Liquidity Effects in Interest Rate Options Markets: Premium or Discount?

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November 2006

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[*]We thank Viral Acharya, Yakov Amihud, Menachem Brenner, Jefferson Duarte, Ludger Hentschel, Apoorva Koticha, Haitao Li, Neil Pearson, Allen Poteshman, Peter Ritchken, Matt Spiegel, Kent Womack, an anonymous referee, and the participants at the Bank of Canada conference on fixed income markets for suggestions and helpful discussions on this topic. We remain responsible for all errors.

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ABSTRACT

This paper examines the effects of liquidity on interest rate option prices. Using daily bid and ask prices of euro (€) interest rate caps and floors, we find that illiquid options trade at *higher* prices relative to liquid options, controlling for other effects, implying a liquidity *discount*. This effect is opposite to that found in all studies on other assets such as equities and bonds, but is consistent with the structure of this over-the-counter market and the nature of the demand and supply forces. We also identify a systematic factor that drives changes in the liquidity across option maturities and strike rates. This common liquidity factor is associated with lagged changes in investor perceptions of uncertainty in the equity and fixed income markets.

JEL Classification: G10, G12, G13, G15

Keywords: Liquidity, interest rate options, euro interest rate markets, Euribor market, volatility smiles.

Liquidity has long been recognized as an important factor driving the prices of financial assets. Since the seminal paper by Amihud and Mendelson (1986), several empirical studies have shown that stocks with lower liquidity have lower prices and command higher expected returns. From a theoretical perspective, Longstaff (1995a) derives an analytical upper bound for the discount in security prices for lack of marketability. In addition, within a partial equilibrium setting, Longstaff (2001) shows that trading frictions can lead to substantial implied liquidity premia. More recently, financial economists have focused on the commonality in liquidity across various assets and its implications for asset pricing. It has been documented that there is significant commonality in liquidity across different stocks and that liquidity risk, captured by the variation in the common liquidity component, is important for explaining the cross-section of stock returns.[1] In the fixed income markets as well, starting with Amihud and Mendelson (1991), the evidence is that illiquidity affects bond prices adversely. Furthermore, common factors have been shown to drive liquidity in the Treasury and corporate bond markets.[2]

Thus far, the literature has identified several stylized facts about liquidity in the stock and bond markets and its impact on the prices and returns of the respective assets. However, relatively little is known about the implications of liquidity for pricing in derivatives markets, such as those for equity or interest rate options. An exception in this relatively sparse literature is the study by Brenner, Eldor and Hauser (2001), who confirm the expected result that non-tradable currency options in Israel are discounted by 21 percent on average, as compared to exchange-traded options.[3] But is this always the case? Are illiquid options *always* priced lower than liquid options, similar to the liquidity effect consistently observed in the underlying asset markets, or does this depend on the institutional structure of the market? Is there a common factor that explains the changes in liquidity across option strikes and maturities? What are the fundamental macro-

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[1] See for example, Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), Amihud (2002) and Chordia et al. (2005), for evidence of the former, and Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) for evidence of the latter. For an extensive review of this literature, see Amihud, Mendelsen and Pedersen (2005).

[2] See Krishnamurthy (2002) and Longstaff (2004) for evidence from the Treasury bond markets, and Elton et al. (2001), Longstaff et al. (2005), De Jong and Driessen (2005) and Nashikkar and Subrahmanyam (2006) and others for evidence from the corporate bond market.

[3] In other related studies, Vijh (1990), George and Longstaff (1993), and Mayhew (2002) examine the determinants of equity option bid-ask spreads, while Bollen and Whaley (2004), Cetin et al. (2006), and Garleanu et al. (2006) examine the impact of supply and demand effects on equity option prices.

economic drivers of this common liquidity factor in the options markets? We raise and answer these important questions using cap and floor data from the over-the-counter interest rate options market, which is one of the largest (and yet least researched) options markets in the world, with about $28 trillion in notional principal and $570 billion in gross market value outstanding as of December 2005.[4]

Contrary to the accepted wisdom in the existing literature based on evidence from other asset markets, we find that more illiquid interest rate options trade at *higher* prices, controlling for other effects. This effect goes in the direction opposite to what is observed for stocks, bonds, and even for some exchange-traded currency options. Our paper is the first to document such a liquidity effect in any financial market, and is also the first one to examine liquidity effects in the interest rate options markets. This result has important implications for incorporating liquidity effects in derivative pricing models, since we show that the conventional intuition, which holds in other asset markets, may not hold in some derivatives markets.

We also characterize a common factor that drives the changes in the liquidity of interest rate options at different strike rates and maturities. This common factor explains about one-third of the variation in liquidity across these options. Our results suggest that this systematic liquidity factor is related to the lagged changes in uncertainty in the equity and fixed income markets. This finding has important implications for the measurement and hedging of aggregate liquidity risk in interest rate option portfolios. Perceptions of greater uncertainty in these markets result in greater illiquidity in the interest rate options markets. We are not aware of any other study that has documented a common liquidity factor in any derivatives market. This investigation of the commonality of liquidity complements similar studies for other markets such as those for stocks, Treasury and corporate bonds.

Our study contributes to the existing literature in several ways. According to the available evidence, the impact of illiquidity on asset prices is *overwhelmingly* presumed to be negative, since the marginal investors typically hold a long position, thereby demanding compensation for

[4] *BIS Quarterly Review*, June 2006, Bank for International Settlements, Basel, Switzerland.

the lack of immediacy they face if they wish to sell the asset. Thus, the liquidity premium on the asset is expected to be positive — other things remaining the same, the more illiquid an asset, the higher is its liquidity premium and its required rate of return, and hence, the lower is its price. For example, in the case of a bond or a stock, which are assets in positive net supply, the marginal investor or the buyer of the asset demands compensation for illiquidity, while the seller is no longer concerned about the liquidity of the asset after the transaction. In fact, within a two-asset version of the standard Lucas economy, Longstaff (2005) shows that a liquid asset can be worth up to 25 percent more than an illiquid asset, even if both have identical cash flow dynamics.

However, derivative assets are different from underlying assets like stocks and bonds. First, there is no reason to presume that liquidity in the derivatives markets is an exogenous phenomenon. Rather, it is the result of the availability and liquidity of the hedging instruments, the magnitude of unhedgeable risks, and the risk appetite and capital constraints of the marginal investors, among other factors. Thus, illiquidity in derivatives markets captures all of the concerns of the marginal investor about the expected hedging costs and the risks over the life of the derivative. In particular, in the case of options, since they cannot be hedged perfectly, the dealers are keen to carry as little inventory as possible. Therefore, the liquidity of the option captures the ease with which a dealer can offset the trade. Consequently, the liquidity of an option matters to the dealers and has an effect on its price. Second, derivatives are generally in zero net supply. Therefore, it is not obvious whether the marginal investor in derivatives holds a long or a short position. In addition, in the case of options, the risk exposures of the short side and the long side are not necessarily the same, since they may have other offsetting positions. Both the buyer and the seller continue to have exposures to the asset after the transaction, until it is unwound. The buyer demands a *reduction* in price to compensate for the illiquidity, while the seller demands an *increase*. Due to the asymmetry of the option payoffs, the seller has higher risk exposures than the buyer. The net effect of the illiquidity, which itself is endogenous, is determined in equilibrium, and one cannot presume *ex ante* that it will be either positive or negative, especially if the motivations of the two parties for engaging in the transaction are different.

This is especially true for interest rate cap and floor markets, which are over-the-counter (OTC) institutional markets with hardly any retail presence. The buyers of caps and floors are typically (buy and hold) corporations wanting to hedge their interest rate risk. The sellers (derivative desks at large commercial and investment banks) are concerned about hedging the risks of the caps and floors that they sell, and hence about the liquidity of these options, which is a proxy for such concerns.[5] Thus, for the purposes of pricing of liquidity, the marginal investor in this market is likely to be net short. The size of individual trades is relatively large, with the contracts being long-dated portfolios of options (up to ten years maturity or more). The long-dated contract creates enormous transaction costs if the seller just hedges dynamically using the underlying spot or derivative interest rate markets. Consequently, the market maker with a net short position may *raise* the price of illiquid options.[6] Hence, illiquidity in this case has a *positive* relationship with the price, rather than the conventional *negative* relationship identified in the literature so far. This is indeed what we find, within an endogenous specification for option liquidity and prices.

Our result can be explained in the context of deviations from the Black-Scholes world, where both the buyer and the seller can hedge costlessly in the underlying market; consequently, illiquidity should not have an effect on the price of an option. However, in the real world, options cannot be exactly replicated, due to stochastic volatility, jumps, discrete rebalancing or transaction costs.[7] There are also limits to arbitrage, as outlined in Shleifer and Vishny (1997) and Liu and Longstaff (2004). In addition, option dealers face model misspecification and biased parameter estimation risk (Figlewski (1989)). These factors result in some part of the risk in options being unhedgeable, leading to an upward sloping supply curve (Bollen and Whaley (2004), Jarrow and Protter (2005) and Garleanu, Pedersen and Poteshman (2006)). In addition,

[5] In recent years, hedge funds have been quite active in this market. Based on our conversations with practitioners in this market, we understand that these players typically have short positions in options.

[6] The results in Brenner at al. (2001), to the effect that illiquid currency options were priced *lower* than traded options, can also be explained by the same argument. In their case, illiquidity had a negative relationship with price. Since these options were auctioned by the Central Bank of Israel, the buyers of these options were the ones who were concerned about illiquidity, and not the seller.

[7] Constantinides (1997) argues that, with transaction costs, the concept of the no-arbitrage price of a derivative is replaced by a range of prices, which is likely to be wider for customized, over-the-counter derivatives (which include most interest rate options), as opposed to plain-vanilla exchange-traded contracts, since the seller has to incur higher hedging costs to cover short positions, if they are customized contracts. In a similar vein, Longstaff (1995b) shows that in the presence of frictions, option pricing models may not satisfy the martingale restriction.

since dealers in this market are net short, they may hit their capital constraints more often if they have to sell more options to make a market (Brunnermeier and Pedersen (2006)). They would, therefore, ask for more compensation for providing liquidity, thus making the supply curve upward-sloping.

Option liquidity is related to the slope of this upward-sloping option supply curve in three ways. First, the time when options become more illiquid coincides with the time the sellers face greater unhedgeable risks relative to their risk appetite and capital. In addition, it becomes more difficult for sellers to reverse their trades and earn the bid-ask spread. They face greater basis risk, since they have to hold an inventory of options that they cannot hedge perfectly. Second, the sellers face greater model risk when there is less liquidity — when there are fewer option trades, the dealers have less data to reliably calibrate their pricing models. Third, as modeled in Duffie, Garleanu, and Pedersen (2005), due to bilateral trading in OTC markets, dealers can have market power; hence search frictions can increase bid-ask spreads as well as liquidity premia.[8] All these factors result in increasing the slope of the option supply curve when there is less liquidity, consistent with Cetin, Jarrow, Protter and Warachka (2006). The impact of a steeper supply curve on option prices and bid-ask spreads can be understood within the theoretical model of Garleanu, Pedersen and Poteshman (2006). Given the inventory of the dealer, a steeper supply curve would result in wider bid-ask spreads, since the difference in prices for a unit positive and a unit negative change in their inventory would be larger. In addition, if the net demand by the end-users is positive (as in the case of interest rate caps and floors), a steeper supply curve will result in higher option prices, since the dealer is net short in the aggregate.[9] In such a scenario, higher bid-ask spreads (lower liquidity) would be associated with higher prices, resulting in a liquidity discount, not a premium. Our empirical results are consistent with these implications, given the structure of the OTC interest rate options markets.

[8] Admittedly, the search costs do not change much on a daily basis. Thus, the contribution of the mechanism in Duffie et al (2005) to our story is possibly only secondary. We are thankful to an anonymous referee for pointing this out.

[9] Garleanu et al (2006) do not specifically examine the relationship between illiquidity and the prices of derivative assets. Their main focus is on the effects of the changes in inventory on prices through movement along the supply curve. However, their set-up is also useful in understanding the changes in the slope of the supply curve and the resultant relationship between illiquidity and option prices.

Although there is a plethora of research on liquidity effects in equity and debt markets, particularly in the United States, there is scant evidence in the case of derivative markets. Using data from the interest rate options markets, our results underscore the fact that the positive relationship between liquidity and asset prices cannot be generalized to other markets without considering the structure of the market and the nature of the demand and supply forces. This fundamental point must be taken into account in both theoretical and empirical research. Since interest rate derivatives form a substantial proportion of the global derivatives markets, our results could potentially provide insights into the broad question of liquidity effects in derivatives markets. In addition, our investigation of the fundamental drivers of liquidity in this market may influence the construction of models for pricing and hedging of interest rate derivatives.

The structure of our paper is as follows: In Section 1 we describe the data set and present summary statistics. After controlling for the term structure and volatility factors, a simultaneous equation system is employed to estimate and examine the relationship between the price (excess implied volatility relative to a benchmark) and the liquidity (relative bid-ask spread) of interest rate options. Section 2 presents the results for this relationship. Section 3 explores the commonality in the changes in liquidity of interest rate options across strike rates and maturities, and links this systematic factor to changes in macro-economic variables. Section 4 concludes with a summary of the main results and directions for future research.

1. Data

The data for this study consist of an extensive collection of euro (€) interest rate cap and floor prices over the 29-month period from January 1999 to May 2001, obtained from WestLB (Westdeutsche Landesbank Girozentrale) Global Derivatives and Fixed Income Group. These are daily bid and offer price quotes over 591 trading days for nine maturities (two years to ten years, in annual increments) across twelve different strike rates ranging from 2% to 8%. On a typical day, price quotes are available for about 30-40 caps and floors, reflecting the maturity-strike combinations that exhibit market interest on that day.

WestLB is one of the dealers that subscribe to the interest rate option valuation service from Totem. Totem is the leading industry source for asset valuation data and services supporting independent price verification and risk management in the global financial markets. Most derivative dealers subscribe to their service. As part of this service, Totem collects data for the entire "skew" of caplets and floorlets across a series of maturities from its set of dealers. They aggregate this information and return the consensus values back to the dealers that contribute data to them. The market consensus values supplied to the dealers include the underlying term structure data, caplet and floorlet prices, as well as the prices and implied volatilities of the reconstituted caps and floors across strikes and maturities. Hence, the prices quoted by dealers such as WestLB that are a part of this service reflect market-wide consensus information about these products. This is especially true for plain-vanilla caps and floors, which are very high-volume products with standardized structures that are also used by dealers to calibrate their models for pricing and hedging exotic derivatives. Therefore, it is extremely unlikely that any large dealer, especially one that uses a market data integrator such as Totem, would deviate systematically from market consensus prices for these "plain vanilla" products. Our discussions with market participants confirm that the bid and ask prices quoted by different dealers (especially those that subscribe to Totem) for "plain vanilla" caps and floors are generally similar.[10]

Another way to assess the representativeness of the data is to consider the competitiveness of the market. The euro OTC interest rate derivatives market is extremely competitive, especially for plain-vanilla contracts like caps and floors. The BIS estimates the Herfindahl index (sum of squares of market shares of all participants) for euro interest rate options (which includes exotic options) at about 500-600 during the period from 1999 to 2004, which is even lower than that for USD interest rate options (around 1,000), compared to a range of 0-10, 000 (where 0 indicates a perfectly competitive market and 10,000 a market dominated by a single monopolist.) The Henfindahl index values indicates that the OTC interest rate options market is a fairly competitive market; hence, it is safe to rely on option quotes from a top European derivatives dealer

[10] The use of market dealer quotations for studying liquidity effects is consistent with several prior studies, including Longstaff et al. (2005).

(reflecting the best market consensus information available with them) like WestLB during our sample period. Given the competitive structure of the market, any dealer-specific effects on the quotes are likely to be small and unsystematic.

This data set allows us to conduct the empirical analysis for caps and floors across strike rates. These caps and floors are portfolios of European interest rate options on the six-month Euribor with a six-monthly reset frequency. In the Appendix, we provide the details of the contract structure for these options. Along with the options data, we also collected data on euro swap rates, and the daily term structure of the euro interest rates, from the same source. These are key inputs necessary for conducting our empirical tests.

Table 1 provides descriptive statistics on the midpoint of the bid and ask prices for caps and floors over our sample period. The prices of these options can be almost three orders of magnitude apart, depending on the strike rate and the maturity of the option. For example, a deep out-of-the-money, two-year cap may have a market price of just a few basis points, while a deep in-the-money, ten-year cap may be priced above 1500 basis points. Since interest rates varied substantially during our sample period, the data have to be reclassified in terms of "moneyness" ("depth in-the-money") to be meaningfully compared over time. In Table 1, the prices of options are grouped together into "moneyness buckets," by calculating the Log Moneyness Ratio (LMR) for each cap/floor. The LMR is defined as the logarithm of the ratio of the par swap rate to the strike rate of the option. Therefore, a zero value for the LMR implies that the option is at-the-money forward, since the strike rate is equal to the par swap rate. Since the relevant swap rate changes every day, the moneyness of the same strike rate, same maturity, option, as measured by the LMR, also changes each day. The average price, as well as the standard deviation of these prices, in basis points, are reported in the table. It is clear from the table that cap/floor prices display a fair amount of variability over time. Since these prices are grouped together by moneyness, a large part of this variability in prices over time can be attributed to changes in volatilities over time, since term structure effects are largely taken into account by our adjustment.

We also document the magnitude and behavior of the liquidity costs in these markets over time, for caps and floors across strike rates. We use the bid-ask spreads for the caps and floors as a proxy for the illiquidity of the options in the market. In an OTC market, this is the *only* measure of illiquidity available for these options. Other measures of liquidity common in exchange-traded markets such as volume, depth, market impact etc., are just not available. In our sample, we do observe the bid-ask spread for each option every day. Therefore, we settle for using this metric as a meaningful, although potentially imperfect, proxy for liquidity.[11]

It is important to note that these are measures of the liquidity costs in the interest rate options market and not in the underlying market for swaps. Although the liquidity costs in the two markets may be related, the bid-ask spreads for caps and floors *directly* capture the effect of various frictions in the interest rate options market, along with the transaction costs in the underlying market, as well as the imperfections in hedging between the option market and the underlying swap market. Furthermore, unlike options on different equities, which are not directly related to each other, caps and floors with different strike rates and maturities all depend on the same underlying yield curve. In addition, the market for underlying swaps is extremely liquid (the typical bid-ask spreads on interest rate swaps are a couple of basis points) with hardly any time variation. Therefore, the transaction costs in the underlying market cannot explain any variation in the liquidity of caps and floors either through time or in the cross-section. In addition, the *current* bid-ask spreads for caps and floors themselves are proxies for the *expected* costs of hedging and the *expected* level of unhedgeable risks, since the dealers set the current bid-ask spreads based on their expectations of these frictions. Therefore, the current bid-ask spread of the option is the liquidity proxy relevant for pricing analysis.

In Table 2, we present the relative bid-ask spreads (RelBAS), defined as the bid-ask spreads divided by the mid price (the average of the bid and ask prices) of the option, grouped together into moneyness buckets by the LMR. Close-to-the-money caps and floors have relative bid-ask

[11] The bid-ask spread has been used as a proxy for liquidity by several prior studies, including Amihud and Mendelsen (1986), and has been shown to be highly correlated with other proxies for liquidity. In addition, in the spot fixed income markets, Fleming (2003) and Goldreich, Hanke and Nath (2005) show that the spread quoted by market makers who supply liquidity better measures the value investors place on immediacy, rather than the actual trade prices or trade sizes.

spreads of about 8-9%, except for some of the shorter-term caps and floors that have higher bid-ask spreads. Since deep in-the-money options (low strike rate caps and high strike rate floors) have higher prices, they have lower relative bid-ask spreads (3-4%). Some of the deep out-of-the-money options have large relative bid-ask spreads — for example, the two-year deep out-of-the-money caps, with an average price of just a couple of basis points, have bid-ask spreads almost as large as the price itself, on average about 80.9% of the price. Part of the reason for this behavior of bid-ask spreads is that some of the costs of the market makers (transaction costs on hedges, administrative costs of trading, etc.) are absolute costs that must be incurred whatever may be the value of the option sold. However, some of the other costs of the market maker (inventory holding costs, hedging costs, etc.) are dependent on the value of the option bought or sold. It is also important to note that, in general, these bid-ask spreads are much larger than those for most exchange-traded options.

For our empirical tests, we pool the data on caps and floors, since this would allow us to obtain a wider range of strike rates, covering rates that are both in-the-money and out-of-the-money, for both caps and floors. Before doing so, we check for put-call parity between caps and floors, and find that, on the average, put-call parity holds in our data set.[12] These parity computations are a consistency check which assures us about the integrity of our data set.

2. How do Liquidity and Option Prices vary?

We use implied volatilities from the Black-BGM model, estimated using mid prices (the average of bid and ask) to characterize option prices throughout the analysis from here on.[13] The raw implied volatility obtained from the Black BGM model removes underlying term structure effects from option prices.[14] Therefore, a change in the implied volatility of an option from one day to

[12] The results of these computations are not reported in the paper, but are available from the authors.

[13] The use of implied volatilities, from a variant of the Black-Scholes model, even though model-dependent, is in line with all prior studies in the literature, including Bollen and Whaley (2004). The details of the calculation of implied volatility are provided in the Appendix.

[14] Our implied volatility estimation is likely to have much smaller errors than those generally encountered in equity options (see, for example, Canina and Figlewski (1993)). We pool the data for caps and floors, which reduces errors due to misestimation of the underlying yield curve. The options we consider are much longer term (the shortest cap/floor has a two-year maturity), which reduces this potential error further. For most of our empirical tests, we do not include deep ITM or deep OTM options, where estimation errors are likely to be larger. Furthermore, since we consider the implied flat volatilities of caps and floors, rather

the next can be attributed to changes in interest rate uncertainty, or other effects not captured by the model, and not simply due to changes in the underlying term structure. We then estimate the *excess implied volatility* (EIV, similar to that used in Garleanu, Pedersen and Poteshman (2006)) as the difference between the implied volatility and a benchmark volatility estimated using a panel GARCH model on historical interest rates. We check for the robustness of our results by estimating the benchmark volatility using several alternative methods. The EIV is a cleaner measure of the *expensiveness* of options, since even the general level of interest rate volatility has been factored out of the implied volatility of each option contract. In addition, in the empirical tests where we use EIV, we control for the shape of the volatility smile (using functions of LMR), and use several term structure variables as well as approximate controls for the skewness and excess kurtosis in the underlying interest rate distribution. In the presence of these controls, the changes in the EIV for a particular option cannot be attributed to changes in the underlying term structure or to changes in the general level of interest rate volatility. Therefore, the EIV can be effectively used to examine factors, such as liquidity, *other than* the underlying term structure or interest rate uncertainty that may affect option prices in this market.[15] In the rest of the paper, we use the EIV as a measure of the expensiveness of the option, for every strike and maturity.

2.1. Panel GARCH Model for Benchmark Volatility

The GARCH models proposed by Engle (1982) and Bollerslev (1986) have been extended to explain the dynamics of the short-term interest rate by Longstaff and Schwartz (1992), Brenner, Harjes, and Kroner (1996), Cvsa and Ritchken (2001), and others. These studies find that for modeling interest rate volatility, it is important to allow the volatility to depend both on the level of interest rates and on unexpected information shocks. The asymmetric volatility effect as modeled in Glosten, Jagannathan, and Runkle (GJR, 1993) has also been found to improve volatility forecasts. In particular, these studies recommend using a GJR-GARCH (1,1) model with a square-root type level dependence in the volatility process.

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than spot volatilities, the errors are even further reduced due to the implicit "averaging" in this computation. The "flat" volatility is the weighted average of the volatilities for all the caplets/floorlets in a cap/floor, while the "spot" volatility is the volatility of an individual caplet/floorlet.

[15] Changes in the EIV, in the presence of these controls, are somewhat analogous to the excess returns used in asset pricing studies.

However, for estimating the relevant benchmark volatilities for caps/floors, we need to model forward rate volatilities. These present an additional challenge, since the volatilities for different forward rate maturities, while being different, are linked together due to the common factors that drive the entire term structure of interest rates. Therefore, the entire term structure of forward rate volatilities must be estimated simultaneously in an internally consistent modeling framework. We extend this literature and develop a panel GARCH model with the following process for forward rates:

$$
\begin{aligned}
&f_{t,T} = \alpha_0 + \alpha_1 f_{t-1,T} + \varepsilon_{t,T}, \qquad \varepsilon_{t,T} \sim N\left(0, h_{t,T}^2\right) \\
&h_{t,T} = \sigma_{t,T}\sqrt{f_{t-1,T}} \qquad\qquad \text{.....(1)} \\
&\sigma_{t,T}^2 = \beta_0 + \beta_1\sigma_{t-1,T}^2 + \beta_2\varepsilon_{t-1,T}^2 + \beta_3\varepsilon_{t-1,T}^2 I_{t-1,T}^-, \qquad I_{t-1,T}^- = 1 \ \mathit{if}\ \varepsilon_{\text{t-1,T}} < 0
\end{aligned}
$$

where $f_{\text{t,T}}$ is the six-month tenor forward rate, T periods forward, observed at time t. This is a panel version of the GJR-GARCH(1,1) model with square-root level dependence. It is a parsimonious yet very flexible model that nests many widely used GARCH models, as well as the continuous time term structure models in the Heath, Jarrow, and Morton (HJM, 1992) framework, including the Cox, Ingersoll, and Ross (CIR, 1985) model. We estimate this panel GARCH model using the maximum likelihood method and the Marquardt-Levenberg algorithm. We have a panel of 19 forward rates of six-month tenor with maturities ranging from six-months to 9.5 years in increments of six months each. For each day, we estimate the GARCH model on the history of the forward rates available up to that day. We impose a minimum requirement of 66 days of data (about three months) which gives us sufficient observations (66 x 19 = 1,254) to estimate this panel GARCH model reliably. Based on the estimated model, we forecast the volatilities of all the forward rates. Using these forward rate volatility forecasts, we price each caplet individually using the Black model, and then invert the resultant at-the-money cap price to obtain the flat implied volatility which is then used as the benchmark volatility in the EIV calculation. The panel

GARCH model is a sophisticated way of extracting information from historical volatility, which we convert into a consistent benchmark through the Black model.[16]

In addition to using this panel GARCH model to estimate the benchmark volatility, we employ two alternative volatility measures as benchmarks to compute the EIV for additional robustness. The first is a simple historical volatility estimated as the annualized standard deviation of changes in the log forward rates of different maturities, using the past 66 days of forward rate data (our results are again robust to different choices of this historical time window). The second alternative volatility measure we use is a comparable implied volatility from the swaption market. We use only the at-the-money "diagonal" swaption volatilities since they are the most actively traded swaption contracts in the market. For example, for the two-year caps/floors, we use the 1x1 swaption (one-year option on the one-year forward swap) volatility as the relevant benchmark, since the 1x1 swaption price reflects the volatilities of forward rates out to two-years in the term structure. Similarly, for the four-year caps/floors, we use the 2x2 swaption volatility as the benchmark volatility. For the three-year caps/floors, we use the average of the 1x1 and the 2x2 swaption volatilities. The other benchmark volatilities are calculated in a similar manner.

It is important to note that the first two benchmark volatility measures (the panel GARCH based volatility and simple standard deviation) are both historical volatility measures. In principle, one could forecast the volatility of forward rates over the life of the cap/floor using the panel GARCH model. However, interest rate options like caps/floors (unlike most equity options) have very long maturities, ranging from two to ten years. In addition, the Euribor data are only available from January 1999 onwards (since the introduction of the Euro). Therefore, forecasting the volatilities of forward rates two to ten years in the future using the panel GARCH model is likely to be unreliable. As a result, we use these two alternative historical volatility measures (panel GARCH and standard deviation) as proxies for the expected volatility. The advantage of the panel GARCH methodology is that it extends to forward rates a model that has been shown to work

[16] We do extensive robustness tests using several alternative specifications of the panel GARCH model (including a specification with a parametric volatility hump similar to the one in Fan, Gupta, and Ritchken (2006)), to ensure that our results are not driven by any particular choice of a model for the benchmark volatility. These results are not presented in the paper to save space, but can be furnished by the authors, upon request..

well for forecasting the short rate volatility. The advantage of the historical standard deviation is its simplicity and freedom from imposition of any model structure. However, both these benchmarks suffer from the fact that they are backward looking whereas option prices are based on forward looking volatilities. The volatility from the swaption market provides us with a measure of the expected volatility in the underlying Euribor market (which is common to both caps/floors as well as swaptions) over the maturity of the cap/floor, but from a different market that is not directly influenced by the liquidity effects in the cap/floor markets. These three benchmark volatility measures, applied separately, complement each other and provide us with confidence about the robustness of our results.

Figure 1 presents the scatter plots for the EIV across moneyness represented by LMR for our three benchmark volatility measures – panel GARCH, standard deviation, and swaption implied volatility. The plots are presented for three representative maturities — two-year, five-year, and ten-year — for the pooled cap and floor data. The plots for the other maturities are similar. These plots clearly show that there is a significant smile curve, across strike rates, in these interest rate options markets. The smile curve is steeper for short-term options, while for long-term options, it is flatter and not symmetric around the at-the-money strike rate. It is also important to note that the range of moneyness observed in this market is much greater than that generally observed in the equity markets. For example, for two-year caps/floor, it is not uncommon to find options that have strike rates that are 40%-50% higher or lower than the at-the-money strike rate. We classify the options that have LMRs between -0.1 and 0.1 as being at-the-money, since the volatility smile is virtually non-existent within this moneyness range.

2.2. How are Liquidity and Price Related in the Interest Rate Options Markets?

As argued in the literature, the relationship between the liquidity of an asset and its price is of fundamental importance in any asset market. For common underlying assets like stocks and bonds usually more liquid assets will have lower returns and higher prices. However, for derivative assets, especially those in zero net supply where it is not clear whether the marginal investor would be long or short, this relationship may go either way. In this subsection, we examine this relationship for Euro interest rate caps and floors.

To gain an initial understanding of this relationship, we first estimate the correlation between the EIV and the RelBAS for all maturities for all three of the benchmark volatility measures. For example, the correlation between the EIV (based on the panel GARCH model) and the RelBAS is about 0.41 for two-year maturity caps/floors, 0.35 for five-year maturity caps/floors, and 0.44 for ten-year maturity caps/floors, which are all statistically significantly greater than zero. Figure 2 presents the sample scatter plots for the two, five, and ten-year maturity options, for all the three benchmark volatility measures. The plots for the other maturities are similar. Across all the nine maturities, we find that the average of the correlation coefficients (between the EIV and the RelBAS) is 0.41 using the panel GARCH based benchmark volatility, 0.44 using the historical standard deviation based benchmark volatility, and 0.43 using the swaption based benchmark volatility. Although these are just "raw" correlations between option expensiveness and illiquidity, they do indicate that, on average, more illiquid options appear to be more expensive across all moneyness buckets and maturities.

Illiquidity, especially for a derivative asset, arises endogenously due to the fundamental frictions in financial markets. In particular, the bid-ask spreads capture the slope of the supply curve of the dealers, which is affected by hedging costs, extent of unhedgeable risks, and the dealers' risk appetite and capital. Liquidity in a broader sense also captures the ease with which the market-makers can find an offsetting trade. Even though the dealers may not find offsetting trades for their entire inventory, they would still prefer to carry as little inventory as possible. Therefore, finding an offsetting trade and hence the liquidity of the options themselves matters to them. To the extent that they cannot find an offsetting trade, they would charge a premium to carry that inventory. In this manner, liquidity could be both a "cause" and an "effect". In fact, in the context of a dynamic trading model, Gallmeyer, Hollifield, and Seppi (2005) show that, especially for long-dated securities, the demand discovery process leads to endogenous joint dynamics in prices and liquidity. Thus, both liquidity and price can have an effect on each other, and it is likely that they are jointly determined by a set of exogenous macro-financial variables. Therefore, we model this endogenous relationship within a simultaneous equation model of liquidity (relative bid-ask spreads) and price (EIV), using macro-financial variables as the exogenous determinants of these two endogenous variables.

2.2.1. Liquidity Effects in ATM Options

Unlike underlying asset markets, options markets have another dimension (the strike price/rate) along which both liquidity and prices change, as shown in the figures above. There is a smile (or a skew) across strike rates in both implied volatilities as well as liquidity. These smiles/skews arise in part due to the skewness and excess kurtosis in the distribution of the underlying interest rates. In order to clearly disentangle liquidity effects from any effects arising due to the volatility smiles/skews observed in this market, we first focus *only* on at-the-money options, with LMRs between -0.1 and 0.1. More precisely, these options are *near-the-money*, instead of being truly at-the-money. However, as shown in figure 1, the volatility smile is virtually flat within this moneyness range, hence the smile effects, if any, are likely to be negligible for these options.[17] In spite of the smile being virtually flat for these at-the-money options, we control for any residual smile effects within this moneyness bucket using an asymmetric quadratic function of LMR that best explains the variation in EIV as well as in RelBAS across strikes.[18] Therefore, we use *LMR*, LMR^2, and ($1_{LMR<0}.LMR$) as controls for any residual strike rate effects for both liquidity and price in the simultaneous equation model.

Our discussions with market participants revealed that the dealers consider the vega and the moneyness of the options while setting bid-ask spreads. As a good approximation, vega can be expressed as a quadratic function of the moneyness of the option. Thus, including these LMR controls in the RelBAS equation also takes care of the dependence of bid-ask spreads on vega and on moneyness. The objective of such LMR controls in both the equations is to filter out any residual dependence of EIV and RelBAS on the moneyness of the option, and examine whether

[17] In additional tests, we find that our results are robust to narrower (LMRs between -0.05 and 0.05) or wider (LMRs between -0.15 and 0.15) LMR ranges for defining options as at-the-money.

[18] This is based on our examination of alternative functional forms using pooled time-series cross-sectional regressions of EIV and RelBAS on various functions of LMR, and is consistent with the appearance of the plots presented in Figure 1. Our results are robust to the exclusion of these LMR controls for the at-the-money options.

there is still any relationship between these two variables, as well as between these two variables and the exogenous variables in the model.

Therefore, we estimate the following equation system for ATM options with LMRs between -0.1 and 0.1:

$$
\begin{aligned}
EIV &= c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + \\
&\qquad c6SwpnVol + c7 * DefSprd + c8 * 6Mrate + c9 * Slope \\
RelBAS &= d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) + \\
&\qquad d6 * SwpnVol + d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd
\end{aligned}
\quad \text{.....(2)}
$$

The two-equation simultaneous-equations model above has two endogenous variables (EIV and RelBAS), a vector of LMR controls, and a vector of exogenous variables in both the equations for model identification. The intuition behind the choice of the exogenous variables is explained below.

The swaption volatility (*SwpnVol*) is included to examine whether the price and the liquidity of these options vary significantly with the level of uncertainty in the interest rate options markets. Although we have already benchmarked the cap/floor implied volatility against various proxies for the expected interest rate volatility, we include the swaption volatility as a control to account for any residual dependence of the EIV on the level of volatility. During more uncertain times, information asymmetry issues, which may influence both price and liquidity, are likely to be more important than during periods of lower uncertainty. If there is significantly greater information asymmetry, market makers may charge higher than normal prices for options, since they may be more averse to taking short positions. This, in turn, will lead to higher excess implied volatilities of options. During times of greater uncertainty, a risk-averse market maker may demand higher compensation for providing liquidity to the market, which would affect the relative bid-ask spreads in the market. The market price of liquidity risk may also be higher during more uncertain times. We use the ATM swaption volatility as an explanatory variable here, since it is not subject to the liquidity effects in the cap/floor markets. The ATM swaption volatility can be interpreted as a general measure of the future interest rate volatility.

We considered including other option Greeks as additional controls but did not do so for two reasons. First, the squared LMR included above is an approximate proxy for the convexity term. Second, introducing other option Greeks explicitly may introduce potential collinearity, since, to a first order approximation, these risk parameters can be modeled as linear functions of volatility and the square root of the time to expiration.[19]

The six-month German Treasury-Euribor Spread (*DefSprd*) is included as a measure of the aggregate default risk of the constituent banks in the Euribor fixing. It controls for any credit risk effects in liquidity and price. This is especially important for caps and floors since these are over-the-counter options not backed by a clearing corporation or an exchange; hence the level of credit risk may affect the pricing as well as the liquidity of these options. The default spread is also included as another proxy for the level of uncertainty in the market, since it goes up during uncertain times. Since model risk is higher when the level of uncertainty is high, it is likely that higher default spread regimes are associated with higher option prices as well as wider bid-ask spreads.

In the first equation of the simultaneous equation model, we include the spot six-month Euribor (*6Mrate*) and the slope of the yield curve (*Slope*, defined as the difference between the five-year and six-month spot rates) as instruments for EIV. These variables are used as proxies for the expectations of the market about the direction in which interest rates are expected to move in future. If interest rates are mean reverting, very low interest rates are likely to be followed by rate increases. Similarly, a steeply upward-sloping yield curve signals rate increases. Thus the yield curve variables are also likely to capture the demand for these interest rate options. The short rate and the slope also proxy for the expectations in the financial markets about future inflation and money supply, which are fundamental determinants of the term structure of interest rates and its volatility. However, it is unlikely that the yield curve variables have a direct effect on the relative bid-ask spreads of these options. Therefore, we use them as instruments for the excess implied

[19] See, for example, Brenner and Subrahmanyam (1994), who provide, in the context of the Black-Scholes model, approximate values for the risk parameters of options that are close to being at-the-money on a forward basis.

volatilities. In our econometric tests later in this paper, we examine whether these instruments are valid from a statistical standpoint.

The ATM volatility and term structure variables act as approximate controls for a model of interest rates displaying skewness and excess kurtosis.[20] Typically, in such models, the future distribution of interest rates depends on the current day's volatility and on the level of interest rates. Thus, by including the contemporaneous volatility and interest rate variables in the regression, we try to capture the relationship between the excess implied volatilities and liquidity without explicitly considering a more detailed structural model for interest rates.

In the second equation of the simultaneous equations model, we include the logarithm of the trading volume of the three-month Euribor futures contract on the London International Financial Futures Exchange i.e. LIFFE (*LiffeVol*) and the spread between the three-month AA financial CP rate and the three-month Treasury bill rate (*CpTbSprd*) as instruments for the relative bid-ask spreads in this market. The Euribor futures volume is a proxy for trading activity due to interest rate hedging demand. There are no volume data available for caps and floors, since they are traded over-the-counter. Most of the trading activity for these options is either by firms attempting to hedge their interest rate exposures or from inter-dealer trades. The Euribor futures volume variable is likely to be positively correlated with the trading volumes (and liquidity) for caps and floors, since, to some extent, they are substitute products for hedging interest rate risk. However, there is no reason for the Euribor futures volume to affect the excess implied volatilities of these options, except through liquidity effects. Therefore, it is likely to be a valid instrument.

The CP to T-bill spread has been used as an instrument for the variation in aggregate liquidity demand in several prior studies, including Krishnamurthy (2002) and Gatev and Strahan (2006). Since the CP market is illiquid in comparison with the T-bill market, the spread between the two rates reflects aggregate liquidity demand. The largest investors in the CP market are banks and money market mutual funds; hence the spread is reflective of the aggregate liquidity demand of

[20] Our results for the ATM bucket are robust to the explicit inclusion of the historical skewness and excess kurtosis of the interest rate distribution as additional controls in the simultaneous equation model.

these institutions. Therefore, this spread is likely to be positively correlated to the bid-ask spreads that these institutions charge for making markets in the instruments in which they are active, to the extent that macro institution-level liquidity may be correlated with micro contract-level liquidity. However, it is unlikely that this spread would affect the excess implied volatilities of these options, except through their effect on liquidity. Hence, it is a valid instrument for the relative bid-ask spreads. Later, we also examine the statistical validity of both these instruments in our econometric tests.

These macro-financial variables, taken together, incorporate most of the relevant information about fundamental economic indicators, such as expected inflation, GDP growth rate, and risk premia. The macro-financial variables along with the LIFFE futures volume also control for the volatility risk premium in this market. Since the fundamental economic variables are available at most monthly, we must rely on daily proxies for the expectations of these economic factors in the financial markets.[21]

This simultaneous equation model is estimated using three-stage least squares, since the residuals in each equation may be correlated with the endogenous variables, and these residuals may also be correlated across the two equations. We use instrumental variables to produce consistent estimates, and generalized least squares (GLS) to account for the correlation structure of the residuals across the two equations. In the first stage, we develop instrumented values for both the endogenous variables, using all exogenous variables in the system as instruments. In the second stage, based on a two-stage least squares estimation of each equation, we obtain a consistent estimate of the covariance matrix of the equation disturbances. Using this covariance matrix of residuals from the second stage, and the instrumented values of the endogenous variables from the first stage, we then perform a GLS estimation as the third stage of the three-stage least squares estimation.

[21] We considered other macro-financial variables as well, such as yields on speculative grade long-term debt, the short term repo rate as a proxy for money supply, and the volatility and stock returns in European equity markets. These variables were eliminated due to collinearity with the variables included in the model.

The results for this model are presented in Table 3. Our primary inference is regarding the sign of the coefficients c2 and d2. Both these coefficients are positive and statistically significant for *all* option maturities. This shows that for ATM options, within the endogenous framework specified above, controlling for potential exogenous drivers of price and liquidity in this market, higher values of EIV are associated with higher values of RelBAS, and vice-versa. In other words, more liquid options are priced *lower*, while less liquid options are priced *higher*, after taking into account the effects of the macro and control variables. This is an important result, and is quite different from the joint behavior of price and liquidity observed in other asset markets, such as those for stocks and bonds. For example, in the equity markets, it has been shown that more liquid stocks have lower returns (higher prices); what we observe here is the opposite; that is, that more liquid options have *lower* prices. Thus in this market, higher liquidity is actually associated with a discount, not a premium.

The primary explanation for this result is the fundamental difference between derivative assets and underlying assets alluded to in the introduction. Derivatives are in zero net supply; therefore, it is unclear whether the marginal investor in these assets would be long or short. In addition, the long and short positions in derivatives have asymmetric risk exposures (especially for options), and present different hedging needs to the counterparties on both sides. As argued by Brenner, Eldor and Hauser (2001), for an asset in zero net supply, both the buyer and the seller are concerned about illiquidity pushing the prices in the opposite directions. Depending on the risk exposure and the hedging needs of each side, either the "buyer-effect" (lower prices for illiquid assets) or the "seller-effect" (higher prices for illiquid assets) could dominate. In this market, we find that the "seller-effect" dominates and the more illiquid options have higher prices. From our discussions with the market participants it is clear that, in general, in this market, the dealers are net sellers of these options whereas the corporate entities are the net buyers. On any given day, a substantial proportion of the trades in the interest rate options market are sell-side trades where banks sell caps and floor to corporate clients.[22] The corporate end-users buy these options to

[22] An experienced market maker, who was the global head of the interest rate options desk at one of the largest banks in the world put the proportion of sell-side trades at between 80% and 90% on any given day. Unfortunately, there are no hard data available to substantiate his estimate.

hedge their other interest rate exposures and are usually not concerned about the liquidity of the options, since they typically hold these options to maturity. On the other hand, the dealers, who are net sellers, are concerned about the liquidity of the options because it captures the effects of imperfect hedging, limited risk appetite and capital constraints (Figlewski (1989) and Garleanu, Pedersen and Poteshman (2006)).[23] Limited risk appetite and capital constraints may be a result of the agency considerations between the dealers and their financiers. Imperfect hedging may be due to the trading frictions in the underlying asset markets, as well as the presence of risks in options that cannot be hedged using the underlying assets (such as unspanned stochastic volatility or jumps). There is basis risk between options of different strikes and maturities, which makes it impossible to exactly offset a short position in an illiquid option by buying a liquid option at a different strike and/or maturity. In such a scenario, when option market-makers sell illiquid options, it is difficult for them to find an offsetting trade and earn the bid-ask spread on the option right away (which is something traders generally try to do). Furthermore, the dealers are exposed to greater model risk, when there is less liquidity in the market, since they have fewer traded prices available to reliably calibrate their pricing models. Therefore, they are exposed to unhedgeable risks for which they demand compensation by way of higher option prices on more illiquid contracts. The resulting increase in the slope of the upward sloping option supply curve (Bollen and Whaley (2004) and Garleanu, Pedersen and Poteshman (2006)) increases the bid-ask spreads as well as the option prices, consistent with our findings. Thus, given the structure of this particular market, it is not surprising that the "seller-effect" dominates and that the illiquid options trade at higher prices.

We can examine the magnitude of the coefficient c2 to determine the economic significance of the responsive of EIV to relative bid-ask spreads. Depending upon maturity, the excess implied

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[23] Garleanu, Pedersen and Poteshman (2006) focus on the effects of changing inventory on the prices of derivatives due to the risk aversion of the dealer and imperfect hedging. However, their set-up is also useful for examining the relationship between the prices and bid-ask spreads *given the level of inventory*. Changing levels of inventory affect the prices through movement along a given supply curve, whereas the relationship between the bid-ask spreads and prices is a result of the changing slope of the supply curve. Our empirical analysis examines the latter relationship rather than the former. Our analysis is not meant to throw any light on the extent to which the changes in the levels of inventory affect prices, but not bid-ask spreads. However, we control for macro-economic variables that capture the changes in the demand for these options, and hence the changing levels of dealers' inventory.

volatilities increase by 25 to 70 basis points for every percentage point increase in the relative bid-ask spreads of these options. Alternatively, we can estimate the absolute price impact of a one standard deviation change in the relative bid-ask spread of an option. For example, the standard deviation of the relative bid-ask spreads of five-year caps and floors is about 2.3%. Therefore, a one standard deviation shock to the liquidity of the ATM five-year caps and floors would translate into a 115 basis point increase in their excess implied volatility, given a c2 coefficient of 0.50. For representative interest rates and ATM volatility, this is equivalent to an increase of about 5% in the absolute price (the price in Euros) of the five-year ATM cap/floor. In general, for other maturities, a one standard deviation shock to the liquidity of a cap/floor translates into an absolute price change of between 4% and 8%, which is an economically significant magnitude.

In addition, we see that the coefficients c2 and d2 are generally increasing in the maturity of these options. This indicates that the longer maturity options exhibit a stronger liquidity effect, perhaps to compensate the seller for the illiquidity over a longer time frame. This is consistent with the effect reported by Goldreich, Hanke and Nath (2005) in the U.S. Treasury securities market, where the average liquidity over the asset's remaining life is found to affect yields, since the expected trading costs to the marginal investor vary with the remaining life of the security. Similar results are reported by Nashikkar and Subrahmanyam (2006) for the U.S. corporate bond market; they estimate that a one standard deviation improvement in liquidity leads to a 8-11 basis points reduction in the non-default component of the yield spread. These results shed some light on the term structure dimension of liquidity effects in this market.

The coefficients of the exogenous variables in the two equations provide important information about the common determinants of price and liquidity in this market. Higher spot rates are generally associated with higher excess implied volatility, implying that when there are inflation concerns and expectations of rising interest rates, the dealers charge even higher prices (and wider bid-ask spreads) for selling these options. Note that these options are all nearly ATM, therefore they are *not* confounded by any smile effects that may be observed in this market. Once the effects of the spot rate are accounted for, the slope of the yield curve has a less significant effect on the EIV. The impact of increasing interest rate uncertainty is similar — when swaption volatilities are higher, the excess implied volatilities are also higher. When there is more uncertainty in fixed income markets, dealers appear to charge even higher prices (and wider bid-ask spreads) for these options. This increase in uncertainty worsens the basis risk and the model risk that dealers face, which in turn adversely affects liquidity, thereby increasing the slope of their supply curve, leading to higher prices and wider bid-ask spreads. Aggregate credit risk concerns, proxied by the default spread, do not appear to be significantly related to either price or liquidity in this market, except for longer maturity options. The futures volume on LIFFE is negatively related to the relative bid-ask spreads on caps and floors, indicating that the demand for hedging interest rate risk is one of the determinants of liquidity in the interest rate options markets. The CP/T-bill spread appears to be weakly related to the bid-ask spreads, since it is statistically significant at the 5% level only for some of the maturities. Aggregate liquidity shocks to financial institutions might play some role in affecting the liquidity of interest rate options, but their role is not very significant, perhaps because the interest rate options business is not a significant proportion of their overall operations.

We use the single equation version of the Hausman test to examine whether the variables assumed to be exogenous in the system are, in fact, uncorrelated with the structural disturbances. We examine the issue for each equation, for each of the exogenous variables. For each exogenous variable individually, we adopt the following procedure. First, we estimate the parameters of the equation using two-stage least squares, treating the variable in question as an exogenous variable. Then we compute an instrumental variables estimate of the same parameters, where the instrumented values of the endogenous variables are estimated using the remaining exogenous variables, excluding the exogenous variable being examined. A Wald test based on the difference of these two estimators examines the null hypothesis that the variable in question is indeed exogenous. In addition, we compute a system-wide statistic for model specification based on the specification test in Greene (2000). This is a likelihood ratio test based on the residuals with respect to the exogenous variables computed using three-stage least squares (a full information estimator) versus those computed using OLS. If the likelihood ratio statistic is below the chi-square critical value (with degrees of freedom equal to the number of over-identifying restrictions), the model specification is not rejected.

The diagnostic tests for the validity of our model show that our overall model specification cannot be rejected. The system-wide likelihood ratio statistic is 1.44, much below the critical value at the 5% level of 5.99. In addition, the Hausman tests for each exogenous variable result in chi-square statistics well below the critical value at the 5% level of 3.84. Therefore, we cannot reject the null hypothesis that the variables assumed to be exogenous are indeed exogenous within the system. These tests give us confidence that our instruments are valid within the overall specification of the simultaneous equation model.

The results so far use the mid prices of ATM options to estimate their EIV. Therefore, positive values of c2 and d2 imply that wider bid-ask spreads are associated with higher mid prices, controlling for other factors. However, in response to lower liquidity, do the dealers just increase their ask prices, keeping their bid prices the same (which would still result in higher mid prices)? Or is there any effect of illiquidity on the bid prices of these options as well? In order to understand this relationship between liquidity and option prices further, we re-estimate the simultaneous equation model separately using EIV computed from ask and bid prices. The results from this analysis are presented in Table 4; to conserve space, we only report the coefficients of c2 and d2. (The size and significance of the other coefficients as well as the R^2 of the regressions are similar to those in Table 3.). On the ask side, the coefficients are positive and significant for all option maturities, indicating that the ask prices are definitely higher in states of the world where the bid-ask spreads are wider. This is consistent with the hypothesis that the dealers charge higher prices for selling these options, when they are more illiquid. The results on the bid side are actually more interesting — the coefficients are positive for almost all maturities, but significant at the 5% level only for the longer maturity caps and floors. This shows that when there is less liquidity, the dealers are also willing to *pay* more for *buying* some of the caps/floors, especially the longer maturity options. This is consistent with the explanation that some part of the risk of options is unhedgeable, and hence the dealers are less willing to hold net short positions, especially in the longer maturity options. When there is less liquidity, the dealers are also *willing* *to pay* more to find a counterparty to reverse their sell-side trades than they are when there is more liquidity. Of course, since the dealers are net short in the aggregate, not all of them are *able* to reverse their sell-side trades.

For robustness, we re-estimate the simultaneous equation model for ATM options using the two alternative benchmark volatility measures – the historical standard deviation of log changes in forward rates, as well as the relevant implied volatility from the swaptions market. The excess implied volatilities are calculated in a similar manner, as the difference between the implied volatility of the cap/floor and the benchmark volatility. The results from these tests are presented in Table 5. In the interest of brevity, we only present the coefficients of interest, c2 and d2, since the other coefficients are of similar sign and significance as before. We again find that across all maturities, the coefficients of EIV and RelBAS are positive and statistically significant, indicating that these ATM options become more expensive when their liquidity reduces, and vice-versa.[24]

2.2.2. Liquidity Effects in Options Across Strikes

In this sub-section, we expand our analysis to options across all strikes. In order to properly control for smile effects, we introduce the skewness and excess kurtosis of the underlying interest rate distribution, interacted with LMR, in the simultaneous equation model as additional controls for the time-varying patterns of volatility smiles in these markets. Skewness and excess kurtosis are estimated on a rolling basis using the historical forward rates data from the most recent 66 days. Much of the volatility smile in options arises as a result of stochastic volatility, jumps in interest rates or both, which manifest themselves in the interest rate distribution as skewness and excess kurtosis. Therefore, controlling for the skewness and excess kurtosis at least partially controls for the daily variation in the volatility smile arising from stochastic volatility or jumps in interest rate. As before, we also have the asymmetric quadratic functional form of LMR as a control for the general shape of the volatility smile observed in these markets. Therefore, the model we estimate is as follows:

[24] We did several further robustness checks on our results, by re-estimating these models for bid and ask prices separately, as well as by using different historical time windows for calculating the standard deviation based reference volatility. These results were similar, and are not reported in the paper, but are available directly from the authors.

$$
\begin{aligned}
EIV = {} & c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) \\
& + c6 * SwpnVol + c7 * DefSprd + c8 * 6Mrate + c9 * Slope \\
& + c10 * LMR * Skew + c11 * | LMR | * Kurt \qquad \text{......(3)} \\
RelBAS = {} & d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) \\
& + d6 * SwpnVol + d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd
\end{aligned}
$$

The skewness is interacted with LMR, as the effect of skewness on the smile is likely to be asymmetric and dependent on the moneyness of the option. Excess kurtosis is interacted with absolute LMR, as the effect of kurtosis on the smile is likely to be symmetric and higher for away-from-the-money options. We estimate this model for bid, mid, and ask prices, for our primary measure of option expensiveness that uses the panel GARCH based volatility as the benchmark. The results for this analysis are presented in Table 6. Again, to conserve space, we again present only the coefficients of interest, c2 and d2, since the size and significance of the coefficients as well as the R^2 are similar to those in Table 3. We find that, across all maturities, more illiquid options are more expensive. They have significantly higher mid and ask prices, while the results for bid prices are somewhat weak for the shorter maturity caps/floors. It is important to note that the liquidity effects that we observe in this sub-section are *incremental* to the general volatility smile that is observed in this market, controlling for the daily changes in the skewness and excess kurtosis in the underlying interest rate distribution, thereby controlling for at least some of the effects of stochastic volatility and jumps in interest rates. In addition, since these effects are also present in the bid prices of these options, at least for the longer maturity caps/floors, it is unlikely that they arise only due to an increase in the ask prices.

As a further robustness check for the liquidity effects across strikes, we re-estimate the simultaneous equation model in equation (3) using the historical standard deviation and swaption implied volatilities as benchmark. These results, presented in Table 7, confirm our results for ATM options that more illiquid options are more expensive, controlling for other effects, regardless of the benchmark volatility used for estimating the excess implied volatilities.

2.3. The Relationship between Changes in Liquidity and Prices

To analyze the relationship between price and liquidity further, we re-estimate the simultaneous equation model using first differences for at-the-money options. If liquidity affects asset prices, then changes in liquidity should also change asset prices (Amihud, Mendelsen, and Wood (1990)). In Table 8, we present the results of the simultaneous equation model, where daily changes in EIV and RelBAS are regressed on each other as well as on changes in LMR functions and macro-financial variables, as follows:

$$\begin{aligned}\Delta EIV &= c1 + c2 * \Delta RelBAS + c3 * \Delta LMR + c4 * \Delta LMR^2 + c5 * \Delta\left(1_{LMR<0}.LMR\right) + \\ &\qquad c6\Delta SwpnVol + c7 * \Delta DefSprd + c8 * \Delta 6Mrate + c9 * \Delta Slope \\ \Delta RelBAS &= d1 + d2 * \Delta EIV + d3 * \Delta LMR + d4 * \Delta LMR^2 + d5 * \Delta\left(1_{LMR<0}.LMR\right) + \\ &\qquad d6 * \Delta SwpnVol + d7 * \Delta DefSprd + d8 * \Delta LiffeVol + d9 * \Delta CpTbSprd\end{aligned} \quad \text{......(4)}$$

This model explicitly tests for the relationship between daily changes in the price and the liquidity of options, as opposed to the relationship between the levels of these variables examined earlier. As before, we estimate this model for ATM options separately for each option maturity. The results in Table 8 are similar to the ones reported in Table 3, although these models have lower explanatory power, which is not surprising since they are estimated based on daily changes. The daily change in EIV is positively associated with the daily change in RelBAS, controlling for changes in option-specific and macro-financial variables. In addition, we find that positive shocks to the changes in uncertainty in the fixed income markets are associated with positive shocks to changes in both price and liquidity of these interest rate options, although these effects are weaker than those observed in the simultaneous equation model, in levels, estimated in the previous subsection. One of the reasons why these effects are weaker could be the nature of the relationship between shocks to liquidity/price and the shocks to these macro-financial variables: If the relationship between them is not contemporaneous, and one affects the other with a lag, we may not observe strong significance in the contemporaneous models estimated above. We deal with the issue of lagged responses in the next section. The overall model specification and Hausman tests again confirm that our instruments are valid. In addition, consistent with our findings in the previous section, we find stronger effects on the ask side than on the bid side, though the coefficients c2 and d2 are positive in both cases. We also obtain similar results when we repeat the tests for options across all strike rates, and when we use the other measures of EIV, calculated using the alternative benchmark volatilities.

The analysis above helps us understand the joint determinants of price and liquidity in this market. However, as the results indicate, there is a term structure element to the liquidity effects in this market; that is, the variation in liquidity is not the same for all maturities. In addition, there are strike rate effects that we have controlled for. Therefore, the natural question is to what extent this liquidity is driven by common factors across different strikes and maturities. In the next section, we explore these common drivers of liquidity in the interest rate options markets.

3. Are There Common Drivers of Liquidity in the Interest Rate Options Market?

We first examine the time-variation in liquidity for caps/floors. Figure 3 presents the time-series plots of the relative bid-ask spreads for each maturity by moneyness. The out-of-the-money bucket contains caps with LMRs less than -0.1 and floors with LMRs greater than 0.1. Similarly, the in-the-money bucket contains caps with LMRs greater than 0.1 and floors with LMRs less than -0.1. The at-the-money bucket contains caps and floors with LMRs between -0.1 and 0.1. For each day and each maturity, the relative bid-ask spreads within each bucket are averaged and then plotted over time. Each plot presents the time-series of the relative bid-ask spreads for the nine option maturities separately for the three moneyness groups. These plots clearly indicate that there is significant time-variation in relative bid-ask spreads, across maturity and moneyness. In addition, within each moneyness group, there appear to be both systematic and unsystematic components (across maturities) to the time variations in the relative bid-ask spreads. Indeed, the extent of commonality in the time-variations in bid-ask spreads in this market is one of the primary questions we investigate in this section.

Next, we examine the average correlations between relative bid-ask spreads across different moneyness groups. Within each moneyness group, we have nine maturities. For each maturity, we have a time-series of relative bid-ask spreads. We compute the correlation between the relative bid-ask spreads across different maturities, within each moneyness group. We average these correlations within the moneyness groups — these are reported as the diagonal elements in Table 9. For example, the OTM/OTM value of 0.68 is the average correlation between the relative bid-ask spreads for nine maturities for OTM options (so it is an average of (9 x 8)/2; that is, 36 correlations). This indicates that the average correlation between relative bid-ask spreads within OTM options, across all maturities, is 0.68. In addition to the average correlations within each moneyness group, we also estimate the average correlations across the moneyness groups. For example, the OTM/ITM value of 0.24 is the average correlation between relative bid-ask spreads for each maturity for the OTM options with those for each maturity for the ITM options (there are 9 x 9, or 81, correlations). This indicates that the average correlation between the OTM and the ITM option relative bid-ask spreads, across all maturities, is 0.24. These correlations indicate some interesting patterns. First, the relative bid-ask spreads appear to be fairly highly correlated across maturities *within* each moneyness group, although this correlation is a bit lower for OTM options. Second, the correlations *across* moneyness groups is considerably lower, especially between OTM options and either ATM or ITM options. It appears that there is significant movement in the relative bid-ask spreads, across maturities and strikes, but the OTM options seem to vary a bit differently from the ATM and the ITM options. The time-series plots of relative bid-ask spreads presented in Figure 3 indicate similar patterns.

These correlations and time-series plots indicate that some part of the variation in the relative bid-ask spreads appears to be systematic. From a market-wide perspective, it is important to understand if there is any systematic component to the liquidity shocks that has an impact on this market. This issue has strong implications for the pricing of liquidity risk in this market, as well as for hedging aggregate liquidity risk in interest rate options. If the liquidity shocks to this market are entirely unsystematic, then they do not create significant liquidity risk concerns, since they can be diversified away in a portfolio of options. However, if there is a systematic component to these liquidity shocks, then there may be liquidity risk concerns in this market, especially during periods of market stress.[25] The structure of such systematic liquidity shocks, and their macro-economic interpretation, can provide important inputs for designing strategies to hedge aggregate liquidity risk in this market.

3.1. Extracting the Common Liquidity Factor

We use a panel regression framework to examine whether the time-series variation in the liquidity of individual options has any systematic market-wide component, after controlling for the changes in option-specific parameters. We divide our options into 27 panels (nine maturities each for the three moneyness groups – OTM, ATM, and ITM), and estimate the following regression model on daily changes:[26]

$$\Delta RelBAS_{it} = c1 + c2 * \Delta EIV_{it} + c3 * \Delta LMR_{it} + c4 * \Delta LMR^2{}_{it} + c5 * \Delta\left(1_{LMR<0}.LMR\right)_{it} + \varepsilon_{it} \quad ..(5)$$

We include fixed effects for each panel to account for any panel-specific effects that may not be captured by the specification above. The intuition behind this regression is to examine the changes in liquidity, and remove the part of those changes that can be explained by changes in option-specific variables, such as option expensiveness (EIV) and functions of LMR. Although the panels are formed based on the three categories for moneyness, we still include the functions of LMR as controls, because even within a moneyness bucket, the LMR of an option can change every day. Hence, we must account for the part of the change in RelBAS that is due to changes in the LMR. In addition, including the change in EIV enables us to take into account the change in any other option-specific information as well as the option-specific impact of changes in any market variable.

We estimate this panel regression model using the Prais-Winsten Full FGLS estimator. The disturbances in this model are assumed to be heteroskedastic and potentially correlated across the 27 panels. In addition, we allow for first-order autocorrelation in the disturbances, within each

[25] This issue has been explored in the context of the equity markets by Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), and with regard to the bond markets by Longstaff (2004) and Longstaff et al. (2005).

[26] In the equity option markets, Jameson and Wilhelm (1992) show that the bid-ask spreads are related to option "Greeks." As explained earlier, our asymmetric quadratic function of LMR acts as an approximate control for these Greeks, in addition to controlling for the shape of the volatility smile.

panel, with the coefficient of the AR(1) process allowed to be different for each panel. Therefore, the standard errors are robust for the error structure specified in the model, and the parameter estimates are conditional on the estimates of the disturbance covariance matrix and on the autocorrelation parameters estimated for each panel. For robustness, we estimate this panel regression using alternative error structures and estimation procedures (including maximum likelihood), and find similar results.

The results for this panel regression are presented in Table 10. Consistent with our results in the previous section, positive changes in excess implied volatilities are associated with positive changes in the relative bid-ask spreads, suggesting that improvements in liquidity are associated with a *decrease* in option expensiveness in this market, controlling for strike rate effects. Since this regression is estimated as a panel over our entire data set, we have a very large number of observations. Across all of these 44,070 observations, we are trying to explain the changes in liquidity in terms of changes in option expensiveness and changes in LMR controls, using only five parameters. The model is statistically significant and explains about 9% of the variation in liquidity changes. Therefore, even though some part of the liquidity changes are statistically significantly associated with changes in option-specific parameters, it appears that a large part of the change in liquidity may have systematic components.

If there are no systematic components in the changes in liquidity in this market, then we should not see any structure in the residuals obtained from this regression. Any structure in these residuals would indicate a missing common systematic factor that affects liquidity changes. Therefore, we examine the principal components of the correlation matrix of these residuals across the panels. We use the correlation matrix, since principal components are sensitive to the units in which the underlying variables are measured. Using the correlation matrix instead of the covariance matrix avoids this potential error.

Since we have 27 panels, we obtain a 27 x 27 correlation matrix, which provides us with 27 principal components, each one of length 27. If the residuals were perfectly correlated, the first eigenvalue would be 27, and a single factor would explain all the variation. If the residuals were uncorrelated, all 27 eigenvalues would be 1. The results of this principal components analysis are presented in Table 11. The first eigenvalue is 9.04, which implies that about 33% (9.04/27) of the variation in these residuals can be explained by the first common factor. This is statistically significant, and indicates that about one-third of the variation in residual changes in liquidity (not explained by the changes in option-specific variables) is accounted for by the first common factor. This strongly suggests that there is a market-wide systematic component to the liquidity shocks that affect these options. The second principal component explains an additional 11% of the variation. The third and subsequent principal components are statistically insignificant.

The structure of these principal components (eigenvectors) reveals the impact of the market wide liquidity shock on options of different maturity and moneyness. The first principal component is a parallel shock across all maturities and moneyness, indicating that a market wide liquidity shock increases the bid-ask spreads of options of all maturity and moneyness. However, the loadings are higher in magnitude for the OTM and the ATM options as compared to the ITM options. This indicates that the most common liquidity shock, that occurs about one-third of the times, widens the bid-ask spreads of primarily the OTM and the ATM options, and has less effect on ITM options.

The second principal component has a negative weight on all the OTM options, and a positive weight on the ATM and the ITM options. Further, the weight on the ITM options is greater than that on the ATM options. Within each moneyness bucket, the weights across maturities are relatively constant. Therefore, the second-order market-wide liquidity shock, which occurs about one-tenth of the time, is opposite in structure to the primary liquidity shock. In particular, the effect of this second liquidity shock is to widen the bid-ask spreads for the ATM and the ITM options, while at the same time narrowing the bid-ask spreads for the OTM options. This may indicate a substitution effect, where the market, when hit by an adverse second-order common liquidity shock, appears to drive liquidity away from the ATM and the ITM options to the OTM options. Since the OTM options are much cheaper than the ATM and ITM options, this finding is intuitive — adverse common liquidity shocks do not just cause the liquidity of these options to dry up across the board. In addition, they partly shift the liquidity from expensive to cheaper options. The loading on the ITM options is even higher than that on the ATM options, which supports this explanation, since it implies that the reduction in liquidity is greater among the ITM options than among the ATM options. The third and subsequent eigenvectors have no significant structure.

3.2. Macro-economic Drivers of the Systematic Liquidity Shocks

Our results, so far, indicate the presence of a significant common factor that drives liquidity changes in interest rate options across strikes and maturities. In this section, we explore the fundamental drivers of this systematic liquidity factor. If changes in macro-economic variables can be linked with contemporaneous or future changes in the systematic liquidity factor, this would have important implications for the measurement of liquidity risk in this market, as well as for hedging aggregate liquidity risk in a portfolio of interest rate options.

Our results in the previous section indicate that changes in the uncertainty in fixed income markets are associated with changes in liquidity for options, across strike rates, for all maturities. In this section, we use this variable along with other macro-financial variables that capture yield curve, default risk and equity market uncertainty to examine how much of the systematic liquidity factor they can explain.

We first construct a daily systematic liquidity factor based on the analysis of the residuals in the previous sub-section. We use only the first principal component, which explains 33% of the residual variation in the relative bid-ask spreads. Using the first eigenvector as weights, for each day, we estimate a weighted-average residual across the 27 maturity-moneyness buckets. This gives us a daily time-series of the unexplained first common factor of liquidity changes in this market. We regress this factor on contemporaneous and lagged daily changes in the five macro-financial variables – the short rate, the slope of the term structure, our measure of the swaption volatility, the default spread, and our index of Euro zone equity market volatility.

As discussed earlier, the default spread (*DefSprd*) captures the credit risk concerns in the economy. It may also proxy for overall uncertainty and model risk. We include the contemporaneous and lagged default spreads to examine whether they have any impact on the systematic liquidity shock experienced in this market. In the simultaneous equation models in section 2.2, the short rate (*6Mrate*) and the slope (*Slope*) of the term structure are found to be valid instruments for EIV. However, in that model, we use the contemporaneous levels of these variables as instruments. In this section, our objective is to understand whether changes in fundamental macroeconomic variables drive the systematic liquidity shocks. Therefore, we examine whether *lagged* changes in the short rate and the slope of the term structure have any impact on the liquidity factor. For completeness, we also test whether contemporaneous changes in these two variables are significant. The short rate and the slope of the term structure are financial variables that proxy for expected inflation and money supply. Therefore, it is important to test whether the liquidity shocks, especially in the interest rate options markets, are fundamentally driven by lagged changes in the expectations about inflation and money supply.

The volatility of the DAX index (*DAXVol*) is an indicator of the level of uncertainty in the Euro zone equity markets, which could also have an impact on market expectations of future liquidity. Since stock prices reflect expectations about future cash flows and discount rates, the average volatility in the equity market is also an index of the level of uncertainty in the expectations about future cash flows and discount rates. Therefore, we include this variable as a proxy for general market uncertainty. In a similar spirit, the swaption volatility (*SwpnVol*) is included as a proxy for uncertainty about future inflation and money supply. We use the Akaike information criterion to determine the appropriate number of lags to include in the regression. The results of this regression model are presented in Table 12.

We find that the improvement in the explanatory power of the model is insignificant beyond the fourth lag in the macro-financial variables. These macro-financial variables together explain about 28% of the unexplained first common liquidity factor in this market. The short rate and the slope of the term structure do not appear to have much effect on contemporaneous and future values of this factor. This implies that current as well as lagged expectations about inflation, money supply, or general business conditions do not appear to have a significant effect on the liquidity of the options in this market. This is also consistent with the use of the contemporaneous values of these variables as instruments only for excess implied volatilities and not relative bid-ask spreads, in the simultaneous equation models in section 2.

Similarly, an increase in aggregate credit risk concerns in the economy, proxied by the default spread variable, does not appear to be related to the systematic liquidity shock that affects the fixed income options markets. In the case of option contracts, only the buyer of the option is exposed to the credit risk of the seller of the option. Since the interest rate options market studied here is an OTC market, the buyers of these caps and floors are exposed to the risk of the dealers defaulting. However, there are two primary reasons why these credit risk effects do not appear to affect the liquidity in this market. First, most of the dealers in this market are investment grade institutions – in fact many of them are high investment grade firms, with very little credit risk.[27] Therefore, cap and floor buyers generally are not unduly concerned about the dealers defaulting on these contracts. Second, and more importantly, this is a dealer-driven market where most of the trades are in the form of dealers selling caps and floors to corporate clients, with the prices being set by the dealers, who have more market power than the typical buyer of these contracts. Therefore, it is not surprising to find that the dealers do not care as much about aggregate credit risk in the economy, since they are mostly on the sell side! It would be interesting to examine the impact of credit risk concerns on the liquidity of options where the buy side is as influential as the sell side in setting prices and bid-ask spreads, as would happen for exchange-traded options.

The uncertainty proxies, both in the fixed income and equity markets, appear to be significant drivers of this systematic liquidity shock in fixed income options markets. Lagged changes in the DAX index volatility up to three days earlier, and lagged changes in the swaption volatility up to four days earlier, are significant in explaining the time variation in the systematic liquidity factor. However, for both of these variables, the coefficient on contemporaneous changes is either insignificant or weakly significant. The coefficients on lagged changes, with a lag of between one and four days, are mostly significant. These results indicate that the traders in fixed income options markets appear to use the levels of uncertainty in both the fixed income and equity

[27] The institution that supplied our data, WestLB, was an AAA-rated institution during the period of this study, since it was *de facto* guaranteed by the German Treasury.

markets to form their own expectations about future liquidity and the liquidity premia. When they observe a positive shock to the uncertainty in these markets, they appear to respond by increasing the prices of caps and floors, while simultaneously widening the bid-ask spreads that they quote. Increased uncertainty makes it more expensive for the dealers to make markets for these options, due to an increase in unhedgeable risks as well as greater model risk. Both these factors may cause the dealers to increase prices as well as bid-ask spreads. From a timing standpoint, it appears that these liquidity effects in the fixed income options markets appear between one and four days *after* the volatility shocks are observed in the fixed income as well as equity markets.

It is important to note that the results here represent the sensitivity of the systematic component of the changes in the bid-ask spreads of these options to the changes in the macroeconomic environment, after controlling for the changes in option-specific parameters, which includes the changes in the expensiveness of the option. They do not represent the relationship between liquidity and *aggregate* price changes in this market. In addition, the systematic liquidity shock that has been extracted from the data is only one of the determinants of the observed bid-ask spreads, and accounts for only about one-third of their common variation. The rest of the variation is partly due to changes in option-specific parameters, especially moneyness. Therefore, these effects constitute only a part of the changes in the bid-ask spread observed in the data.

As a robustness check, we return to the panel regression model of equation (5), and re-estimate the model, after including the contemporaneous value and four lags for each of the five macro-financial variables. The intuition behind this exercise is to check whether the macro-financial variables, identified as being related to the unexplained systematic variation in relative bid-ask spreads, do indeed help in explaining the variation in the relative bid-ask spreads of these options across strike rates and maturities. We find that this augmented panel regression model explains about 16.9% of the variation in the relative bid-ask spreads, up from about 9% that was explained only by the changes in option-specific variables. Therefore, introducing contemporaneous and lagged changes in these macro-financial variables nearly doubles the explanatory power of the panel regression model, which attempts to jointly model the time variation in the relative bid-ask spreads across different strike rates and maturities, using over 44,000 observations. Further, the first principal component of the correlation matrix of residuals from this augmented panel regression model accounts for about 11.3% of the variation in the residuals. While statistically significant, it is much lower than the 33% explained by the first principal component of residuals from the panel regression model, without the macro-financial variables. Therefore, adding the macro-financial variables explains a large part of the common factor that drives liquidity in this market. In fact, the macro-financial variables, especially the volatilities in the equity and fixed income markets, remove most of the structure in the part of the variation in liquidity in caps and floor unexplained by changes in option-specific variables. The remaining unexplained variation in the liquidity of these options is largely unsystematic.

Our results in this section provide important insights into the fundamental drivers of liquidity in this market. It appears that uncertainty about macroeconomic conditions plays a much more important role than the direction of the expectations about what may happen in the future, in determining the liquidity of caps and floors in the euro interest rate markets. Our results complement the findings of Chordia, Sarkar and Subrahmanyam (2005), who report that volatility shocks are informative in predicting liquidity shifts in the stock and bond markets. However, unlike their paper, we do not find any link between proxies for macro liquidity and transaction level liquidity. This difference is likely due to the fact that our paper focuses on options that are zero net supply assets, unlike stocks and bonds, which are in positive net supply, where macro liquidity may play a greater role.

In addition, our results indicate a certain level of predictability in the systematic liquidity shock that affects the fixed income options markets. This has important implications for forecasting the liquidity risk in this market, as well as for the hedging of aggregate liquidity risk in portfolios of caps and floors. Since the liquidity factor in this market is related to lagged changes in volatilities, from a risk measurement perspective, a GARCH-type model could be used for forecasting the volatility in the equity and fixed income markets, and the systematic liquidity shock estimated based on these volatility shocks. From a risk management perspective, institutions holding portfolios of caps and floors could construct macro-hedges against the liquidity risk in these options by taking appropriate positions in the volatilities in equity and fixed income markets. For both of these objectives, it is crucial to understand the extent of commonality in liquidity in this market, and the primitive structure of this systematic liquidity factor.

4. Concluding Remarks

The liquidity of an asset has an important influence on its market price. In recent years, this influence has been analyzed extensively in the U.S. equity markets, and, to a lesser extent, in the U.S. Treasury, corporate bond, and some foreign exchange options markets. Two important facts have emerged from these investigations: illiquidity suppresses the price of an asset, resulting in higher expected return; and there is a common factor in liquidity across various assets.

In contrast to this work on the underlying stock and bond markets, there is very little work on the influence of liquidity in the derivatives markets, particularly the interest rate derivatives markets. This gap is striking for three reasons. First, derivatives markets are an important segment of the global financial markets, and thus need to be taken into account in assessing the overall liquidity in financial markets. Second, the effect of liquidity on the prices of derivatives is, by no means, clear cut. With zero net supply, both the buyers and sellers of derivatives are exposed to its illiquidity. In addition, in the case of derivatives, it is not obvious whether the marginal investor would be long or short. It would depend on the risk exposures and the hedging needs of either side. Thus, the prices of illiquid derivatives could be higher or lower, as compared to the prices of derivatives that are more liquid. Third, the interest rate derivatives market is an OTC market, with a structure quite different from exchanges, and with contracts that are generally more illiquid compared to many exchange traded contracts. Therefore, the inferences drawn from studies on the liquidity effects in exchange-traded contracts may not be readily extendable to OTC contracts.

The liquidity and the price of an asset are fundamentally endogenous variables. Therefore, we examine the liquidity effects in the euro interest rate options markets within a simultaneous equation model that endogenizes both liquidity and price, thereby modeling liquidity both as a cause and an effect. Our results show that more illiquid interest rate options are more *expensive*, controlling for other determinants of liquidity and price. Thus, this result is in sharp contrast to earlier findings in the stock and bond markets and in some exchange-traded currency options markets. As our results indicate, the relationship between illiquidity and asset prices cannot be generalized based on evidence from just the stock and the bond markets.

Our second result, on the commonality in liquidity across options, is similar to what has been found in other markets. We find that there is a significant common component to the liquidity of interest rate options at various strike prices and maturities. We also find that this common movement is explained by the shocks to the volatility in the equity and interest rate markets. An increase in uncertainty in the equity and interest rate markets appears to cause a negative liquidity shock in the interest rate options market. In terms of more primitive macro-economic factors, it is not the expectations about inflation or growth that seem to affect the liquidity in interest rate options; it is the uncertainty about these expectations that affects the liquidity in this market.

Our results have important implications for the role of liquidity in the pricing of derivative instruments. It would be worthwhile to explore this effect in other derivatives markets and for derivative instruments other than options, to see if this influence is similar, especially in different market settings. It would also be interesting to focus on crisis periods, such as the aftermath of the Russian default in 1998 and the LTCM failure that followed thereafter, to examine the issue of liquidity in such an extreme scenario. A related question that has not been explored in the literature so far is the interplay between the liquidity effects in the underlying asset market versus the market for derivatives. The key question is whether and how the commonality in the liquidity factor affects the interactions and the lead-lag relationships between these two markets.

Another important direction for future research based on our results is the development of models where one could include these drivers of liquidity in the pricing kernel itself. Furthermore, since interest rate options are much harder to price, with large pricing and hedging errors in general, liquidity-adjusted models could provide better pricing and hedging. Given the enormous size of this market, there are systemic effects of mispricing and/or inaccurate hedging - understanding such liquidity effects can help reduce such systemic risks. We leave these questions for future research.

Appendix: Implied Volatility in the Black Model for Caps and Floors

The standard model used for dealer quotations for interest rate caps and floors is the Black (1976) model of pricing of options on futures and forward contracts. The model is a variant of the basic Black and Scholes (1973) option pricing model. Applied to the interest rate option context, the model assumes that interest rates are log-normally distributed and relates the price of a European call option (*C*) and a put option (*P*), at time *0*, on an interest rate forward rate agreement (FRA), to the underlying variables as follows:[28]

$$
\begin{aligned}
C &= \left[fN(d_1) - kN(d_2) \right] \times m \times B_{0,t+m} \\
P &= \left[-kN(-d_2) - fN(-d_1) \right] \times m \times B_{0,t+m} \\
d_1 &= \frac{\ln {f}/{k} + {t\sigma^2}/{2}}{\sqrt{t}\sigma} \\
d_2 &= d_1 - \sqrt{t}\sigma
\end{aligned}
\tag{A.1}
$$

where

$f =$ forward interest rate for the period t to $t+m$,

$\sigma =$ annualized volatility of the forward interest rate t on the maturity date,

$m =$ maturity period of the underlying loan,

$t =$ maturity date of the option,

$k =$ strike rate of the option, and

$B_{0,t+m} =$ the zero bond price at time *0,* for the bond maturing at date $t+m$.

Of course, the key variable in the above equations, which is not observable, but about which market participants may have differing views, is the volatility. Given all other parameters, a price of an option can be inverted to obtain an implied volatility. Thus implied volatility is a manifestation of price of the option.

[28] This formula is also consistent with the model proposed by Brace, Gatarek and Musiela (1997) [BGM] and Miltersen, Sandmann and Sondermann (1997), which is popular among practitioners. BGM derive the processes followed by market quoted rates within the HJM framework, and deduce the restrictions necessary to ensure that the distribution of market quoted rates of a *given* tenor under the risk-neutral forward measure is log-normal. With these restrictions, caplets of that tenor satisfy the Black (1976) formula for options on futures and forward contracts.

An interest rate cap (floor) is a collection of caplets (floorlets). A caplet (floorlet), in turn, is a single European call (put) option on a reference interest rate, expiring on a specific date. Hence, a cap (floor) can be regarded as a portfolio of European call (put) options on interest rates, or equivalently, put (call) options on discount bonds. Typically, an interest rate cap is an agreement between a cap writer and a buyer (for example, a borrower) to limit the latter's floating interest payments to a specific level for a given period of time. The cap is structured on a specific reference rate (usually the three- or the six-month Libor (London Interbank Offer Rate) or Euribor (Euro Interbank Offer Rate)) at a predetermined strike level. The reference rate is reset at periodic intervals (usually three or six months). In a similar manner, an interest rate floor contract sets a *minimum* interest rate level for a floating rate *lender*. The cap and floor contracts are defined on a pre-specified principal amount.[29]

A caplet with maturity t_i and strike rate k pays at date t_i, an amount based on the difference between the rate (r_i) at time t_i and the strike rate, if this difference is positive, and zero otherwise. The amount paid is based on the notional amount and the reset period of the caplet and is paid on a discounted basis at time t_i. The payoff of this caplet at date t_i on a notional principal of €A is:

$$c_{t_i} = A(t_{i+1} - t_i)\max\left[\frac{r_i - k}{1 + r_i(t_{i+1} - t_i)}, 0\right] \tag{A.2}$$

The payoff from a floorlet can be described in a similar manner.

Since the interest rate over the first period is known, there is no caplet corresponding to the first period of the cap. For example, a two-year cap on the six-month Euribor rate, with four semiannual periods over its life, would consist of three caplets, the first one expiring in six months, and the last one in one year and six months. Thus, the underlying interest rate for the first period is the six-month Euribor rate on the date six months from initiating the cap contract.

Each caplet or floorlet has to be valued separately, using a valuation model such as the Black or BGM model in equation (A.1) (the same model that is generally used by the market for quotation

[29] Interest rate caps and floors for various maturities and reference rates in all the major currencies are traded in the over-the-counter (OTC) markets. The most common reference rate is the three-month Libor for USD caps/floors, and the six-month Euribor in the euro markets.

purposes), with the price of the cap or floor being the sum of these prices. The volatilities used for each caplet or floorlet, which are generally different, across strike rates and maturities, are sometimes called *spot volatilities*. The market quotation for interest rate caps and floors, however, is based on the *same* volatility for all the caplets in a particular cap (or the floorlets in a particular floor). In other words, the market price of a cap (or floor) can be derived by plugging in this constant volatility for all the component caplets (or floorlets) in the contract. This constant volatility is referred to as the *flat volatility* for the particular cap (or floor) and varies with the maturity of the contract. Since caps are portfolios of caplets, the implied flat volatilities of caps reflect some average of the implied spot volatilities of individual caplets. In this paper, our primary objective is to examine liquidity effects in interest rate options. For doing that, we need to focus on traded assets, which are caps and floors. Therefore, we use the flat volatilities of caps and floors, since spot volatilities would correspond to caplets and floorlets, which are untraded assets.

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Table 1

Descriptive Statistics for Cap and Floor Prices

This table presents descriptive statistics on euro (€) interest rate cap and floor prices, across maturities and strike rates, over the sample period from January 1999 to May 2001. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All prices are averages, reported in basis points, with the standard deviations of these prices in parenthesis.

Maturity	Caps					Floors				
	Deep OTM	OTM	ATM	ITM	Deep ITM	Deep ITM	ITM	ATM	OTM	Deep OTM
	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3
2-year	2.1	11.1	43.2	107.7	250.5	250.5	153.7	55.5	13.6	3.6
	(0.5)	(5.8)	(19.8)	(30.9)	(58.8)	(48.1)	(50.7)	(25.4)	(7.9)	(2.0)
3-year	10.7	37.7	91.9	209.6	481.3	529.1	285.3	111.3	32.7	6.9
	(10.0)	(20.0)	(33.8)	(52.3)	(133.4)	(114.2)	(74.7)	(44.6)	(18.0)	(4.6)
4-year	22.3	72.6	152.7	311.3	674.4	728.3	406.4	176.1	62.1	12.0
	(12.5)	(32.2)	(49.7)	(78.3)	(193.1)	(138.7)	(98.9)	(64.8)	(27.8)	(7.9)
5-year	42.7	119.4	221.7	409.1	872.3	910.8	519.5	244.7	94.3	19.2
	(16.3)	(48.6)	(67.2)	(95.4)	(252.2)	(161.2)	(122.5)	(84.5)	(35.2)	(13.9)
6-year	66.9	163.7	286.6	507.9	1,006.6	1,093.1	663.8	323.7	128.6	27.2
	(20.2)	(64.4)	(84.6)	(109.5)	(257.4)	(173.2)	(133.1)	(101.9)	(43.5)	(18.7)
7-year	93.7	210.9	355.8	610.8	1206.4	1,239.0	809.3	393.3	164.1	36.9
	(25.4)	(82.2)	(99.3)	(125.3)	(275.5)	(147.0)	(127.5)	(115.2)	(51.9)	(33.0)
8-year	123.9	264.2	433.2	706.8	1,248.2	1,284.7	924.7	425.2	199.2	46.8
	(31.4)	(98.1)	(115.9)	(162.8)	(253.4)	(120.8)	(139.3)	(108.3)	(59.6)	(32.8)
9-year	152.1	309.6	509.9	811.8	1,310.3	NA	997.1	482.3	235.0	58.9
	(35.6)	(103.2)	(128.7)	(172.2)	(205.3)		(150.2)	(120.9)	(69.6)	(41.5)
10-year	179.6	347.8	598.0	881.3	1,493.4	NA	815.5	541.7	242.9	71.3
	(39.8)	(106.7)	(140.0)	(153.4)	(275.3)		(31.1)	(139.6)	(61.9)	(50.1)

Table 2

Relative Bid-Ask Spreads for Caps and Floors

This table presents summary statistics on the bid-ask spreads for euro (€) interest rate caps and floors, scaled by the average of the bid and ask prices for the options, across strike rates, for different maturities, expressed as percentages. The statistics are presented for the sample period from January 1999 to May 2001. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All the spreads are averages, with the standard deviations of the relative spreads in parentheses.

Maturity	Caps					Floors				
	Deep OTM	OTM	ATM	ITM	Deep ITM	Deep ITM	ITM	ATM	OTM	Deep OTM
	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3
2-year	80.9%	32.4%	14.7%	7.1%	3.8%	2.5%	4.5%	13.3%	30.8%	77.2%
	(21.2%)	(14.3%)	(4.8%)	(2.4%)	(0.5%)	(1.3%)	(1.3%)	(7.9%)	(11.7%)	(24.1%)
3-year	44.2%	19.0%	11.4%	7.0%	3.8%	2.9%	4.7%	11.2%	31.6%	72.0%
	(22.9%)	(5.7%)	(3.2%)	(2.5%)	(0.6%)	(1.1%)	(1.1%)	(6.1%)	(18.1%)	(25.2%)
4-year	26.1%	14.4%	9.1%	6.2%	4.1%	2.9%	4.5%	8.4%	22.2%	59.9%
	(9.4%)	(4.7%)	(2.5%)	(2.2%)	(1.0%)	(1.0%)	(1.0%)	(2.5%)	(14.5%)	(28.7%)
5-year	20.0%	12.6%	8.6%	6.1%	4.1%	3.1%	4.7%	8.2%	19.8%	59.5%
	(5.5%)	(3.9%)	(2.3%)	(2.1%)	(0.9%)	(1.0%)	(1.1%)	(2.3%)	(13.2%)	(27.4%)
6-year	18.3%	12.1%	8.5%	5.7%	4.1%	3.3%	4.7%	7.9%	15.8%	50.2%
	(4.8%)	(3.6%)	(2.2%)	(1.4%)	(0.9%)	(0.9%)	(1.2%)	(2.0%)	(7.5%)	(24.6%)
7-year	17.6%	11.5%	8.4%	5.5%	4.1%	3.4%	4.6%	7.8%	14.0%	45.3%
	(4.4%)	(3.4%)	(2.1%)	(1.3%)	(3.9%)	(0.9%)	(1.1%)	(1.9%)	(5.0%)	(24.6%)
8-year	17.1%	11.1%	8.3%	5.6%	4.0%	3.2%	4.5%	8.1%	14.0%	42.3%
	(3.8%)	(3.3%)	(2.0%)	(1.1%)	(0.3%)	(1.0%)	(1.1%)	(2.0%)	(5.1%)	(21.9%)
9-year	17.1%	11.0%	8.3%	6.0%	4.2%	NA	4.8%	8.3%	14.0%	40.0%
	(3.4%)	(3.1%)	(1.9%)	(0.7%)	(0.3%)		(1.0%)	(2.0%)	(5.2%)	(20.8%)
10-year	17.1%	11.2%	7.9%	6.2%	4.1%	NA	4.7%	8.1%	14.9%	38.6%
	(2.9%)	(3.0%)	(1.8%)	(0.6%)	(0.3%)		(1.2%)	(2.2%)	(5.5%)	(20.6%)

Table 3

Determinants of Excess Implied Volatility and Bid-Ask Spreads in ATM Caps and Floors

This table presents the results for a simultaneous equation model, for near-the-money options with LMRs between -0.1 and 0.1, where the excess implied volatility of euro (€) interest rate caps/floors and relative bid-ask spreads are determined endogenously as a function of each other and of other exogenous variables, for the sample period from April 1999 to May 2001.

$$EIV = c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + c6SwpnVol + c7 * DefSprd + c8 * 6Mrate + c9 * Slope$$

$$RelBAS = d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) + d6 * SwpnVol + d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd$$

EIV is the excess implied volatility of the mid-price of the cap/floor relative to the benchmark volatility estimated using a panel GARCH model on historical interest rates. RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. 6Mrate is the six-month Euribor rate. Slope is the difference between the five-year and six-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the six-month Euribor and the six-month Treasury rate. LiffeVol is the logarithm of the trading volume of three-month Euribor futures on the LIFFE. CpTbSprd is the spread between the three-month AA Financial Commercial Paper rate and the three-month T-bill rate. Only the coefficients of interest are presented in this table.

Panel A: EIV as the dependent variable

Maturity	c2	c6	c7	c8	c9	Obs	R^2
2-year	0.24**	0.43**	0.00	1.45*	1.96	1100	0.19
3-year	0.28**	0.58**	0.00	0.54*	1.34**	1392	0.22
4-year	0.52**	1.28**	0.04	0.87**	0.48*	1448	0.24
5-year	0.50**	0.52**	0.08	0.35**	0.01	1430	0.21
6-year	0.69**	0.73**	0.12	0.75**	0.68*	1468	0.38
7-year	0.71**	0.93*	0.22*	0.90**	-1.12	1386	0.26
8-year	0.78**	0.91**	0.18*	0.86**	0.11	1237	0.22
9-year	0.48**	0.81*	0.25*	0.84*	0.07	1202	0.31
10-year	0.71**	0.43**	0.56*	0.10*	0.14	887	0.31

Panel B: RelBAS as the dependent variable

Maturity	d2	d6	d7	d8	d9	Obs	R^2
2-year	0.50**	0.46*	0.00	-0.03*	0.25	1100	0.26
3-year	1.43**	0.89**	0.00	-0.02**	0.19**	1392	0.17
4-year	1.24**	0.60**	0.03	-0.10**	0.31*	1448	0.22
5-year	1.19**	0.63**	0.06*	-0.20**	0.44**	1430	0.32
6-year	1.32**	0.68**	0.13**	-0.42**	0.71**	1468	0.32
7-year	1.38**	0.62**	0.04*	-0.27**	0.64*	1386	0.32
8-year	1.29**	0.65**	0.34**	-0.60**	0.76*	1237	0.32
9-year	1.41**	0.45**	0.37**	-0.86**	0.66**	1202	0.42
10-year	1.46**	0.32**	0.64**	-0.90**	0.51**	887	0.42

** implies significance at the 5% level; * implies significance at the 10% level.

Table 4

Bid and Ask Side Determinants of Liquidity and Price (ATM)

This table presents the results for a simultaneous equation model, for near-the-money options with LMRs between -0.1 and 0.1, estimated separately using bid and ask prices, where the excess implied volatility of euro (€) interest rate caps and floors and the relative bid-ask spreads are determined endogenously as a function of each other, and other exogenous variables, for the sample period from April 1999 to May 2001:

$$EIV = c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) +$$
$$c6SwpnVol + c7 * DefSprd + c8 * 6Mrate + c9 * Slope$$
$$RelBAS = d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) +$$
$$d6 * SwpnVol + d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd$$

EIV is the excess implied volatility of the bid or the ask price of the cap/floor relative to the benchmark volatility estimated using a panel GARCH model on historical interest rates. RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. 6Mrate is the six-month Euribor rate. Slope is the difference between the five-year and six-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the six-month Euribor and the six-month Treasury rate. LiffeVol is the logarithm of the trading volume of three-month Euribor futures on the LIFFE. CpTbSprd is the spread between the three-month AA Financial Commercial Paper rate and the three-month T-bill rate. Only the coefficients of interest are presented in this table.

Maturity	Bid Side		Ask Side	
	c2	d2	c2	d2
2-year	0.08*	0.18*	0.28**	0.74**
3-year	0.07*	0.21*	0.31**	1.66**
4-year	0.21*	0.17*	0.92**	1.39**
5-year	0.34*	0.54**	0.71**	1.58**
6-year	0.41**	0.43*	0.75**	1.89**
7-year	0.31*	0.71*	0.82**	2.01**
8-year	0.33**	1.22**	0.91**	1.54**
9-year	0.22**	0.67**	0.65**	1.79**
10-year	0.44**	0.91**	0.82**	1.72**

** implies significance at the 5% level; * implies significance at the 10% level.

Table 5

Determinants of Liquidity and Price Using Alternative Volatility Benchmarks (ATM)

This table presents the results for a simultaneous equation model, for near-the-money options with LMRs between -0.1 and 0.1, where the excess implied volatility of euro (€) interest rate caps and floors and the relative bid-ask spreads are determined endogenously as a function of each other and other exogenous variables, for the sample period from April 1999 to May 2001:

$$EIV = c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + c6SwpnVol + c7 * DefSprd + c8 * 6Mrate + c9 * Slope$$

$$RelBAS = d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) + d6 * SwpnVol + d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd$$

EIV is the implied volatility of the mid price of the cap/floor relative to the alternative benchmark volatilities (historical standard deviation of changes in log rates and swaption volatilities of comparable maturity). RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. 6Mrate is the six-month Euribor rate. Slope is the difference between the five-year and six-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the six-month Euribor and the six-month Treasury rate. LiffeVol is the logarithm of the trading volume of three-month Euribor futures on the LIFFE. CpTbSprd is the spread between the three-month AA Financial Commercial Paper rate and the three-month T-bill rate. Only the coefficients of interest are presented in this table.

Maturity	Historical Standard Deviation		Swaption Volatility	
	c2	d2	c2	d2
2-year	0.39**	1.05**	0.19*	0.44**
3-year	0.41**	1.56**	0.22**	0.99**
4-year	0.55**	1.91**	0.31**	1.05**
5-year	0.68**	2.05**	0.52**	1.51**
6-year	0.74**	1.77**	0.45**	1.24**
7-year	0.61**	2.15**	0.61**	1.33**
8-year	0.79**	2.08**	0.44**	1.28**
9-year	0.81**	1.64**	0.57**	1.69**
10-year	0.83**	1.79**	0.63**	1.42**

** implies significance at the 5% level; * implies significance at the 10% level.

Table 6

Determinants of Liquidity and Price Across Strikes

This table presents the results for a simultaneous equation model estimated separately using bid, mid and ask prices across all available strikes, where the excess implied volatility of euro (€) interest rate caps and floors and the relative bid-ask spreads are determined endogenously as a function of each other and other exogenous variables, for the sample period from April 1999 to May 2001:

$$EIV = c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + c6 * SwpnVol$$
$$+ c7 * DefSprd + c8 * 6Mrate + c9 * Slope + c10 * LMR * Skew + c11 * |LMR| * Kurt$$
$$RelBAS = d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) + d6 * SwpnVol$$
$$+ d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd$$

EIV is the implied volatility of the bid or the ask price of the cap/floor relative to the benchmark volatility estimated using a panel GARCH model on historical interest rates. RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. 6Mrate is the six-month Euribor rate. Slope is the difference between the five-year and six-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the six-month Euribor and the six-month Treasury rate. Skew is the skewness of the historical distribution of interest rates. Kurt is the excess kurtosis of the historical distribution of interest rates. LiffeVol is the logarithm of the trading volume of three-month Euribor futures on the LIFFE. CpTbSprd is the spread between the three-month AA Financial Commercial Paper rate and the three-month T-bill rate. Only the coefficients of interest are presented in this table.

Maturity	Bid Prices		Mid Prices		Ask Prices	
	c2	d2	c2	d2	c2	d2
2-year	0.04	0.07	0.18*	0.92*	0.33**	1.19**
3-year	0.09*	0.03	0.31**	0.78*	0.61**	1.75**
4-year	0.13*	0.08*	0.54**	1.07**	0.54**	1.22**
5-year	0.10*	0.15*	0.21**	0.99**	0.78**	1.49**
6-year	0.22*	0.49*	0.46**	1.22**	1.02**	1.85**
7-year	0.25**	1.03**	0.55**	1.10**	0.91**	2.42**
8-year	0.19*	0.57*	0.63**	1.45**	0.76**	1.66**
9-year	0.29**	0.89**	0.72**	1.51**	0.82**	2.34**
10-year	0.34**	1.24**	0.57**	1.09**	0.93**	1.73**

** implies significance at the 5% level; * implies significance at the 10% level.

Table 7

Determinants of Liquidity and Price Across Strikes Using Alternative Volatility Measures

This table presents the results for a simultaneous equation model estimated separately using mid prices across all available strikes, where the excess implied volatility of euro (€) interest rate caps and floors and the relative bid-ask spreads are determined endogenously as a function of each other and other exogenous variables, for the sample period from April 1999 to May 2001:

$$EIV = c1 + c2 * RelBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + c6 * SwpnVol$$
$$+ c7 * DefSprd + c8 * 6Mrate + c9 * Slope + c10 * LMR * Skew + c11 * |LMR| * Kurt$$
$$RelBAS = d1 + d2 * EIV + d3 * LMR + d4 * LMR^2 + d5 * \left(1_{LMR<0}.LMR\right) + d6 * SwpnVol$$
$$+ d7 * DefSprd + d8 * LiffeVol + d9 * CpTbSprd$$

EIV is the implied volatility of the mid price of the cap/floor relative to the alternative benchmark volatilities (standard deviation of changes in log rates as well as swaption volatilities of comparable maturity). RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. 6Mrate is the six-month Euribor rate. Slope is the difference between the five-year and six-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the six-month Euribor and the six-month Treasury rate. Skew is the skewness of the historical distribution of interest rates. Kurt is the excess of the historical distribution of interest rates. LiffeVol is the logarithm of the trading volume of three-month Euribor futures on the LIFFE. CpTbSprd is the spread between the three-month AA Financial Commercial Paper rate and the three-month T-bill rate. Only the coefficients of interest are presented in this table.

Maturity	Historical Volatility		Swaption Volatility	
	c2	d2	c2	d2
2-year	0.32*	0.88*	0.19*	0.49*
3-year	0.45**	1.05**	0.52*	0.78**
4-year	0.61**	1.31**	0.44**	1.55**
5-year	0.52**	1.07**	0.29**	1.41**
6-year	0.49**	2.11**	0.22**	1.29**
7-year	0.65**	1.55**	0.54**	2.67**
8-year	0.79**	1.71**	0.65**	2.51**
9-year	0.68**	2.03**	0.27**	2.13**
10-year	0.72**	1.48**	0.33**	1.45**

** implies significance at the 5% level; * implies significance at the 10% level.

Table 8

Determinants of Changes in Excess Implied Volatility and Bid-Ask Spreads (ATM)

This table presents the results for a simultaneous equation model, for near-the-money options with LMRs between -0.1 and 0.1, where daily changes in the excess implied volatility of euro (€) interest rate caps and floors and daily changes in the relative bid-ask spreads are determined endogenously as a function of each other and of changes in other exogenous variables, for the sample period from April 1999 to May 2001:

$$\Delta EIV = c1 + c2 * \Delta RelBAS + c3 * \Delta LMR + c4 * \Delta LMR^2 + c5 * \Delta\left(1_{LMR<0}.LMR\right) + c6\Delta SwpnVol + c7 * \Delta DefSprd + c8 * \Delta 6Mrate + c9 * \Delta Slope$$

$$\Delta RelBAS = d1 + d2 * \Delta EIV + d3 * \Delta LMR + d4 * \Delta LMR^2 + d5 * \Delta\left(1_{LMR<0}.LMR\right) + d6 * \Delta SwpnVol + d7 * \Delta DefSprd + d8 * \Delta LiffeVol + d9 * \Delta CpTbSprd$$

EIV is the implied volatility of the mid-price of the cap/floor relative to the benchmark volatility estimated using a panel GARCH model on historical interest rates. RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. 6Mrate is the six-month Euribor rate. Slope is the difference between the five-year and six-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the six-month Euribor and the six-month Treasury rate. LiffeVol is the logarithm of the trading volume of three-month Euribor futures on the LIFFE. CpTbSprd is the spread between the three-month AA Financial Commercial Paper rate and the three-month T-bill rate. Only the coefficients of interest are presented in this table.

Panel A: Changes in EIV as the dependent variable

Maturity	c2	c6	c7	c8	c9	Obs	R^2
2-year	0.47*	0.59*	0.00	-1.37	1.36*	1090	0.07
3-year	1.09*	8.40	0.01	-2.30*	1.03	1364	0.05
4-year	0.63*	0.87**	0.07*	-4.55*	2.69	1439	0.06
5-year	0.93**	1.04*	0.03	-3.79*	2.23	1404	0.07
6-year	1.80*	1.04*	0.02**	-1.29*	3.48	1429	0.09
7-year	3.29**	2.84	0.01*	-3.68	4.77	1367	0.11
8-year	3.75*	5.14*	0.01	-0.81**	1.55	1149	0.12
9-year	4.87*	4.11*	0.09	-6.47*	-1.00	1112	0.14
10-year	2.87**	3.92*	0.01	-1.11**	-3.16	886	0.09

Panel B: Changes in RelBAS as the dependent variable

Maturity	d2	d6	d7	d8	d9	Obs	R^2
2-year	2.86	1.14**	0.00	-0.08	0.01	1090	0.03
3-year	2.84**	0.08**	0.00	-0.03	0.05	1364	0.09
4-year	0.14**	0.58*	0.07	-0.02*	0.01	1439	0.07
5-year	0.44*	0.82**	0.03*	-0.02**	0.00	1404	0.06
6-year	0.56*	0.76**	0.03	-0.03**	-0.01	1429	0.06
7-year	1.65**	0.98**	0.08	-0.04**	-0.01	1367	0.09
8-year	0.22**	1.34**	0.03*	-0.02**	0.00	1149	0.13
9-year	0.45**	1.02**	0.02*	-0.01**	0.00	1112	0.13
10-year	0.12**	0.88*	0.04	-0.01**	0.00	886	0.12

** implies significance at the 5% level; * implies significance at the 10% level.

Table 9

Correlations Among Relative Bid-Ask Spreads

This table presents average time-series correlations between relative bid-ask spreads across moneyness buckets (at-the-money (ATM), in-the-money (ITM) and out-of-the-money (OTM) of euro (€) interest rate caps and floors. The numbers below are averaged across the correlations between the nine maturities within each moneyness bucket. For example, the OTM/OTM value is the average correlation between bid-ask spreads for each of the nine maturities within OTM options (so it is an average of (9 x 8)/2; that is, 36 correlations). The OTM/ITM value is the average correlation between bid-ask spreads for each maturity for OTM options with those for each maturity for ITM options (so it is an average of 9 x 9, or 81, correlations), and so on. The correlations are calculated for the sample period from January 1999 to May 2001.

	Average Correlations		
	OTM	ATM	ITM
OTM	0.68		
ATM	0.34	0.86	
ITM	0.24	0.65	0.78

Table 10

Determinants of Changes in Bid-Ask Spreads

This table presents results of a panel regression of changes in relative bid-ask spreads on changes in excess implied volatility of mid-price and changes in the moneyness variables of euro (€) interest rate caps and floors.

$$\Delta RelBAS_{it} = c1 + c2 * \Delta EIV_{it} + c3 * \Delta LMR_{it} + c4 * \Delta LMR^2{}_{it} + c5 * \Delta\left(1_{LMR<0}.LMR\right)_{it} + \varepsilon_{it}$$

EIV is the implied volatility of the mid-price of cap/floor relative to the benchmark volatility estimated using a panel GARCH model on historical interest rates. RelBAS is the bid-ask spread scaled by the mid-price. LMR (Log Moneyness Ratio) is the log of the ratio of the swap rate to the strike rate of cap/floor. Δ indicates the first difference. There are 27 groups (i = 1 to 27) in the panel (nine maturities - two-year to ten-year X three moneyness groups - at-the-money, out-of-the-money and in-the-money for each maturity). The table presents GLS estimates for the sample period from April 1999 to May 2001.

	c1	c2	c3	c4	c5	Obs	Adj R^2	p-value for F statistic
Coefficient	-0.024	0.248	0.423	0.163	-0.556	44070	0.0907	0.000
t-stats	-2.60	4.52	3.94	1.90	-2.69			

Table 11

Commonality in Changes in Bid-Ask Spreads

This table presents the structure of the principal components of the correlation matrix of the residuals obtained from the panel regression:

$$\Delta RelBAS_{it} = c1 + c2 * \Delta EIV_{it} + c3 * \Delta LMR + c4 * \Delta LMR^2 + c5 * \Delta\left(1_{LMR<0}.LMR\right) + \varepsilon_{it}$$

EIV is the implied volatility of the mid-price of the option relative to the benchmark volatility estimated using a GARCH model on historical interest rates. RelBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of the swap rate to the strike rate of the option. Δ indicates first difference. There are 27 groups in the panel consisting of nine maturities (two years to ten years) for the three moneyness groups (at-the-money, out-of-the-money and in-the-money). The regression is estimated for the sample period from April 1999 to May 2001, based on data on euro (€) interest rate caps and floors.

		Principal Component		
		1	2	3
Eigenvalue		9.04	2.87	1.41
% explained		0.33	0.11	0.05
Moneyness	Maturity	Eigenvectors		
OTM	2-year	0.18	-0.06	0.29
OTM	3-year	0.23	-0.15	0.17
OTM	4-year	0.28	-0.18	0.00
OTM	5-year	0.27	-0.10	0.05
OTM	6-year	0.30	-0.11	0.04
OTM	7-year	0.29	-0.02	0.08
OTM	8-year	0.33	-0.00	0.02
OTM	9-year	0.31	-0.06	-0.02
OTM	10-year	0.26	-0.01	0.05
ATM	2-year	0.13	0.03	0.05
ATM	3-year	0.17	0.04	0.07
ATM	4-year	0.20	0.00	-0.11
ATM	5-year	0.14	0.05	-0.13
ATM	6-year	0.21	0.15	0.10
ATM	7-year	0.19	0.12	0.07
ATM	8-year	0.24	0.07	-0.15
ATM	9-year	0.26	0.06	0.11
ATM	10-year	0.14	0.14	-0.22
ITM	2-year	0.01	0.11	0.55
ITM	3-year	0.00	0.36	0.28
ITM	4-year	0.06	0.41	-0.02
ITM	5-year	0.01	0.40	0.00
ITM	6-year	0.00	0.42	0.07
ITM	7-year	0.08	0.36	0.09
ITM	8-year	0.11	0.19	-0.52
ITM	9-year	0.07	0.16	-0.28
ITM	10-year	0.13	0.07	0.02

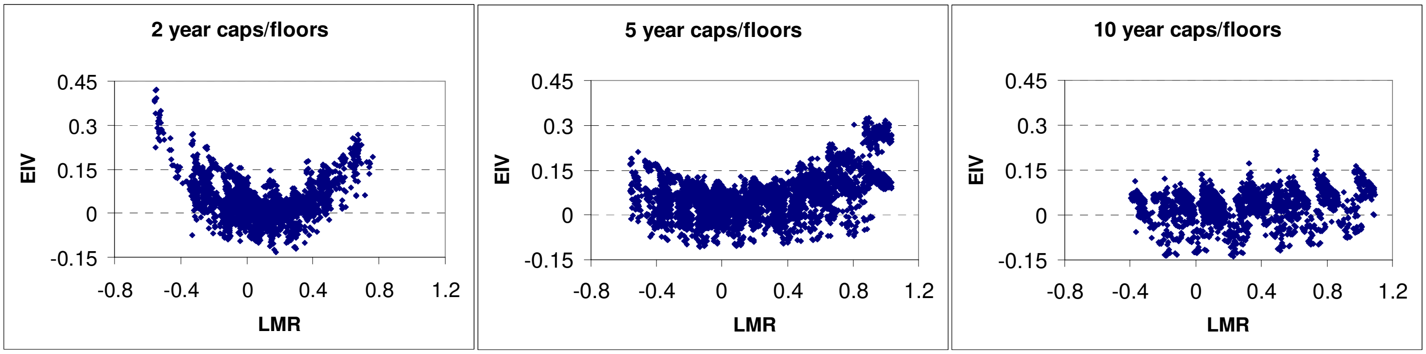
Table 12

Macro-Economic Determinants of the Systematic Liquidity Factor

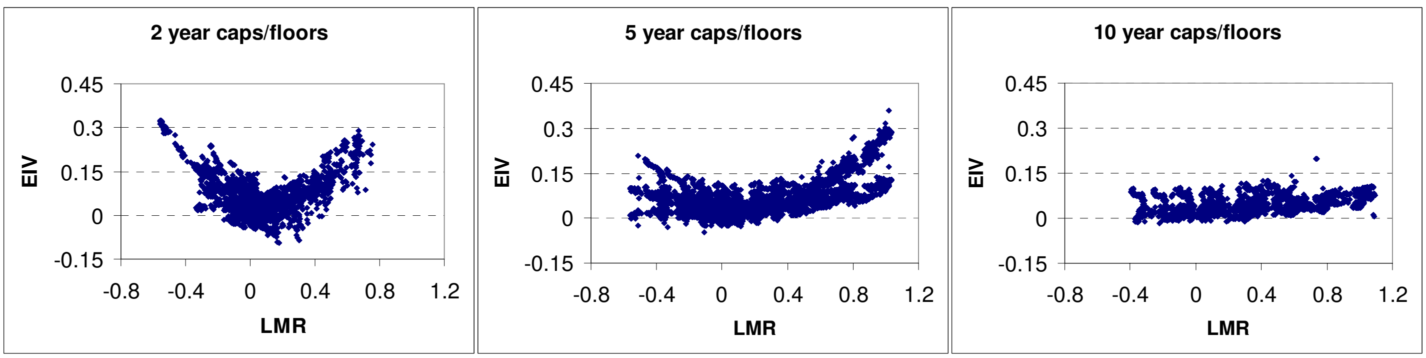
This table presents the results of the regression of the first principal component of the correlation matrix of residuals (from the panel regression in Table 10) on contemporaneous and lagged changes in macro-financial variables. 6Mrate is the six-month Euribor. Slope is the difference between the five-year and six-month Euribor rates. DefSprd is the difference between six-month Euribor and the six-month Treasury rate. DAXVol is the implied volatility of DAX index options. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. The regression is estimated for the sample period from April 1999 to May 2001, based on data on euro (€) interest rate caps and floors.

	Coefficient	t-stats
Constant	-0.11	-1.02
6Mrate		
Contemporaneous	0.19	0.71
Lag 1	0.55	0.30
Lag 2	4.21	1.01
Lag 3	1.98	0.45
Lag 4	5.15	0.62
Slope		
Contemporaneous	0.22	0.30
Lag 1	1.49	0.89
Lag 2	4.61	1.44
Lag 3	2.35	1.17
Lag 4	2.88	1.22
DefSprd		
Contemporaneous	0.49	0.62
Lag 1	-1.33	-1.08
Lag 2	-2.15	-1.26
Lag 3	1.50	0.95
Lag 4	1.37	1.33
DAXVol		
Contemporaneous	0.05	0.79
Lag 1	0.23	1.87
Lag 2	0.31	2.33
Lag 3	0.19	2.12
Lag 4	0.08	1.75
SwpnVol		
Contemporaneous	0.17	1.72
Lag 1	0.35	2.12
Lag 2	0.52	2.78
Lag 3	0.45	2.65
Lag 4	0.29	1.99
R^2		0.28
p-value for F-stat		0.00

Panel A: Panel GARCH volatility as benchmark



Panel B: Historical standard deviation as benchmark



Panel C: Swaption implied volatility as benchmark

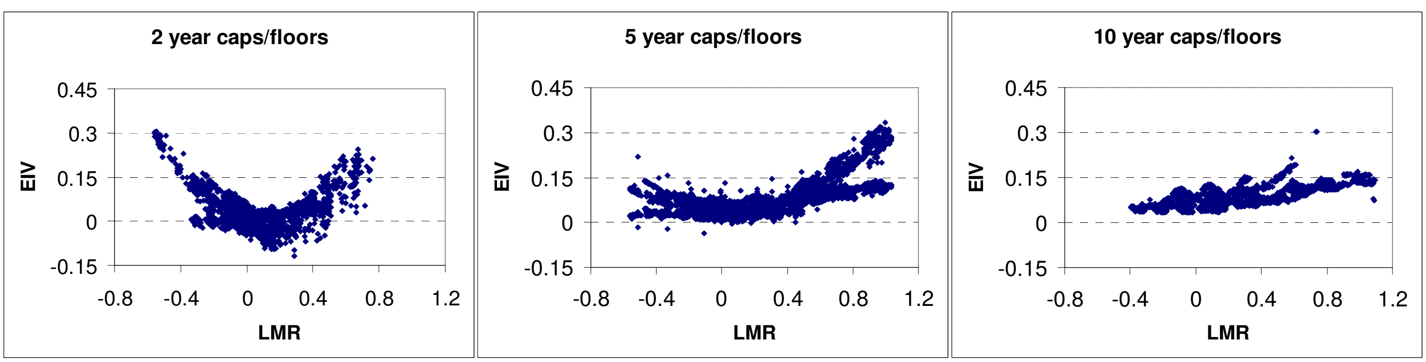
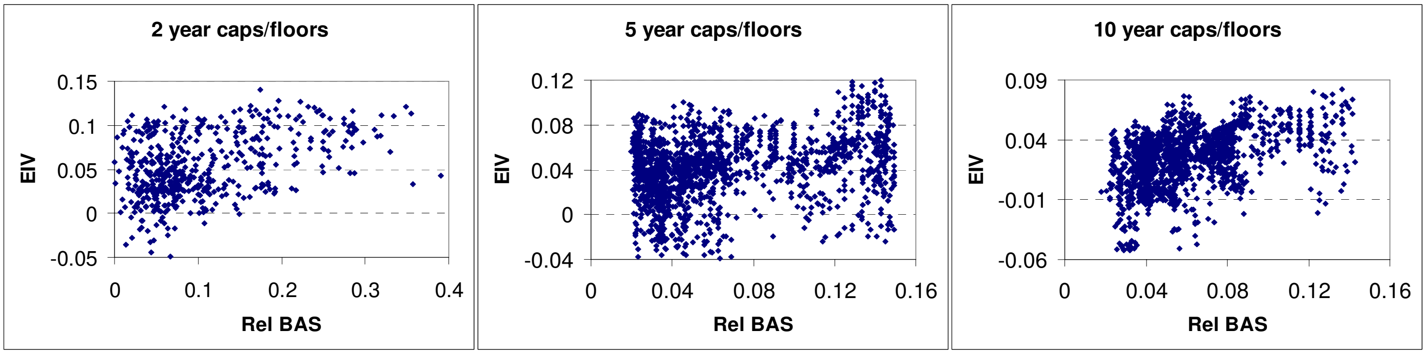
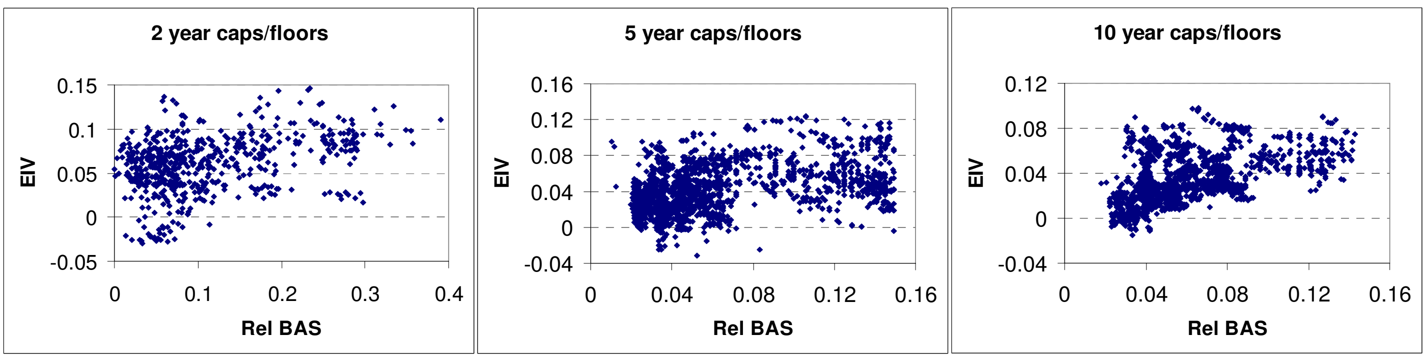


Figure 1. Volatility smiles using alternative benchmark volatilities. This figure presents scatter plots showing the shape of the volatility smiles for the excess implied flat volatilities of euro (€) interest rate caps and floors, using three alternative benchmark volatilities - panel GARCH volatility, historical standard deviation and swaption implied volatility - over the sample period April 1999 to May 2001. The plots are presented for three representative maturities – two, five, and ten years. The plots for the other maturities are similar.

Panel A: Panel GARCH volatility as benchmark



Panel B: Historical standard deviation as benchmark



Panel C: Swaption implied volatility as benchmark

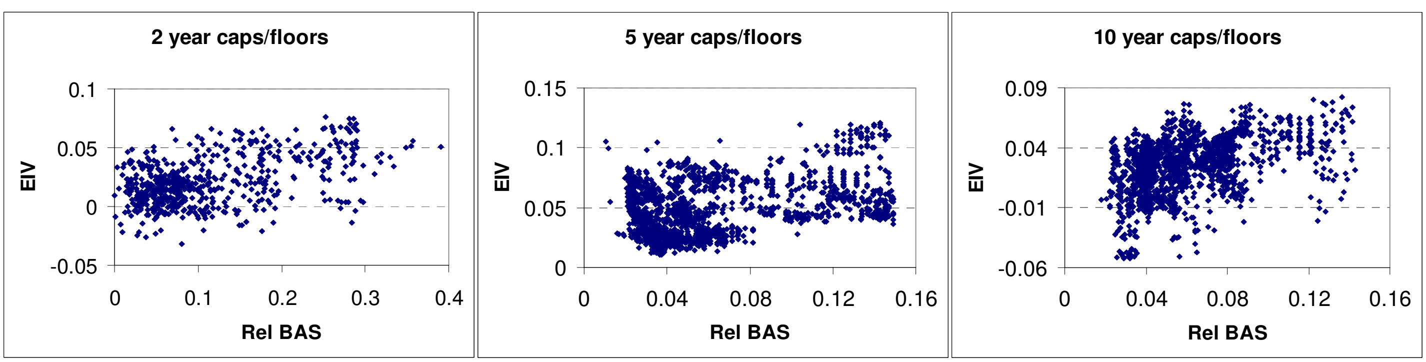


Figure 2. Plots of excess implied volatility versus liquidity. This figure presents three sample scatter plots of the excess implied volatility of euro (€) interest rate caps and floors for the three benchmark volatilities - panel GARCH volatility, historical standard deviation and swaption implied volatility. The graphs show the relationship between the EIV and the RelBAS for two, five, and ten-year maturity caps and floors. The plots for other maturities are similar. The plots are constructed using data for euro (€) interest rate caps and floors over the sample period April 1999 to May 2001.

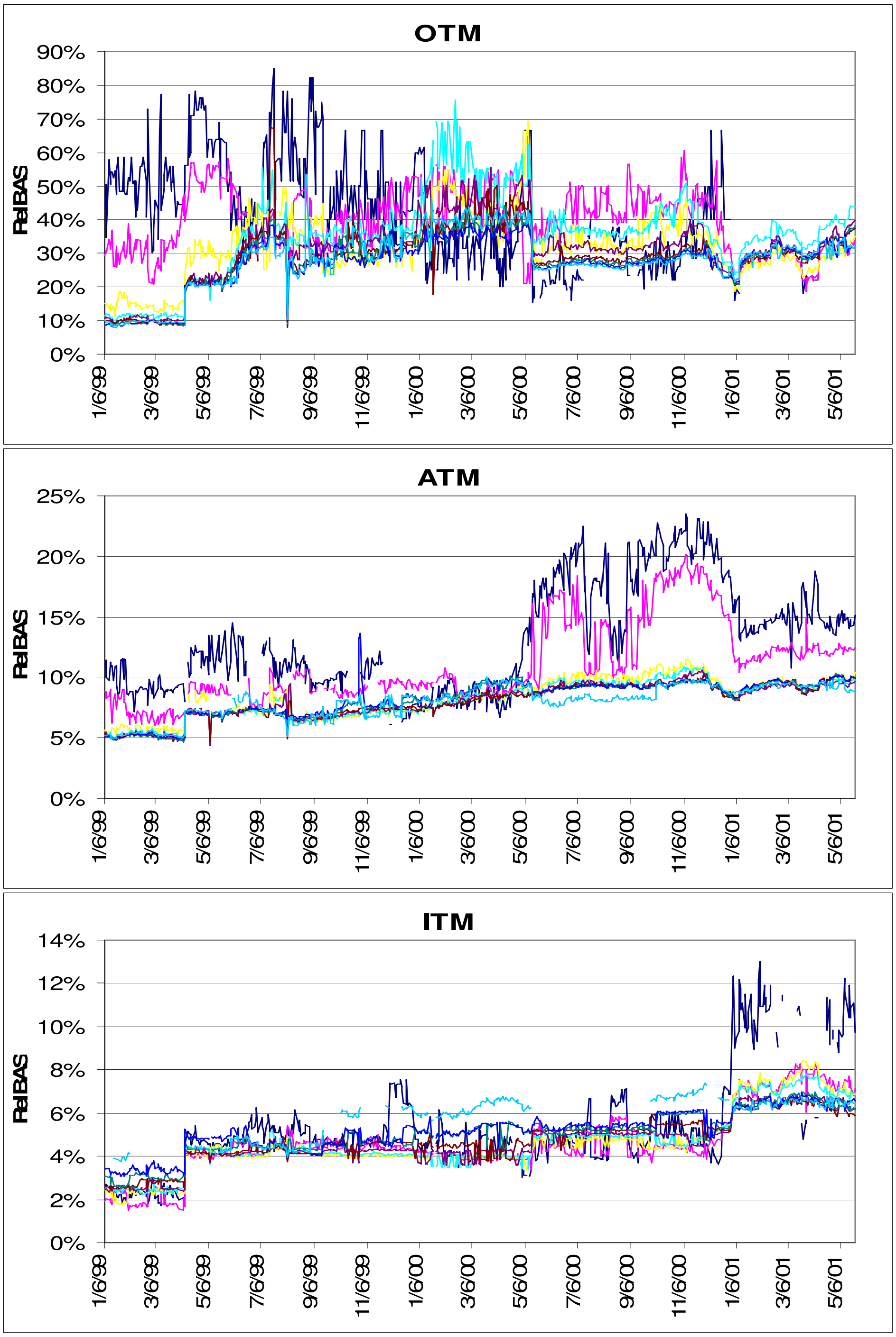


Figure 3. Time variation in relative bid-ask spreads. This figure presents the time-series plots of the relative bid-ask spreads of euro (€) caps and floors for each of the nine maturities (from two years to ten years), separately by moneyness, over the sample period January 1999 to May 2001. Each plot has nine time series representing the nine option maturities. The out-of-the-money (OTM) bucket contains caps with LMRs less than -0.1 and floors with LMRs greater than 0.1. The in-the-money (ITM) bucket contains caps with LMRs greater than 0.1 and floors with LMRs less than -0.1. The at-the-money (ATM) bucket contains caps and floors with LMRs between -0.1 and 0.1.