Liquid Metal Flows in Circular Insulated Ducts in Nonuniform Magnetic Fields

S. Molokov[1], C. B. Reed[2]

[1]*Coventry University, School of Mathematical and Information Sciences, Priory Street, Coventry CV1 5FB, UK*

[2]*Argonne National Laboratory, Technology Division, 9700 South Cass Avenue, Argonne, IL 60439, USA*

Abstract

Magnetohydrodynamic flows in insulated circular ducts in nonuniform magnetic fields are studied with reference to liquid metal blankets and divertors of fusion reactors. Particular emphasis is made on C-MOD. The ducts are supposed to be straight, while the gradient of the magnetic field to be inclined by an angle α to the duct axis. The results are presented for the values of the Hartmann numbers, *Ha*, of 1000 and 100. Three-dimensional pressure drop, development length, three-dimensional length and nonuniformities of the velocity profiles have been evaluated. It has been shown that for *Ha* = 1000 the three-dimensional effects are of considerable importance, while for *Ha* = 100 they may be neglected.

[1] Corresponding author: *Tel: +44-24-76888601, Fax: +44-24-76888585, e-mail: s.molokov@coventry.ac.uk*

1. Introduction

Liquid metal flows in circular ducts play a fundamental role in liquid metal blankets and divertors for fusion reactors. Concerning blankets, both inlet and outlet pipes have circular cross-section. The flow in these pipes is fully three-dimensional, since liquid metal flows in a strong, nonuniform magnetic field. Similar situation occurs in circular ducts supplying liquid metal to divertors elements [1], [2].

When a liquid metal flows in a strong magnetic field, electric currents are induced. These currents in turn interact with the magnetic field and the resulting electromagnetic force induces a high MHD pressure drop and significant nonuniformities of the velocity profile in the duct cross-section. The pressure drop in particular is considered to be one of the most critical issues for self-cooled blankets. The magnitude of the electromagnetic force with respect to viscous and inertial forces is determined by two parameters, the Hartmann number, Ha, and the interaction parameter, N, respectively. The range of typical parameter values for various machines, based on lithium flow in circular, insulated ducts (or jets) of 5-10 mm radius, is shown in Table 1.

As the three-dimensional effects in blanket/divertor elements are of considerable importance, laminar, inertialess MHD flow in an insulating circular duct in a strong, nonuniform magnetic field is studied in this paper. The emphasis is on the range of parameters relevant to C-MOD. Some estimates are given for NSTX as well, though it should be emphasised that the flow regime in this machine is different (Table 1). The flow regime is determined by the parameter Ha/Re [3], where Re is the Reynolds number. According to the experimental data for circular insulating ducts [3], for $Ha/Re > 0.025$ the flow is laminar, while for $Ha/Re < 0.025$ it is turbulent. The latter regime is characteristic for NSTX.

Both the flow geometry studied here and the Cartesian co-ordinate system (x, y, z) are shown in Figs. 1 and 2. Cylindrical co-ordinates (r, θ, x) are also used, which are defined as follows (see Fig. 2): $z = r\sin\theta$, $y = r\cos\theta$. The magnetic field $\mathbf{B}^* = B_0^* B(x,z)\hat{\mathbf{y}}$ is supposed to have a single component, out of the plane of the figure, where B_0^* is the characteristic value of the magnetic field in the upstream region, i.e. for $x \to -\infty$. Dimensional quantities are denoted by letters with asterisks, while their dimensionless counterparts - with the same letters, but without the asterisks.

The inlet/outlet pipes may enter/leave the tokamak area at a certain, small angle α to the gradient of the toroidal field (Fig. 1). One of the aims of the current study is to estimate sensitivity of the three-dimensional pressure drop Δp_{3D} to the variation of α.

The family of the magnetic fields studied here is $B(\xi) = \frac{1}{2}(B_d + B_u) + \frac{1}{2}(B_d - B_u)\tanh\gamma\xi$, where $\xi = x\cos\alpha + z\sin\alpha$. The field induction varies between the constant values of B_u = 1 to the left of Line 1 and B_d to the right of Line 2 (see Fig. 1). The field is nonuniform between these lines. The field gradient is defined by γ. For α = 0° one gets ξ = x, thus the field gradient is aligned with the duct axis.

The aim of the study is to estimate the values of Δp_{3D}, the development length, as well as nonuniformities in the velocity profiles owing to the nonuniform magnetic field.

2. Formulation

Consider a steady flow of a viscous, electrically conducting, incompressible fluid in a straight, insulating, circular duct in the x-direction (Figs. 1, 2). The characteristic values of the length, the fluid velocity, the electric current density, the electric potential, and the pressure are

a^* (the duct radius), v_0^* (average fluid velocity), $\sigma v_0^* B_0^*$, $a^* v_0^* B_0^*$, and $a^* \sigma v_0^* B_0^{*2}$, respectively. In the above, σ, ρ, ν are the electrical conductivity, density and kinematic viscosity of the fluid.

It is assumed that the flow is inertialess, which requires $N >> Ha^{1/2}$ [4], where $Ha = a^* B_0^* (\sigma / \rho\nu)^{1/2}$ is the Hartmann number, which expresses the ratio of the electromagnetic to the viscous force, and $N = a^* \sigma B_0^{*2} / \rho v_0^*$ is the interaction parameter, which expresses the ratio of the electromagnetic to the inertial force.

The dimensionless, inductionless, inertialess equations governing the flow are [5], [6]:

$$Ha^{-2}\nabla^2 \mathbf{v} + \mathbf{j}\times\mathbf{B} = \nabla p , \quad \mathbf{j} = -\nabla\phi + \mathbf{v}\times\mathbf{B} , \tag{1a,b}$$

$$\nabla \cdot \mathbf{v} = 0 , \quad \nabla \cdot \mathbf{j} = 0 , \tag{1c,d}$$

where **v** is the fluid velocity, **j** is the electric current density, ϕ is the electric potential, and p is the pressure.

The boundary conditions at the duct wall are the no-slip- and the insulating-wall- conditions:

$$\mathbf{v} = 0, \quad j_r = 0 \quad \text{at } r = 1, \tag{1e}$$

where j_r is the radial component of current.

Far upstream and far downstream the flow is fully developed, which requires

$$\partial p/\partial z \to 0 , \quad \partial\phi/\partial x \to 0 \quad \text{as } x \to \pm\infty . \tag{1f,g}$$

Finally, the solution is normalized using the condition of a fixed average velocity:

$$2\int_{-\pi/2}^{\pi/2} d\theta \int_0^1 r u dr = \pi , \tag{1h}$$

where u is the x-component of velocity.

3. High-*Ha* flow model

In a sufficiently strong magnetic field the flow region splits into the following subregions (Fig. 2): the core *C*, the Hartmann layer *H* of thickness $O(Ha^{-1})$ at the wall, and the Roberts layers *E* with dimensions $O(Ha^{-1/3})\times O(Ha^{-2/3})$ at $\theta = \pm\pi/2$, $r = 1$. The details of the high-*Ha* model, valid for terms up to $O(Ha^{-1})$ are given in [7] (see also [6], [8], [9]), and thus are omitted here. The analysis leads to two two-dimensional partial differential equations for the core pressure $P(x,z)$ and the wall electric potential $\Phi(x,z)$. These equations, subject to appropriate boundary conditions, are solved numerically on a non-equidistant grid, using a finite-difference method described in [6]. For a typical calculation we use 257 points in the *x*-direction and 43 points in the *z*-direction. The length of the computational domain is $l_{comp} = 100$ (see Fig. 1), while the other parameters are: $\gamma = 0.8$, $B_d = 0.2$, $B_u = 1$ (flow out of the intense-field region).

4. Results

The three-dimensional effects are assessed using such characteristics as the three-dimensional pressure drop and the three-dimensional length.

The *three-dimensional pressure drop,* Δp_{3D} is defined as follows:

$$\Delta p_{3D} = \Delta p - \tfrac{3}{16}\pi Ha^{-1} l_{comp}\left(B_u - B_d\right), \qquad (2)$$

where Δp is the total pressure drop between points $x = -\, l_{comp}/2$ and $x = l_{comp}/2$. In this definition the fully developed pressure gradients far upstream and far downstream given by the expressions $dp/dx\big|_{u,d} = -\tfrac{3}{8}\pi Ha^{-1} B_{u,d}$ are extended up to $x = 0$.

The *three-dimensional length*, d_{3D}, is defined as follows [6], [7]:

$$d_{3D} = -\frac{\Delta p_{3D}}{dp/dx|_u} = \frac{8Ha}{3\pi}\Delta p_{3D} . \qquad (3)$$

3.1 Flow for $a = 0^o$

Far upstream and far downstream from the nonuniform-field region the flow is fully developed. It is driven by the pressure gradients $dp/dx|_{u,d}$, respectively. In these regions both the core velocity $u_{Cu,d}$ and the wall electric potential may be approximated to $O(1)$ by the expressions

$$u_{Cu,d} = \tfrac{3}{8}\pi\sqrt{1-z^2} , \quad \Phi_{u,d} = \tfrac{3}{16}\pi B_{u,d}\left[z\sqrt{1-z^2} + \arcsin z\right], \qquad (4a,b)$$

respectively (see [6]-[9]).

Since $B_u \neq B_d$, from Eq. (4b) follows that for any fixed value of $z \neq 0$ there is a difference in the values of potential upstream and downstream. This axial potential difference drives axial electric currents and causes the three-dimensional effects.

The interaction of the magnetic field with the axial current pushes the fluid from the center of the duct to the sides in the upstream region, and peaks of axial velocity appears at the side regions (Fig. 3). The development of the axial velocity profiles at $y = 0$ along the duct axis and at the side region for $Ha = 1000$ and for $Ha = 100$ is shown in Fig. 3. Since the fluid is pushed towards the side regions, a zone with reduced velocity develops in the center of the duct (Fig. 3). The flow in this zone is stagnant for $Ha = 1000$.

For Ha =1000 the development lengths in the upstream and the downstream regions are: $l_{dev,u} = 9.5$ and $l_{dev,d} = 7.5$, respectively. Thus the total development length is $l_{dev} = 17$ duct radii.

The development of the core pressure along the duct along the axis and in the side region for *Ha* = 1000 and for *Ha* = 100 is shown in Fig. 4. For *Ha* = 1000 the pressure values deviate from the fully developed ones in the region $-6 \leq x \leq 3$ owing to the three-dimensional effects. There is a partial pressure recovery at $z = 0$ in the region $-0.5 \leq x \leq 3$ owing to the returning current.

The three-dimensional pressure drop Δp_{3D}, shown in Fig. 4, is 1.48 10^{-2} for *Ha* = 1000 and 4 10^{-2} for *Ha* = 100, respectively. This gives the respective values of the three-dimensional length, d_{3D}, of 12.5 and 3.4. This means that for *Ha* = 1000 the three-dimensional effects are significant, while for *Ha* = 100 they can be neglected.

3.2 Flow for $\alpha \neq 0^o$

For $\alpha \neq 0^o$ the flow becomes non-symmetric with respect to z. Both pressure and potential at $z \approx 1$ drop sooner than those at $z \approx -1$. This is because for a fixed x the magnetic field at $z = 1$ is lower. As a result, in the nonuniform field region more fluid flows at $z = 1$ than at $z = -1$. For *Ha* = 1000 the peaks of velocity in these regions are 7.23 and 6.65, respectively (Fig. 5). However, these values are only marginally lower than 7.33 for the flow for $\alpha = 0^o$.

Variation of Δp_{3D} with α for *Ha* = 1000 is shown in Fig. 6. It is seen that Δp_{3D} decreases with increasing α. However, it remains almost constant for $\alpha \leq 30^o$. Overall, most of the flow features remain the same as those for $\alpha = 0^o$ up to 30^o.

5. Conclusions

For values of *Ha* relevant to C-MOD the three-dimensional effects in insulating circular ducts in a nonuniform magnetic field are expected to be of considerable importance. The three-dimensional pressure drop is equivalent to the extension of ducts with fully developed flow by 12 duct radii. The nonuniformity of the fluid velocity is significant as well with velocity peaks reaching the value of about 7 times the average one. The development length resulting from the nonuniform field is about 17 duct radii. This means that the three-dimensional effects from various blanket/divertor elements may overlap, and the whole device may need to be treated as a single piece. Nevertheless, owing to much lower Hartmann numbers, three-dimensional effects in C-MOD will be of less importance than in large-scale machines, such as ARIES [6], [9]. Flows for $\alpha < 30^o$ are qualitatively the same as for $\alpha = 0^o$.

Acknowledgement

This work has been supported by the Office of Fusion Energy Sciences, U.S. Department of Energy, under Contract No. W-31-109-ENG-38.

References

[1] R. Mattas et al., ALPS – advanced limiter-divertor plasma-facing systems, Fus. Eng. Des. 49-50 (2000) 127-134.

[2] B.G. Karasev and A.V. Tananaev, Liquid metal fusion reactor systems, Plasma Devices and Operations 1 (1990) 11-30.

[3] R.A. Gardner and P.S. Lykoudis, Magneto-fluid-mechanic pipe flow in a transverse magnetic field. Part 1. Isothermal flow, J. Fluid Mech. 47 (1971) 737-749.

[4] R. Holroyd and J.S. Walker, A theoretical study of the effects of wall conductivity, non-uniform magnetic fields and variable-area ducts on liquid metal flows at high Hartmann number, J. Fluid Mech. 84 (1978) 471-495.

[5] U. Müller and L. Bühler, Magnetofluiddynamics in channels and containers, Springer, 2001.

[6] S. Molokov and C.B. Reed, Liquid metal flow in an insulated circular duct in a strong nonuniform magnetic field. Part 1: Flow in a straight duct and the benchmark problem, Argonne National Laboratory Report, ANL/TD/TM01-18, 2001.

[7] S. Molokov and C.B. Reed, Liquid metal flow in an insulated circular duct in a strong nonuniform magnetic field. Part 2: Inclination of the field gradient to the duct axis, Argonne National Laboratory Report, ANL/TD/TM01-19, 2001.

[8] T.Q. Hua and J.S. Walker, Three-dimensional MHD flow in insulating circular ducts in non-uniform transverse magnetic fields, Int. J. Engng Sci. 27 (1989) 1079-1091.

[9] S. Molokov and C.B. Reed, Parametric study of the liquid metal flow in a straight insulated circular duct in a strong nonuniform magnetic field, Fusion Technol. (submitted).

Table 1 Typical values of parameters and flow regimes for various machines

	LARGE-SCALE MACHINES, E.G. ITER, ARIES	**C-MOD**	**NSTX**
Hartmann number, *Ha*	~10^3-10^5	~500-3000	~50-400
Interaction parameter, *N*	~10^3-10^4	~50-200	~1
Reynolds number, *Re*	~10^4-10^5	~10^4-10^5	~10^4-10^5
Criterion of transition to turbulence, *Ha/Re*	~0.1	~0.03	~0.003
Most likely flow regimes	Laminar MHD flow	Laminar MHD flow; possibly turbulent MHD flow in some elements of the blankets/divertors	Turbulent MHD flow

Figure captions

Fig. 1 Schematic diagram of the flow in a straight circular duct: (a) nonuniform magnetic field and (b) projection of the duct on the (x,z)-plane

Fig. 2 Cross-section of a circular duct and flow subregions for high *Ha*.

Fig. 3 Axial velocity at $y = 0$ for different values of z and for $\alpha = 0^o$.

Fig. 4 Variation of pressure with x for different values of z and the three-dimensional pressure drop for $\alpha = 0^o$.

Fig. 5 Axial velocity at $y = 0$ for $Ha = 1000$ and for $\alpha = 20^o$.

Fig. 6 Variation of the three-dimensional pressure drop with α.

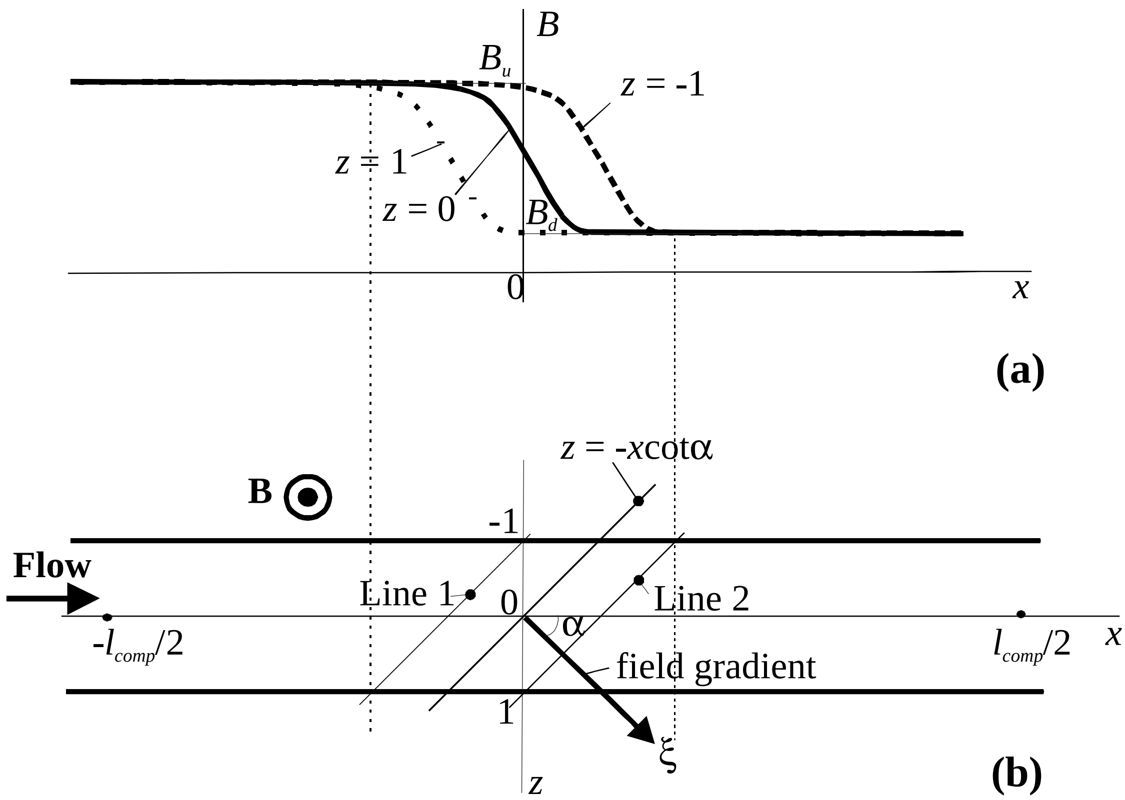

Fig. 1

Molokov and Reed. Liquid Metal flows...

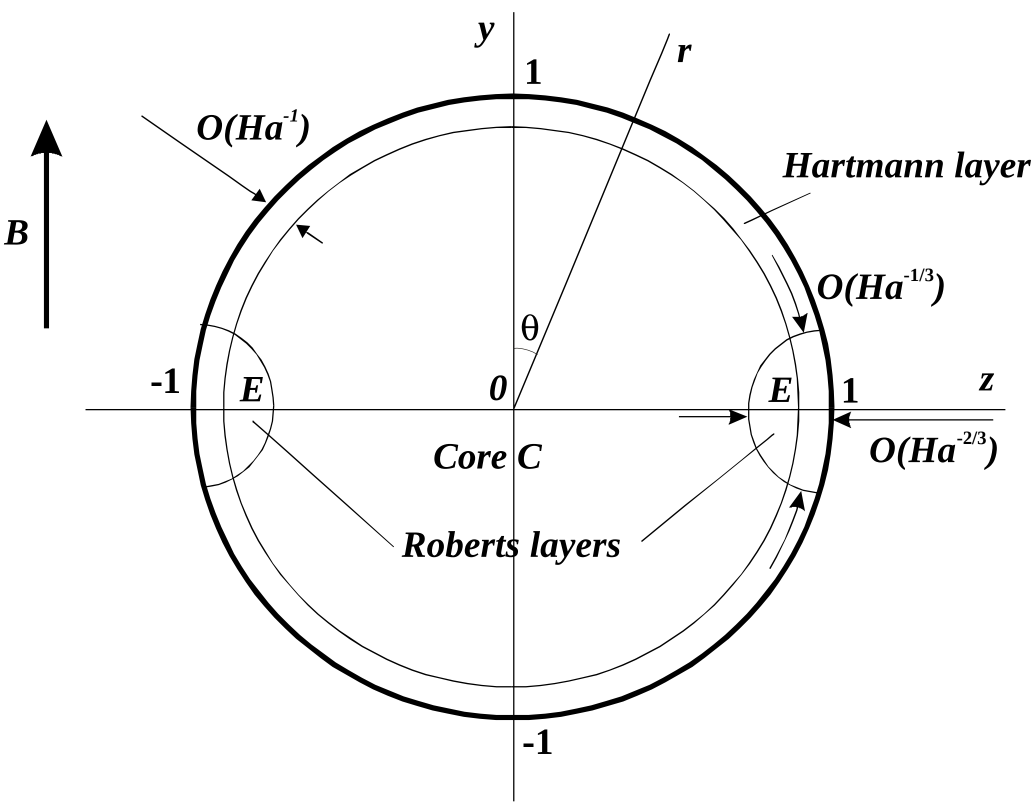

Fig. 2

Molokov and Reed. Liquid Metal flows...

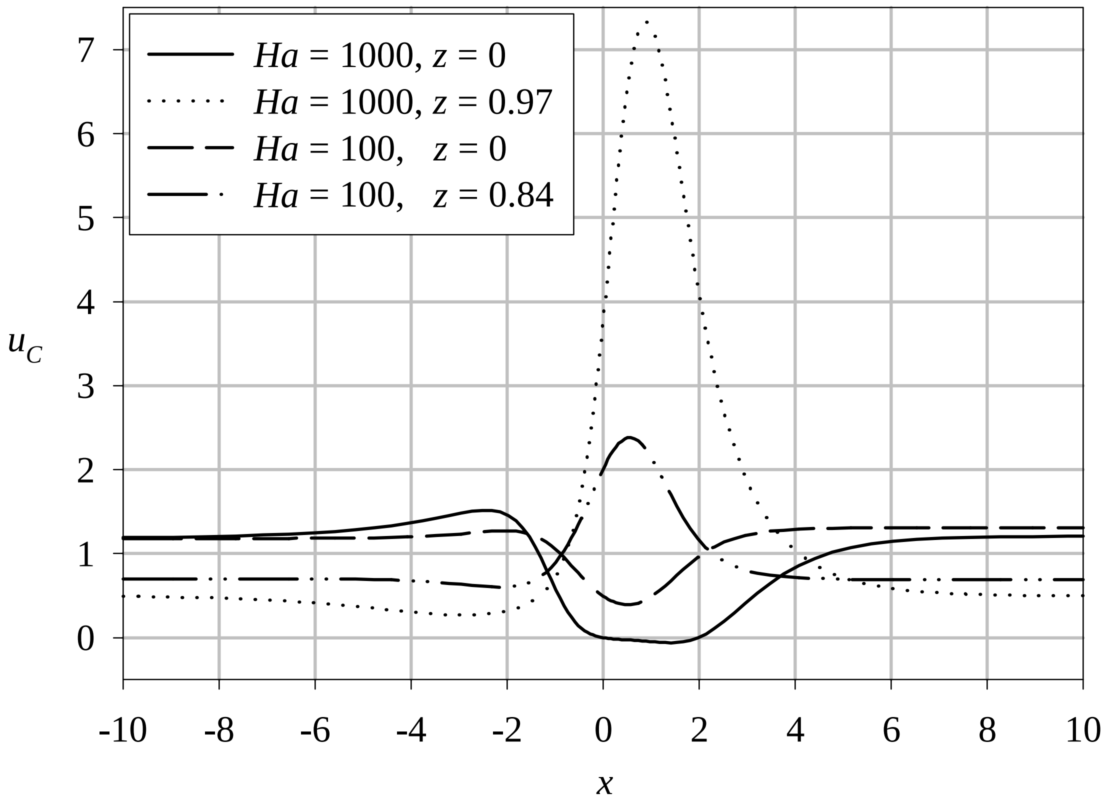

Fig. 3

Molokov and Reed. Liquid Metal Flows...

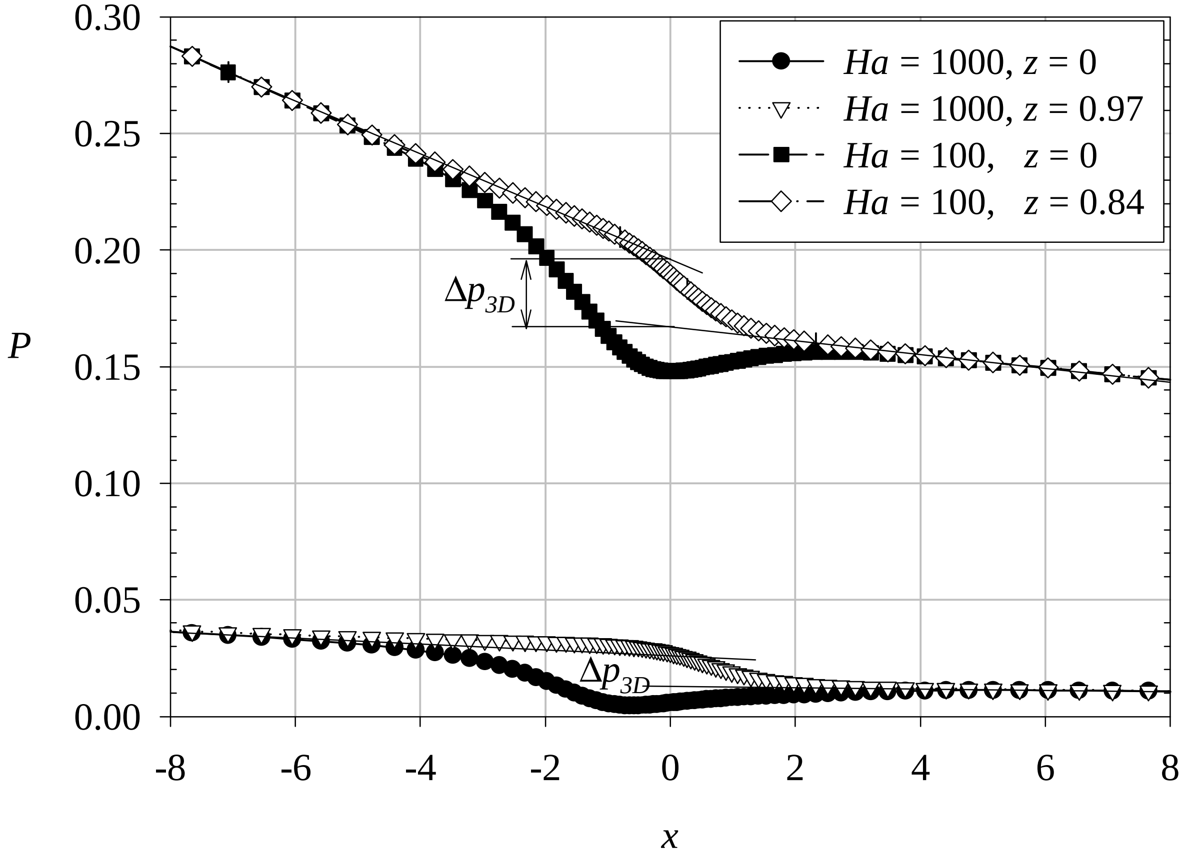

Fig. 4

Molokov and Reed. Liquid Metal Flows...

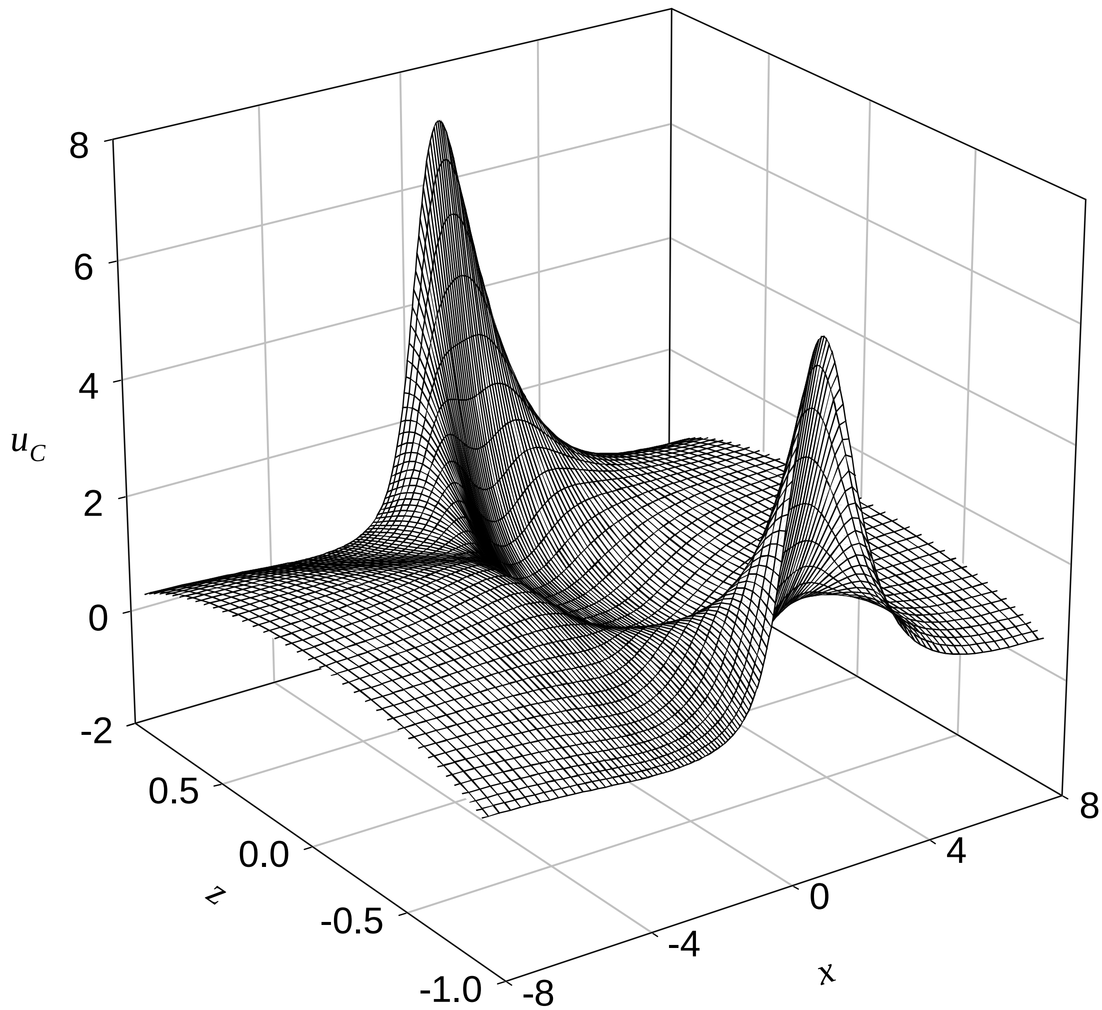

Fig. 5
Molokov and Reed. Liquid Metal Flows...

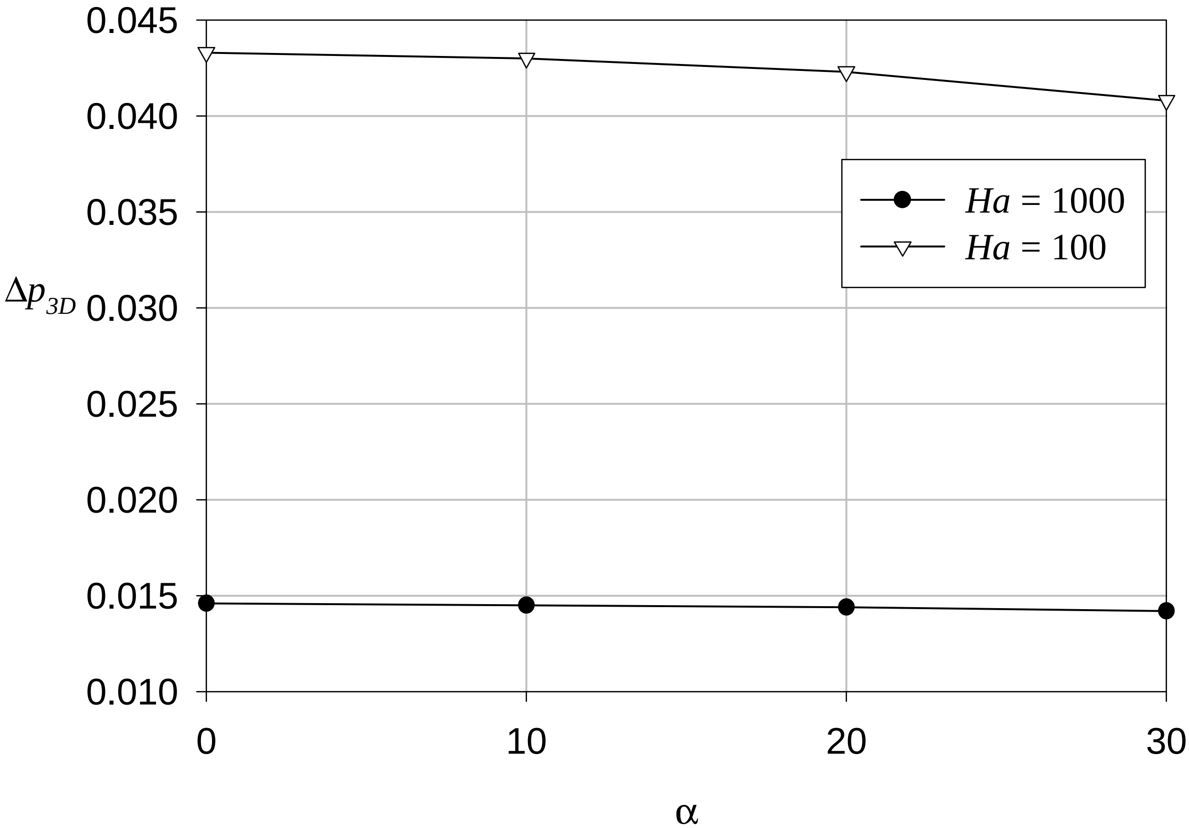

Fig. 6

Molokov and Reed. Liquid Metal Flows...