Liquid Argon
Maximum Convective Heat Flux
vs Liquid Depth

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D0 EN 237

DEPTH IN LIQUID ARGON VERSUS MAXIMUM CONVECTIVE HEAT FLUX

by Tom Peterson
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(report on work done in February 1988)

Introduction.

In order to help answer questions about the magnitude of heat flux to the liquid argon in a liquid argon calorimeter which could cause boiling (bubbles), calculations estimating the heat flux which can be removed by free convection were made in February, 1988. These calculations are intended to be an estimate of the heat flux above which boiling would occur. No formal writeup was made of these calculations, although the graph dated 3 Feb 88 and revised (adding low-velocity forced convection lines) 19 Feb 88 was presented in several meetings and widely distributed. With this description of the calculations, copies of the original graph and calculations are being added to the D0 Engineering Note files.

Assumptions.

The liquid argon surface is in equilibrium with argon vapor at a pressure of 1.3 bar, so the surface is at 89.70 K. The liquid is entirely at this surface temperature throughout the bulk of the volume, except locally where it is warmed by a solid surface at a higher temperature than the bulk liquid. This surface temperature is taken to be the boiling temperature of argon at the pressure corresponding to 1.3 bar plus the liquid head; hence it is a function of depth below the surface. The free and forced convection correlations used are from Kreith, "Heat Transfer", for heated flat plates in a large (i.e., no other objects nearby enough to disturb the flow) uniform volume of fluid. Heat flux is a function of plate size, really length along the flow path (since a boundary layer increases in thickness starting from the leading edge of the plate), and orientation (i.e., vertical or horizontal).

Method.

A table (on page 1) was made using delta-T above the surface temperature of 89.70 K as the independent variable. From this the saturation (boiling) pressure corresponding to 89.70 K plus delta-T and depth below the surface were found. It can be seen that the liquid density at a constant (surface) temperature is practically constant with depth, and this density was used to calculate depth for a given pressure. A density at the elevated temperature (saturation temperature) also has to be tabulated since it is the difference between this and the bulk density that drives the free convection. Other fluid properties (thermal conductivity, viscosity, and Prandtl number) are found for the pressure at the depth and an average of the saturation (surface) and bulk temperatures. Grashof number includes a factor L cubed (L is plate length), so Grashof divided by L cubed is calculated first.

The tables on pages 2 and 3 contain Grashof number, Nusselt number, and convection coefficients for the various lengths and orientations, tabulated as a function of head. The product of delta-T and the convection coefficient gives the heat flux, tabulated on page 4.

Convection coefficients are also calculated for low-velocity forced convection for comparison.

The resulting heat fluxes were then plotted as a function of distance below the liquid surface. Note that depth is on the vertical axis with zero at the top, and heat flux is on the horizontal axis.

Results.

The maximum heat flux which can be carried away by free convection (i.e., the heat flux above which boiling occurs) is .001 W/sq.cm. at 4 inches below the surface and 0.1 to 0.2 W/sq.cm. 15 feet below the surface. Forced convection over a 1 cm plate with a fluid velocity of 1 cm/sec, or a 10 cm plate at 10 cm/sec, is about like free convection. The line for much higher heat flux is 10 cm/sec flow over a 1 cm plate.

Discussion.

The two key assumptions here are that the bulk of the liquid is at the surface temperature, and that the threshold of boiling is when the solid surface is at the saturation temperature of the liquid. It is possible that experiments would give free convection heat fluxes much lower or much higher than these results if these assumptions are in error.

Much higher heat fluxes via free convection, especially near the surface where I used very small delta-T's, might be possible if nucleation of boiling occurs at some surface temperature significantly (like a degree) above the saturation temperature of the liquid. Nucleation would not occur at a lower delta-T than was used here, since it was just the delta-T to the saturation temperature.

So this calculation took the most pessimistic possible assumption (in terms of avoiding boiling) regarding the onset of nucleation.

Conversely, the limits of free convection might be much lower than calculated here, again especially near the surface, since the mechanism of heat dissipation from the bulk involves warming of the bulk liquid above the surface temperature and transfer of heat through a boundary layer to the surface where evaporation takes away the heat (Atkinson, et. al., "Heat and Evaporative Mass Transfer Correlation at the Liquid-vapour Interface of Cryogenic Liquids", ICEC 10, 1984). Based on that paper, the predicted heat flux to EC, and the surface area for evaporation in EC, I calculate that the liquid would be superheated to a depth of about 3 feet. This free convection calculation would result in no allowable heat flux to that depth since the liquid is already at or above saturation temperature.

A third factor which could cause reality to differ from these calculations is that the geometry is not a small heated flat plate suspended in fluid. Corners, edges and irregularities will disrupt the boundary layer and enhance free convection heat transfer. But a hot spot on the inner vessel wall will have no leading edge, so the velocity profile will be different from what is assumed here. It may be best approximated by the 10 cm plate in these calculations since that is mostly covered by a thicker boundary layer than the 1 cm plate, but the total heat added is small enough that flow is still laminar. The vertically oriented 100 cm plate has a higher heat flux to the liquid than the 10 cm plate since for the 100 cm plate flow is turbulent rather than laminar.

Conclusions.

As I have indicated in the above discussion, there is considerable uncertainty in these results. Near the surface I can imagine a difference from these calculations of two orders of magnitude in either direction. At 15 feet deep I have much more confidence in these predictions; I would expect them to be within a factor of two of experimental results for the onset of boiling for heated objects in the argon.

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SUBJECT DEPTH IN LIQUID ARGON VERSUS MAXIMUM CONVECTIVE HEAT FLUX

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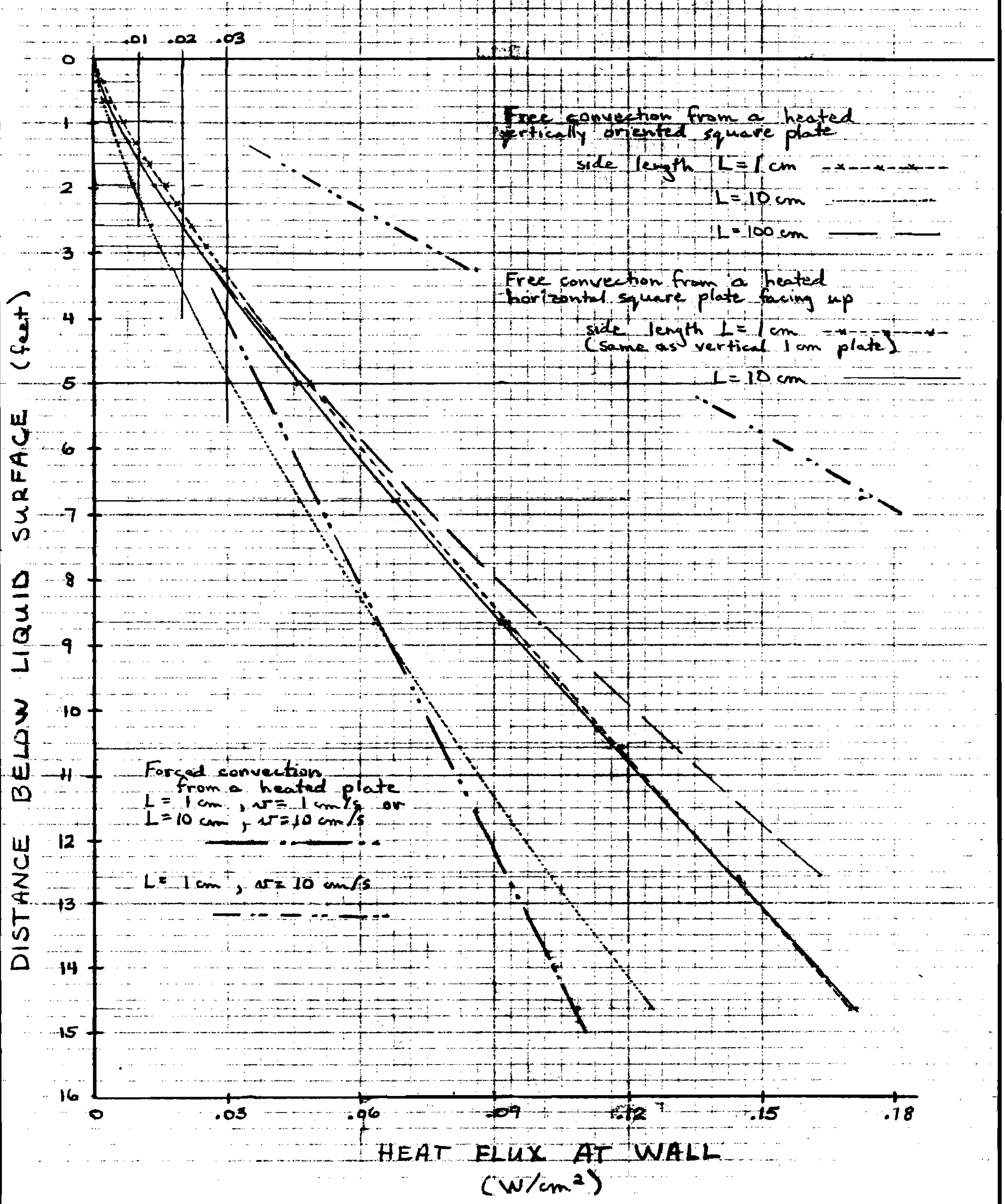

TJP
5 Feb 88

Map out Heat Flux vs Depth for Various Plate Sizes, Laminar and Turbulent Flow

Under a 1.3 bar (89.70 K) surface

k_{avg} wall (sat) to at depth W/cmK	ΔT (K)	ρ_{sat} at $T+\Delta T$ (g/cm³)	ρ_∞ at T_{surf}, P_{depth} (g/cm³)	μ_{avg} (wall to ∞ at depth) (g/cm s)	Gr/L^3 (cm⁻³)	P_{depth} (P_{sat} at T_{wall}) (bar)	head (ft) (using ρ_∞)	Pr_{avg} wall (sat) to at dep
1.241×10^{-3}	.10	1.3773	1.3780	2.419×10^{-3}	1.616×10^{5}	1.314	0.34	2.40
1.242×10^{-3}	.20	1.3767	1.3780	2.415×10^{-3}	3.009×10^{5}	1.327	0.66	2.40
1.242×10^{-3}	.30	1.3761	1.3780	2.412×10^{-3}	4.407×10^{5}	1.340	0.97	2.39
1.243×10^{-3}	.40	1.3755	1.3780	2.408×10^{-3}	5.816×10^{5}	1.353	1.29	2.39
1.244×10^{-3}	.50	1.3749	1.3780	2.404×10^{-3}	7.232×10^{5}	1.367	1.63	2.38
1.245×10^{-3}	.60	1.3742	1.3780	2.401×10^{-3}	8.883×10^{5}	1.380	1.94	2.38
1.246×10^{-3}	.70	1.3736	1.3780	2.397×10^{-3}	1.032×10^{6}	1.393	2.26	2.37
1.247×10^{-3}	.80	1.3730	1.3780	2.394×10^{-3}	1.175×10^{6}	1.407	2.60	2.36
1.247×10^{-3}	.90	1.3724	1.3780	2.390×10^{-3}	1.319×10^{6}	1.420	2.91	2.36
1.248×10^{-3}	1.00	1.3717	1.3780	2.386×10^{-3}	1.489×10^{6}	1.434	3.25	2.35
1.253×10^{-3}	1.50	1.3686	1.3780	2.370×10^{-3}	2.246×10^{6}	1.505	4.98	2.34
1.257×10^{-3}	2.00	1.3654	1.3781	2.349×10^{-3}	3.082×10^{6}	1.580	6.80	2.32
1.261×10^{-3}	2.50	1.3623	1.3781	2.336×10^{-3}	3.868×10^{6}	1.657	8.67	2.31
$.266\times10^{-3}$	3.00	1.3591	1.3781	2.322×10^{-3}	4.697×10^{6}	1.736	10.59	2.30
$.270\times10^{-3}$	3.50	1.3559	1.3782	2.309×10^{-3}	5.562×10^{6}	1.818	12.58	2.29
1.275×10^{-3}	4.00	1.3527	1.3782	2.292×10^{-3}	6.439×10^{6}	1.903	14.64	2.28

$$\frac{Gr}{L^3} = \frac{\rho_{sat}\underset{(T+\Delta T)}{(\text{at depth})} \times g\left[\rho\left(T_{surface}, P_{depth}\right) - \rho_{sat}\right]}{\mu_{avg}^2}$$

(w/cm^{2}K)

head (ft)	Gr L=1 cm 10 cm 100 cm	Gr Pr L=1 10 100	Nu (vert plate) L=1 10 100	Nu (hor plate) L=1 10 100	$\overline{h}$ (vert plate) L=1 10 100	$\overline{h}$ (hor plate L=1 10 100
0.34	1.616 $\times 10^{5}$ 1.616 $\times 10^{8}$ 1.616 $\times 10^{11}$	3.878 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	13.8 77.9 907	13.5 102.1	.0171 .00967 .0113	.0168 .0127
0.66	3.009 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	7.222 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	16.2 91.0 1163	15.7 125.6	.0201 .0113 .0144	.0195 .0156
0.97	4.407 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	10.53 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	17.8 100.0 1353	17.3 142.4	.0221 .0124 .0168	.0215 .0177
1.29	5.816 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	13.90 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	19.1 107.2 1512	18.5 156.2	.0237 .0133 .0188	.0230 .0194
1.63	7.232 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	17.21 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	20.1 113.0 1646	19.6 167.8	.0250 .0141 .0205	.0244 .0209
1.94	8.883 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	21.14 $\times 10^{5}$ $\times 10^{8}$ $\times 10^{11}$	21.2 119.0 1788	20.6 179.7	.0264 .0148 .0223	.0256 .0224
2.26	1.032 $\times 10^{6}$ 1.032 $\times 10^{9}$ 1.032 $\times 10^{12}$	2.446 $\times 10^{6}$ 2.446 $\times 10^{9}$ 2.446 $\times 10^{12}$	21.9 123.4 1895	21.4 188.6	.0273 .0154 .0236	.0267 .0235
2.60	1.175 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	2.773 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	22.6 127.4 1993	22.0 196.7	.0282 .0159 .0249	.0274 .0245
2.91	1.319 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	3.113 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	23.3 131.7 2087	22.7 204.4	.0291 .0164 .0260	.0283 .0255
3.25	1.489 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	3.499 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	24.0 138.0 2187	23.4 212.5	.0300 .0172 .0273	.0292 .0265
4.98	2.246 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	5.256 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	26.6 162.4 2573	25.9 243.4	.0333 .0203 .0322	.0325 .0305
6.80	3.082 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	7.150 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	28.7 183.6 2910	27.9 269.7	.0361 .0231 .0366	.0351 .0339
8.67	3.868 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	8.935 $\times 10^{6}$ $\times 10^{9}$ $\times 10^{12}$	30.3 200.8 3182	29.5 290.5	.0382 .0253 .0401	.0372 .0366

head (ft)	Gr L = 1 cm 10 cm 100 cm	Gr Pr L = 1 10 100	Nu (vert) L = 1 10 100	Nu (hor) L = 1 10 100	$\overline{h}$ (vert) L = 1 10 100	$\overline{h}$ (hor) L = 1 10 100
10.59	4.697×10^6	10.80×10^6	31.8	31.0	.0403	.0392
	$\times 10^9$	$\times 10^9$	217	309.5	.0275	.0392
	$\times 10^{12}$	$\times 10^{12}$	3432		.0434	
12.58	5.562×10^6	12.74×10^6	33.2	32.3	.0422	.0410
	$\times 10^9$	$\times 10^9$	231	327.0	.0293	.0415
	$\times 10^{12}$	$\times 10^{12}$	3667		.0466	
14.64	6.439×10^6	14.68×10^6	34.4	33.4	.0439	.0426
	$\times 10^9$	$\times 10^9$	245	342.8	.0312	.0437
	$\times 10^{12}$	$\times 10^{12}$	3881		.0495	

Vertical plate correlations: (Kreith, <u>Heat</u> <u>Transfer</u>, fig 7-4)

$$Nu = .555\,(Gr\,Pr)^{1/4} \qquad (10^5 < Gr < 1.3\times10^9)$$
(laminar flow)

$$Nu = .021\,(Gr\,Pr)^{2/5} \qquad (1.3\times10^9 < Gr < 10^{12})$$
(turbulent flow)

Horizontal plate correlations: (Kreith, equs 7-24, 7-25)

$$Nu = .54\,(Gr\,Pr)^{1/4} \qquad (10^5 < Gr < 2\times10^7)$$
(laminar flow)

$$Nu = .14\,(Gr\,Pr)^{1/3} \qquad (2\times10^7 < Gr < 3\times10^{10})$$
(turbulent flow)

$$Nu \equiv \frac{hL}{k}$$

TJP
22 Jan 88

Low Velocity Forced Convection in Liquid Argon over a Heated Plate

First check what flat plate length and fluid velocities result in turbulent flow:

$$Re = \frac{\rho v x}{\mu}$$

ρ (1.3 bar liquid argon) = 1.4 g/cm^3

μ (1.3 bar liquid argon) = 2.4×10^{-3} g/cm sec

$$Re = \frac{(1.4\ g/cm^3)\ v x}{(2.4 \times 10^{-3}\ g/cm\ sec)}$$

for $Re \geq 5 \times 10^5$ need $vx \geq 857$

looking at $v \leq 10$ cm/sec, x arbitrary so start with laminar flow equation.

Use x = 10 cm as for free convection calc.

The local convective-heat-transfer coefficient is

$$h_{cx} = 0.332 \frac{k}{x} Re_x^{1/2} Pr^{1/3} \quad \text{(Krieth pg 268)}$$

k (sat liq at 1.3 bar) = 1.2399×10^{-3} W/cm K

ρ (" " " " ") = 1.378 g/cm^3

μ (" " " " ") = 2.4186×10^{-3} g/cm sec

Pr (" " " " ") = 2.397

$$h_{cx} = (0.332) \frac{(1.2399 \times 10^{-3}\ W/cm K)}{(10\ cm)} \left[\frac{(1.378\ g/cm^3)(v\ \frac{cm}{s})(10\ cm)}{2.4186 \times 10^{-3}\ g/cm s} \right]^{1/}$$

$$\times (2.397)^{1/3}$$

$$= \left(4.158 \times 10^{-3} \frac{W}{cm^2 K}\right) \left(v \frac{cm}{sec}\right)^{1/2}$$

For $v = 10$ cm/sec get $h_{cx} = 13.15\times10^{-3}$ W/cm² K

$v = 1$ cm/sec get $h_{cx} = 4.16\times10^{-3}$ W/cm² K

$v = 0.1$ cm/sec get $h_{cx} = 1.315\times10^{-3}$ W/cm².

∴ Free convection heat transfer coefficients from the horizontal plate are like 0.2 cm/sec to 2 cm/sec flow velocities with forced convection.

Liquid Argon head is ~185" at bottom or h = 469.9 cm

$$\rho g h = (1.4\ \text{g/cm}^3)(980.665\ \text{cm/s}^2)(469.9\ \text{cm}) \times$$
$$\times\ 1.45\times10^{-5}\ \frac{\text{psi}}{\text{g/cms}^2} = 9.355\ \text{psid} = 0.64\ \text{bar}$$

so could have boiling temp at bottom of 93.9K (1.94 bar) and 89.7 K (1.30 bar) at top, for a ΔT of 4.2K.

ΔT of 2.1 K subcooling at center is possible, but on sides where free convection works better and velocities are higher.

For bottom, 2 cm/sec flow $h_c = 6\times10^{-3}$ W/cm².

and ΔT = 4.2K get $\boxed{2.5\times10^{-2}\ \text{W/cm}^2\ \text{max heat flux}}$

On our 10 cm × 10 cm square this is $\boxed{2.5\ \text{W}}$

So convectively can safely remove .03 W/cm² but boil at 0.03 W/cm²

Second Look at Low Velocity Forced Convection

$$\overline{Nu}_L = 0.664\ Re_L^{1/2}\ Pr^{1/3} = .664\left(\frac{\rho_{avg}\, v\, L}{\mu_{avg}}\right)^{1/2}\left(Pr_{avg}\right)^{1/3}$$

$$Re_L = \frac{\rho v L}{\mu}$$

where avg is wall to ∞ at depth.

Use data on 5 Feb 88 table for avg μ and ρ under 1.3 bar surface at various dept

ΔT (k)	head (ft)	Nu for $vL =$ .1	1	10	100	1000
.10	0.34	6.71	21.22	67.09	212.16	670.9
.20	0.66					
.30	0.97					
.40	1.29					
.50	1.63	6.71	21.21	67.08	212.13	670.8
.60	1.94					
.70	2.26					
.80	2.60					
.90	2.91					
1.00	3.25	6.70	21.19	67.01	211.9	670.1
1.50	4.98					
2.00	6.80	6.72	21.24	67.17	212.42	671.7
2.50	8.67					
3.00	10.59	6.73	21.28	67.29	212.79	672.9
3.50	12.58					
4.00	14.64	6.75	21.33	67.45	213.31	674.54

want $L = 1, 10, 100$ cm

and $v = 0.1, 1, 10$ cm/sec

$\therefore \quad vL = .1, 1, 10, 100, 1000$

for $L = 1 \quad 1 \text{ or } 10 \quad 1, 10, 100 \quad 10, 100 \quad 100.$

$$\bar{h} = \frac{Nu\, k_{avg}}{L}$$

$$q\ (w/cm^2) = \bar{h}\ \Delta T$$

$$.664 \left[\left(\frac{\rho_{avg}}{\mu_{avg}} \right) v L \right]^{1/2} \left(Pr_{avg} \right)^{1/3}. = Nu$$

h (w/cm^{2}k)

ΔT (k)	head (ft)	v = 0.1			v = 1.0			v = 10 cm/s		
		L = 1 cm	10	100	1	10	100	1	10	100
.1	0.34	.008	.003	.001	.026	.008	.003	.083	.026	.008
.5	1.63									
1.0	3.25									
2.0	6.83									
3.0	10.59									
4.0	14.64	.0086	.0027	.0009	.0272	.0086	.0027	.086	.0272	.0086
		low	low	low		low	low			low