the stack of branes in Fig. 3 takes place, resulting in an increase of the central-charge deficit

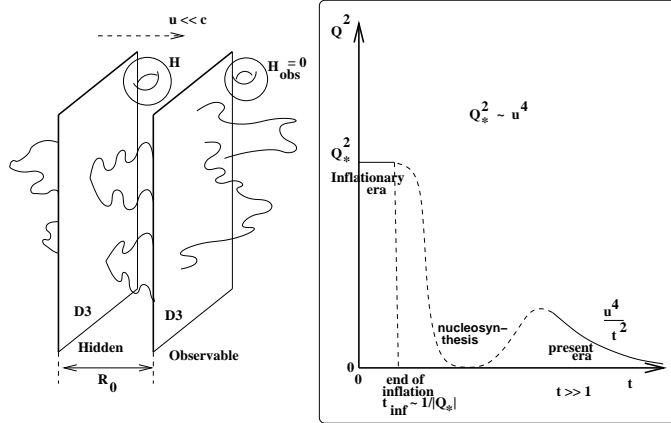


Figure 4: A nucleosynthesis-friendly Liouville cosmology scenario, according to which the second collision of a moving brane world with the (left) stack of branes in Fig. 3 occurs after the phase where the brane almost stops due to radiation. This scenario provides a relaxation model for the cosmological vacuum energy (central-charge deficit of the Liouville  $\sigma$  model), which passes first through a metastable phase where it almost vanishes (up to magnetic field contributions) during the nucleosynthesis era, and then raises again, as a result of the second collision, but at a much lesser height than in the initial collision.

(vacuum energy), but at a much lesser height than in the initial collision (due to the much smaller velocities involved), as seen in Fig. 4). Eventually the central charge relaxes again to zero asymptotically, providing the vacuum energy in the present epoch.

The question then arises as to what precisely causes the current energy density of the Universe in such a case. So far we have argued that a linear (in the string frame) dilaton may act as a quintessence field, in accordance with current cosmological phenomenology. However, it is our opinion that, in order to answer this question completely, one should also incorporate in the above discussion the *recoil* fluctuations on the brane world, which echo the initial brane collision. As mentioned above, such effects would provide *positive* contributions to the present-era central charge deficit of the corresponding stringy  $\sigma$  model, for asymptotically long times after the initial collision. The recoil contributions depend on the recoil velocity of the branes during the (adiabatic) bouncing inflationary phase, but they diminish with the cosmic time, relaxing towards either zero or some other small positive (equilibrium) value, determined for instance by the internal magnetic field (c.f., (99) or (100), (107) respectively). These recoil contributions may be responsible for parts of the dark energy density of the observable Universe, which exceed those due to the linear-dilaton quintessence. They could also be in accordance with current astrophysical observations [9, 10]. Such recoil contributions may overcome any negative thermal contributions in the bulk, so that the bulk energy never becomes negative, in contrast to the negative brane energy for non-zero velocities.

These and other related issues are currently under investigation, and we hope to be able to report some more complete results soon.

## 5 Conclusions and Outlook

We have examined in this work various cosmological models based on non-critical Liouville strings - Q-Cosmologies - with various asymptotic configurations of the dilaton, and have speculated on the inflationary phase, on the possibility of exit from it and reheating, as well as the large-times eras of the Universe (current and future). A particularly interesting case from a physical point of view is that of a linear dilaton that is asymptotically linear in cosmic time, which is known to correspond to a true conformal field theory [12]. In such a model we have observed that the string coupling is identified (up to irrelevant constants of order one) [8] with the deceleration parameter of the Universe, through equation (56). We have argued that the present-era phenomenology of the model, including matter, is compatible with the astrophysical data in a quite natural way, for suitable values of the adjustable parameters in the model.

We stress once more the importance of being non-critical in order to arrive at (56). In critical strings, which usually assume the absence of a four-dimensional dilaton, such a relation cannot be obtained, and the string coupling is not directly measurable with cosmological data. The logarithmic variation with the cosmic time of the dilaton field at late times implies a slow variation of the string coupling (56),  $\dot{g}_s/g_s = 1/t_E \sim 10^{-60}$  in the present era, and hence a corresponding variation of the gauge coupling constants. However, this variation is too small to be seen currently.

The use of Liouville strings to describe the evolution of our Universe is natural, since non-critical strings are associated with non-equilibrium situations which undoubtedly occurred in the early Universe. We have discussed in this framework the phase diagram of a Liouville cosmological string model of two colliding-brane worlds. We have seen that, immediately after the collision, the bulk string Universe passes through a hot, metastable phase, before entering an inflationary cold phase. On the other hand, the brane Universe (our world) remains thermalized throughout the two phases, at a relatively high temperature, causing gravitational radiation from the brane to the bulk, which tends to equilibrate the temperature, which eventually decelerates the motion of the brane world in the bulk. From the point of view of an observer on the brane, however, the brane Universe may at present seem to be accelerating, with the acceleration provided by the dilaton field of the string multiplet, as mentioned above.

Exit from the inflationary phase is still an unresolved issue, although scenarios have been conjectured, involving for instance a second collision of the brane world of the model of [21] with the stack of D-branes in Fig. 3. This could provide extra contributions to the reheating of the brane world, as a result of the gravitational collapse of D-particle populations to form bulk black holes, which subsequently emit Hawking radiation.

There are many phenomenological tests of this class of cosmologies that can be performed, which the generic analysis presented here is not sufficient to encapsulate. Tensor perturbations in the cosmic microwave background radiation is one of them. The emission of gravitational degrees of freedom from the hot brane to the cold bulk, during the inflationary and post-inflationary phases is something to be investigated in detail. A detailed knowledge of the dependence of the equation of state on the redshift is something that needs to be looked at in the context of specific models. The constant equation of state obtained here is only an asymptotic feature of an era where the gravitational sector dominates. Moreover, issues regarding the delicate balance of the expansion of the Universe and nucleosynthesis, which requires a very low vacuum energy, must be resolved in specific, phenomenologically semi-realistic models, after

proper compactification to three spatial dimensions, in order that the conjectured cosmological evolution has a chance of success.

Finally, the compactification issue *per se* is a most important part of a realistic stringy cosmology. In our discussion above, we have presented a rather simplified compactification on magnetized internal manifolds, in Type-II five-brane models, which also provided phenomenologically realistic ways of breaking target-space supersymmetry in cold Universes, compatible with the very small value of the vacuum energy that has been reported in the Universe today. However, in the context of the model of [21], involving eight-branes and orientifolds, such compactifications may present subtleties that require extra attention [49].

We hope to be able to report on these and other related issues in future work. We are far from claiming a detailed understanding in this framework of several important facts of modern cosmology, such as the Universe's current acceleration, dark energy, the various phase transitions in the past history of the cosmos, *etc.*. Nevertheless, we believe that Liouville strings are probably the only viable way, in the context of string theory, to discuss rigorously cosmological string backgrounds, especially those involving accelerated Universes and, in general, dark-energy contributions to the Universe's energy budget.

In this last respect, we stress once more that the non-equilibrium Liouville approach to cosmology advocated in this article is based exclusively on the treatment of target time as an irreversible dynamical renormalization-group scale on the world sheet of the Liouville string (the zero mode of the Liouville field itself). This irreversibility is associated with fundamental properties of the world-sheet renormalization group, which lead in turn to the loss of information carried by two-dimensional degrees of freedom with world-sheet momenta beyond the ultraviolet cutoff [14] of the world-sheet theory. This fundamental microscopic time irreversibility may have other important consequences, associated with fundamental violations of CPT invariance [24, 17, 57] in both the early Universe and the laboratory, providing other tests of these ideas.

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