

Liquid Crystal Elastomers

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PREFACE

Liquid crystals are unusual materials. As their name suggests, they inhabit the grey area between liquids and solids. They have long range orientational order, typically of the unique axes of their component rod-like or plate molecules. Spatial variations of this average direction of molecular orientation are resisted by so-called curvature (Frank) elasticity. On the other hand liquid crystals can flow, albeit as anisotropic liquids.

Polymers too are unusual materials. Above the glass transition, the physics is mostly dominated by the high entropy inherent in the disorder of their component long chain molecules. Resistance to molecular shape change arises mostly from the imperative to maintain high entropy. Viscoelastic flow and rubber elasticity are macroscopic manifestations of this principle. Thus rubber, where the long molecules are linked together, also inhabits the grey region between liquids and solids. Though nominally a solid, rubber is capable of very high deformations, greater than any other type of solid. Its internal molecular motion is rapid, as in a liquid, with the resulting amorphous solid being highly extensible rather than glassy. If it were not for the few crosslinks holding the chains into a percolating network, rubber would flow under stress, as ordinary polymers and other liquids do. The bulk (compression) modulus of typical rubber is of the same order as that of all liquids, and solids, but the shear modulus is about $10^{-4} - 10^{-5}$ times smaller. Thus rubber essentially deforms as a liquid, that is by shearing at constant volume. It is a weak solid and therein lies its enormous technological importance.

This book is concerned about the phenomena arising when these two marginal materials, liquid crystals and polymers, are combined into one even more mysterious material – polymer liquid crystals. For two compelling reasons we shall concentrate on such polymers crosslinked into networks, that is, on elastomers and gels made from polymer liquid crystals:

1. Liquid crystal elastomers exhibit many entirely new effects that are not simply enhancements of native liquid crystals or polymers. We shall see their thermal phase transformations giving rise to spontaneous shape changes of many hundreds of per cents, transitions and instabilities induced by applied mechanical stress or strain, and some unusual dynamical effects. Strangest of all, we shall see elastomers under some conditions behaving entirely softly, deforming as true liquids do without the application of stress. All these new forms of elasticity have their genesis in the ambiguities between liquid and solid that are present in liquid crystals and polymers, but are only brought to light in a crosslinked rubbery network.
2. A molecular picture of rubber elasticity is now well established. Since the late 1930s its entropic basis has been understood and turns out to be as universal as, say, the ideal gas laws. The rubber shear modulus, μ , is simply $n_s k_B T$ where n_s counts the number of network strands per unit volume, and temperature T enters for the same entropic reason it does in the gas laws. There is no mention of the

chemistry of chains or other molecular details and the picture is thus of great generality. We call this the classical theory, to which various complexities such as crosslink fluctuations, entanglements and nematic interactions have later been added.

By contrast to simple polymers, which change shape only in response to external forces, liquid crystal polymers do so *spontaneously* when they orientationally order their monomer segments. Can one nevertheless create a picture of their rubber elasticity of the same generality as that of classical rubber? It turns out that one can, with the sole extra ingredient of chain shape anisotropy (a single number directly measurable by experiment). We shall treat this anisotropy phenomenologically and find we can explore it at great length. One could go into many theoretical complexities, taking into account effects of finite chain extensibility, entanglements and fluctuations – however, in all cases, the underlying symmetry of spontaneously anisotropic network strands enters these approaches in the same way and the new physical phenomena are not thereby radically influenced.

Alternatively, one could try to calculate the polymer chain anisotropy that appears in the molecular picture of rubber elasticity. There is, however, no universal agreement about which way to do this. A further complication is that polymer liquid crystals can be either main chain or side chain variants, where the rod-like elements are found respectively in, or pendant to, the polymer backbone. Nematic and smectic phases of considerable complexity and differing symmetry arise according to the molecular geometry. For instance side chain fluids can exist in 3 possible uniaxial nematic phases, N_I , N_{II} and N_{III} , with still further biaxial possibilities.

In this book, by concentrating on *Liquid Crystal Elastomers*, rather than polymer liquid crystals *per se*, we relegate these theoretical uncertainties in the understanding of polymer liquid crystals to a subsidiary role. Key physical properties of crosslinked elastomers and gels are established without any detailed knowledge of *how* chains become spontaneously elongated or flattened. When more molecular knowledge is required, an adequate qualitative understanding of nematic and smectic networks can be obtained by adopting the simplest molecular models of polymer liquid crystals. In contrast, a treatise on polymer liquid crystals would have to address these issues rather more directly.

These two reasons, the existence of novel physical phenomena and their relative independence from the details of molecular interactions and ordering, explain the sequence of arguments followed by this book. We introduce liquid crystals, polymers and rubber elasticity at the rather basic level required for the universal description of the main topic – Liquid Crystal Elastomers. Then we look at the new phenomena displayed by these materials and, finally, concentrate on the analysis of key features of nematic, cholesteric and then smectic rubbery networks.

Rubber is capable of very large extensions. Many important new phenomena of nematic origin only occur at extensions of many tens of percents and are themselves highly non-linear. Linear continuum theory is utterly incapable of describing such a regime and this inadequacy is a motivation for our molecular picture of nematic rubber

elasticity. However, it is clear that in liquid crystal elastomers we have not only the Lamé elasticity of ordinary solids and the Frank curvature elasticity of liquid crystals, but also novel contributions arising from the coupling of the two. The richness and complexity of this new elasticity are such that it is worthwhile also analysing it using the powerful and general methods of continuum theory. There is a second motivation for studying continuum theory – for smectic elastomers there is not yet any underlying molecular theory and phenomenological theory is the best we can do. Because of their important technological applications, for instance in piezo- and ferroelectricity, an understanding of smectic elastomers is a vital priority. The latter chapters of our book are devoted to this, addressing the linear continuum approaches to elastomers with more complicated structure than simple uniaxial nematics. We also build a bridge between the elasticity methods of rubber and the application of continuum theory into the non-linear regime. At this point we revisit the symmetry arguments which explain why ‘soft elasticity’ is possible and why it cannot be found in classical elastic systems.

We were tempted to take ‘Solid Liquid Crystals’ as our title. This would have been apt but obscure. We hope that this book will illuminate the peculiar materials that merit this description.

Mark Warner and Eugene Terentjev
26 February 2003

Figure Acknowledgments

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CONTENTS

1	A bird's eye view of liquid crystal elastomers	1
2	Liquid crystals	9
2.1	Ordering of rod and disc fluids	9
2.2	Nematic order	11
2.3	Free energy and phase transitions of nematics	15
2.4	Molecular theory of nematics	20
2.5	Distortions of nematic order	22
2.6	Transitions driven by external fields	25
2.7	Anisotropic viscosity and dissipation	29
2.8	Cholesteric liquid crystals	33
2.9	Smectic liquid crystals	38
2.9.1	Smectic A liquids	39
2.9.2	Smectic C liquids	43
3	Polymers, elastomers and rubber elasticity	46
3.1	Configurations of polymers	47
3.2	Liquid crystalline polymers	51
3.2.1	Shape of liquid crystalline polymers	53
3.2.2	Frank elasticity of nematic polymers	60
3.3	Classical rubber elasticity	61
3.4	Manipulating the elastic response of rubber	66
3.5	Finite extensibility and entanglements in elastomers	69
4	Classical elasticity	74
4.1	Deformation tensor and Cauchy–Green strain	74
4.2	Non-linear and linear elasticity	77
4.3	Geometry of deformations and rotations	82
4.3.1	Rotations	82
4.3.2	Shears and their decomposition	83
4.3.3	Square roots and polar decomposition of tensors	89
4.4	Compressibility of rubbery networks	90
5	Nematic elastomers	94
5.1	Structure and examples of nematic elastomers	95
5.2	Stress-optical coupling	98
5.3	Polydomain textures and alignment by stress	100
5.4	Monodomain ‘single-crystal’ nematic elastomers	104
5.4.1	Spontaneous shape changes	105
5.4.2	Nematic photoelastomers	107
5.5	Field-induced director rotation	110

5.6	Applications of liquid crystalline elastomers	114
6	Nematic rubber elasticity	119
6.1	Neo-classical theory	119
6.2	Spontaneous distortions	122
6.3	Equilibrium shape of nematic elastomers‡	128
6.4	Photo-mechanical effects	130
6.5	Thermal phase transitions	136
6.6	Effect of strain on nematic order	138
6.7	Mechanical and nematic instabilities	143
	6.7.1 Mechanical Freedericks transition	146
	6.7.2 The elastic low road	149
6.8	Finite extensibility and entanglements	149
7	Soft elasticity	154
7.1	Director anchoring to the bulk	155
	7.1.1 Director rotation without strain	155
	7.1.2 Coupling of rotations to pure shear	158
7.2	Soft elasticity	159
	7.2.1 Soft modes of deformation	160
	7.2.2 Principal symmetric strains and body rotations	164
	7.2.3 Forms of the free energy allowing softness	166
7.3	Optimal deformations	167
	7.3.1 A practical method of calculating deformations	167
	7.3.2 Stretching perpendicular to the director	169
7.4	Semi-soft elasticity	173
	7.4.1 Example: random copolymer networks	174
	7.4.2 A practical geometry of semi-soft deformations	175
	7.4.3 Experiments on long, semi-soft strips	177
	7.4.4 Unconstrained elastomers in external field	178
7.5	Semi-soft free energy and stress	179
7.6	Thermomechanical history and general semi-softness	183
	7.6.1 Thermomechanical history dependence	184
	7.6.2 Forms of the free energy violating softness	185
8	Distortions of nematic elastomers	188
8.1	Freedericks transitions in nematic elastomers	189
8.2	Strain-induced microstructure: stripe domains	195
8.3	General distortions of nematic elastomers	202
	8.3.1 One-dimensional quasi-convexification	203
	8.3.2 Full quasi-convexification	206
	8.3.3 Numerical and experimental studies	208
8.4	Random disorder in nematic networks	211
	8.4.1 Nematic ordering with quenched disorder	213
	8.4.2 Characteristic domain size	214

8.4.3	Polydomain-monodomain transition	216
9	Cholesteric elastomers	221
9.1	Cholesteric networks	222
9.1.1	Intrinsically chiral networks	222
9.1.2	Chirally imprinted networks	223
9.2	Mechanical deformations	228
9.2.1	Uniaxial transverse elongation	229
9.2.2	Stretching along the pitch axis	234
9.3	Piezoelectricity of cholesteric elastomers	237
9.4	Imprinted cholesteric elastomers	243
9.5	Photonics of cholesteric elastomers	246
9.5.1	Photonics of liquid cholesterics	247
9.5.2	Photonics of elastomers	250
9.5.3	Experimental observations of elastomer photonics	252
9.5.4	Lasing in cholesterics	254
10	Continuum theory of nematic elastomers	257
10.1	From molecular theory to continuum elasticity	258
10.1.1	Compressibility effects	258
10.1.2	The limit of linear elasticity	259
10.1.3	The role of nematic anisotropy	261
10.2	Phenomenological theory for small deformations	263
10.3	Strain-induced rotation	266
10.4	Soft elasticity	270
10.4.1	Symmetry arguments	270
10.4.2	The mechanism of soft deformation	272
10.5	Continuum representation of semi-softness	275
10.6	Unconstrained director fluctuations	278
10.7	Unconstrained phonons	281
10.8	Light scattering from director fluctuations	285
11	Dynamics of liquid crystal elastomers	293
11.1	Classical rubber dynamics	294
11.1.1	Rouse model and entanglements	296
11.1.2	Dynamical response of entangled networks	298
11.1.3	Long time stress relaxation	301
11.2	Nematohydrodynamics of elastic solids	303
11.2.1	Viscous coefficients and relaxation times	305
11.2.2	Balance of forces and torques	306
11.2.3	Symmetries and order parameter	308
11.3	Response to oscillating strains	309
11.4	Experimental observations	313
11.4.1	Oscillating shear	314
11.4.2	Steady stress relaxation	318

12 Smectic elastomers	322
12.1 Materials and preparation	322
12.1.1 Smectic A elastomers	324
12.1.2 Smectic C and ferroelectric C* elastomers	326
12.2 Physical properties of smectic elastomers	327
12.2.1 Smectic-A elastomers	327
12.2.2 Smectic-C elastomers	331
12.3 A molecular model of Smectic-A rubber elasticity	332
12.3.1 The geometry of affine layer deformations	334
12.3.2 Response to imposed λ_{xx} deformation.	336
12.3.3 Imposed in-plane shear λ_{xz}	338
12.3.4 Imposed extension λ_{zz} along the layer normal	338
12.3.5 Imposed shear λ_{zx} out of plane	342
12.3.6 General deformations of a SmA elastomer	343
12.4 Comparison with experiment	345
12.4.1 Microstructure after the CMHH transition	346
13 Continuum description of smectic elastomers	349
13.1 Continuum description of smectic A elastomers	349
13.1.1 Relative translations in smectic networks revisited	350
13.1.2 Nematic -strain, -rotation and -smectic couplings	352
13.2 Effective smectic elasticity of elastomers	354
13.3 Effective rubber elasticity of smectic elastomers	359
13.4 Layer elasticity and fluctuations in smectic A elastomers	363
13.5 Layer buckling instabilities: the CMHH effect	370
13.6 Quenched layer disorder and the N-A phase transition	373
13.7 Smectic C and ferroelectric C* elastomers	377
References	382
Index	393
Author Index	399
Online Appendices	
1 Nematic order in elastomers under strain	406
2 Biaxial soft elasticity	412
3 Stripe microstructure	416
4 Couple-stress and Cosserat elasticity	423
5 Expansion at small deformations and rotations	429

A BIRD'S EYE VIEW OF LIQUID CRYSTAL ELASTOMERS

Liquid crystal elastomers bring together, as nowhere else, three important ideas: *orientational order* in amorphous soft materials, *responsive molecular shape* and *quenched topological constraints*. Acting together, they create many new physical phenomena that are the subject of this book. This bird's eye view sketches how these themes come together and thereby explains the approach of our book.

In the early chapters we introduce the reader to liquid crystals and to polymers since they are our building blocks. One could regard the first part of our book as a primer for an undergraduate or graduate student embarking on a study of polymer or liquid crystal physics, or on complex fluids and solids. Then elastomers are discussed both from the molecular point of view, and within continuum elasticity. We need to understand how materials respond at very large deformations for which only a molecular approach is suitable. Also one needs to understand the resolution of strains into their component pure shears and rotations, the latter also being important in these unusual solids. We also provide a primer for the basics of these two areas that are otherwise only found in difficult and advanced texts.

Classical liquid crystals are typically fluids of relatively stiff rod molecules with long range orientational order. The simplest case is nematic – where the average ordering direction of the rods, the director \mathbf{n} , is uniform. Long polymer chains, with incorporated rigid anisotropic units can also order nematically and thus form liquid crystalline polymers. By contrast with rigid rods, these flexible chains elongate when their component rods align. This results in a change of average molecular shape, from spherical to spheroidal as the isotropic polymers become nematic. In the prolate anisotropy case, the long axis of the spheroid points along the nematic director \mathbf{n} , Fig. 1.1.

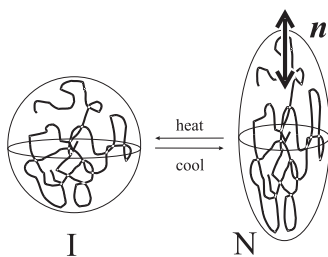


FIG. 1.1. Polymers are on average spherical in the isotropic (I) state and elongate when they are cooled to the nematic (N) state. The director \mathbf{n} points along the principal axis of the shape spheroid. (The mesogenic rods incorporated into the polymer chain are not shown in this sketch, only the backbone is traced.)

So far we have no more than a sophisticated liquid crystal. Changes in average molecular shape induced by changes in orientational order do little to modify the properties of this new liquid crystal. Linking the polymer chains together into a gel network fixes their topology, and the melt becomes an elastic solid – a rubber. Radically new properties can now arise from this ability to change molecular shape while in the solid state. To understand this we have to consider rubber elasticity.

In rubber, monomers remain highly mobile and thus liquid-like. Thermal fluctuations move the chains as rapidly as in the melt, but only as far as their topological crosslinking constraints allow. These loose constraints make the polymeric liquid into a weak, highly extensible material. Nevertheless, rubber is a solid in that an energy input is required to change its macroscopic shape (in contrast to a liquid, which would flow in response). Equivalently, a rubber recovers its original state when external influences are removed. Systems where fluctuations are limited by constraints are known in statistical mechanics as ‘quenched’ - rigidity and memory of shape stem directly from this. It is a form of imprinting found in classical elastomers and also in chiral solids, as we shall see when thinking about cholesteric elastomers.

Can topology, frozen into a mobile fluid by constraints, act to imprint liquid crystalline order into the system? The expectation based on simple networks would be ‘yes’. This question was posed, and qualitatively answered, by P-G. de Gennes in 1969. He actually asked a slightly more sophisticated question: Crosslink conventional polymers (not liquid crystalline polymers) into a network in the presence of a liquid crystalline solvent. On removal of the solvent, do the intrinsically isotropic chains remember the anisotropy pertaining at the moment of genesis of their topology?¹ The answer for ideal chains linked in a nematic solvent is ‘no’! Intrinsically nematic polymers, linked in a nematic phase of their own making, can also elude their topological memory on heating. How this is done (and failure in the non-ideal case) is a major theme of this book.

Second, what effects follow from changing nematic order and thus molecular shape? The answer is new types of thermal- and light-induced shape changes.

The third question one can ask is: While in the liquid-crystal state, what connection between mechanical properties and nematic order does the crosslinking topology induce? The answer to this question is also remarkable and is discussed below. It leads to entirely new effects – shape change without energy cost, extreme mechanical effects and rotatory-mechanical coupling. We give a preview below of these effects in the form of a sketch – details have to await the later chapters of the book.

Rubber resists mechanical deformation because the network chains have maximal entropy in their natural, undeformed state. Crosslinking creates a topological relation between chains that in effect tethers them to the solid matrix they collectively make up. Macroscopic deformation then inflicts a change away from the naturally spherical average shape of each network strand, and the entropy, S , falls. The free energy then rises, $\Delta F = -T\Delta S > 0$. This free energy, dependent only on an entropy change itself driven by molecular shape change, explains why polymers are sometimes thought of as

¹ G. Allen saw the similarity of this question to that of crosslinking in the presence of a mechanical field, a great insight considering how monodomain liquid crystal elastomers are made today.

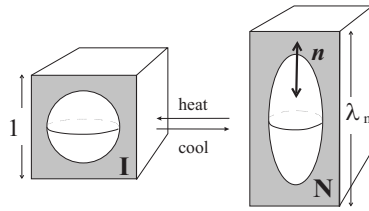


FIG. 1.2. A unit cube of rubber in the isotropic (I) state. Embedded in it is shown the average of the chain distribution (spherical). The block elongates by a factor λ_m on cooling to the nematic (N) state, accommodating the now elongated chains.

‘entropic springs’. Macroscopic changes in shape are coupled to molecular changes. In conventional rubber it is always the macroscopic that drives the molecular; the induced conformational entropy of macromolecules offers the elastic resistance.

Nematic polymers suffer spontaneous shape changes associated with changing levels of nematic (orientational) order, Fig. 1.1. One now sees a reversal of influence: changes at the molecular level induce a corresponding change at the macroscopic level, that is induce mechanical strains, Fig. 1.2: a block of rubber elongates by a factor of $\lambda_m > 1$ on cooling or $1/\lambda_m < 1$ on heating. This process is perfectly reversible. Starting in the nematic state, chains become spherical on heating. But mechanical strain must now accompany the molecular readjustment. Very large deformations are not hard to achieve, see Fig. 1.3. Provided chains are in a broad sense ideal, it turns out that chain shape can reach isotropy both for the imprinted case of de Gennes (on removal of nematic solvent) and for the more common case of elastomers formed from liquid crystalline polymers (on heating). Chains experiencing entanglement between their crosslinking points also evade any permanent record of their genesis. Many real nematic elastomers and gels in practice closely conform to these ideal models. Others are non-ideal – they retain some nematic order at high temperatures as a result of their order and topology combining with other factors such as random pinning fields and compositional fluctuations. They still show the elongations of Fig. 1.3, but residues of non-ideality are seen in the elastic effects we review below.

This extreme thermomechanical effect, and the phenomena of Figs. 1.5 and 1.7, can only be seen in monodomain, well aligned samples. Without very special precautions

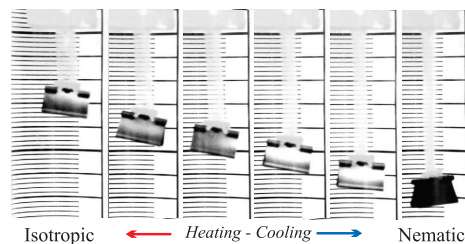


FIG. 1.3. A strip of nematic rubber extends and contracts according to its temperature. Note the scale behind the strip and the weight that is lifted!

during fabrication, liquid crystal elastomers are always found in polydomain form, with very fine texture of director orientations. The great breakthrough in this field, developing a first method of obtaining large, perfect monodomain nematic elastomers, was made by Küpfer and Finkelmann in 1991.

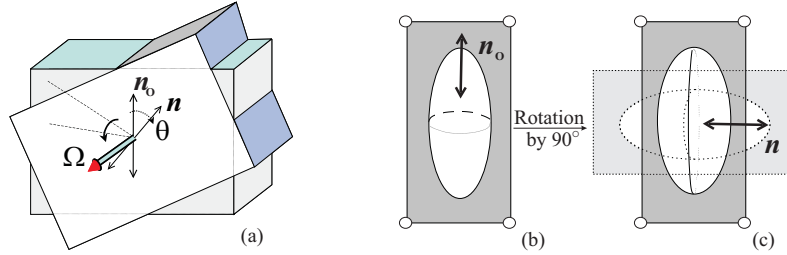


FIG. 1.4. (a) Rotations of the director and matrix by angles θ and Ω , respectively. From (b) to (c) the director, and thus chain shape distribution, is rotated by 90° from \mathbf{n}_o to \mathbf{n} . The rubber is mechanically clamped and hence the chains in (c) that would be naturally elongated along \mathbf{n} must be compressed: the dotted spheroid in (c) is compressed to the actual solid spheroid.

Nematic-elastic coupling was the third question we posed and gives rise to new rotational phenomena ubiquitous in liquid crystal elastomers. It is possible to rotate the director and the rubber matrix independently, see Fig. 1.4 (a). Such relative rotations of the body and of its internal anisotropy axis show that nematic elastomers are not simply exotic, highly-extensible, uniaxial crystals. Such materials belong to a class displaying so-called Cosserat elasticity, but with the distinction that deformations and rotations can be large in elastomers. Imagine now rotating the director while clamping the body so its shape does not change, Figs. 1.4(b) and (c). The natural, prolate spheroidal distribution, when rotated by 90° to be along \mathbf{n} , has a problem. Chains do not naturally fit, since the clamped body to which they are tethered is not correspondingly elongated along \mathbf{n} to accommodate their long dimensions. Chains in fact must have been compressed to fit, at considerable entropy loss if they were very anisotropic. A rotation of 180° recovers the initial state, so the free energy must be periodic, and turns out to be $F = \frac{1}{2}D_1 \sin^2(\theta - \Omega)$. The rotational modulus, D_1 , was first given by de Gennes in the infinitesimal form $\frac{1}{2}D_1(\theta - \Omega)^2$. A rotation of the director in Fig. 1.4(b) would lead to a ‘virtual’ intermediate state depicted by dotted lines in Fig. 1.4(c). Subsequent squeezing to get back the actual body shape demanded by the clamp condition (full lines) of Fig. 1.4(c) costs an energy proportional to the rubber modulus, μ , and to the square of the order, Q , (since Q determines the average chain shape anisotropy). Thus $D_1 \sim \mu Q^2$. In contrast to ordinary nematics, it costs energy to uniformly rotate the director independently of the matrix.

In liquid nematics it is director gradients that suffer Frank elastic penalties, and thus long-wavelength spatial variations of the rotation angle cost vanishingly small energy. Thermal excitation of these rotations causes even monodomain nematic liquids to scat-

ter light and to be turbid. Not so monodomain nematic elastomers which are optically clear because even long wavelength director rotations cost a finite rubber-elastic energy $\frac{1}{2}D_1\theta^2$ and cannot be excited, see Fig. 1.5. The excitations have acquired a mass, in the language of field theory.



FIG. 1.5. A strip of monodomain ‘single-crystal’ nematic rubber. It is completely transparent and highly birefringent (image: H. Finkelmann).

Local rotations, so central to nematic elastomers, yield a subtle and spectacular new elastic phenomenon which we call ‘soft elasticity’. Imagine rotating the director but now *not* clamping the embedding body, in contrast to Figs. 1.4(b) and (c). One simple response would be to rotate the body by the same angle as the director, and this would clearly cost no energy. However, contrary to intuition, there is an infinity of other ways by mechanical deformation to accommodate the anisotropic distribution of chains without its distortion as it rotates. Thus the entropy of the chains does not change, in spite of macroscopic deformations. Figure 1.6 illustrates the initial and final states of a 90° director rotation. They are separated by a path of states, characterised by an intermediate rotation angle θ and by a corresponding shape of the body, one of which is shown. This θ -state is shown in the sketch (b) accommodating the spheroid without distorting it. A special combination of shears and elongations/compressions is required, but it turns out not very difficult to achieve in experiment!

One of the traditional ways to rotate the director in liquid crystals is by applying an electric (or magnetic) field and generating a local torque due to the dielectric anisotropy. Due to the nematic-elastic coupling, the director rotation is very difficult if an elastomer sample is mechanically constrained. Apart from a few exceptions (all characterised by a very low rubber-elastic modulus, such as in highly swollen gels) no electrooptical response can occur. However, if the elastomer is mechanically unconstrained, the situation changes remarkably. In a beautiful series of experiments, Urayama (2005,2006)

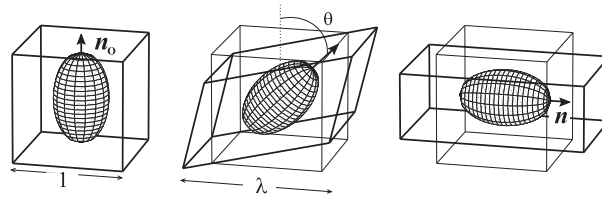


FIG. 1.6. Rotation of chain shape distribution, from \mathbf{n}_0 to \mathbf{n} , with an intermediate state θ shown. The unconstrained rubber deforms to accommodate the rotating director without distorting the chain distribution.

has confirmed the prediction of soft elasticity: that the field-induced director rotation has no energy cost, can easily reach 90° rotation angles and has associated mechanical strains that almost exactly follow the sketch in Fig. 1.6.

Practically, when dealing with rubbers, one might instead impose a mechanical distortion (say an elongation, λ , perpendicular to the original director) and have the other components of strain, and the director orientation, follow it. The result is the same – extension of a rubber costs no elastic energy and is accompanied by a characteristic director rotation. The mechanical confirmation of the cartoon is shown in stress-strain curves in Fig. 1.7(a) and the director rotation in Fig. 1.7(b).

We have made liquid crystals into solids, albeit rather weak solids, by crosslinking them. Like all rubbers, they remain locally fluid-like in their molecular freedom and mobility. Paradoxically, their liquid crystallinity allows these solid liquid crystals to change shape without energy cost, that is to behave for some deformations like a liquid. Non-ideality gives a response we call ‘semi-soft’. There is now a small threshold before director rotation (seen in the electrooptical/mechanical experiments of Urayama (2005,2006), and to varying degrees in Fig. 1.7); thereafter deformation proceeds at little additional resistance until the internal rotation is complete. This stress plateau, the same singular form of the director rotation, and the relaxation of the other mechanical degrees of freedom are still qualitatively soft, in spite of a threshold.

There is a deep symmetry reason for this apparently mysterious softness that Fig. 1.6 rationalises in terms of the model of an egg-shaped chain distribution rotating in a solid that adopts new shapes to accommodate it. Ideally, nematic elastomers are rotationally invariant under separate rotations of both the reference state and of the target state into which it is deformed. If under some conditions, not necessarily the current ones, an

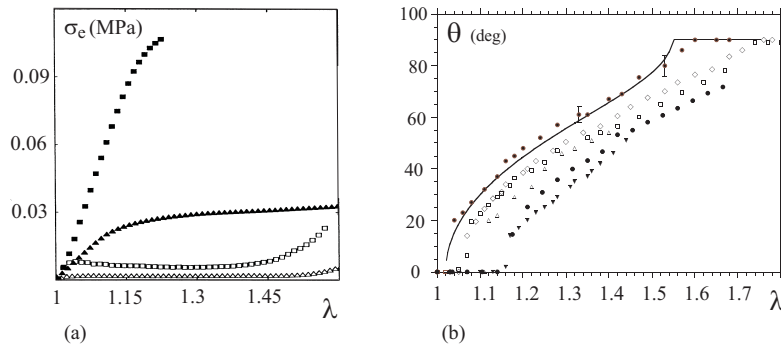


FIG. 1.7. (a) Stress-deformation data of K upfer and Finkelmann (1994), for a series of rubbers with the same composition and crosslinking density, but differing in preparation history: some show a normal elastic response while others are remarkably soft. (b) The angle of director rotation on stretching nematic elastomer perpendicular to the director for a variety of different materials, from Finkelmann *et al.* (1997). The solid line from, theoretical modeling, accurately reproduces singular points and characteristic shape of data.

isotropic state can be attained, then a theorem of Golubović and Lubensky shows that in consequence soft elasticity must exist. It is a question of care with the fundamental tenet of elasticity theory, the principle of material frame indifference. We shall examine this theorem and its consequences many times in this book, including what happens when the conditions for it to hold are violated, that is when semi-softness prevails.

Elastic softness, or attempts to achieve it, pervade much of the elasticity of nematic elastomers. If clamps or boundary conditions frustrate uniform soft deformation trajectories, microstructures will evolve to allow softness with the cost of interfaces being a relatively smaller price to pay. There are similarities between this so-called 'quasi-convexification' and that seen in martensite and other shape-memory alloys.

Cholesteric liquid crystals have a helical director distribution. Locally they are very nearly conventional nematics since their director twist occurs typically over microns, a much longer length scale than that associated with nematic molecular ordering. They can be crosslinked to form elastomers which retain the cholesteric director distribution. Several phenomena unique to cholesterics emerge: Being locally nematic, cholesteric elastomers would like on heating and cooling to lose and recover orientational order as nematic elastomers do. However, they cannot resolve the requirement at neighbouring points to spontaneously distort by λ_m , but in different directions. Accordingly, their chains cannot forget their topologically imprinted past when they attempt to reach a totally isotropic reference state (the second de Gennes' prediction of 1969). Thus cholesteric rubbers also cannot deform softly in response to imposed strains. Their optical and mechanical responses to imposed stress are exceedingly rich as a result. They are brightly coloured due to selective reflection and change colour as they are stretched – their photonic band structure changes with strain. They can emit laser radiation with a colour shifted by mechanical effects. Further, the effect of topological imprinting can select and extract molecules of specific handedness from a mixed solvent. Such rubbers can act as a mechanical separator of chirality – a new slant on a problem that goes back to Pasteur.

We have sketched the essentials of nematic (and cholesteric) rubber elasticity. This survey leaves out many new phenomena dealt with in later chapters, for instance electromechanical Fredericks effects, photo-elastomers that drastically change shape on illumination, rheology and viscoelasticity that crosses between soft and conventional depending upon frequency and geometry, and so on.

Smectics are the other class of liquid crystal order. They have plane-like, lamellar modulation of density in one direction (SmA), or additionally a tilt of the director away from the layer normal (SmC). Many other more complex smectic phases exist and could also be made into elastomers. In many smectic elastomers, layers are constrained not to move relative to the rubber matrix. Deformations of a rubber along the layer normal are thus resisted by a layer spacing modulus, B , of the order of 10^2 times greater than the shear modulus of the matrix. Distortions in plane, either extensions or appropriate shears, are simply resisted by the rubber matrix. Thus SmA elastomers are rubbery in the two dimensions of their layer planes, but respond as hard conventional solids in their third dimension. Fig. 1.8 shows this behaviour. Such extreme mechanical anisotropy promises interesting applications.

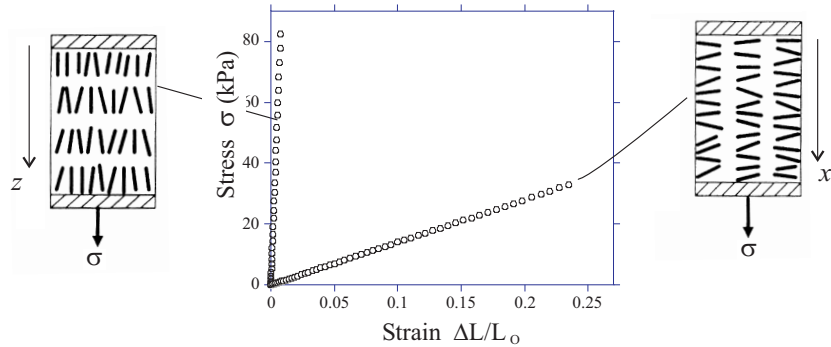


FIG. 1.8. In-plane fluidity and parallel rigidity in a smectic A elastomer (Nishikawa *et al.*, 1997). The Young modulus parallel and perpendicular to the layer normals differ very greatly - the rubber elasticity is two-dimensional.

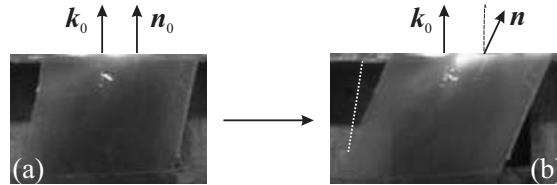


FIG. 1.9. (a) A SmA elastomer (Hiraoka *et al.*, 2005). (b) Spontaneous shear λ_{xz} in achieving the SmC state.

The director tilt associated with the transition from SmA to SmC induces distortion in the polymer chain shape distribution. Since chain shape is coupled to mechanical shape for an elastomer, one expects, and sees in Fig. 1.9, spontaneous distortion. This response to order change is analogous to the elongations associated with orientational order of chains on entering the nematic state, but here we instead have shear. The amplitude is also large, of the order of 0.4 in the figure. As in the nematic case, the broken symmetry suggests a mechanism for SmC solids richer still than that of SmA elastomers, including SmC soft elasticity equivalent to that of Fig. 1.6.

The tilted, SmC, liquids also exist in chiral forms which must, on symmetry grounds be ferroelectric. Their elastomers are too. Ferroelectric rubber is very special: mechanically it is soft, about 10^4 times lower in modulus than ferro- and piezoelectrics because, as sketched above, its molecules are spatially localised by topological rather than energetic constraints. Distortions give polarisation changes comparable to those in ordinary ferroelectrics. But the response in terms of stress must necessarily be 10^4 times larger than in conventional materials.

We end our preview as we started – solids created by topological constraints are soft and highly extensible. Liquid crystal elastomers share this character with their important cousins, the conventional elastomers. But their additional liquid crystalline order gives them entirely new kinds of elasticity and other unexpected phenomena.