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Liquefaction Potential: Developments Since 1976

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Liquefaction Potential: Developments Since 1976

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SYNOPSIS Developments in the analysis and assessment of liquefaction potential since 1976 are critically reviewed. The major development is the emergence of dynamic effective stress analysis and its gradual introduction into practice. Other topics reviewed include constant volume cyclic simple shear testing, quantitative analysis of the effects of system compliance on the results of cyclic loading tests, probabilistic methods and the effects of overconsolidated, previous stress history, static shear stresses and geological aging on liquefaction potential.

INTRODUCTION

Liquefaction of saturated sands during earthquakes has become a major topic in soil dynamics since the earthquakes in Niigata, Japan and in Alaska in 1964 caused widespread damage by inducing liquefaction of the ground. The quantitative study of seismically induced liquefaction began with the publication by Seed and Lee (1966) of their pioneering work on the cyclic triaxial test. Sparked by this development, the study of liquefaction grew rapidly.

The next ten years saw the development of new laboratory tests for measuring liquefaction potential; the cyclic simple shear test (Peacock and Seed, 1968; Finn, Pickering and Bransby, 1971), the torsional cyclic simple shear test (Ishihara and Li, 1972; Ishibashi and Sherif, 1974), and large scale shake table tests with precise control on loading and drainage conditions (Finn, Emery and Gupta, 1970, 1971; Finn, 1973; De Alba, Chan and Seed, 1975). Total stress methods of dynamic response analysis were developed for determining earthquake induced shear stress histories in soil deposits and for deducing from these histories, with the aid of laboratory data, seismically induced porewater pressures in saturated sands. Outstanding among these developments were the simplified analysis of liquefaction potential (Seed and Idriss, 1971), the equivalent linear method of analysis for determining seismic shear stresses which has been incorporated in the computer program, SHAKE (Schnabel, Lysmer and Seed, 1972) for onedimensional analysis of level ground, and procedures for converting an irregular seismic shear stress history to an equivalent number of cycles of uniform shear stress of specified magnitude (Lee and Chan, 1972; Seed, Idriss, Makdisi and Banerjee, 1975). The latter development allows the prediction of the porewater pressures caused by the seismic shear stress history using data from conventional laboratory cyclic loading tests at constant stress amplitude. Empirical methods for estimating liquefaction potential in the field were developed also, in part, because of their convenience in engineering practice and, in part, because of uncertainties and difficulties with the more sophisticated

methods. The empirical methods are based on records of the results of standard penetration tests at a variety of sites and the performance of these sites during earthquakes. As a result of these developments, a fundamental understanding of the mechanism of liquefaction and of the influence of material, loading and environmental conditions on liquefaction potential has emerged.

All these developments were reviewed by Seed (1976) in a very important and comprehensive assessment of the state-of-the-art in research and practice in the field of liquefaction. The present review examines developments subsequent to Seed's study with some exceptions that require an explanation. Seed reviewed many important developments published in research reports which were available to him but not in general circulation at the time. Since 1976, these reports have been published in the geotechnical literature, some as recently as 1979. These contributions are not reviewed formally again. However, some repetition has been necessary on occasion in order to put a recent development in perspective. The impact of a development in any field can be gauged only against some generally accepted background of information and understanding. The background depicted by Seed (1976,1979) is accepted widely and recent developments are assessed from this perspective. It will be seen that the more recent developments expand and clarify that background. Although some revision in concepts or practice may be suggested by the new developments they do not contradict the general picture drawn by Seed in any fundamental way.

FACTORS INFLUENCING LIQUEFACTION POTENTIAL

Seed (1976) listed the major factors affecting the liquefaction potential of saturated cohesionless soils as (a) relative density, (b) method of soil formation (soil structure), (c) grain characteristics, (d) lateral pressure coefficient and overconsolidation, (e) period under sustained load (aging), and (f) previous strain history. Nothing new can be said about the roles of (a) relative density or (b) soil structure in liquefaction. There have been some useful developments in understanding of the roles of the other variables.

Grain Characteristics

Lee and Fitton (1968) showed a strong dependence of liquefaction potential on mean grain size. Their data indicated that as the mean grain size increased, the resistance to liquefaction increased. The data were obtained on 2.8 in (7.1 cm) diameter samples in undrained cyclic triaxial tests. Wong and Seed (1975) using 12 in (30.5 cm) diameter samples showed a much weaker dependence on mean grain size. Analyses by Martin, Finn and Seed (1978) of the effects of compliance in undrained test systems indicate that the effects of mean grain size noted by Lee and Fitton (1968) may be due almost entirely to the effects of membrane penetration, which would have a much stronger influence in the 7.1 cm diameter samples tested by Lee and Fitton than in the 30.5 cm diameter samples tested by Wong and Seed. The data from Lee and Fitton, and Wong and Seed will be presented in a later section on system compliance together with the same data corrected for compliance.

Overconsolidation and Lateral Pressure Coefficient

The stress ratios required to cause initial liquefaction are influenced significantly by the overconsolidation ratio OCR and the value of the lateral pressure coefficient at rest K_o . This was demonstrated experimentally, first, by Seed and Peacock (1971) and confirmed later by Ishibashi and Sherif (1974). These effects were predicted in analytical studies by Finn, Pickering and Bransby (1971), Seed and Peacock (1971) and Castro (1975).

An investigation into the effects of OCR and K_{o}

was carried out at the University of British Columbia by Bhatia (1980) using the constant volume cyclic simple shear test equipment developed by Finn and Vaid (1977,1978). Data on the effects of OCR were obtained similar to that of Seed and Peacock (1971) and are shown in Fig. 1.

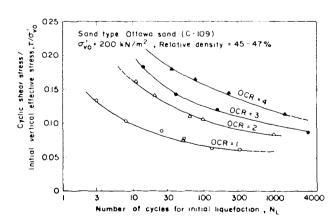


Fig. 1 Effect of Overconsolidation on Liquefaction Resistance.

During these tests the lateral stress was measured giving the initial value K_0 and the current value K = ratio of effective horizontal $to vertical stress <math>\sigma_{n}^{\prime}/\sigma_{v}^{\prime}$ during testing. K_0 increases with increasing OCR. The increase in liquefaction resistance with increasing OCR is attributed to the increase in K_0 which reflects the increase in mean confining effective stress, σ_{m0}^{\prime} (Ishihara et al, 1977,1978). The mean confining stress is derived directly from the measured vertical and lateral effective stresses or from the formula, $\sigma_{m0}^{\prime} = \sigma_{v0}^{\prime} (1+2K_0)/3$, where σ_{v0}^{\prime} = initial vertical confining pressure.

The data in Fig. 1 are replotted in Fig. 2 using

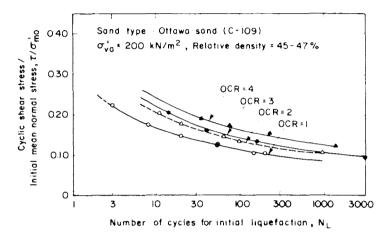


Fig. 2 Liquefaction Resistance of Overconsolidated Samples in Terms of Mean Normal Confining Stress.

 σ_{mo}^{+} instead of σ_{vo}^{+} to define the cyclic stress ratio π/σ^{+} where $\pi = cyclic$ shear stress. The resulting plot indicates that the entire effect of overconsolidation cannot be explained simply in terms of changes in K_o or the mean confining pressure although, clearly, they are responsible

for a major part of the effect. Overconsolidation apparently generates other changes, probably related to grain structure or grain contacts which have a further beneficial effect on liquefaction characteristics.

It is of interest to trace the changes in K during cyclic loading. As shown in Fig. 3, the K of a normally consolidated sample increases during cyclic loading while the K of an overconsolidated sample decreases. The rates of increase or decrease depend on the amplitude of the cyclic shear strains. The pattern of volume changes during cyclic loading of sands is connected closely with their potential for deve-loping porewater pressure under undrained loading conditions (Martin, Finn and Seed, 1975). The volume change characteristics of overconsolidated sands are quite different from those of normally consolidated sands. Volume changes in a sand with OCR = 4 in a strain-controlled cyclic simple shear test with maximum shear strain $\gamma = 0.3$ % are shown in Fig. 4. Note that initially volume expansion occurs which apparently modifies the structure induced by overconsolidation and releases some of the lateral

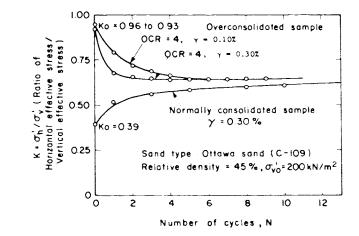


Fig. 3 Variations in the Lateral Pressure Coefficient during Cyclic Loading.

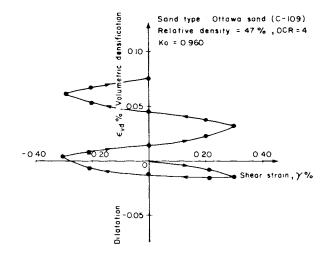


Fig. 4 Volume Changes in an Overconsolidated Sand during Cyclic Loading.

Thereafter, volume contraction occurs. stress. The volumetric strains, $\epsilon_{\rm vd}$ %, in Ottawa sand at a relative density, $D_r = 45\%$, and having a range in OCR from 1 to 4 are shown in Fig. 5 for various cyclic strain amplitudes in simple shear tests. Note the significant decreases in volumetric strains induced by a given cyclic loading with increasing OCR. Using the Martin, Finn and Seed (1975) porewater pressure model, it is easy to predict the reduced porewater pressures in overconsolidated materials from the data on volumetric strains. The different rates of porewater pressure development in a normally consolidated and overconsolidated Ottawa sand at) = 46% are shown in Fig. 6. They are remarkably similar to porewater pressure curves for samples with and without previous strain history (Finn, Bransby and Pickering, 1971). Previous strain history increases the value of K and, therefore, its effects are very similar to verconsolidation.

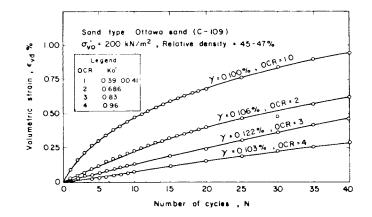


Fig. 5 Volume Changes in Sands with Different Overconsolidation Ratios.

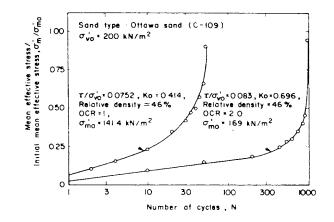


Fig. 6 Rates of Development of Porewater Pressure in a Normally and Overconsolidated Sand.

Effects of Aging on Liquefaction Potential

Ohsaki (1969) appears to have been the first to suggest that the liquefaction potential of deposits in the field is influenced by the age of the deposit. The most startling example occurred in the city of Niigata during the 1964 earthquake. Alluvial sands and hydraulic fills placed since the Meiji restoration of the late 19th century all liquefied. Much older sand deposits in the same city did not. Seed (1976) confirmed this result by laboratory studies in which identical samples were subjected to sustained pressure for periods ranging from 0.1 to 100 days prior to testing so as to simulate to a limited extent the effects of geological aging. Even over this brief period of time the samples showed increased resistance to liquefaction. When measured in terms of the cyclic stress ratio to cause liquefaction, the increase in resistance for the oldest samples was about 25%. Extrapolation of this data base to longer times was accomplished by comparing the resistance to liquefaction of undisturbed field samples with that of freshly deposited samples of the same

sand. The results are very significant; increases in the resistance to liquefaction of the order of 75% due to geological aging appear to be feasible. Donovan (1976) also has found substantially higher resistance to liquefaction in undisturbed samples of Valdez rockfill which had been consolidating in the field for one year under the weight of a fill than in samples of the same material consolidated to the same pressure for the first time prior to testing.

Youd and Perkins (1978) have greatly increased the data base on the effects of geological aging. In a comprehensive study of ground failures caused by liquefaction they showed that such failures seemed to be limited to certain geological settings. This study complements an earlier study by Youd and Hoose (1977). The studies show that for cohesionless material the older the deposit the greater the resistance to liquefaction. Their data and corresponding data from many Japanese earthquakes indicate that saturated loose hydraulic fill nearly always liquefies and that deposits of cohesionless material of Holocene age are susceptible to liquefaction. The incidence of liquefaction in Pleistocene deposits is much rarer and is very rare indeed in pre-Pleistocene deposits. These conclusions are based on world-wide data on liquefaction during earthquakes to 1975 and have been confirmed again by the incidences of liquefaction during two major earthquakes in Japan in 1978, Miyagi-ken-oki (JSCE, 1980) and Off-Oshima (Ishihara et al, 1978) and the Rumanian earthquake of 1977 (Fukuoka et al, 1977).

Very recently, fundamental studies were begun in Japan to determine the differences between sands of different geological ages and very interesting preliminary results have been obtained by Tolno (1975). It has been shown that the older the depooit the greater the density. Figure 7 shows the increase in density (and the decrease in void ratio) from late Holocene to the very

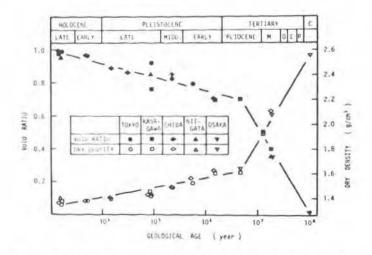


Fig. 7 Effect of Geological Age on the Void Ratio and Dry Density.

old early Tertiary deposits. The very marked density increases in the pre-Pleistocene deposits indicates clearly why these do not liquefy. These increases in density would be detected by increased N-values. Equally interesting are the very different kinds of contacts between grains of sand in the younger and older deposits. Finer particles tend to separate the larger sand grains in hydraulic fills and younger deposits as shown in the electron-micrographs, Fig. 8a and 8b. This leads to more unstable deposits

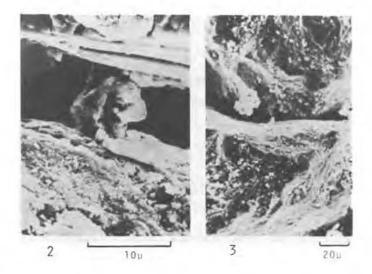


Fig. 8 Grain Contacts in Hydraulic Fill (2) and Holocene Deposit (3).

which compact more readily under shaking. In the older Pleistocene deposits these finer particles tend to be squeezed out and there seem to be more substantial contacts between the more stable sand grains (Figs. 9a and 9b). The data

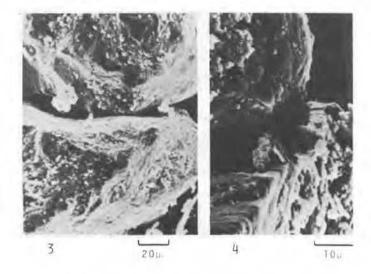


Fig. 9 Grain Contacts in Holocene (3) and Late Pleistocene (4).

on the effects of geological aging have provided a means of zoning for liquefaction potential.

Zoning for Liquefaction Potential

Youd and Perkins (1978) produced zoning maps for liquefaction by combining geological and seismic information. The estimates of liquefaction when plotted on geological maps produce a liquefaction susceptibility map. Generalized susceptibility maps delineate sedimentary deposits with relatively different likelihoods of containing sediments susceptible to liquefaction. They do not show absolute susceptibility. Since some measure of strong shaking is necessary to cause liquefaction in susceptible soils, a measure of the potential for sufficiently strong shaking must be mapped also. An indication of the level of shaking required to cause liquefaction is provided by the data of Kuribayashi and Tatsuoka (1975) who determined the distances at which significant liquefaction could occur from the sources of earthquakes of various magnitudes. Their data indicate that recent alluvial deposits with the water table within a few metres of the surface, will liquefy within a radius of R of an earthquake of magnitude M>6, given by the equation

$$\log_{10} R = 0.8M - 4.5 \tag{1}$$

This equation represents the mean of the field data. A map showing the relative potential for sufficiently strong shaking has been called a ground failure or liquefaction opportunity map. A map showing the combined information on both the susceptibility and opportunity maps gives a relative measure of liquefaction potential.

Ishihara and Ogawa (1978) have produced a liquefaction zoning map for downtown Tokyo. The map is based on a maximum ground acceleration of 0.25 g for the downtown area and the water table is assumed to be within 1 m of the surface. The liquefaction susceptibility of the soils for the stated conditious is defined primarily by threshold values of the standard penetration resistance N given by N = 15 + D where D is in metres. The threshold values were established by a combination of data from simplified liquefaction analysis (Ishihara, 1977) and field experience in previous earthquakes.

Liquefaction potential or zoning maps are a recent development. A major benefit of such zoning maps is that they will alert developers and structural engineers to the possibility of liquefaction and the need for specialized studies of the problem in some locations.

Effect of Strain History

The influence of previous strain history on the liquefaction potential of sands was pointed out by Finn et al (1970). Their study showed that an increased resistance to liquefaction occurred when small cyclic shear strains were applied to a saturated undrained sample and drainage was allowed prior to measuring the resistance of the sample to liquefaction in an undrained cyclic loading test. On the other hand, prior large shear strains were found to cause a reduction in resistance to liquefaction. In current terminology this process of imparting a strain history to the samples is called preshearing. With the instrumentation available at the time it was not possible to determine accurately the limiting values of shear strain beyond which liquefaction resistance was reduced by preshearing although the data indicated that such a limit was probably less thau l% and depended on the number of cycles. The pore water pressure increase during the preshearing also was considered as an index of preshearing and it was found that strength increases occurred for pore water pressures up to those representing initial liquefaction provided the preshearing was stopped before significant strains were developed. Bjerrum (1973) extended the concept of preshearing to offshore engineering to include the beneficial effects of small storms on the resistance of seafloor sands to liquefaction during large storms.

Seed, Mori and Chan (1977) conducted an extensive investigation of the effects of preshearing or prior seismic history. To avoid any boundary effects associated with small samples, they used a shaking table to induce cyclic shear stresses on large scale samples under simple shear conditions. The extensive data from this investigation conclusively demonstrated the beneficial effects of small preshearing on the resistance to liquefaction of saturated sands in laboratory cyclic loading tests. Furthermore, the data provided estimates of the possible increases in resistance (Fig. 10). The reason for the increase in resistance, however, is still unknown

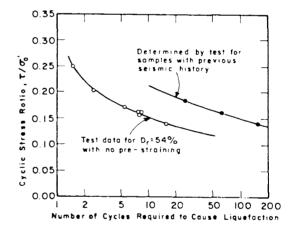


Fig. 10 Effect of Previous Seismic History on Liquefaction Potential (Seed, Mori and Chan, 1977).

although it may be attributed plausibly to increases in the lateral pressure co-efficient K_{O} during preshearing and to stabilization of the existing structure by minor adjustments at grain contacts (Finn et al, 1970).

The preshearing in Seed's tests was conducted at levels to simulate the effects of low magnitude earthquakes. No attempt was made to define the limits beyond which preshearing might weaken the resistance to liquefaction. Two further studies have probed this aspect of the problem. Singh, Donovan and Park (1980) have studied the effects of prior seismic history in cyclic triaxial tests and have suggested that the effect of preshearing is beneficial if it generates pore water pressures less than 60% of the confining pressure. For some sands this is the level of pore water pressure at which strains begin to increase rapidly during cyclic loading. Therefore, this conclusion is in general accord with previous results.

Ishihara and Okada (1978) also have conducted an extensive investigation of the effects of prior seismic history. They interpreted their data in terms of plasticity theory and formulated a precise criterion to define the limits between beneficial and deleterious preshearing. This criterion is illustrated in Fig. 11. Any degree

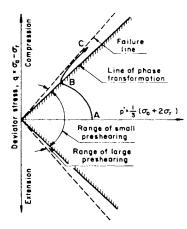
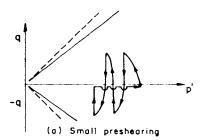


Fig. 11 Phase Transformation Lines for Liquefying Sand.

of preshearing that results in effective stress paths that lie within the phase transformation lines shown in Fig. 11 will be beneficial (Fig. 12a); preshearing which results in effective stress paths which go outside these lines will be deleterious (Fig. 12b). The phase



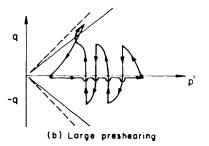


Fig. 12 Stress Paths Resulting in (a) Small Strains: (b) Large Strains.

transformation lines are defined by Ishihara, Tatsuoka and Yasuda (1975) as the loci of points such as B (Fig. 11) at which the stress path undergoes abrupt changes in curvature and direction. Under effective stress paths within these lines a sample does not develop large strains and behaves like a solid plastic material. If the stress path crosses these lines, a sample develops large strains and tends to deform more as a viscous fluid. The phase transformation line has been used also as an index of initial liquefaction (Ishihara et al, 1975). Thus, the Ishihara criterion defining the limits of beneficial preshearing are very similar to the conclusious reached by Finn et al (1970) in their original study of the problem.

Despite the thorough investigations of the effects of prior seismic history, there is as yet no way to treat seismic history as a discrete and separate variable in the analysis of liquefaction potential in engineering practice. However, the results of the studies indicate clearly the importance of determining liquefaction potential on undisturbed rather than reconstituted samples. Only by using good quality "undisturbed" samples from the field can there be some assurance that the influence of variables such as prior seismic history and aging are reflected in a reasonable way in test data. De Alba, Chan and Seed (1976) have shown that sampling destroys most of the effects of previous strain history in samples taken from large shaking table specimens of pluviated sands. Whether sampling would have the same effect on sands of considerable geological age in the field is not known.

Static Shear

In most earth structures, soil elements along potential failure surfaces are subjected to appreciable static shear stresses. Consequently, the porewater pressures on these potential failure surfaces are most closely modelled by cyclic simple shear tests with initial static shear stresses (Finn and Byrne, 1976) or by cyclic triaxial tests on anisotropically consolidated samples (Lee and Seed, 1967). The static anisotropic consolidation stresses generate initial static shear stresses on potential failure planes and the cyclic deviator stresses superimpose cyclic shear stresses.

Two distinct cases of cyclic loading must be considered for anisotropically consolidated samples (Seed et al, 1973). When the cyclic stress ratio $\sigma_{dcy}/2\sigma_{3c} < 0.5$ (K_c-1), there will be no reversal in the direction of application of resultant shear stresses on potential failure planes during cyclic loading; that is, the net axial stress will continue to be the major principal stress. When the cyclic stress ratio $\sigma_{\rm dcy}/2\sigma_{\rm 3c}$ > 0.5 (K_c-1) there will be a shear stress reversal during each cycle of loading. Seed et al (1969,1973) and Finn and Byrne (1976) have shown that stress reversal is required if a condition of initial liquefaction is to be achieved. Whether there is shear stress reversal or not, the anisotropically consolidated samples strain progressively during cyclic loading in contrast to isotropically consolidated samples in which significant strains only develop when the porewater pressure reaches

about 60% of the confining pressure. In anisotropically cousolidated samples without shear stress reversal the porewater pressures generated during cyclic loading do not increase sufficiently to produce a state of initial liquefaction (Seed et al, 1973; Finn and Byrne, 1976) but stabilize at values that depend on the anisotropic consolidation ratio and the magnitude of the cyclic shear stress ratio.

For a study of the seismic stability of tailings dams Finn, Lee, Maartman and Lo (1978) conducted a detailed investigation into the development of porewater pressures in anisotropically consolidated samples of a single cycloned tailings sand in cyclic triaxial tests. It was found that the data on porewater pressures for both isotropic and anisotropically consolidated samples could be represented by equation (2) similar to that developed by Seed et al (1969) for isotropically consolidated samples.

$$\frac{u_{CY}}{\sigma_{3c}} = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \left[\left(\frac{N}{N_{50}} \right)^{1/\alpha} - 1 \right]$$
(2)

in which N₅₀ is the number of cycles to generate a porewater pressure u equal to one half of the confining stress σ'_{3c} and

 $\alpha = \alpha_1 K_c + \alpha_2$

At $D_r = 50\%$, $\alpha_1 = 3$ and $\alpha_2 = -2$ for the tailings sand. Non-dimensional porewater pressure curves for values of $K_c = 1$ through $K_c = 3$ computed using eqn. (2) are given in Fig. 13. The patterns of development of porewater pressure

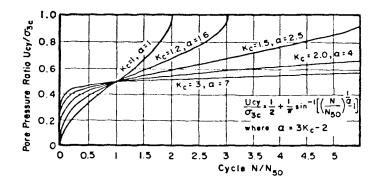


Fig. 13 Normalized Porewater Pressure Curves for Various K_c.

indicated by these curves, especially in the range N/N₅₀ < 2, is very similar to those determined by Finn and Byrne (1976) for double cycloned tailings sand in cyclic simple shear tests. All the <u>theoretical</u> curves eventually reach a porewater pressure corresponding to $u/\sigma_{3c} = 1$, although this condition will not always be achieved as discussed previously.

However, the number of stress cycles to reach initial liquefaction on the theoretical curves becomes inordinately large with increasing values of $\rm K_{c}$. In practical cases, for the

number of stress cycles under likely consideration, the porewater pressures given by the analytical curve will be within the upper possible limit of porewater pressure.

For the tailings sand tested, eqn. (2) has been found to be satisfactory in the range of relative densities from $D_r = 35\%$ to $D_r = 63\%$. Computed and measured porewater pressures for various values of N/N₅₀ are shown in Fig. 14 for

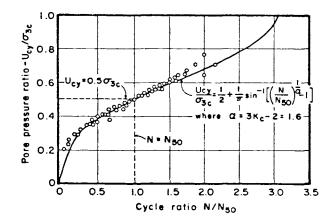


Fig. 14 Comparison of Computed and Measured Porewater Pressures for $K_c = 1.2$.

 $K_c = 1.2$ and, therefore, for $\alpha = 1.6$. The average relative density was $D_r = 54$ %. The agreement between computed and measured porewater pressures is quite good.

More recently, Vaid and Finn (1979) have conducted a detailed investigation of the effects of static shear on the response to cyclic loading in the cyclic simple shear test. The results of their study support substantially the previous findings with a few exceptions. Lee and Seed (1967) showed that the slightest degree of stress reversal during cyclic loading in triaxial tests would lead ultimately to a state of initial liquefaction. The simple shear data indicated that a significant level of stress reversal was required. The prevalent belief that the presence of static shear stress always increases the resistance of sand to cyclic loading was found to require some qualification. The cyclic loading resistance, expressed in terms of the amplitude of cyclic shear stress ratio, 'cy/'vo' required to cause a fixed amount of strain in a given number of cycles, was found to increase, decrease or remain unaltered, depending on the relative density of the sample, the shear strain level of interest and the magnitude of initial static shear stress (Fig. 15a and b). However, if the cyclic loading resistance was measured in

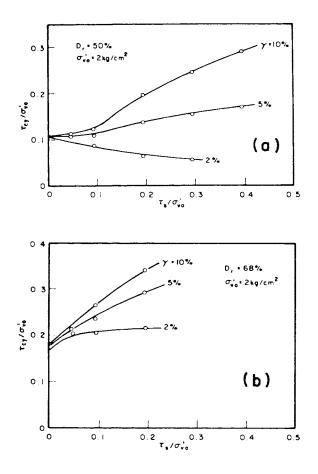


Fig. 15 Effect of Static Shear on Resistance to Cyclic Loading for Various Strength Criteria.

terms of the maximum shear stress, (cyclic + static), an increase in resistance with increase in static shear stress was always noted irres-pective of the relative density or shear strain level of interest.

EFFECT OF SYSTEM COMPLIANCE ON LIQUEFACTION POTENTIAL

The implicit assumption in all liquefaction tests performed on saturated sands under undrained conditions is that volume changes occuring during the test are zero (except for the very small reduction in volume of the water phase as the pore water pressure increases). However, if the volume of the system confining the sand sample increases as the porewater pressure rises, the apparent bulk modulus of the water phase is decreased and measured porewater pressures become less than those that would have occurred with a confining system of zero compliance. In the case of simple shear tests, such compliance may occur due to changes in membrane thickness, slight expansion of the sample space and compliance in the pore pressure measuring system. In the case of triaxial samples, a major source of compliance may be due to reduction in membrane penetration into peripheral voids as the porewater pressure increases (Lade and Hernandez, 1977; Kiekbusch and Schuppener, 1977).

Compliance results in an increased number of load cycles to cause initial liquefaction compared to that occurring for the same loading conditions in situ, and thus the test errors induced by compliance are on the unsafe side. Similar errors also occur if the effective bulk modulus of the water phase is decreased by air bubbles, i.e., where degrees of saturation are less than 100%.

Martin, Finm and Seed (1978) have presented a theoretical framework for the analysis of the effects of both system compliance and the degree of saturation on liquefaction potential based on their incremental model for porewater pressure generation (1975) given by

$$\Delta u = \frac{\overline{E}_{r} \Delta v}{nE_{r}} \frac{1}{K_{w}}$$
(3)

in which Au, Ar $_{\rm Vd}$ are the increments in porewater pressure and plastic volume change respectively, $\bar{\rm E}_{\rm r}$ = rebound modulus, n = porosity and ${\rm K}_{\rm w}$ = bulk modulus of water.

If the increase of porewater pressure is accompanied by an increase in volume of the system confining the sand sample, porewater pressure increments become less than those given by eqn. 3. The effect may be considered quantitatively by defining a compliance modulus, K_c , in which K_c = pressure required for unit volume increase of the confining system. Then we have, in terms of a compliance ratio $C_r = \bar{E}_r/K_c$, the porewater pressure increment

$$= \frac{\overline{E}_{r}^{\Lambda} vd}{1 + C_{r}}$$
(4)

Simple Shear Tests

Δu

The effects of system compliance on porewater pressure development and the number of cycles to cause initial liquefaction in cyclic simple shear tests may be estimated using the effective stress method of analysis (Finn, Lee and Martin, 1976, 1977) with a range of values of C_r . The computed numbers of cycles to cause initial liquefaction are shown plotted in Fig. 16 as a function of τ_{hv}/τ_{vo} for values of $C_r = 0$, 0.25 and 0.5. The resulting errors in τ_{hv}/τ_{vo} ratios required to cause initial liquefaction after a given number of cycles N may be obtained from Fig.16 for the various values of C_r , and are shown plotted in Fig. 17. The errors for $C_r = 1$, for example, are expressed by the ratio

$$\frac{\frac{1}{2}hv}{\frac{1}{2}vo} (C_{r} = 1) - \frac{\frac{1}{2}hv}{\frac{1}{2}vo} (C_{r} = 0)$$

$$\frac{\frac{1}{2}hv}{\frac{1}{2}vo} (C_{r} = 0)$$
(5)

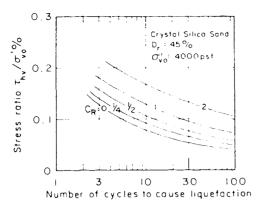


Fig. 16 Effect of Compliance on Liquefaction Potential.

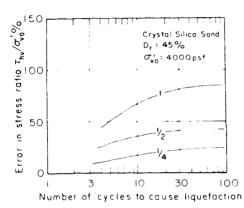


Fig. 17 Errors due to Compliance in Simple Shear Tests.

for the given values of N. Note that the magnitude of the errors plotted in Fig. 17 would vary with sand type, grading, and relative density. However, the curves shown in Fig. 17 give an indication of the order of magnitude of the errors that may be expected to occur.

Unfortunately, measurements of system compliance were not made at the time of testing for the types of equipment associated with published simple shear test results and, thus, an accurate assessment of errors in published data cannot be made. However, recent test results using the University of British Columbia simple shear apparatus give values of the compliance ratio up to 0.25 for stress ratios associated with liquefaction in about 20 cycles of loading. No compliance data are available for the other simple shear devices used for cyclic simple shear tests.

Large-scale simple shear tests on saturated sand samples have also been performed using shaking tables (De Alba et al, 1976) to induce cyclic shear stresses. However, compliance problems are also difficult to eliminate with this type of equipment, despite the large sample volumes. Measurements of system compliance in shaking table tests are described by De Alla (1974) and the data indicate average compliance ratios of the order of 0.5 and corrections to measured liquefaction test data were necessary to compensate for compliance. The corrections were made on a theoretical basis using a procedure similar to that described previously, and average errors in measured liquetaction test data were computed to be of the order of 45% in the range 10-30load cycles to cause initial liquefaction.

Triaxial Tests

The effects of membrane penetration on measured porewater pressures during undrained triaxial tests on saturated sand, and on measured volume changes during drained tests were first recognized by Newland and Allely (1959). To make corrections for the effects of membrane penetration, methods have been developed to measure the magnitude of volume changes due to membrane penetration as a function of effective confining pressure for a given test configuration (De Alba, 1974, Raju et al, 1974).

Test results obtained by Raju's method for isotropic unloading tests on 1.4-in. (36-mm) diameter triaxial samples of Crystal Silica and Ottawa sands, having a relative density of 45% showed that volume changes arising from membrane penetration may be of the same order as those arising from deformation of the sand skeleton, with membrane penetration volume changes increasing with increased particle size. These tests and others carried out by various investigators (El-Sohby, 1969; El-Sohby and Andrawes, 1972; Roscoe et al, 1963) have indicated that for uniform sands and a given sample size, membrane peneration characteristics are primarily a function of particle size (as characterized by the mean grain size $D_{r,0}$), and are reasonably independent of sample density.

The significance of membrane penetration, as it affects cyclic liquefaction tests, may be examined by defining a membrane compliance ratio, $C_{\rm RM}$, equal to the ratio of the average slope of the rebound curve of the sand skeleton to that of the membrane penetration volume change curves. Membrane penetration volume change curves tend to be geometrically similar, and the ratio is reasonably independent of initial effective confining stress. Computed errors in the cyclic stress ratio to cause liquefaction in 30 cycles are shown in Fig. 18 as functions of the mean grain diameter, D_{50} -mm.

To assess the significance of the error functions shown in Fig. 18, experimental data showing the influence of D_{50} and sample size on the cyclic stress ratio $-\frac{1}{dc}/2$ to cause initial liquefaction were studied. Data reported by Lee and Fitton (1968) and Wong, Seed and Chan (1975) for uniform sands and gravels of medium density are shown plotted in Fig. 19. It is of interest to use the error functions to apply corrections to the experimental data shown in Fig. 19. To cover the larger gravel sizes, the error functions were extrapolated assuming the observed linear behavior with D_{50} continued to the larger sizes. The corrected stress ratio curves for the 12-in and 2.8-in (305-mm and 71-mm) diameter

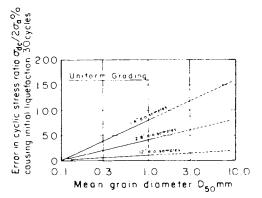


Fig. 18 Errors due to Compliance in Triaxial Tests.

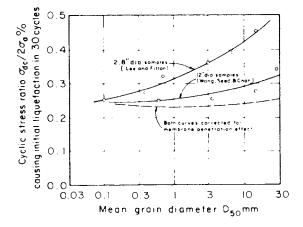


Fig. 19 Influence of Membrane Penetration on Liquefaction Potential.

samples were almost coincident and the mean curve is shown plotted in Fig. 19. The horizontal trend and coincidence of the curves suggests that experimentally observed differences in initial liquefaction resistance with grain size and sample diameter, could be due primarily to the effects of membrane compliance. The experimental curves also suggest that the effects of membrane compliance become negligible for values of $D_{50} < 0.1$ mm as indicated by the theoretical functions of Fig. 18. The curves in Fig. 19 also indicate that the effects of membrane compliance are less important for large diameter samples.

It should be noted that the experimental data and theoretical calculations presented previously have been for uniformly graded sands. The effects of membrane compliance for well-graded sand samples would be expected to be less severe due to the reduction in the size of peripheral voids. The computed estimates of the errors in liquefaction data given in this section are presented to illustrate the importance of system compliance. They should not be used in a quantative analysis without a careful study of the assumptions underlying their conjutation; these are discussed in detail in the original paper.

Partial Saturation

It is widely recognized that 100° saturation of test samples is an essential prerequisite for reliable cyclic liquefaction test results. Using the basic liquefaction theory previously outlined for simple shear tests, the effect of compliance arising from partial saturation on the results of liquefaction tests can be evaluated.

Pore pressure increments during undrained partially saturated tests may be computed using Eq.3 by using appropriate values of the bulk modulus K_w corresponding to the degree of saturation S_r .

A simplified expression developed by Koning (1963) may be used.

$$\frac{1}{K_{w}} = \frac{S_{r}}{K_{w0}} + \frac{1-S_{r}}{u}$$
(6)

in which K_{wo} = bulk modulus of water with no air bubbles, K_w = bulk modulus of water containing air bubbles, S_r = degree of saturation, and u = porewater pressure (absolute).

Using the incremental calculations previously described for simple shear tests, the time history of porewater pressure increases may be computed for various initial degrees of saturation S_{ro} (the initial pore pressure is assumed equal to atmospheric pressure). The stress ratios $\sum_{hv} / \frac{1}{vo}$ required to cause initial liquefaction are plotted versus number of cycles for various degrees of saturation in Fig. 20.

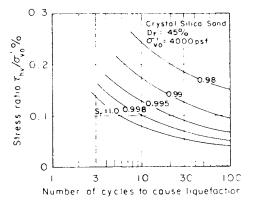


Fig. 20 Effect of Saturation (%) on Liquefaction Potential.

It may be seen that only a slight reduction in degree of saturation results in a significant increase in the stress ratio to cause initial liquefaction in a given number of cycles, a fact that reinforces the necessity for full saturation in liquefaction testing.

CONSTANT VOLUME CYCLIC TESTING

Errors due to system compliance are common to all undrained liquefaction testing. The magnitude of the compliance is primarily a function of the type of test equipment and the grain size of the sand. An alternative procedure for determining the undrained behaviour of sand without the complications of compliance is to conduct constant volume tests. In these tests, the changes in confining pressures to maintain constant volume are equivalent to the changes in porewater pressure in the corresponding un-drained test. Constant volume triaxial testing of sand as an equivalent method of obtaining its behaviour under undrained loading was first suggested by Taylor (1948). Application of the constant volume test to simple shear conditions was proposed by Pickering (1973) in a technical note and he presented some preliminary results on the constant volume liquefaction of saturated Ottawa sand.

The University of British Columbia (UBC) simple shear apparatus, described by Finn et al (1971), has been modified by Finn and Vaid (1977,1978) to permit cyclic shear testing at constant sample volume. The modified apparatus is shown in Fig. 21. The two components of linear horizontal strain are identically zero in this

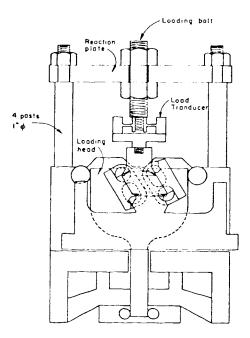


Fig. 21 Constant Volume Cyclic Simple Shear Apparatus.

simple shear appartus. Thus, a constant volume condition is achieved by clamping the loading head to prevent vertical strain. A horizontal reaction plate is clamped to four vertical posts which are threaded into the body of the simple shear apparatus. A stiff vertical load transducer is attached to the sample loading head and carries on its upper side a heavy loading bolt which passes through a central hole in the

reaction plate. The desired vertical pressure on the sample is applied by tightening the loading bolt nut on the underside of the reaction plate. Simultaneously, the loading head is clamped in position by tightening the loading bolt nut on the top side of the reaction plate. Two small stiff pressure transducers (350-kPa capacity and full scale deflection of 0.0015-cm) are permanently mounted on one of the moveable lateral boundaries in order to monitor the lateral stresses during cyclic loading.

Maximum gross volume change introduced at the onset of liquefaction in this so called constant volume test is very small and arises as a result of the recovery of elastic deformation in the vertical loading components when the load on the clamped loading head is reduced to zero. The use of a thick reaction plate, heavy vertical posts and loading bolt, and a very stiff load transducer reduces the vertical movement of the clamped head to a negligible amount. For lique- $/cm^2$

faction tests with initial
$$\phi_{\rm VO}^* = 2 \text{ kg/cm}$$

(196-kPa) this movement amounted to a maximum of

5 x 10^{-4} cm which was only 5% of the movement of the floating head due to system compliance in liquefaction tests on saturated undrained samples in the same equipment and is equivalent to a total vertical strain of the order of 0.02%. Thus, a more accurate evaluation of liquefaction potential can be made using the new test.

A series of conventional liquefaction tests was carried out on saturated undrained samples. Constant values of initial confining stress and cyclic shear stress (:/ \cdot ' = 0.13) were used and only the sample density was varied. The results from this series of tests are shown in Fig. 22. Corresponding results obtained by constant volume liquefaction tests are also shown

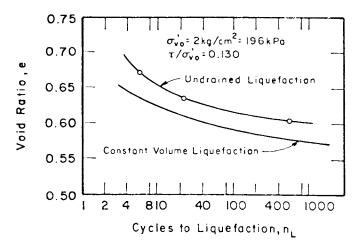


Fig. 22 Comparison of Liquefaction Potentials Measured in Conventional and Constant Volume Cyclic Simple Shear Apparatus.

for comparison. It is clear from Fig.22 that for identical methods of sample preparation the resistance to liquefaction measured in conventional cyclic simple shear tests on undrained saturated samples is consistently larger than

that measured in the constant volume tests. Since the test samples were prepared in an identical manner, the increased resistance to liquefaction in undrained tests is attributable directly to the system compliance in the undrained test.

The constant volume test is very quick and easy to carry out and has none of the difficulties associated with undrained cyclic tests. There is an extraordinarily high degree of reproducibility and consistency in the test results.

ENDOCHRONIC DESCRIPTION OF LIQUEFACTION DATA

Endochronic theory was developed by Valanis (1971) to describe non-linear material response. A key element in this theory is the concept of intrinsic time. This is a parameter which defines the sequence of events leading to successive states of a material. The endochronic method is a "black box" approach that attempts to express important parameters of response to loading as monotonically increasing functions of suitable transformed variables. Such a method has the potential to describe the increases in porewater pressures in sands during cyclic loading.

Bazant and Krizek (1976) developed an endochronic constitutive law for the liquefaction of sand. They used an endochronic description of the stress-strain relations and represented densification or volumetric strains caused by cyclic shearing in terms of endochronic variables. Zienkiewicz, Chang and Hinton (1978) also used endochronic variables to describe the volumetric strains caused by cyclic loading. Finn (1979,1980) and Finn and Bhatia (1980a,b) have investigated the possibility of representing porewater pressures and volumetric strains in a variety of sands by endochronic variables. They found that the porewater pressure response of a sand at a given relative density in cyclic loading tests could be represented by a single porewater pressure function over the ranges of stresses or strains of usual interest. The response function is a super-efficient representation of porewater pressure data, replacing the usual family of curves by a single curve.

The Porewater Pressure Function

Data on porewater pressures, u, developed in Ottawa sand at a relative density $D_r = 45$ % during undrained constant strain cyclic loading tests in simple shear are shown in Fig. 23 for 4 different shear strain amplitudes. For each strain amplitude the non-dimensional porewater pressure ratio, u/o', is plotted against the number of uniform cycles, N, of shear strain γ giving a smooth curve. It is clear that the porewater pressure ratio is a function of shear strain amplitude, γ , and the number of cycles, N, or

$$u/\sigma_{VO}' = f(\gamma, N)$$
(7)

An alternative to N in describing the strain history of the sample is the length, ξ , of the strain path corresponding to N cycles of γ . The variable ξ is a monotonically increasing and

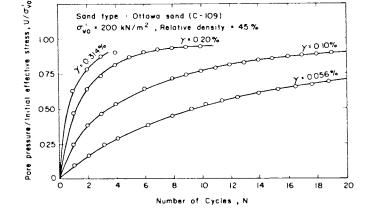


Fig. 23 Porewater Pressure vs. Strain Cycles.

continuous variable. Various definitions of the length of the strain path are possible. A definition which equates an increment in the length of strain path with an increment in deviatoric strain is

$$dt_{j} = \left| \left(\frac{1}{2} dt_{ij} \cdot dt_{ij} \right) \right|^{\frac{1}{2}}$$
(8)

For the simple shear test i, j = 1, 2 and

$$d\varepsilon = |\varepsilon_{12}| = \frac{1}{2} |dy|$$
(9)

The porewater pressure ratio in constant strain tests may now be defined by

$$u/\sigma_{VO}' = g(\gamma, \lambda)$$
(10)

The data in Fig. 23 are shown in Fig. 24 with

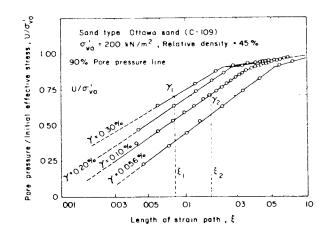


Fig. 24 Porewater Pressure vs. Length of Strain Path.

 $u/\sigma'_{\rm VO}$ plotted against the strain length ξ . A natural logarithmic plot is used to expand the plotted path length at small values of ξ .

The number of cycles, N, in eqn. (7) has been replaced by the continous variable ξ . However, before constant strain data can be generalized to irregular strain patterns the explicit dependence on the shear strain amplitude, γ , must be removed. Ideally, u/σ'_{vo} should be expressible

as a monotonically increasing function of a single variable κ which can be defined unambiguously for both constant strain and irregular strain histories. The variable κ , therefore, must represent all the parameters defining the strain history including varying strain amplitudes and numbers of cycles. Since the effect of the strain history is to induce porewater pressure and weaken the resistance of the sand to deformation, κ is called a damage parameter.

The porewater pressure ratio u/σ_{VO}^* may be expressed as a function of κ if a transformation T exists such that for $\kappa = T\xi$

$$u/\sigma_{VO}^{\prime} = G(\kappa) \tag{11}$$

The transformation T and the function G are found using the data in Fig. 24. If the data are to be explicitly independent of shear strain amplitude then the transformation T should collapse all the curves in Fig. 24 into a single curve giving u/σ'_{VO} as a function of κ , i.e., $u/\sigma'_{VO} = G(\kappa)$. It is reasonable to assume that if such a transformation exists it will be a function of γ .

Referring to Fig. 24, consider a particular value of u/σ'_{vo} which occurs at ξ_1 , for a shear strain amplitude γ_1 and at ξ_2 for a shear strain amplitude γ_2 . Can this value of u/σ'_{vo} be associated with the value κ_1 of a new variable κ such that

$$\kappa_1 = T\xi_1 = T\xi_2 \tag{12}$$

for all (γ_1, ξ_1) and (γ_2, ξ_2) ? If so, then all the curves can be collapsed into one curve giving u/σ'_{vo} as a function of κ . Consider

$$T = e^{\lambda \gamma}$$
(13)

Then

$$\kappa_1 = \xi_1 e^{\lambda \gamma_1} = \xi_2 e^{\lambda \gamma_2}$$
(14)

$$e^{\lambda(\gamma_1 - \gamma_2)} = \xi_2 / \xi_1$$
(15)

or

λ

$$= \ln(\xi_2/\xi_1) / (\gamma_1 - \gamma_2)$$
(16)

The existence of a unique porewater pressure function G(κ) requires a unique value of λ for a given sand at a given relative density. However, when λ is applied to many different data pairs ($\gamma_1, \xi_1; \gamma_2, \xi_2$) in Fig. 24 a range of values of λ is obtained with a mean value $\lambda = 4.99$. Using this value of λ each data point $(u/\sigma'_{VO}, \gamma, \xi)$ was converted to a data point $(u/\sigma'_{VO}, \kappa)$ using the transformation

$$\kappa = \xi e^{4.99\gamma}$$
(17)

The new data points are shown plotted in Fig. 25

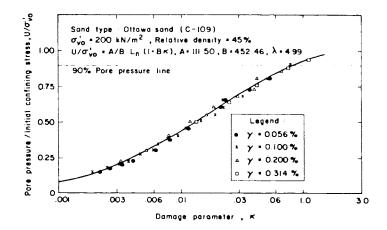


Fig. 25 Porewater Pressure vs. Damage Parameter, κ (Ln. Scale).

against the natural logarithm of κ . Because a unique value of λ does not exist, the plotted points define a narrow band rather than a single curve.

For any sand a wide range in the values of λ corresponding to different data pairs (γ_1 , ξ_1 ;

$$\gamma_2, \xi_2)$$
 may be obtained especially if a wide

range is covered by the test data. However, despite the range in λ , the use of the mean value of λ for a variety of sands consistently has yielded data points falling within a narrow band such as that shown in Fig. 25. A nonlinear least squares curve fitting method was used to determine the curve shown in Fig. 25 describing the relationship between u/σ'_{VO} and κ . The

equation of this curve is

$$u/\sigma_{VO}^{*} = G(\kappa) = (A/B) \ln(1+B\kappa)$$
(18)

with A = 111.50, and B = 452.46.

The same data are plotted in Fig. 26 to a natural scale. The curve shown in Fig. 26 was fitted to the data also by the non-linear least squares curve fitting method. The equation of this curve is

$$u/\sigma'_{VO} = \kappa (D\kappa + C) / (A\kappa + B)$$
(19)

with A = 79.42, B = 0.93, C = 93.58, and D = 71.86.

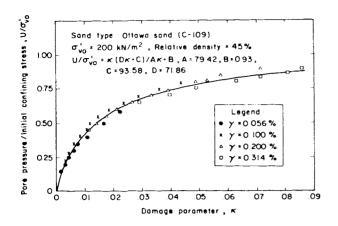
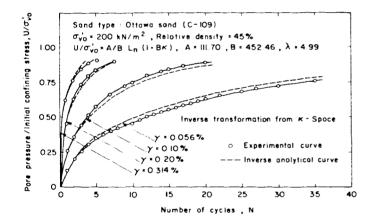
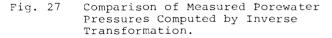


Fig. 26 Porewater Pressure vs. Damage Parameter, κ (Natural Scale).

The accuracy with which the basic test data in Figs. 23 and 24 is represented by eqns. (18) and (19) may be tested by using the inverse transformation of T to transfer points from the curves defined by eqns. (18) and (19) back to Figs. 23 and 24 and comparing computed results with the original test data. Inverse analytical and experimental porewater pressure curves are shown in Fig. 27 plotted against N. It is clear that





over the range of strain amplitudes typically of interest in earthquake engineering the porewater pressure function $G(\kappa)$ gives a very good representation of the test data. It should be emphasized that $G(\kappa)$ represents not just the 4 curves shown but any test curves that might be determined within the same range. The function $G(\kappa)$ blankets the entire strain amplitude range.

If the representation is not considered satisfactory in any particular strain range additional values of λ should be computed in this region. These additional values will weight a new mean value of λ towards this strain range and improve the accuracy of data representation. Experience to-date indicates that, provided a reasonable number of data pairs are used initially in determining the mean λ , no further adjustments are necessary.

The same procedure is followed in order to represent data from constant stress amplitude cyclic simple shear tests. In this case, the length of the stress path is defined as

$$d\xi = (d\sigma_{ij} \cdot d\sigma_{ij})$$
(20)

in which σ_{ij} are deviatoric stresses and for simple shear i, j = 1, 2.

The transformation T for cyclic simple shear tests is given by

$$T = e^{\lambda \tau / \sigma_{VO}}$$
(21)

in which τ is the cyclic shear stress amplitude for σ_{VO}^{i} the initial vertical effective pressure. For these tests eqn. (20) gives

$$d\xi = |d\tau| \tag{22}$$

For cyclic triaxial tests

$$T = e^{\lambda \sigma_{d}/2\sigma_{o}'}$$
(23)

in which σ_d is the cyclic deviator stress and σ_Q' is the initial mean effective stress.

Verification

The first step in verifying the generality of this procedure was to check whether it applied to data from constant strain cyclic loading tests on other sands. Porewater pressure data were obtained on 6 other sands; a Crystal Silica sand described by Silver and Seed (1971) and 5 Japanese sands, Bandajima, Toyoura, Nakashima, Shiomi and Sengenyama. For each sand, using the procedures outlined above, a porewater pressure function $u/\sigma'_{VO} = G(\kappa)$ was obtained which gave an accurate representation of the porewater pressure data for all strain amplitudes tested. The next step in verification of the new procedure

for data representation is to determine whether it can be used to predict porewater pressures for irregular strain histories.

The irregular strain history shown in Fig. 28a was applied in an undrained cyclic simple shear test on Ottawa sand at $D_r = 45$ °. The porewater pressures generated by this strain history are given by the curve labelled "experimental" in Fig. 28b. In applying the new procedure, the strain history in Fig. 28a was converted incrementally to the continuous variable, the strain path length ξ by eqn. (8). The variable ξ was then transformed to the variable κ by the transformation T given by eqn. (13) using the value $\lambda = 4.99$ determined from the constant strain amplitude test data. Then the porewater pressures due to the increasing κ were computed using either eqn. (18) or (19). The computed

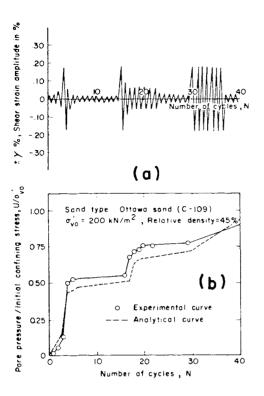


Fig. 28 Comparison of Computed and Measured Porewater Pressures for Irregular Strain History.

porewater pressures are shown by the curve labelled "analytical" in Fig. 28b. The calculated and measured values of porewater pressure are in close agreement. Similar results have been obtained for the other sands. Therefore, it appears that the procedure outlined above may be used to predict directly the porewater pressures due to irregular strain histories from data on uniform cyclic strain histories.

The general applicability of the new procedure to predicting the porewater pressure response of saturated sands under irregular shear stress loadings using data from constant cyclic stress tests has also been verified. However, the accuracy is always less than for constant strain tests. This is due to the fact that experimental scatter in constant stress test data is always greater than for constant strain data. The increased scatter occurs primarily at high porewater pressures exceeding 70% of effective overburden pressure.

Application in Dynamic Analyses

The development of the porewater pressure function $u/\sigma'_{VO} = G(\kappa)$ from basic test data greatly simplifies both total and effective stress methods of dynamic analysis.

In dynamic total stress analyses computed stress or strain histories are transformed to values of the damage parameter κ and the magnitude of porewater pressure is determined directly from eqns. (18) or (19). There is no longer any need to run additional analyses to convert the irregular stress or strain histories to equivalent uniform excitations before using data from laboratory tests.

In dynamic effective stress analysis a porewater pressure model based on fundamental properties of the soil skeleton and the water is no longer required. During any time increment Λ t in the dynamic analysis, the increment in porewater pressure can be determined from the incremental change in κ . The soil properties now may be modified for this change in porewater pressure and the analysis continued for the next time increment Λ t. The use of the new procedure means that no special laboratory tests are required to get parameters for dynamic effective stress analysis. It now can be based entirely on routine laboratory tests.

PROBABILISTIC METHODS

The application of probabilistic methods in soil dynamics is still in a rudimentary state of development. Progress to 1979 has been summarized by Christian (1980). Many of the attempts to apply probabilistic methods have focussed on the liquefaction potential of level ground and involve the introduction of probabilistic descriptions of some of the significant variables in Seed and Idriss' (1971) simplified method for estimating liquefaction potential.

The first significant contribution was made by Donovan (1971) with the introduction of the cumulative damage method. An assessment of this method compared with other approaches has been presented by Valera and Donovan (1977).

The simplified cumulative damage procedure is based on the concept that the effect of cyclic loading on soil is analogous to fatigue effects and therefore uses Miners' fatigue law to determine the cumulative effect of the cycles of shear stress. A Rayleigh-type distribution function corresponding to a narrow band process has been used to describe the distribution of peak shear stresses during an earthquake. The distribution is defined by the sigma ratio, the ratio of the peak acceleration to the root mean square value. The ratio has a range between 3 and 6 when computed for the effective duration of a strong motion record as defined by Husid et al (1969) and Trifunac and Brady (1975). If the total duration of the record is used the ratio may be as large as 10 (Donovan and Singh, 1978). In effect, Donovan treats the soil properties as deterministic and the earthquake related variables as probabilistic. Faccioli (1973) has developed a somewhat similar approach based on fatigue theory but treats the earthquake statistics differently.

Halder and Tang (1979), on the other hand, treat all the variables in the simplified Seed and Idriss procedure as random variables. The uncertainties associated with these variables are described by their variances. Halder and Tang investigated the sensitivity of the probability of liquefaction to levels of uncertainty and tried to determine the relative significance of the various parameters. Calculated probabilities of liquefaction showed substantial agreement with the recorded behaviour of various sites during earthquakes. Yegian and Whitman (1978) place the problem of liquefaction in the context of overall seismic risk analysis and attempt to assess the probability of foundation failure due to liquefaction. In their risk analysis, they characterize the intensity of the earthquake by parameters related to the earthquake rather than the maximum acceleration of the site; they use the magnitude M and the hypocentral distance R rather than the epicentral distance. The hypocentral distance appears to be more appropriate at closer distances, especially for deep earthquakes. For shallow earthquakes and at greater distances, the two measures of distance are quite similar.

For a given earthquake E, characterized by M and R, the probability of liquefaction ${\rm P}_{\rm E}\,({\rm F}_{\rm L})$ can be expressed as

$$P_{E}(F_{L}) = P(F_{L} | MR) \cdot P(MR)$$
(24)

where $P(F_L | MR)$ is the probability that lique-faction will occur if the earthquake occurs and P(MR) is the probability of earthquake occurrence.

To evaluate P(F_L|MR) an empirical expression characterizing the resistance of a site to liquefaction was developed from observation of liquefaction and non-liquefaction in the field during earthquakes. The conditional probability P(F_L|MR) is then combined with P(MR) to estimate

the overall risk.

Probabilistic approaches to the assessment of liquefaction are not yet common in engineering practice. However, Donovan's method was used to establish liquefaction criteria for the Alaskan oil pipeline (Donovan and Singh, 1978).

POREWATER PRESSURE MODELS

It has long been a fundamental principle of soil mechanics that deformations are controlled by effective stresses and effective stress methods of stability analysis are widely used in static problems. Formerly, however, all dynamic analyses including those for liquefaction studies were carried out in terms of total stresses because of the lack of a model for predicting the porewater pressures developed by cyclic or seismic loading which could be conveniently coupled with a procedure for dynamic analysis. The first such model was proposed by Martin, Finn and Seed (1974,1975). This model is based on the physical properties of the sand skeleton and the water and the volumetric strain potential of the coupled fluid-sand system. Because of its fundamental nature the model has proved very useful not only for dynamic effective stress analysis but for investigating the influence of environmental and test conditions on porewater pressure response (Martin et al, 1978).

A very different kind of model has been proposed by Ishihara, Tatsuoka and Yasuda (1975) based on the effective stress path during cyclic loading. A modified form of this model has been proposed by Ghaboussi and Dikmen (1978) as part of a comprehensive procedure for the analysis of liquefaction. Stress path models for predicting changes in porewater pressures or effective stresses during cyclic and seismic loadings are based on the premise that the effective stress path during undrained cyclic loading can be established with sufficient accuracy on the basis of certain simplifying assumptions about the behaviour of sands under load. The reliability of the models depends on how closely the assumed behaviour of sands approximates reality.

The stress path in q-p' space (q = shear stress; p' = mean normal stress) during plastic deformation under undrained loading conditions is assumed to be of a simple geometrical shape and thus can be specified by a simple mathematical function. In the original model the stress path was assumed to be a circle. Ghaboussi and Dikmen (1978) assume that the effective stress path is one quarter of an ellipse. The stress path deviates abruptly from the assumed shape as the critical state or failure line is approached. The points at which various stress paths change direction abruptly are assumed to lie on a line called the phase transformation line (Fig. 11) and the stress path model in its basic form is assumed to apply only for the region of stress space between the phase transformation lines in compression and extension. The phase transformation lines are assumed to correspond to initial liquefaction and any cyclic loading beyond this stage quickly results in very low effective stresses and complete liquefaction. Unloading from any point within the phase transformation lines is assumed to result in purely elastic deformations and to cause no changes in porewater pressures or effective stresses. In the q-p' plane, therefore, unloading is represented by vertical straight lines parallel to the q-axis. Finn (1979) has given a critical review of the stress path porewater pressure models.

Ishihara (1980) has made some significant changes in his original model. The stress path is now assumed to be parabolic and it is defined in terms of the horizontal shear cyclic stress τ , and normal effective stress σ'_V in level ground during an earthquake (Fig. 29). More importantly,

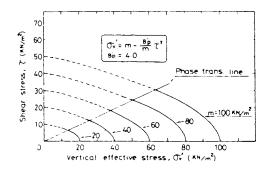


Fig. 29 Parabolic Stress Paths with Varying m-values (Ishihara, 1980).

the assumption that no porewater pressures are developed during unloading has been abandoned. The shape of the unloading curve is now defined in such a way that there is an empirically controlled development of porewater pressure during unloading (Fig. 30). Ishihara (1980) has also defined the stress path between the phase transformation and failure lines (Fig. 11), an area ignored in the original version of the model.

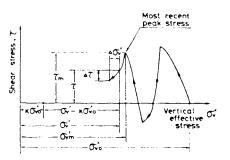


Fig. 30 Pore Pressures during Unloading (Ishihara, 1980).

The Martin, Finn and Seed model is finding increasing use in practice, not only in various procedures for dynamic response analysis, but also to explain the pore pressure response under cyclic loading. For these reasons, the model has been under continuous study. Since its development, equipment and test procedures have emerged which allow the individual assumptions underlying this model to be verified directly. The results of comprehensive study of the model have been reported by Finn and Bhatia (1980c) and some of the results will be reported here.

Verification of the Model

The model is defined by eqn. 3, repeated below.

$$\Delta u = \frac{E_{r} \Delta e_{vd}}{1 + \frac{n\bar{E}_{r}}{K}}$$
(3)

In its simplest form, the model assumes that the water is an order of magnitude stiffer than the soil skeleton and therefore that the net volumetric strain during undrained cyclic loading is negligible. This is the usual assumption that undrained tests are constant volume tests. The consequence of this assumption is that the plastic and elastic volumetric strains that occur during cyclic loading must be equal and of opposite sense. This, for example, is a fundamental assumption of the critical state theory as applied to undrained loading (Schofield and Wroth, 1968). Since residual (or permanent as opposed to transient) porewater pressures are generated during undrained cyclic loading of sands, there must be a tendency for plastic volumetric strains to occur. These strains are absorbed by elastic rebound in the soil skeleton due to the reduction in effective stresses and "constant" volume is preserved, but they are recoverable on allowing the sample to drain. The increment in porewater pressure, Au, during a short time interval, At, of cyclic loading is then assumed to be given by

$$\Lambda u = \tilde{E}_r \Lambda \epsilon_{vd}$$
(25)

in which $\Lambda \varepsilon_{\rm vd}$ is the elastic (or plastic) volumetric strain increment that occurs due to the increment in porewater pressure and $\overline{\rm E}_{\rm r}$ is the rebound modulus appropriate for the effective stress state in the sand at the beginning of the time increment, Δt .

The porewater pressure model given by eqn. 25 posed immediately the important question; how was $\Delta \varepsilon_{\rm vd}$ to be obtained? It was assumed that the plastic volumetric strain $\Delta \varepsilon_{\rm vd}$ which developed during one cycle of uniform shear strain γ in an undrained simple shear test would be the same as the volumetric strain in a drained simple shear test. Therefore, a fundamental assumption of the pore pressure model is that there is a unique relationship between volumetric strains in drained tests and porewater pressures in undrained tests on a given sand at corresponding strain histories. The following relationship, independent of effective stress level, was found between $\Delta \varepsilon_{\rm vd}$ and γ , provided

no crushing of the sand grains occurred,

$$\Delta \varepsilon_{vd} = C_1 (\gamma - C_2 \varepsilon_{vd}) + C_3 \varepsilon_{vd}^2 / (\gamma + C_4 \varepsilon_{vd})$$
(26)

in which C_i , i=1,4 are experimentally determined constants.

The measurement of $\bar{\rm E}_{\rm r}$ posed difficult problems. Rebound occurs during undrained cyclic loading but direct measurement of $\bar{\rm E}_{\rm r}$ under such conditions was not possible at that time. Therefore, $\bar{\rm E}_{\rm r}$ was measured during static rebound in a consolidation ring. Test data obtained at the time indicated that $\bar{\rm E}_{\rm r}$ measured in this way resulted in satisfactory predictions of liquefaction strength curves. As will be seen later, $\bar{\rm E}_{\rm r}$ measured in this way results for earlier test data primarily because compliance in the early simple shear equipment absorbed the excess porewater pressure generated by the excessive stiffness of $\bar{\rm E}_{\rm r}$.

The verification of the basic assumptions of the model consists of providing adequate and experimentally based answers to these questions:

- (i) In constant strain cyclic loading tests, is there a unique relationship between volume changes in drained tests and porewater pressures in undrained tests?
- (ii) Can \tilde{E}_r be measured statically?
- (iii) Can the model accurately predict the history of development of porewater pressures and not just the liquefaction strength curve under both uniform and non-uniform loading conditions?

Experimental Verification

Volumetric strains were measured in drained constant strain cyclic simple shear tests on Ottawa sand at relative densities $D_r = 45\%$ and $D_r = 60\%$.

Porewater pressures were also measured in undrained cyclic tests at the same relative

densities and initial effective confining pressures. Silver and Seed (1971) have shown that volumetric strains are independent of effective vertical confining pressure so the requirement of equal confining pressures in both kinds of tests was not required. However, since uniformity in procedure is conducive to uniformity in samples, the initial effective confining pressures were kept identical for all tests. Volumetric strains are shown plotted against porewater pressures in Fig. 31 for $D_r = 45$ %.

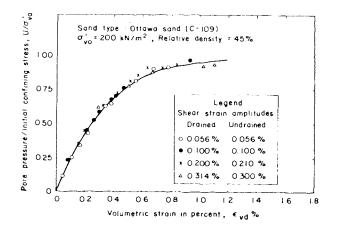


Fig. 31 Relationship between Volumetric Strains and Porewater Pressures in Constant Strain Cyclic Simple Shear Tests.

Ideally, each point should represent corresponding values of these variables for a given number of cycles with equal cyclic strain amplitudes. It will be noticed that there are slight deviations from equality in the applied shear strain amplitudes. The deviations are not considered to be important. The data indicates an apparently unique relationship between the volumetric strains and the porewater pressures. A unique relationship was also found for $D_r = 60$ % (Fig. 32).

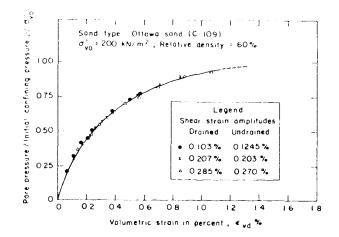


Fig. 32 Relationship between Volumetric Strains and Porewater Pressures in Constant Strain Cyclic Simple Shear Tests.

The slope of the curve representing the relationship between porewater pressure normalized with respect to confining pressure and volumetric strains is the dynamic rebound modulus K_d , normalized with respect to the initial confining pressure. This value of the rebound modulus is less than that measured under static conditions. This agrees with the observations of Seed, Pyke and Martin (1979) that the recoverable strains after cyclic loading are greater than under static conditions. The reasons underlying this difference in response to unloading are being researched further.

The original pore pressure model was based entirely on tests on normally consolidated sands. It is well-known that overconsolidated sands have a greater resistance to liquefaction, the resistance increasing with overconsolidation ratio, OCR. Both drained and undrained constant strain cyclic simple shear tests were conducted on Ottawa sand at OCR = 2,3 and 4 to check the effects of overconsolidation on the model parameters and to investigate whether the model could successfully predict the porewater pressure response for overconsolidated sands. In terms of the model parameters, the chief effect of overconsolidation was reduced volume change due to a given drained cyclic loading compared with that in normally consolidated Ottawa sand (Fig. 5). The reduced volumetric strain potential of the overconsolidated samples was reflected in lower values of the volume change constants C_i (i=1,4).

The unique relationship between volumetric strains in drained cyclic simple shear tests and porewater pressures in undrained tests after similar strain histories established for normally consolidated sands was found to hold also for the overconsolidated sands. Porewater pressures and corresponding volumetric strains for Ottawa sand at $D_r = 47\%$ and OCR = 3 shown in Fig. 33. The data fall very closely on a single

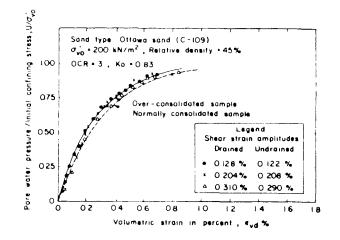


Fig. 33 Relationship between Volumetric Strains and Porewater Pressures in Constant Strain Cyclic Simple Shear Tests.

curve. The corresponding curve for OCR = 1 is shown for comparison. Similar relationships between porewater pressures and corresponding volumetric strains were also found for OCR = 2 and 4. On the basis of this evidence, it may be claimed that the fundamental assumption of the model that a unique relationship exists between porewater pressures in undrained cyclic simple shear tests and volumetric strains in drained cyclic simple shear tests holds for both normally and overconsolidated sands. Furthermore, a single curve may be used to define the relationship between porewater pressures and volumetric strains over the range of OCR covered by the test program without significant error.

The ability of the model to predict porewater pressures in overconsolidated sands was tested by predicting the liquefaction strength curves for various OCR and comparing the results with experimental curves. The strength curves are plots of the cyclic stress ratio τ/σ'_{VO} versus the number of cycles to liquefaction N; τ being the applied cyclic shear stress. Figure 34 shows

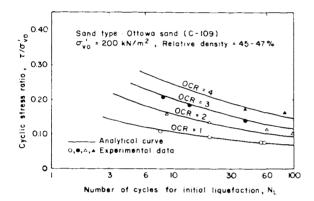


Fig. 34 Cyclic Stress Ratio vs. Number of Cycles for Initial Liquefaction for Various OCR-Ratios.

liquefaction strength curves with OCR = 1,2,3 and 4 computed using eqns. 25 and 26 for Ottawa sand at $D_r = 45$ %. The points are experimental data from undrained constant volume cyclic simple shear tests. The initial effective vertical pressure σ'_{vo} in all tests after the OCR was established was $\sigma'_{vo} = 200 \text{ kN/m}^2$.

The comparison between the computed and measured liquefaction strengths is very good. It seems that the porewater pressure model originally developed for normally consolidated sands, may also be used for overconsolidated sands at least up to the OCR = 4 for which direct verification is available.

It is crucial to verification of the model to test its predictive capability under test conditions other than those under which the model parameters are derived. Since constant strain cyclic tests were used to determine the model parameters, the predictive capability of the model was first tested under constant stress cyclic test conditions. These loading conditions result in an irregular strain history as the porewater pressures develop. The shear strains were measured electronically and used in conjunction with eqns. 25 and 26 to compute the development of porewater pressure. In these calculations $\bar{\mathrm{E}}_{\mathrm{r}}$ = K_d. The comparisons between predicted and measured porewater pressures are shown in Fig. 35 for D_r = 45% and in Fig. 36

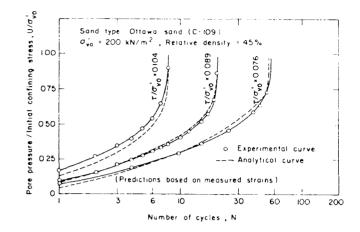


Fig. 35 Predicted and Measured Porewater Pressures in Constant Stress Cyclic Simple Shear Tests, $D_r = 45$ %.

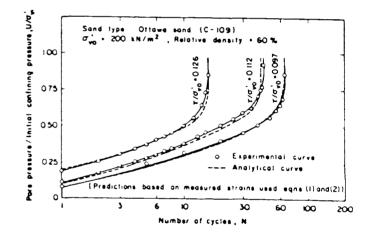


Fig. 36 Predicted and Measured Porewater Pressure in Constant Stress Cyclic Simple Shear Tests, D_r = 60%.

for $D_r = 60$ %. The predictive capability of the model appears to be satisfactory.

When the porewater pressure model is used in dynamic effective stress analysis in practice, the shear strains to be used in eqn. 26 must be computed using an appropriate stress-strain law. Finn, Lee and Martin (1976,1977) adopted a non-linear hysteretic effective stress-strain law with Masing behaviour in unloading and reloading in their method of dynamic effective stress analysis. In the verification study this stress-strain law was used to compute the shear strains required in eqn. 26 for calculating the volumetric strains. The porewater pressures in constant stress tests were then determined by eqn. 25. Typical results are shown in Fig. 37. The agreement between measured and computed porewater pressures is quite

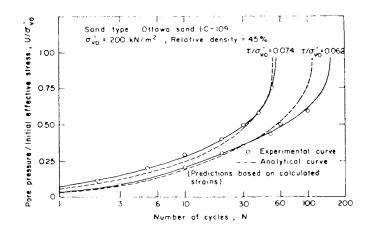


Fig. 37 Predicted and Measured Porewater Pressures in Constant Stress Cyclic Simple Shear Tests, D_r = 45%.

good but not as good as when measured shear strains rather than computed were used in eqn. 26 as might be expected.

The effective stress approach combining the porewater pressure model and the stress-strain relationship was also used to investigate the effects of previous loading history. Sample A, at $D_r = 45$ %, was first subjected to 70 cycles of a stress-ratio $1/\sigma'_{VO} = 0.066$ and then allowed to drain. The rate of development of porewater pressure under this loading is shown in Fig. 38 by curve A. Sample B, also at $D_r = 45$ %, was

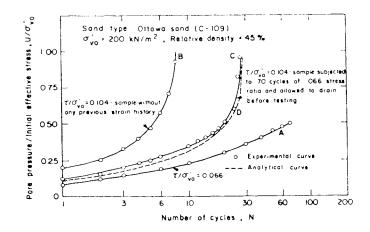


Fig. 38 Predicted and Measured Porewater Pressures in a Sand with Previous Loading History.

subjected to a cyclic stress ratio of $\tau/\sigma'_{vo} = 0.104$. The rate of development of pore-water pressure in this sample is shown in Fig.

38 by curve B. Sample A is now subjected to the cyclic stress ratio $:/_{vo}$ = 0.104 and the rate of development of porewater pressure is given by curve C in Fig. 38. This rate of increase in porewater pressure is considerably less than that generated in sample B which had no previous stress history by the same cyclic stress ratio. The porewater pressures predicted by the effective stress model are given by curve D in Fig. The comparison between predicted and measured porewater pressures is good. The results of the verification study indicate that when the necessary soil properties can be measured accurately, very good correspondence can be obtained between computed and measured porewater pressures over a wide range of loading conditions for both normally and overconsolidated sands.

It must be stressed that the model was developed and verified on the basis of simple shear data. Therefore, it may be used with confidence only for the analysis of cyclic loading problems in the field for conditions which can be approximated by simple shear conditions. This implies more or less level ground underlain by horizontal soil layers and excited by shear waves propagating vertically. Extensions of the model to 2 and 3 dimensions are being explored.

The Model in Practice

For convenience the rebound modulus \overline{E}_r measured in static rebound tests is expressed as

$$\bar{E}_{r} = (\sigma_{v}^{\prime})^{1-m} / mK_{2}(\sigma_{vo}^{\prime})^{n-m}$$
(27)

in which σ'_{vo} is the initial value of the vertical effective stress, $\sigma_{\mathbf{v}}^{*}$ the current value and K₂, m and n are experimental constants for the given sand. Thus, in applying the model in dynamic effective stress analyses 7 constants are measured, the 4 C, and the 3 constants specifying the rebound characteristics of the sand. The C-constants are measured in constant strain cyclic simple shear tests and many commercial laboratories still do not have the capability to conduct these tests. But the greatest difficulties are associated with the measurement of \tilde{E}_r . As was shown above, the magnitude of the rebound modulus $\overline{E}_{\rm r}$ is less under cyclic loading conditions than for unloading under static conditions. The measurement of \bar{E}_r under cyclic loading conditions is time-consuming.

Over the last 3 years the porewater pressure model has been used in engineering practice in the computer programs DESRA-1 and DESRA-2 (Lee and Finn, 1975,1978). The programs are based on the method for non-linear dynamic effective stress analysis developed by Finn, Lee and Martin (1976,1977) and are similar except that DESRA-2 contains an energy transmitting boundary. A procedure has evolved from this practical experience which avoids the direct measurement of the constants. The rebound modulus for static unloading is used (or a typical value selected from the literature) and the volume change constants are obtained by a regression analysis on conventional laboratory data. The values of the constants are selected to ensure that the porewater pressure model predicts the field liquefaction strength curve satisfactorily and gives a rate of porewater pressure development that fits that observed in the conventional laboratory tests. This procedure has proven both satisfactory and efficient in practice. Thus, in most practical problems it is not necessary to measure the model constants directly.

An alternative approach can be used when the volume change constants are known or easily measured. In this case, the $\rm K_2$ constant in the

expression for the rebound modulus is adjusted to give the proper liquefaction strength curve or rate of porewater pressure generation.

The porewater pressure model can be bypassed entirely by establishing a direct link between the porewater pressure and the dynamic response parameters such as the strain or stress history. Endochronic variables supply that link as described earlier. DESRA-1 and DESRA-2 are now being reprogrammed in terms of endochronic variables and dynamic effective stress analysis may then be based entirely on data from conventional undrained cyclic loading tests.

Limiting Strain Concept

The porewater pressure model clearly demonstrates the key role played by shear strains in the generation of porewater pressures. Stoll and Kald (1976) advanced the concept that there might be a limiting shear strain below which no porewater pressures would develop regardless of the number of loading cycles. On the basis of limited test data they suggested that the threshold shear strain was of the order of 10^{-4} or 10^{-2} %. Investigations by Dobry and Grivas (1978) and Dobry and Swiger (1979) support this value for the threshold strain.

Dobry, Powell, Yokel and Ladd (1980) have proposed the use of a threshold shear strain to estimate liquefaction potential of saturated sands. In this approach, the simplified method of Seed and Idriss (1971) is cast in terms of shear strains rather than shear stresses so that ultimately a threshold of peak acceleration is defined in terms of the threshold strain. If the anticipated peak acceleration is greater than the threshold liquefaction may be possible. The method, therefore, appears to establish the susceptibility to liquefaction rather than determining whether for given field data liquefaction will or will not occur.

NON-LINEAR DYNAMIC EFFECTIVE STRESS ANALYSIS

One of the major developments since 1976 has been the emergence of dynamic effective stress analyses and their gradual introduction into engineering practice. A variety of methods are now available based on the porewater pressure models described in the previous section and all incorporating true non-linear stress-strain relations. These methods were developed by Finn, Lee and Martin (1976), Bazant and Krizek (1976), Ghaboussi and Dikmen (1978), Zienkiewicz, Chang and Hinton (1978), Ishihara (1980) and Blazquez, Krizek and Bazant (1980). All of these methods can qualitatively predict the phenomena noted in saturated sands during earthquakes. Most of the models are so new that there is little practical experience with their use or little opportunity to check their predictive capability by either laboratory tests or field data. Therefore, there are few grounds for making a critical assessment of the relative merits of these different approaches. There are prospects that elastic-plastic cap models, well established in practice for static analysis, may be extended for use in cyclic loading. The work of Baladi and Rohani (1979) which has resulted in 3-dimensional isotropic elastic-plastic constitutive modul for saturated granular material would seem to provide the necessary foundation for that development.

Even a casual review of the cited references on non-linear dynamic effective stress analysis reveals that these analyses are fairly complicated and, in some cases, require the measurement of parameters strange to geotechnical practice. Are such sophisticated methods necessary and is the effort devoted to their development justified? In order to answer this question, it is necessary to examine the capability of existing methods of analysis.

Engineering practice is dominated by total stress equivalent linear analyses. A well-known example is the program, SHAKE, developed by Schnabel, Lysmer and Seed (1972). Finn, Martin and Lee (1978) have conducted a comparative study of equivalent linear and non-linear methods of total stress dynamic analysis and of total stress and effective stress methods of dynamic analysis using a hypothetical site of saturated sand 15 m deep with the water table near the surface. The site was excited by the first 10 sec of the N-S acceleration component of the El Centro earthquake of 1940 scaled to 0.1 g. The programs SHAKE, based on equivalent linear analysis, CHARSOIL, a non-linear program (Streeter, Wylie and Richart, 1973) and DESRA-1, a non-linear program which can be operated in either the total or effective stress mode were used.

Acceleration and Stress Response

The acceleration response spectra of ground motions at a sandy site 15 m (50 ft) deep which were computed by SHAKE, CHARSOIL and DESRA are shown in Fig. 39. The spectra all show strong response around a period of 0.5 secs but SHAKE shows much stronger response than the non-linear programs. This stronger response is also re-flected in the magnitudes of computed maximum dynamic shear stresses at various depths in the deposit (Fig. 40). This tendency towards resonant response in analyses based on the equivalent linear method has been noted in a number of comparative studies. Resonance occurs when the fundamental period of the input motion corresponds to the fundamental period of the site as defined by the final set of compatible properties in the iterative equivalent linear method of analysis. Since the analysis is carried out with this constant set of properties for the entire duration of the earthquake, there is time for resonant response to build up. In the non-linear methods this tendency is controlled by the constantly changing stiffness properties. When strong resonant response is a function

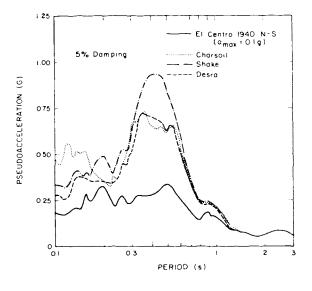
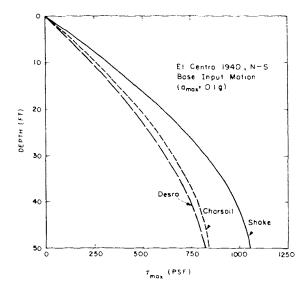
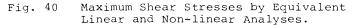


Fig. 39 Acceleration Spectra by Equivalent Linear and Non-linear Analyses.





primarily of the method of analysis it is called pseudo-resonance. Pseudo-resonance may lead to exaggerated dynamic response.

Seed and Martin (1979) have analysed the same site using the non-linear program MASH (Martin and Seed, 1978). The results are shown in Fig. 41 and demonstrate a remarkable degree of agreement.

Estimation of Liquefaction Potential

SHAKE uses a total stress analysis in conjunction with laboratory test data to assess liquefaction potential. To allow a comparison on common terms, therefore, both CHARSOIL and DESRA were also used in the total stress mode in conjunction with data on liquefaction resistance.

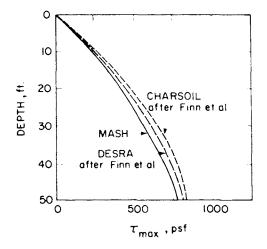


Fig. 41 Shear Stresses by 3 Non-linear Methods (Martin and Seed, 1978).

The variation in the factor of safety against liquefaction with the magnitude of base acceleration as computed by the various methods is shown in Fig. 42. The results for CHARSOIL and DESRA

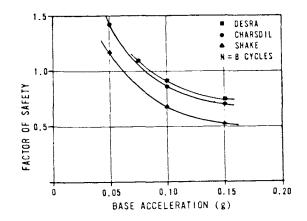


Fig. 42 Factors of Safety Against Liquefaction for $N_{eq} = 8$ (Total Stress Method).

are very similar as might be expected since they give similar shear stress histories. Since SHAKE yielded higher values for the maximum dynamic shear stress it is not surprising that it indicates lower factors of safety against liquefaction for this site.

Effective stress analyses were carried out using DESRA under three different drainage conditions:

- no internal redistribution of porewater pressure,
- 2. internal redistribution but no dissipation across external drainage boundaries, and
- 3. with dissipation.

The analyses were conducted assuming a uniform permeability K = 0.003 ft/sec (0.1 cm/sec).

This permeability is representative of medium sands. Effective stress estimates of liquefaction potential are compared with the previous estimates based on total stress analyses using the same input motions. The factors of safety against liquefaction computed for both total and effective stress methods for various levels of maximum acceleration are shown in Fig. 43.

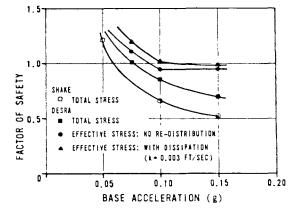


Fig. 43 Factors of Safety Against Liquefaction by Total and Effective Stress Methods.

The effect of porewater pressures is evident in the pseudo-acceleration spectra shown in Fig. 44. The spectrum for total stress analysis indicates a maximum at a period determined only by the final strain compatible shear moduli. The effect

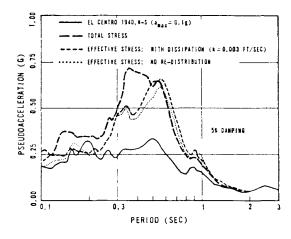


Fig. 44 Effect of Porewater Pressures on Fundamental Period.

of increasing porewater pressure on the moduli is not included. The porewater pressure leads to a further softening or reduction in shear moduli and hence an increase in the fundamental period. This shift in period is indicated by the spectra determined by effective stress analyses. In many situations total and effective stress methods may be expected to yield similar results provided the magnitudes of the porewater pressures generated during shaking are such that they do not have a significant effect on the computed dynamic response or significant pseudo-resonance does not develop.

CONCLUSION

An attempt has been made to give a critical review of developments in liquefaction studies over the period 1976-1980. The period is so short that critical appraisal has been possible only for evolutionary rather than radically different developments. However, it is hoped that the review provides a useful overview of the present state of liquefaction studies.

ACKNOWLEDGEMENTS

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