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Literature Review on Aircraft Structural Risk and Reliability Analysis

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ABSTRACT

Aircraft structural risk and reliability analysis has gained considerable interest in the area of aircraft structural integrity and fleet management in recent times. In this report, a literature review of the current approaches and methodologies that have been utilised in the area of structural risk and reliability analysis for aircraft structures and components is conducted.
The structural risk and reliability approach discussed here deals mainly with the probability of failure due to aircraft structural fatigue. It reviews the engineering techniques and probabilistic methodologies that have been reported in the literature. It also discuss the advantages, disadvantage and the limitations of this approach and the reason for its need in aircraft structural and component integrity. The application of structural risk and reliability analysis is not limited to structural fatigue problem, its implication in areas other than airframe and component fatigue are also presented. This review shows that risk and reliability analysis can be a very useful tool for fleet management and it has potential implications for structural design.

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Executive Summary

Nowadays, the life of an aircraft fleet is no longer governed by its original design life. To a great extent, it is determined by the capability, the maintenance cost and the economic considerations required for the fleet to continue its operational requirements. With the need to maintain aircraft flying longer in an environment of continuous reducing funding levels, aircraft operators have recognised that the current methods of aircraft structural integrity cannot provide adequate information for assisting airframe and component management decisions. An additional tool or a more advance approach is required.

Probabilistic Damage Tolerant approach or Structural Risk and Reliability Analysis has been identified as the potential tool for satisfying these requirements. It is capable of identifying the sources of variables effecting the fatigue life and fatigue strength of the structure in terms of risk. It has also been proven that probabilistic method can be extended to provide very useful information to help managers in making decisions regarding the operation and inspection time of the fleet in order to maintain airworthiness. Especially in ageing aircraft problems, where inspection timing and cost effectiveness is important for life extension programs in allowing the fleet to continue operation by exceeding their design life in the most economical manner. For these reasons, the probabilistic approach is beginning to gain new interests in civil and military aircraft operators.

In this report, a literature review of the current approaches and methodologies that have been utilised in the area of structural risk and reliability analysis for aircraft components and structures is conducted.

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1. Introduction

Structural risk and reliability analysis is the probabilistic engineering approach, also known as the "Reliability method", for supporting the structural integrity analysis of structures and components under service loads. The mathematical basis for this reliability method was established in the late sixties and early seventies [1]. However, due to its requirement for high computational power and what was then unavailable data this method was unattractive for use in structural integrity analysis of airframe structures. As a result the damage tolerance method, which was deterministic, was selected rather than the probabilistic method [2].

In recent times, a new challenge had arisen in the aircraft structural integrity program with the need to maintain aircraft longer in an environment of reduced funding levels. As a result, the aircraft operator found that the current methods of aircraft structural integrity were not adequate. A more advanced approach is required. For these reasons, the probabilistic approach is beginning to gain new interests in civil and military aircraft operators.

In the past, the safe life approach also utilised statistical distributions and scatter factors for determining the safe life and the probability of failure for airframe structures and aircraft components. One can see that reliability methods were not used for determining the scatter factors but rather, conservative assumptions were made to make allowance for the variability in fatigue life and to ensure the desired level of safety is achieved. The probabilistic approach is capable of identifying the sources of variables affecting the fatigue life and fatigue strength of the structure in terms of risk, while eliminating the over-conservatism that maintains safety. It has also been proven that the probabilistic method can be extended to provide very useful information to help managers in making decisions regarding the operation and inspection time of the fleet in order to maintain airworthiness. Especially in ageing aircraft problems, inspection timing and cost effectiveness is important for life extension programs in allowing the fleet to continue operation exceeding by their design life in the most economical manner.

In this report, a literature review of the current approaches and methodologies that has been utilised in the area of structural risk and reliability analysis for aircraft components and structures is conducted.

2. The Need For Structural Risk And Reliability Analysis

In the early days of aircraft fatigue studies, the safe life approach was used to ensure integrity of airframe structures from fatigue failure. In this approach the mean fatigue life of the structure was estimated and then divided by a scatter factor to give a safe operating life (safe life). The scatter factor was primarily used to cover the variability in fatigue performance of the structure and to ensure that the probability of failure was acceptably low [1]. The safe life must ensure that the cumulative probability of failure is less than 1 in 1000 over the safe life of operation.

The disadvantage of this approach was clearly that a very large percentage (99.9%) of the population was retired from service long before their useful life had been reached unless inspections and maintenance are applied. This led to the development of the fail-safe philosophy and marked the beginning of Aircraft Structural Integrity Program (ASIP) [3]. The ASIP defines all of the structurally related activities on an aircraft from initial development until retirement. It is a program that is used in aircraft acquisition as well as for ageing aircraft [3].

This approach allowed structure to remain in service until fatigue cracks were detected by a planned inspection procedure before they reached a dangerous extent and that the structure must have sufficient residual strength to provide safety until cracks are detected from routine inspections [4]. It is these requirements which led to the demand and development of residual strength analysis and NDI techniques.

However, as the design and development of airframes continued to move towards more high performance aircraft, the fail-safe approach to ASIP became more and more inadequate. Structural failures due to fatigue were the cause of the high failure rate in these high performance aircraft during the 1970's. As a result, the fail-safe approach was retired and the damage tolerance philosophy was adopted [3].

The adoption of the damage tolerance approach to ASIP has shown a great success since its employment in mid 1970's. This approach to aircraft structural integrity forced a better understanding of the stresses and operational loads on aircraft, allowing predictions of crack propagation rates and fatigue lives and hence, allowing structural maintenance plans to maintain safe operations for critical components.

This approach assumed a deterministic process for fatigue fracture where a deterministic crack growth function, constant material properties and a specified initial flaw size are employed. It was found very useful and adequate for maintaining a higher level of safety for operational aircraft and it can be readily applied to the design process of new aircraft structure. However, the stochastic nature of fatigue crack growth continues to be a well-recognised problem in the deterministic approach. The variation in crack growth rate, material properties and fracture toughness, and the distribution of crack sizes often causes difficulties in providing an appropriate and economical structural maintenance plan for maintaining safety especially in older aircraft. Nowadays the life of an aircraft fleet is no longer governed by the flight hours specified by the design life. It tends to be determined more by its inherent operational capability and maintenance costs required for its continual operation [5]. In many instances, damage tolerance analysis had shown that it could not provide the conservatism necessary for operating the aircraft below the acceptable risk level and the deterministic approach is not capable of providing any measurement of safety for the operators to manage their fleet of aircraft [6].

Although there are some inherent problems with this current design and ASIP approach, ultimately this philosophy was shown to be a success in providing safe operation for airframe components from structural failure. Today, the result of this success has caused new challenges to be raised from aircraft operators in the need to maintain aircraft flying longer in a condition with reducing funds. This requires a more sophisticated approach or additional tool for assisting and determining the most economical and effective maintenance decisions, regarding inspection timing and inspection methods whilst maintaining a high level of safety. The probabilistic approach has proved to have the potential in meeting all of these new challenges and is currently an area of interest for aircraft structural integrity and for aircraft fleet operators.

The special interest gained in the probabilistic approach is that this approach has significant advantages over the deterministic approach for the structural integrity assessments of ageing aircraft. It is capable of taking into account the variability in material properties, the stochastic crack growth behaviour, and state of damage of the structure via Probability Density Functions (PDF). The primary output of the reliability approach is the Single flight/hour probability of fracture/failure and the total probability of failure. The single flight probability of failure is probabilistic, and it is defined as the probability that failure will occur on a single flight of an aircraft selected randomly from the population, providing quantitative information for the management and assessment of structural safety and maintenance intervals. The probabilistic method is a unique technique which takes many qualities of the safe-life and damage tolerance approach, and at the same time is capable of addressing and providing information regarding inspection, life extension problems and for assessing economic life.

The secondary output of probabilistic analysis can be information regarding the expected costs of completing maintenance scenarios. It can assist managers to determine the most economical and effective maintenance decisions, regarding inspection timing and inspection methods whilst maintaining high level of safety. It is clear that probabilistic damage tolerance analysis is capable in addressing the major issues and challenges in aircraft structural integrity today and have significant advantages over the deterministic approach.

The USAF is the current leader in the application of reliability methodology for aircraft structural integrity. The probabilistic approaches for the solution of ageing aircraft problems have been in use for approximately 18 years. Lincoln [9] has stated that the probabilistic methods have proven useful for several scenarios.

1 Potential-cracking problems has been revealed and the aircraft are beyond their deterministic damage tolerance limits.

2 Aircraft experienced cracking to the extent that deterministic damage tolerance derived inspection intervals need to be shortened in order to preserve safety.

3 Aircraft have been designed to be fail safe, but widespread fatigue damage has degraded the structure such that fail safety of the structure has been compromised.

In the remaining section of this report, an insight into risk and reliability methodologies and approaches that have been used in the past and presently utilised for aircraft structural integrity from the literature are presented. In section 3, the discussion of the interpretation of the acceptable risk is reviewed, followed by the literature review of the probabilistic analysis methodologies and the application of probability to the stochastic fatigue data in section 4 and 5 respectively. Section 6 discusses the application of structural risk and reliability assessment beyond the area of fatigue, such as for corrosion and economic life determination. Section 7 relates the areas in airframe structures, engine components and helicopter components that are considered to be important for the application of structural risk and reliability analyses. Section 8 and 9 contains the discussions and concluding remark for this literature review.

3. Interpretation of Risk

The question that puzzle many unfamiliar users and needs to be answered is what is risk and how does one define the acceptable quantity of risk? Risk is usually interpreted as the chance or the probability of a failure event occurring within a population of details over a period of time. The quantity risk is often difficult to define, especially in defining what is acceptable. In this section, we will attempt to summarise the interpretation of risk that had been utilised in the past and the risk that is acceptable in the current practice of structural risk and reliability for aircraft components.

Some risk of failure exists in all engineering structures. It is important that the risk of failure is kept below the allowable level of risk acceptable for continuous safe operations. The specification of the required safety level is essential to the probabilistic approach. A required probability of survival is already contained indirectly in some airworthiness requirements, which specify safe lives at three standard deviations below the mean fatigue life. However, it was realised that the probability of failure commonly begins at zero and then rises with increasing rapidity to unacceptable values at the end of the service life. So the disadvantage of this risk definition was that it made no indication on the risk of failure per hour or per flight '*r(N)*' [7] on each operation. Now, what aircraft operators and aircraft fleet managers are becoming more interested in, is the risk of failure per hour or the failure rate per hour of their aircraft. Section 3.1 and 3.2 defines in more detail these two risk interpretations.

3.1 Cumulative Risk

Traditionally in the safe life philosophy, to protect airframe structures and engine components against fatigue fracture, a scatter factor is used to limit the operational life of components and structures to a safe life. The variation of fatigue life in airframe structures is assumed to be log normally distributed. The scatter factor is a factor which when applied to the mean of the distribution gives the predicted safe life which ensures that the probability of failure from fatigue will be less than 1 in 1000 [7]. Hence, an acceptable cumulative probability of failure from fatigue is 10^{-3} per aircraft per safe life. This definition is very useful when considering a fleet of aircraft, since the risk defines the fraction of the total population that would be expected to suffer fatigue failure over the safe life period of the component [1].

Helicopter manufacturers use a higher safety requirement for determining the safety factor or safe life for their components. It utilises the TOS/(μ-3σ) method, that is the "top-of-scatter" loads representing 99.9% of the operational loads, and weak component (μ-3σ) representing 99.9% of the population of strength method, to ensure a probability of survival of 0.999999 is achieved [8]. The 0.999999 probability of survival is commonly known as $.9_6$ reliability. Although the method of safe life utilises an assumed statistical distribution in addressing the variability in fatigue strength of the material and the variability in operating spectrum, it does not take into account the change of probability of failure due to cumulative operating damage. It simply relies on its conservative assumptions regarding loads and material properties to achieve the one in a million probability of failure over the retirement period [8]. However, these statements have provided the baseline for what is the acceptable cumulative risk of failure for aircraft structures and components for future reliability assessments.

The disadvantage of the cumulative probability of failure is that it takes no account of the operating life and hence of the time of exposure to risk. In other words, it does not indicate the change of risk rate over the time of operation. The instantaneous failure rate per hour/flight can overcome this inadequacy, and has become the quantity of risk of interest for operators today.

3.2 Instantaneous Risk

Risk is usually interpreted as the instantaneous chance or the instantaneous probability of a failure event occurring within a population of details. For aircraft failures, the instantaneous rate is often defined in terms of per hour or per flight. The interpretation of the risk quantity is often difficult to define, especially in defining what is the acceptable failure rate. One basis for interpretation of this quantity is by means of a survey of accident statistics.

The following statistics were taken from reference [1]. For the United Kingdom and United States civil aircraft, Freudenthal and Payne found an average structural failure rate of 3×10^{-7} per hour for ultimate load failure and 2×10^{-7} per hour for fatigue failure. Pugsley reports a structural accident rate of 10^{-7} per hour in the United Kingdom for military aircraft in the 1930's. Lundberg also suggests 10^{-8} per hour as an average failure rate for structural fatigue with 10^{-9} per hour as a target in view of the rapid growth taking place in civil air transport. For the USAF, Lincoln [9] reports that the overall failure rate for all systems due to structural faults is one aircraft lost in more than ten million flight hours, which is a probability of failure of 10^{-7} per flight.

Another interpretation presented by Lincoln [10], is to relate a quantity of risk we accept in our everyday living. For example, the risk of a major accident that we accept in driving an automobile to work and back home is of the order of 10^{-6}. The USAF has worked with a set of specifications for measuring safety of their aircraft. They have even further categorised the risk into intervals to limit the exposure of aircraft when risk is approaching or have exceeded the acceptable risk limits. Lincoln [10] states that, for most military systems, a single flight probability of failure of 10^{-7} or less is considered adequate to ensure safety for long-term operations. For single flight probability of failure greater than 10^{-7}, consideration should be given to limiting the exposure by modification or replacement. If this quantity is 10^{-5} or greater for an extended period of time, the failure rate should be considered unacceptable. This is the risk quantity that is currently accepted by the USAF.

From the consideration of the above, the proposed safety conditions for airframe structures and engine components are therefore [4]:

Civil operations:

$$r(t)\leq 10^{-7} \ \textit{per hr or} \ P(t)\leq 0.001$$

Military operations

$$r(t)\leq 10^{-6} \ \textit{per hr or} \ P(t)\leq 0.001$$

For helicopter components, the cumulative probability of failure acceptable is:

$$P(t)\leq 10^{-6}$$

These requirements had been documented in the DEFSTAN 970 [11]. Without inspection programs, the use of structural risk and reliability analysis will also mean 999 out of 1000 components for engines and airframe structures and 999,999 out of 1,000,000 components for helicopter components will be retiring earlier than they need to be due to the possibility of one failing. This is similar to the safe-life approach and it is seen as very wasteful.

4. Variability in Structural Fatigue

For the damage tolerant approach, the deterministic functions such as the crack growth curve, the load spectrum and the residual strength function are the fundamental functions that allow accurate future prediction. These input functions are commonly derived through fracture mechanics analysis using finite element and crack growth analysis methods. These functions are also the major input for risk and reliability assessment. For this reason, risk and reliability analysis of structural fatigue process can be thought of simply as the extension of the current damage tolerance analysis. The extension allows the stochastic effect and the variability of fatigue properties to be taken into account using probabilistic methods.

Therefore, the key to assess the reliability of structures and components is to realistically describe the assessment input variables and their variations [12]. In this section, the important random variables in aircraft structural fatigue problems are reviewed and discussed. These input variables are summarised into four groups:

1. The variability in material properties
2. The initial fatigue quality of the component
3. The variability in the crack growth rate and
4. The reliability of the inspection

4.1 Variability in Material Properties

4.1.1 Variability in Material Strength

The material residual strength, '$\sigma_{r.s}$', and the material fracture toughness, 'K_{IC} or K_C', are the most influential material properties when addressing failure due to fatigue. These properties are often used to represent the State of Violation or the 'Limit-State', in structural fatigue reliability problems and hence, are crucial information in structural reliability assessments. These material properties are also subjected to variations and are considered random variables that can be statistically represented.

The variation in material property is typically assumed normally distributed for it is a reasonable model for many natural processes or physical properties [5,13]. Table 1 is taken from reference [14] indicating the typical representative value of the coefficient of variation for several aerospace materials.

Other distribution function such as the Weibull distribution has been suggested [15] for representing the distribution of ultimate strength.

4.1.2 Variability in Material Fracture Toughness

The scatter in the fracture toughness that is observed in nominally identical material might be taken into account using statistical distribution. The coefficient of variation for material fracture toughness is difficult to obtain from the literature. Damage tolerant Design Handbook [16] is an example of where this type of data may be available from. However, due to the small number of specimens tested, the standard deviation calculated based on the available specimens may not be of high reliability. Data pooling is one way of increasing the reliability of the distribution. Statistical models have been derived in the United States to predict fracture toughness reference curve from Charpy data [17].

Hovey *et. al.* [5,6] had considered the material fracture toughness to be a physical property and had assumed it to be normally distributed. Johnson [17] showed that the lognormal and Weibull distributions also gave a reasonable fit to the experimental fracture toughness data.

Table 1: Representative value of coefficient of Variation 'V' for several types of aircraft structural materials.

Material and Type	Representative value of coefficient of variation V
Aluminium Alloy Bars and Extrusions	
Failing Strength	0.01 to 0.05
Proof Strength	0.01 to 0.05
Aluminium Alloy Sheet	
Failing Strength	0.01 to 0.05
Proof Strength	0.01 to 0.05
Aluminium Alloy Tube	
Failing Strength	0.03
Proof Strength	0.05
Magnesium Alloy Bar	
Failing Strength	0.02 to 0.04
Proof Strength	0.07
Steel Bar	
Failing Strength	0.01 to 0.02
Proof Strength	0.01 to 0.03

The variation of material fracture toughness can often be approximated using the variation of material strength, for these two material properties are closely related to one another. High material strength often indicates low material fracture toughness and vice versa. Therefore, in the case where the distribution of fracture toughness is unknown, it can be acceptable to use the values given in Table 1 to represent the coefficient of variation for material fracture toughness, provided a normal distribution is assumed. Care must be taken when non-normal distributions are employed.

4.1.3 Bimodal Distribution in Material Properties

The bimodal distribution can be useful when a single distribution is not sufficient in representing the variation. It is often observed from manufacturing processes that a small percentage of components periodically result in a "bad patch" defective material properties. In a risk and reliability analysis, the PDF of the material property must take into account this possible variability. The technique to take the probability of such event occurring into account for structural risk and reliability assessment has been shown by Matthews & Neal [18]. This technique involves bimodal probability density functions and can be determined by combining the appropriate proportion 'P' of two probability density functions '$f(x)$', which represent the normal quality '$f_1(x)$' and the defective quality '$f_2(x)$' respectively. Equation 1 displays the generalised formulation for this technique.

$$f_T(x) = P_1 \cdot f_1(x) + P_2 \cdot f_2(x) = \sum_{n=1}^{N} P_n \cdot f_n(x) \qquad 1$$

4.2 Initial Fatigue Quality

Structural components inevitably suffer from flaw or crack like defects, such as surface scratches, surface roughness or weld defects of random sizes, which usually occur during the manufacturing and handling process [12]. Thus, the initial flaw quality of the material can be considered as a material property, and to be more accurate, it should be considered as a material process property. These defects are shown to have a detrimental effect on the fatigue life of the structural components by promoting crack initiation sites. In order to make reliable predictions, data regarding the initial flaw qualities must be known. However, the NDT methods presently available cannot provide us with the adequate information concerning the statistical distributions of initial flaws. In addition, it has been found that not all initial flaws, notches or scratches present on the surface of the components represent crack initiation sites. As a result, two concepts have been developed and have been proven to be useful design tools for making life predictions for aircraft structural reliability problems. They are 1) the equivalent initial flaw size (EIFS) distribution and 2) the distribution of time-to-crack initiation (TTCI).

4.2.1 EIFS

Initial flaws of a high quality structure are not detectable. Furthermore, not all flaws are propagated from an initial defect. For this reasons, the equivalent initial flaw size concept was introduced by Gray and Rudd [19] and developed by Yang and Manning [20] as an analysis technique to be used to represent the initial quality of structural details in the durability analysis.

The EIFS is an artificial crack size, which is derived from the distribution of fatigue cracks occurring later on during the service life. The distribution of EIFS is determined by back-extrapolating this distribution of fatigue cracks according to a master crack growth function to zero time or some reference time serving to represent the initial time of the assessment. So the EIFS will result in an actual fatigue crack at a point in time when it is grown forward.

The distribution of fatigue cracks at a particular time can be difficult and costly to determine. This kind of information usually requires a tear down inspection, possibly following a full-scale fatigue test or from retired airframes. Fatigue cracks detected during in-service inspections of structural components or fatigue cracks obtained in laboratory coupon testing may also serve as a starting point for developing the EIFS distribution.

4.2.2 TTCI

From an engineering standpoint, crack initiation is considered to be one of the two major periods in the fatigue life of a component or structure. The period of crack initiation or the time to crack initiation (TTCI) is defined as the time in cycles or flights or flight hours it takes for a non detectable crack from the beginning of fatigue loading to grow to a reference crack size. This crack initiation distribution is physically observable and can be obtain by experiments and tests results.

In some instances, the TTCI period makes up a large proportion of the crack growth life of a structural component and this is especially the case for jet engine discs components [21]. The reference-crack-size is commonly selected on the basis of a detectable crack by the NDI technique. The three-parameter Weibull distribution has been used to characterise TTCI [20], and it is given by:

$$F_T(t) = P[T \le t] = 1 - \exp\left[-\left(\frac{t-\varepsilon}{\beta}\right)^{\alpha}\right]; \qquad t \ge \varepsilon \tag{2}$$

where T = $TTCI$, α = shape parameter, β = scale parameter and ε = lower bound of T. The parameters are determined by test results. Heller and Stevens [22] improved the TTCI by using a Bayesian technique and considering that cracks may be missed during inspection. The lognormal distribution function has also been used to characterise TTCI [23].

It must be noted that the TTCI distribution cannot be considered as a material quality distribution like the EIFS distribution. This is because the TTCI distribution is derived for the given test spectrum.

4.2.3 Distribution of Initial Quality or Crack Size

The initial fatigue quality of a durability critical component is often characterised by the equivalent initial flaw size (EIFS). The EIFS as described in section 4.2.1 is an artificial crack, which will result in an actual fatigue crack in time [5,10,12,19,20,24,25]. In order to determine the equivalent flaw size distribution, the test results of the TTCI are produced and through transformation give the EIFS distribution, as mentioned in section 4.2.1.

The relation between the TTCI distribution and the EIFS distribution can be visualised as in Figure 1. Yang *et al.* [20] has demonstrated this existence of compatibility between the TTCI distribution function and the EIFS distribution function for the Weibull and the lognormal distributions [12]. Since accurate crack growth is almost impossible to predict at these small crack size, a power law matching the crack growth rate is used to reflect the crack growth law transforming the TTCI distribution function back to the y-axis at zero time to produce a compatible EIFS distribution.

It can be found from the literature that the Weibull distribution is the most commonly used for representing the crack size distribution. Many applications indicated that this distribution fits particular well to the tail of the large crack size data found in teardown inspections and maintenance inspections. However, the selection of the probability distribution function should be based on how well it fits the data, not because it is the most commonly used. A good approximation to the largest crack sizes has profound importance in representing the initial crack size distribution, for the largest crack sizes are the dominant driver of the probability of failure calculations.

Lincoln [24] fitted the crack size distribution from the B-707 tear down inspection data using a Weibull distribution. The Weibull distribution was fitted only to the largest crack sizes to ensure a good fit could be obtained for the largest crack size. Similar work was performed again by Lincoln [10] in a structural risk assessment on USAF trainer aircraft. The lognormal and Weibull probability distributions were used to try to fit the inspection data. The Weibull distribution proved to be significantly superior to the lognormal distribution. The Weibull parameters were derived based on the largest crack sizes for the large cracks are the only ones that significantly contribute to the probability of failure, so consequently the small crack portion was ignored.

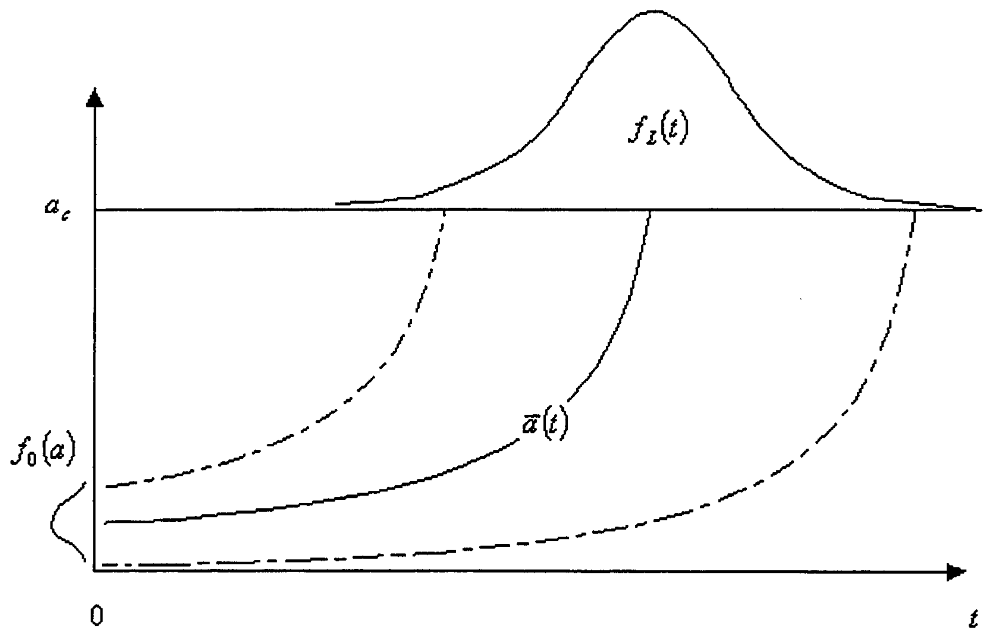

Figure 1: The process showing the compatibility between TTCI and EIFS distribution.

However, these risk assessments looked at predicting the risk of failure of the airframe over a future period of operation, where inspection or maintenance procedures were not incorporated in the assessment. In the case when inspection and maintenance techniques are involved, this approximation to the crack size distribution may cause considerable inaccuracy to the failure probability prediction. This is because although the large crack sizes of the population dominates the failure probability at the beginning, with the application of inspection and repair, the large crack sizes are often eliminated within the first few inspections. In the long term, the small crack sizes may have the most dominant effect on the failure probabilities.

Yang *et. al.* [25] recognised that the initial defect size at critical structural details in aircraft components is affected by many factors, including the material quality, manufacturing quality, notches, notch geometry, stress gradient, structural geometry, crack geometry, etc. As a result, they have proposed a mechanistic-based analysis methodology for determining the EIFS distribution for any given critical location, based on limited experimental test results, such as S-N data. This methodology takes into account the size (volume) effect so that the EIFS distribution for different critical locations can be established from test results using different specimen sizes and geometries. High stress regions (determined by high stress gradient) have a significant effect on EIFS. The stress gradient effect has also been accounted for in the analysis methodology so that test results from smooth specimens can be used to determine the EIFS distributions at notches. This is an extremely useful technique for estimating EIFS distribution when detail experimental data are not obtainable.

4.3 Variability in the Crack Growth Rate

The variation of fatigue life between identical specimens subjected to identical loadings is one of the well-known stochastic variations of fatigue process and it is one of the most important source of variation for determining the probability of failure due to fatigue in structural risk and reliability assessments.

There are many factors that can contribute to the variability in crack growth rate. However, these factors can be summarised in two categories, 1) variation in material properties and 2) variation in service conditions. However, it must be kept in mind that the variability in experimental data on fatigue crack growth kinetics reflects contributions from material property variations, environmental and other uncontrolled variables. Wei *et. al.* [26] suggested that this variation is also affected by measurement precision used in determining the primary data and the subsequent data processing procedures in determining rates from primary data.

Several approaches had been employed to take into account the variability of crack growth rates in fatigue for structural reliability analysis by:

1) the estimation of the overall distribution of fatigue life and,
2) randomising the crack growth rate in the crack growth model (Probabilistic Fracture Mechanics Model).

These two approaches are discussed below.

4.3.1 Distribution of fatigue life

In the 1950s, the determination of this *spread* or *scatter* in fatigue lives played a vital role in the safe life philosophy to ensure that the probability of structural failure is acceptably low. This requirement led the variation of a component's fatigue life to be represented statistically. The research in this area can provide valuable information for estimating the fatigue life distribution for structural risk and reliability assessment when actual data are scarce. This is particularly important since a relatively large number of test results are required for determining a reliable distribution, when in reality, at most, only a few full scale specimens will be available for test. However, there are some questions on the direct use of scatter factors for representing the variation of fatigue lives in reliability problems. This is because scatter factors were conservative factors, or even over-conservative factors, that were taken to ensure the failure probability is small. In reliability problems, the variation of a random variable needs to be accurately presented rather than conservatively presented. Therefore, care must be taken on the use of such data and information as it can have a marked effect on the failure probability prediction.

From the literature, the most common assumption is that the time to failure distribution is lognormally distributed [27]. It was found that the standard deviation for tests on simple unnotched specimens is, in general, greater than that for tests on complex built-up structures of the same material. Table 2 has been taken from reference [27], indicating the variation of standard deviation with specimen type for aluminium alloys.

Table 2: Variation of Standard Deviation with Specimen Type for Aluminium Alloys

Type of Specimen	Standard Deviation of Log-Life
Simple, Unnotched Specimens	0.300
Built-up Aircraft Structures	0.176

Australia derived scatter factors on a fully statistical basis for their aircraft. Other national authorities utilised different values of scatter factors. The U.K. practice utilised a range of scatter factors for deriving the aircraft's safe life, and the selection of a specific scatter factor depends largely on the number of specimens tested. Table 3 lists the total scatter factor and the corresponding standard deviation of log-life used by Australia and the U.K. Note, if the scatter factor ensures that less than 1 in 1000 failure over the safe life of the fleet, then the scatter factor and the standard deviation of the lognormal distribution is related by a simple equation shown in Table 3 and also reference [27].

Virkler *et al.* [28] considered six distributions to describe the variability in da/dN. The test was performed on centre-cracked panels of 2024 – T34 aluminium under constant amplitude loading. The three-parameter lognormal distribution was shown to fit the crack growth data the best.

Table 3: Fatigue life scatter factors and standard deviation for fatigue log-life for Australia and U.K.

	Australia		U.K.
	Canberra	Mirage	Av.P.970
Total Scatter Factor, L_S	5.7	7.2	3.33 to 5.0
Standard Deviation of Log-life, = $\log(L_S)/3.10$	0.244	0.277	0.169 to 0.226

Although the lognormal distribution is the one commonly assumed for airframe structures, others have been suggested. Perhaps the main alternative to the lognormal distribution is the Weibull, also known as the extreme value distribution of the Third Type [27] and it is commonly used by engine component manufacturers for representing the fatigue life distribution of components [29]. Weibull[30], Freudenthal[31,32], Gumbel[33] also suggested that the Weibull distribution better agrees with physical consideration of fatigue failures than the lognormal distribution. Reference [34] displayed a technique on the used of the three-parameter Weibull distribution for estimating the fatigue life distributions.

Smith, Saff and Christian [35] suggested a method for determining the fatigue life distribution from field service inspection data. Inspection data are reported for each critical point in the aircraft. The data will indicate either a crack of a specific length or no crack. The crack length may be either less than, equal to, or greater than the critical size for that location. Non-critical length cracks are projected to failure using the crack growth characteristics for that location to find the life when it will be at critical length. Greater than critical length cracks are projected back to determine the life at failure, that is when it was at critical length. These projected points, along the critical length cracks are used to determine the failure distribution and a three-parameter Weibull distribution was used to statistically represent the randomness of these data. Figure 2 displays this technique. It can be noted that this technique is similar to the equivalent initial flaw size technique discussed in section 5.1.1.

4.3.2 Stochastic Crack Growth Modelling

In structural risk and reliability assessments the crack growth function is the primary input to allow future prediction of crack size, which in turn allows prediction of the failure probability. The traditional fracture mechanics analysis assumes a deterministic crack growth function, although in reality, crack growth rates also exert a large stochastic behaviour, which form the fatigue life distribution at the critical crack size. Several researchers have proposed stochastic crack growth rate models and results showed good correlation with many tests results.

Yang [36] took the statistical variability of crack growth rate into account by randomising the crack growth rate equation, proposed a general lognormal random process model. The approximation has been found to be extremely effective for representing crack growth from fastener hole specimens under spectrum loading. The randomised crack growth rate equation is given in the form of:

$$\frac{da(t)}{dt} = X(t)Qa(t)^b \qquad 3$$

where a is crack length, t is flight hours, da/dt is the crack growth rate, Q and b are constants which depend on material and spectrum loading. $X(t)$ is the stochastic process with a mean equal to unity.

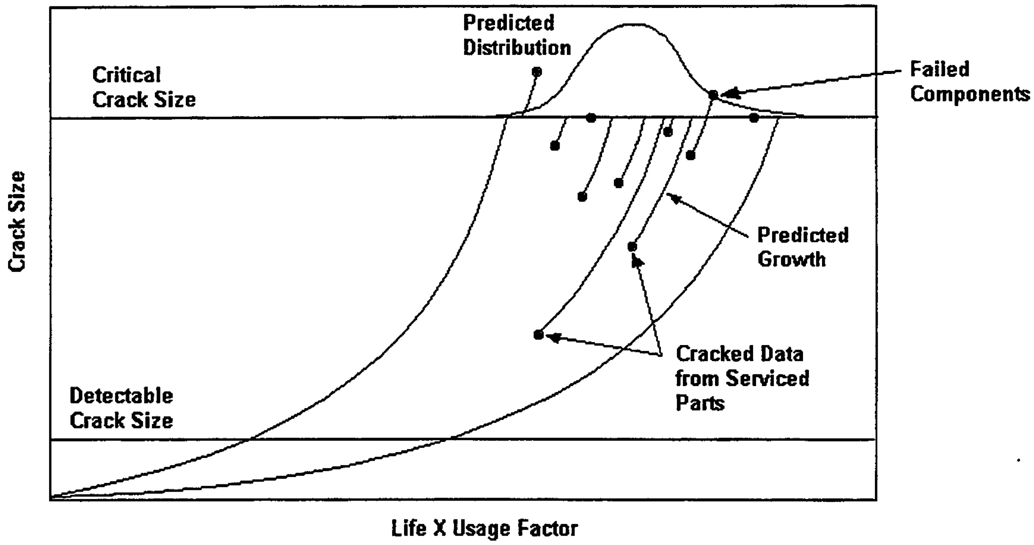

Figure 2: Determination of failure distribution from field service inspection data.

Yang *et al.* [36,37,38,39&40] have also developed probabilistic crack growth models for engine components subjected to either constant amplitude load histories or block type spectrum loadings, using the Pratt and Whitney Aircraft hyperbolic Sine (SINH) crack growth rate model given in the form of:

$$\log\left(\frac{da}{dN}\right) = C_1 \sinh\left[C_2\left(\log(\Delta K) + C_3\right)\right] + C_4 + Z(X) \qquad 4$$

where ΔK is the stress intensity range, da/dN is the crack growth rate, C_1 is a material constant and C_2, C_3 and C_4 are a function of temperature T, loading frequency v, stress ratio R. $Z(X)$ is a randomised parameter with zero mean. Yang [36,37,38] assumes that the crack growth rate followed a lognormal distribution so the randomised parameter was assumed lognormally distributed. Integrating these curves results in a set of crack size, a, vs cycle, n, curves which describe the distribution of time to a given crack size [36].

4.3.3 Usage Severity

For airframes and engine components, the variability of the operating spectrum in the fleet is not as significant as for helicopter usage. Aircraft life varies with the severity of the usage, therefore the number of flight hours for a particular aircraft must be modified by its usage factor to obtain a normalised life which can be compared with that from the other aircraft in the fleet [35]. In some cases, the extreme service conditions are utilised for representing the aircraft fleet to ensure the prediction contains some degree of conservatism for the fleet. Therefore, a single usage spectrum is often adequate for representing fleet wide usage, and hence a single Stress Exceedance Function is sufficient. This conservatism is often seen as poor fleet management by many aircraft operators. This is because the exceedance function is a direct input for deriving the master crack growth function, and for this reason, significant error may result in the prediction of future failure probabilities. However, this technique provides conservative results and is able to avoid a further probability distribution function to be applied on the exceedance function or individual aircraft tracking systems.

For the safe life philosophy of helicopter component manufacturers, this technique of the most severe spectrum is still being utilised to implicitly achieve the 9_6 reliability. This has been recognised as a major cost burden for helicopter operators and results in strong support for damage tolerance philosophy to be introduced for helicopter components. However, helicopter tracking is essentially non-existent, which is the greatest barrier towards the crack growth modelling of helicopter components. However, the procedures in developing the stress spectrum for helicopters are similar for the airframe and rotor systems [41].

4.3.4 Probability distribution of stress

The number of stress exceedances per flight hour/flight function gives the probability of exceeding a given stress at a critical location in a single flight hour/flight. The exceedance function is often used as input for damage tolerance analysis and is therefore readily available for airframes. For gas turbine engine components and helicopter rotor components, this may not be readily available. It is even more difficult for helicopters, since tracking systems virtually do not exist on helicopter components.

From the literature, several methods had been utilised in constructing the probability exceedance function and these methods derive slightly different probability exceedance functions. One method was proposed by Lincoln which was demonstrated in reference [10] and utilised for the USAF Joint STARS B-707 assessment [24]. Another method is utilised in the methodology of NERF [42] and it is similarly purposed by Hovey *et al.* [5] and utilised in the PROF methodology.

The method for describing the probability of stress exceedance function in NERF and PROF is based on statistical techniques. The method described by Lincoln assumes that stress levels occurring once or more per flight have a probability of exceedance of 1, truncating exceedances greater than one to equal one. No real probabilistic techniques were used for this transformation. The comparison of the probability exceedance functions of these two methods showed that the method presented by Lincoln is more conservative over the method proposed by NERF and PROF. This comparison can be found in Appendix Appendix A: .

4.4 Reliability of the Inspection

The simplest structural risk and reliability assessments do not require maintenance or inspection information for their analysis. Thus the failure probability simply provides an indication or a prediction of risk of failure as a function of operating hours for the given operating conditions. It provides the information regarding the number of operating hours required from the initial evaluation state to reach an unacceptable risk of failure, allowing prediction to be made for the forthcoming inspection or replacement schedule for the fleet.

Structural risk and reliability methodologies can be readily extended for evaluating the effects of inspections, repair and replacement on the failure probability of the aircraft or fleet. The additional output capability from such analyses includes the percentage and size of cracks found, the effect of maintenance on the aircraft or fleet failure probability and even long term maintenance plan if reliable input data allows.

Two input functions are necessary for extending structural risk and reliability assessment to allow evaluation with inspection, repair and replacement techniques in the current methodology. They are (1) the Inspection probability of detection (POD) and (2) the equivalent repair crack size distribution. In this section, the literature that addresses these two areas is discussed.

4.4.1 Inspection probability of detection

In the fail-safe and damage tolerance philosophy, the crack size at 0.90 probability of detection with 95 percent confidence was used to define and rate the capability of an inspection technique. This 0.90 probability of detection with 95 percent confidence was somewhat arbitrary selected at the time of establishment to define the inspectable (detectable) crack size [44]. Even today, researchers continue to face difficulties in establishing this crack size for the particular NDI technique due to the numerous factors contributing large influences in its validation. Such factors are the differences in laboratory and depot (or field) conditions, but the one that is most difficult to justify is the human factor influence.

The probabilistic approach, which is currently gaining new interest, is particularly difficult because the complete probability of detection (POD) function must be determined [44]. The choice of POD function can have a marked effect on the failure probabilities after inspections and the expected number of cracks to be found. The literature, such as references [45 & 46], contains a large database of POD curves evaluated for a number of different NDI techniques that were performed for various geometry of laboratory specimens. These data contain valuable information on the detection capability of a NDI technique under a given condition. However, the POD function for a NDI technique is different when inspection is conducted at a different location and different geometry.

This difficulty in obtaining a representative POD function for a particular NDI technique and for a given location forces approximation techniques and functions to be developed [47,48,49]. The "Log-Odds" function is one which has been developed, and it has been widely accepted for approximating POD functions. Several researchers [12,39,47] have shown that this function is very useful in representing POD functions. Its form is shown in the equation below:

$$F_D(a) = POD(a) = \left\{1 + \exp - \left[\frac{\pi}{\sqrt{3}}\left(\frac{\ln(a - a_{\min}) - \mu}{\sigma}\right)\right]\right\}^{-1} \qquad 5$$

where 'a' is the crack size, 'μ' is the natural logarithm of the median detectable crack size 'a_{50}' minus the minimum detectable crack size 'a_0 or a_{min}', ie $\mu = \ln(a_{50}\text{-}a_0)$, and '$\sigma$' is the scale parameter.

An example of where this POD approximation function has been utilised is in the '*PROF*' software package [5] for structural risk assessment and it has also been utilised by Artley [12] and Yang [39]. However, care must be taken when applying this function to ensure some degree of conservatism is taken into account, for example, in the case when this function is required to approximate an actual POD function. A weighted-fit to the actual POD function must be used to ensure that the most dominant crack sizes and their respective probability of detection are fitted with high accuracy. Since the largest crack size is the dominant driver of the failure probability, the analyst needs to recognise that the error caused by over or under estimating the POD function decreases with decreasing crack size, and it is important that the probability of detection of the larger cracks is approximated well. The "Log-Odds" function given in Eq. (4) can be reparametrised to provide a linear form in transformed variables:

$$F_D(a) = \frac{\exp(\alpha^* + \beta^* \ln(a))}{1 + \exp(\alpha^* + \beta^* \ln(a)} \qquad 6$$

where α^* and β^* are constants derived from maximum likelihood estimators. The parameters μ and σ in Eq. (4) and α^* and β^* are related by;

$$\mu = \frac{-\alpha^*}{\beta^*} \qquad \text{and} \qquad \sigma = \frac{\pi}{\beta^* \sqrt{3}}$$

Another analytical POD distribution proposed by Lewis [51] follows a three-parameter Weibull distribution function.

$$POD = 1 - \exp\left\{-\left[\frac{a - a_0}{\lambda - a_0}\right]^{\beta}\right\} \qquad 7$$

where a_0 is the minimum detectable crack size, λ and β are regression parameters. In general, this function, shown in equation 7, cannot provide the adequate representation for POD functions that are typically used for aircraft structural reliability problems.

It is important to appreciate the information that is provided by the POD curves. The POD diagram is the result of a large number of NDI measurements. They give useful information on the sizes of cracks, which can be found and about the sizes of cracks that could be missed [50]. It also gives information about possible scatter. This graph is a kind of a certificate on the performance and the merits of the NDI inspection technique for the relevant NDI environment [51].

4.4.2 Quality of Repair

Once a crack is detected on inspection, it is either repaired or remains in service and is repaired at a later maintenance time if damage accumulation is allowed. Both cases require the crack to be repaired sooner or later to prevent the crack from reaching a crack size that can cause catastrophic failure. Flawless repair is practically impossible to achieve. Repair removes the crack tip of the propagation crack yet it often leaves behind potential initiation sites for cracks to grow on future operation. For this reason, assuming perfect structure after an inspection/repair would create some unconservatism. The risk analysis computer software, "NERF' [42], utilises this technique. It assumes that the population of cracked structures when detected are replaced by perfect structures so further crack growth is not thereby possible, removing them from the analysis. It is seen as a deficiency in the NERF methodology.

Hovey *et. al.* [5,6] utilised the equivalent repair quality to quantify the possible repair flaws inherited in the structure after repairs. This quantification technique is known as the Equivalent Repair Crack Size distribution, and it is analogous to the Equivalent Initial Flaw Size distribution concept as discussed in section 4.2.1 and 4.2.3. Using this technique, one can assume a perfect repair by restricting the repair crack size distribution to extremely small flaw sizes. In some cases when the damaged components are replaced by new components regardless of crack sizes, such as those for rotor discs of gas turbine engines and helicopter dynamic components, an equivalent initial quality can therefore be used to define the repair crack size distribution. Similarly, if the repair quality is found to be of poor quality, the repair crack size distribution is capable of characterising this quality by assuming a large proportion of residual flaws still remains.

Although, the effect of the repair crack size distribution is not immediate, it can have a major effect on the failure probability of the structure and the number of cracks detected at later inspections of the aircraft.

4.5 Discussion

Section 4.1 - 4.4 described and reviewed several common stochastic variables that are of major influence in structural fatigue. In general, risk analysis methodologies remove the use of scatter factors by introducing known variation of material properties, loadings and the variation of structural damage. The purpose of structural risk and reliability analysis requires that these stochastic variables be modelled representatively. Often, the collection of representative data for the modelling of the variables makes up the most important and the most time-consuming part of the risk and reliability assessment.

5. Risk and Reliability Analysis Methodologies

There are many stochastic elements in the reliability assessment of aircraft structures due to fatigue. Some of the major stochastic elements were briefly mentioned in section 4. To simulate the failure probability, the appropriate probabilistic model must be employed so that it can take into account the influential elements.

Many probabilistic analysis methodologies are generic and are therefore applicable for assessing the probability of failure of aircraft components. Examples of four commonly used probabilistic analysis techniques are the strength-load interference method, Conditional Reliability technique, the FORM/SORM and Monte Carlo simulation. In this section, a brief review of the four most common methods and their advantages and disadvantages are presented. More detail of these reliability techniques can be find in reliability texts [13,52].

5.1 Strength-Load Interference Method

The basic structural reliability problem considers only one load effect S resisted by one resistance R. Each is described by a known probability density function, $f_s(\sigma)$ and $f_R(\sigma)$ respectively. The structural element will be considered to have failed if its resistance R is less than the stress resultant S acting on it. This is known as the strength-load interference method.

The *'Numerical Evaluation of Reliability Function - NERF'* [42,53] computer software package is an example of a package that utilised this probabilistic technique in evaluating the failure probabilities due to structural fatigue. It is capable of addressing the variability in fatigue life, residual strength, initial flaw size distribution and inspections.

The probability of failure p_f of the structural element can be stated in any of the following ways:

$$p_f = P(R \leq S) \quad (8)$$

$$= P(R - S \leq 0) \quad (9)$$

$$= P\left(\frac{R}{S} \leq 1\right) \quad (10)$$

The failure probability becomes:

$$p_f = P(R - S \leq 0) = \int_D \int f_{RS}(r,s)\,drds \quad (11)$$

where $f_{RS}(r,s)$ is the joint probability density function of f_R and f_S, and D describes the failure domain.

When R and S are independent, $f_{RS}(r,s) = f_R(r)f_S(S)$, then (*11*) becomes:

$$p_f = P(R - S \leq 0) = \int_{\infty}^{\infty} \int_{\infty}^{s \geq r} f_R(r) f_S(s) dr ds \qquad 12$$

where (4.1.2) can be further written in the single integral form

$$p_f = P(R - S \leq 0) = \int_{\infty}^{\infty} F_R(x) f_S(x) dx \qquad 13$$

This is also known as a 'convolution integral'. Figure 3 graphically demonstrates this method. Analytical solutions of well-known probability density function are often used for the representation of the stress and strength distributions to allow direct integration for determining the failure probability. With the high powered computing capability available today such procedures have become unnecessary, where direct numerical solution procedure using tabulated results can be used to compute the failure probability and removing the assumptions in fitting an analytical expression.

This method of failure probability analysis is one of the oldest methods in structural reliability analysis, and it continues to be popular due to its simplicity and its ease of use. The major disadvantage is the assumption that strength and load are statistically independent, which may not be valid for some problems when load redistribution is a known effect.

5.2 The Conditional Reliability Technique

The conditional reliability technique is very similar to the 'Strength - Load Interference Technique'. The difference is that the conditional reliability technique recognised that the fundamental variables are defined not only in the form of load and resistance, but also characteristics, for example in terms of fatigue problems, such as fatigue properties, initial damage state, fatigue life or crack growth rate etc. In addition, the limit state function is not always clear cut. In other words, the distinction between failure and safety is not defined with a clear boundary and nor is it as simple as 0 or 1 for safe or failure respectively, where values between zero and one may be more representative.

An example of the structural risk assessment methodology that utilises this technique is the '*Probability of Failure - PROF*' [5,6] computer software package developed by the USAF. The generalised form of the probabilistic formulation is:

$$p_f = P\{\overline{H}[\sigma(\mathbf{X})]\} = \int \int_{\mathbf{X}} \{\overline{H}[\sigma(\mathbf{X})] \cdot f_{\mathbf{X}}(\mathbf{X})\} \cdot d\mathbf{X} \qquad 14$$

where $\overline{H}(\;) = 1 - H(\;)$ is the *conditional* probability of failure function defining the violation of the limit state, $\mathbf{X}$ represents all the basic variables involved in the problem and $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability for the n-dimensional vector $\mathbf{X}$ of basic variables. Reference (5) presents an illustration and detail explanation of such problem.

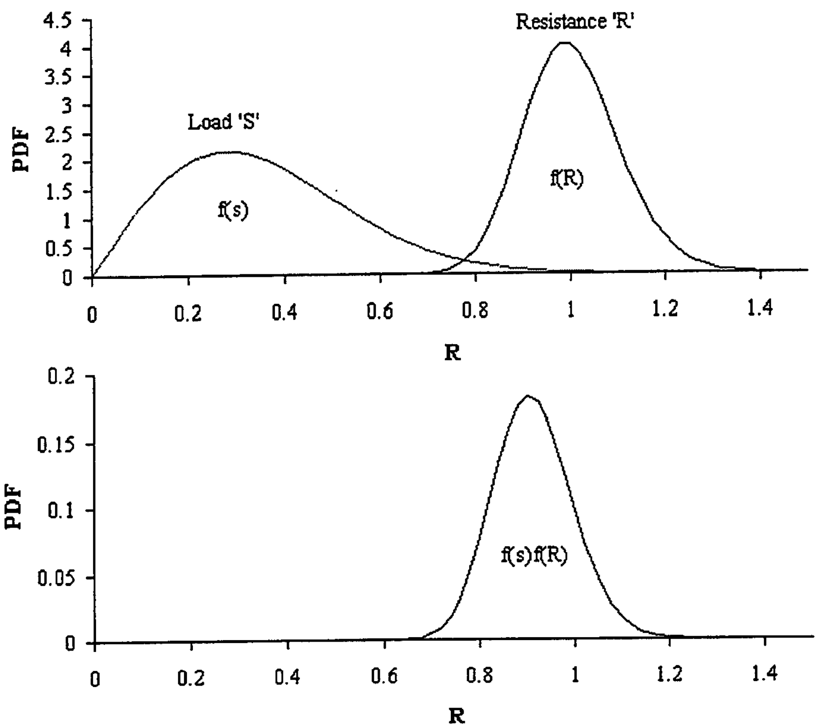

Figure 3: The joint probability density function ($f_R(R)f_S(S)$) of f_R and f_S.

5.3 Monte Carlo Simulation

The Monte Carlo simulation techniques involve 'sampling' at 'random' to simulate artificially a large number of experiments and to observe the result. In the simplest approach, it involves sampling each random variable X_i randomly to give a sample value $\hat{x}_i$. The limit state function $G(\hat{x}_i)=0$ is then checked. If the Limit State is violated, the structure or structural element has failed. The experiment is repeated many times, each time with a randomly chosen sample value. If N trials are conducted, the probability of failure is given approximately by

$$p_f = \frac{n(G(\hat{x}_i)\leq 0)}{N} \qquad 15$$

where $n(G(\hat{x}_i)\leq 0)$ denotes the number of trials n for which the limit state is violated (i.e. $G(\hat{x}_i)\leq 0$).

It is clear that in the Monte Carlo simulation a game of chance is constructed from known probabilistic properties in order to solve the problem many times over, and from that to deduce the required result, ie. probability of failure.

Obviously the number N of trials required is related to the degree of accuracy and degree of confidence level required for p_f. Broding *et al.* [13] suggested that a first estimate of the number N of simulations for a 95% confidence level in the failure probability must be three times greater than the inverse of the cumulative failure probability. Thus, for 95% confidence level and P_f = 10^{-3}, the required number of simulations is more than 3,000, ie:

$$N > \frac{3}{10^{-3}} = 3 \times 10^3 \quad (3{,}000) \qquad 16$$

Other authors have suggested an even greater number of simulations, but it also depends on the function being evaluated [54]. In many cases, the large number of simulations required in order to obtain a certain degree of accuracy made the Monte Carlo method unattractive. In principle, Monte Carlo methods are only worth exploiting when the number of trials or simulations is less than the number of integration points required in numerical integration [13].

Yang *et. al.* [21] demonstrated this simulation technique, used to assess the probability of failure of jet engine discs. The purpose of this assessment was to determine the fatigue reliability of gas turbine engine discs under scheduled inspection maintenance in service for supporting the engine Retirement For Cause philosophy.

Rohrbaugh *et. al.* [55] also utilised the Monte Carlo simulation to simulate the failure probability of fastener holes in flat panels such as those for aircraft fuselage lap joints. A computer code for risk assessment of aircraft structures developed by Cavallini *et. al.*, ref. [56], also utilises the Monte Carlo simulation to determine the probability of failure. The computer code is called "Probabilistic Investigation for Safe Aircraft, - PISA".

5.4 First Order/Second Order Reliability Methods

The 'Second-Moment' methods are approximation methods which are very popular due to their inherent simplicity. In this technique, the probability density functions of each variable are simplified by representing them only by their first two moments (ie, mean and variance), and hence, it is denoted the 'Second-Moment' method. Higher moments, such as skewness and flatness of the distribution, are ignored. This type of representation ultimately assumes the probability density function of the random variables can be described by the normal distribution function, since the first two moments only describe the normal distribution exactly. Then a further useful step is to transform these variables to their standardised normal distribution with zero mean and unit variance. This transformation and approximation of the random variables via standard normal distribution completely simplifies the integration procedures in determining the failure probability, and hence, all the useful properties of the normal distribution function can be utilised. The 'Limit-State' is often non-linear, but as it can be linearised to allow further simplification, the technique is known as the 'first-order' method. Thus, the First Order Limit State and Second Moment random variables approximation techniques are brought together to give the First Order Second Moment (FOSM) reliability method, and it is the basis of this reliability method.

Evidently, this technique yields the exact probability of failure p_f when 1) the random variables are normally distributed and when 2) the Limit-State is linear. However, it is not the case in most reliability problems. Therefore, the failure probability determined from this method is commonly taken as the nominal failure probability rather than the 'true' probability of failure. However, it has been proven that this reliability method can provide good approximation of the probability of failure in most cases, and only in rather extreme situations has this approximation technique been seen to fail [13].

The extensions of the Second Moment and Transformation method offers a range of modelling techniques and transformation techniques for overcoming the disadvantages seen in the FOSM method and hence improving its approximation accuracy. These include the extended FOSM, also known as the 'First-Order' Reliability (FOR) method which allows non-normal distributed functions to be incorporated into the technique. Also the 'Second-Order' Reliability (SORM) method does not require the limit state function to be linearised. Further details of these techniques can be seen in Melchers [39].

The FOSM/FOR/SORM technique advantage over the numerical integration and Monte-Carlo simulation is clearly its simplicity in calculating or approximating the probability of failure. In addition, there will be many basic random variables describing the structural reliability problem. In many cases, n could be very large indeed. Evidently this will create a problem for integration methods. However, this is not so critical for the first order second moment (FOSM) technique.

6. Applications to Aircraft and Helicopter Components

The previous sections described and discussed some of the commonly used techniques and methods for assessing risk and reliability problems for airframe and aircraft structural components. It can be realised that the mathematical background of an aircraft structural risk and reliability analysis is no different to any other reliability problems. The methodologies in assessing structural risk and reliability of aircraft structures, engine components and helicopter components are generically similar. However, the selection of model must be able to sufficiently and realistically model the problem and be able to take into account the important sources of variations. This is particularly important because the operating conditions for airframe structures, engine components and helicopter components may be vastly different, where one source of variation may have insignificant effect on the life of an aircraft structure but have a detrimental effect for engine components and/or helicopter components. It is important that these sources of variations are identified and modelled for a particular assessment. Fracture mechanics analysis and experimental fatigue tests are capable of identifying and addressing some of these sources of variations.

The purpose of this section is to identify some common variables that need to be addressed in the structural risk and reliability assessment for airframe structures, engine components and helicopter components.

6.1 Airframe Structures

Damage tolerance analyses on airframe structures have been the method of design and ensuring safety of airframe structures for over 25 years. Damage tolerance analysis accompanying the Aircraft Structural Integrity Program (ASIP) was able to identify the critical locations in the airframe structure and make fatigue life predictions and inspection intervals to ensure catastrophic failure does not occur.

The predominant problem with fixed wing aircraft is fastener holes. In the literature of structural risk and reliability assessment of airframe structures, considerable attention had been placed on fatigue of fuselage lap joints and wing lower skin-stringer attachments, given by examples in references [2,5,6,7,10,19,24,36,58,59,60,61]. Particularly, references [2,5,7,10,24,58,61] deal with the application of structural risk and reliability assessments on realistic airframe structures.

Damage tolerant designed airframe structures are designed to sustain significant amount of macro-cracking. It has been observed frequently that different initiation sites occur at several critical locations in the same structure or component, such as those at the stringers-skin and fuselage lap joint attachments. This effect has been termed widespread fatigue damage and it has been shown to cause a significant decrease in the residual strength of the structure. The detrimental effect of widespread fatigue damage commonly referred to as multiple site damage (MSD) has become an area of interest in the recent literature by many researchers such as, Hovey *et. al.* [6], Rohrbaugh *et. al.* [55], Cavallini *et. al.* [56], Brot [59] and Balzano *et. al.* [62]. Rohrbaugh *et. al.* [55] investigated the effect of MSD in simple panels with various numbers of fastener holes using probabilistic (Monte Carlo) technique. His results showed MSD significantly reduces the fatigue life of the components. Hovey *et. al.* [6] indicated that risk and reliability analysis has a significant application for predicting widespread fatigue damage in airframe structures.

In the case of inaccessible structure and fail-safe structure, redundant load paths are designed to allow load redistribution in damaged structure so safety can be assured until the next inspection. The load redistribution effect has not been accounted for in the literature. However, redundant structure has a load reduction effect, which is considered conservative in neglecting its effect.

Corrosion/corrosion fatigue is another common problem in airframe structures. Corrosion often acts as an acceleration agent for crack propagation or promotes crack initiation sites for fatigue crack growth. This effect will be discussed in more detail in section 7.1.

6.2 Engine Components

Rotor discs, which include compressor fan discs and turbine discs, of a gas turbine engine are the major areas of concern for engine structural integrity. Traditionally, rotor discs are limited by their design life, which is commonly defined by the time to reach a cumulative failure probability of 0.001. Under this approach, the useful life of rotor discs is not utilised effectively. The enormous replacement cost created a huge cost burden to aircraft operators. The USAF employed the Retirement For Cause (RFC) approach and later adopted the Engine Structural Integrity Program (ENSIP) in an attempt to resolve this problem. The latter approach utilised the damage tolerance approach for predicting crack growth rates in engine components, which forced a better understanding of the material properties, stresses and operating conditions of these engine components. A specification on engine damage tolerance requirements, listed as MIL-STD-1783, has been developed.

This approach to engine components life assessment can be considered as a recent development and has not been widely adopted by most aircraft operators. The potential of this approach had resulted in a research effort called the AGARD Engine Disc Cooperative Test Programme [29], constructed to determine the fatigue behaviour of engine discs materials under realistic engine conditions.

Considerable research contributions have been made by other researchers, such as Yang *et. al.* [21,63,64,65,39&40], and he stated [65] that crack propagation of gas turbine engine materials under operational environments is different to that of aircraft structures in the following aspects;

(1) the statistical distribution of the crack growth damage accumulation is influenced by many parameters such as temperature T, loading frequency v, stress ratio R, holding time T_h, etc.

(2) the number of specimens tested under a single environmental condition mentioned in (1) is very small, and

(3) a homogenous data base does not exist, since each specimen was usually tested using different specimen geometry, maximum load, initial crack size and final crack size.

In the literature, to resolve the lack of data problem, data-pooling methods are commonly utilised [66] to estimate the crack growth data for engine material.

Pratt and Whitney Aircraft, General Electric Co. and Yang [39] have proposed an analytical formulation for predicting crack growth rates for engine components by taking into account the influential parameters, such as listed in (1) above. Yang [39] had taken these formulations a step further to propose probabilistic crack growth rate equations to account for the stochastic effects of crack growth. These techniques had been demonstrated to show good agreement with the experimental results

Pratt and Whitney Aircraft utilise the hyperbolic sine (SINH) model to describe the crack growth rate data. The stochastic crack growth rate equation was proposed by Yang [39], and it was previously described in section 5.3.2.

Yang et. al. [39] proposed another stochastic crack growth model by modifying the Paris model. The statistical crack growth model is shown in equation (6) [39]:

$$\frac{da}{dt} = ZQa^{b} \qquad 17$$

where Z is a random variable accounting for the contributions to the crack growth rate, given by:

$$Z = H_1 H_2 H_3 H_4 S^{V} \qquad 18$$

where H_1, H_2, H_3, H_4, and S are random variables denoting the contributions to the statistical variability of material properties, crack geometry, crack modelling, crack growth damage due to equivalent cycle and service loads, temperature, stress ratios and holding time respectively.

For components with low cycle fatigue life (LCF), such as gas turbine engine components, the initial fatigue quality (IFQ) in the fatigue analysis is particularly important. The initial fatigue quality can be represented by the statistical distribution of the Time-To-Crack-Initiation (TTCI) model, as suggested by Yang *et. al.* [39,40]. The Equivalent-Initial-Crack-Size (EIFS) model can be difficult, because the quality control of engine components is better than airframe components [41].

6.3 Helicopter Components

The current design philosophy for helicopter rotary component is still based on the safe life approach, where manufacturers impose a life limit on their production parts. Similarly to engine operators, owners of helicopters are experiencing a significant cost burden for replacement parts. It was mentioned previously that helicopter manufacturers continue to use a fatigue methodology based on Miner's rule of linear cumulative damage. Most helicopter manufacturers assume that at the endurance limit the fatigue strength is normally distributed and use a "working" S-N curve that is three standard deviations below the mean to describe the fatigue strength of the material. In addition, they assume an extreme load spectrum, which is three standard deviations above the mean, implicitly achieving the reliability of 9_6 at the retirement life of their components. Retiring 999,999 components due to the possibility of a single component failing is very wasteful. This method is the so-call "*TOS/μ–3σ*" methodology. However, it can be readily shown that this 9_6 reliability is simply describing the probability that the assumed fatigue property (working S-N function) and assumed load spectrum (TOS load spectrum) occurring or exceeded is less than 1 in one million. It does not describe the reliability regarding the fatigue life of the population of the components.

The "*TOS/μ–3σ*" methodology is no real reliability method. To describe this technique correctly, it simply replaces the use of scaling factors and safety factors by employing statistical distribution functions to determine the conservative fatigue property and the loads spectrum experienced by the component so the fatigue life determined is conservative for the population been considered.

Continuous efforts had been made in an attempt to reduce the over-conservatism in the helicopter component life prediction. Zion [8] addresses this problem by utilising explicit statistical methods, incorporating the variables of fatigue strength, flight loads and usage to reduce the overconservative assumptions that were historically made on helicopter components by the "*TOS/μ–3σ*" methodology. Again, the disadvantage of this method is that it does not provide operators with an indication of the instantaneous risk of failure for better management of their helicopters or helicopter components.

The use of damage tolerance analysis for helicopter component design or structural integrity purposes is almost non-existent prior to recent times. In the USAF, Lincoln suggested that the success of the damage tolerance philosophy in airframe structures and engine component indicates a similar result could be obtained for helicopter components [41]. The introduction of the damage tolerance analysis to helicopter components would allow better understanding of the loading and stress on the component, hence possible optimisation of the design process for components. The establishment of the damage tolerance analysis on helicopter components is necessary for the reliability assessment of helicopter components to be carried forward. This is because in order to predict the probability of failure due to structural fatigue in future operations, damage tolerance analysis is necessary for providing an accurate crack growth function and residual strength analysis at the critical locations. Considerable efforts are being made in the industry in recent times on the damage tolerant design of rotorcraft components but progress is slow [67].

From his work, Lincoln [41] pointed out that the area of most deficiency for helicopter component damage-tolerance-assessment is operational data from service usage for stress spectrum development. Helicopter tracking data are essentially non-existent prior to recent time. Therefore, an enormous effort was made by the USAF to establish a baseline spectrum for their assessments. He also indicated that the work performed on the HH-53C and HH-60A helicopters by the USAF has shown that the damage tolerance approach is viable for helicopters and damage tolerance methods applied to helicopters will be as successful as they have been for fixed wing aircraft and engines. In addition, damage tolerance applied to helicopter components will allow a better understanding of stress and loading on these dynamic components, and allow optimisation of components that had originally a short fatigue life and shortening the life of components that had originally extremely long life to allow additional weight saving. The paper in ref [41] contained the discussion and recommendations that are given for future damage tolerance assessment of existing helicopters and on the incorporation of damage tolerance capability in new designs. However, it can be seen that a considerable amount of work would be required in establishing the capability for structural risk and reliability assessment of helicopter components.

Reference [68] presents a study which compared the original safe-life design to the theoretical damage tolerance design for four critical components. This study involved theoretical redesigns of the four selected components to potentially meet damage tolerance requirements. This study showed that the damage tolerance methodology is highly feasible for the certification of helicopter dynamic components.

Another deficiency in the application of helicopter structural risk assessment is the fact that macro-crack propagation represents only a negligible fraction of the life of components subjected to high - or very high cycle fatigue loadings. These components are subjected to near threshold stress intensity factors, where the period of short crack growth and mechanically short crack (Crack Initiation) dominates most of the components fatigue lives. The failure of linear elastic fracture mechanics (LEFM) in predicting crack growth in the short crack regime is a well-recognised problem and it is an area of interest for many researchers. This indicates that the current LEFM theory may not be adequate or sufficient enough in supporting helicopters components, and in general, dynamic component life assessment. A large effort is required before helicopter component risk assessment based on crack growth can be applied.

Before DTA can be adequate for assessing High Cycle Fatigue components, a traditional approach can be used to provide the necessary information for a risk and reliability assessment.

6.3.1 Probabilistic Model Based Cumulative Fatigue Damage

Rotorcraft dynamic component lifing has traditional been based on Miner's rule for cumulative fatigue damage, where components fail when the cumulative damage equal 1. When the cumulative damage is plotted against number of cycles, a cumulative damage function that is similar to the crack growth curve is obtained. This function is useful for predicting the damage in the component in terms of operating time. Fatigue test results have also shown that failure does not occur at exactly 1. Experiments and fatigue tests show that failure occurs randomly between 0.3 to 3, which can be statistically represented. This effect is by definition the fatigue life distribution.

Similarly, the EIFS distribution can be replace with an equivalent initial damage distribution. Finally, the residual strength function will need to be approximated as a function of damage instead of crack size. Providing, these information regarding the structure or the component are known, the application of risk and reliability assessments of dynamic component can be feasible. However, we must keep in mind the accuracy of the input functions and variables reflect the overall reliability of the risk assessment. The question that needs to be asked is how well does the cumulative damage model represent the real damage of the component due to service loading. This is an important question that needs to be answered before the output of this proposed probabilistic approach are utilised for realistic purposes.

7. Future Applications

The previous sections deal mainly with the application of structural risk and reliability analysis to structural fatigue. However, structural risk and reliability analysis is not restricted to predicting the probability of failure due to structural fatigue damage. It can be useful for evaluating the probability of failure of many other stochastic processes, such as corrosion and fretting, which are other detrimental problems in aircraft structural and component integrity. Incorporating Probability of Detection and maintenance into the risk assessment can provide useful output for estimating the cost of maintenance and for predicting the economical life for the aircraft. These are the areas that can be important tools for aircraft fleet management in the future.

In this section, a review is made on the three potential areas of risk and reliability assessment. They are:

1. Corrosion/Corrosion fatigue
2. Cost of Maintenance and
3. Economic life estimation.

7.1 Corrosion

The chemical and thermal environment to which a component is subjected can significantly influence crack growth under both static and cyclic loading known as stress corrosion cracking and corrosion fatigue respectively. Corrosion is another form of wear out process which is a common problem in ageing aircraft and has received considerable attention in the history of aircraft operation and in the literature.

In the USAF, the corrosion damage to ageing aircraft has been shown to cause the most significant cost burden of any structurally related item. It was found that the costs of corrosion to the USAF could be conservatively estimated at US$700 million per year [9]. This is the largest maintenance cost of any structurally related item and Lincoln [44] stated that corrosion damage is the major factor in the decision to retire an aircraft. The USAF Joint STARS B-707 [24] structural integrity assessment found that corrosion damage in the structure was more significant than predicted. Corrosion and corrosion fatigue damage in aircraft fuselage lap splice joints are critical areas which have caused some major concerns to investigators. Large efforts such as at the Institute for Aerospace Research of National Research Council Canada (IAR/NRCC) and the USAF for several years have investigated the fatigue characteristics of fuselage splices containing Multiple site fatigue damage (MSD) [23]. The objective of this work is to propose an accurate and cost effective methodology for probabilistic analysis of lap splices under corrosion damage.

Nevertheless, it is clear from the experiences of aircraft operation that corrosion has became an accepted economic problem and a cost burden, not a safety problem, for failures due to corrosion act purely as an accelerating agent for promoting crack growth and crack initiation sites. Failure caused by corrosion is almost non-existent in the past twenty years of aircraft failures. Therefore, the push for better corrosion detection techniques or corrosion analysis techniques has not been large over the past years. However, a recent sample of corrosion problems in USAF aircraft fleets has uncovered that corrosion damage can raise safety concerns in promoting corrosion fatigue damage in originally non fatigue critical areas [70]. From these studies it has been recognised that consistency with the damage tolerance and risk and reliability approach would be required for corrosion assessments to maintain safety [70,71].

As previously mentioned, corrosion can have a detrimental effect on the integrity of aircraft structures by promoting fatigue crack initiation and accelerated growth, and it deteriorates the strength of a critical component [69]. By recognising these mechanisms, it can be seen that the mechanism of corrosion and fatigue is very similar and can have an interacting effect on each other. For this reason, the interaction between fatigue cracking and corrosion from a quantitative point of view needs to be well understood. This form of wear out process can be treated similarly to that of fatigue crack growth, where the corrosion or corrosion fatigue data could be expressed in a crack growth rate format or the crack length versus time format. In this case the crack growth rate is expressed in units of length per time, instead of length per cycle as for fatigue. The reduction of area causes a strength reduction of the component and a similar residual strength can be derived for corrosion cracking. Grandt [60] has looked into methodologies for stress corrosion cracking. The data showed that the $\log(da/dt)$ versus $\log(K)$ curve assumes a sigmodal shape between a lower K_{ISCC} (ISCC denote Mode I stress corrosion cracking threshold) and upper K_C asymptote. These data could be represented by an empirical equation similar to that of fatigue crack-growth-rate equation. Different crack geometries and material property curves can be treated in a manner analogous to fatigue crack growth. Thus this approach can assess the interaction of corrosion and fatigue under realistic operating conditions.

Other approaches to investigate the effect of corrosion on the fatigue life of specimens are by simulating the effect of corrosion by percentage of thickness loss in the specimens [6,23]. Thus, this combination of corrosion and fatigue assumes that corrosion/fatigue interaction occurs only in the context of pre-existing corrosion and in dry specimens, and it has been suggested to be a reasonable approximation [23]. Some Boeing service bulletins allow operators to continue operating the aircraft with corrosion in lap joints provided that it is less than 10% of the original sheet thickness.

Undoubtedly, the rate of corrosion suffers from a stochastic process and can be treated probabilistically. There are many variables that can have a significant influence on corrosion cracking or corrosion fatigue. Examples are the environment, temperature, material, cyclic frequency etc and research has shown that the scatter is considerably larger for corroded specimens than for non-corroded specimens. These sources of variations can be treated as random variables and take part as input variables in the reliability analysis. Koch *et. al.* [69] showed that pitting initiation is a statistical process, and the number of pits formed on the surface occurs in a random fashion and can be described statistically by a lognormal distribution. Again it is important to keep in mind that techniques used to represent the problem realistically is the key to reliability analysis.

Structural risk and reliability analysis due to the effect of corrosion has not been considered widely for ASIP purposes. It can have significant potential in reducing the cost of corrosion damage in airframe structures. Hovey *et. al.* [6] attempted a structural risk assessment on the fatigue on lap-joint specimens incorporating some effects of corrosion into analysis using the updated structural risk assessment software, PROF [6]. Their techniques modelled corrosion severity in terms of various levels of uniform percentage corrosive thinning, requiring proportionate adjustments of the stress levels, crack growth predictions and residual strength analysis [6]. This analysis required multiple runs of the updated PROF for the various combinations of scenarios and the total failure probability is the combined probability of failure of the proportion of the population under each scenario. In general, their results displayed the somewhat non-conservative risks of the predicted failure probabilities in comparison to the observed results. This example clearly demonstrated that it is possible to extend structural risk and reliability analysis to include the effect of corrosion fatigue and in general, other probabilistic descriptions that influence fatigue life. However, further research is required to ensure conservative probabilistic prediction could be achieved.

7.2 Cost Analysis & Risk Management

The new challenge raised from ASIP is the need to maintain aircraft flying longer in a condition with reduced funding. The cost of maintenance includes the costs of failure, inspection, repair, rework, replacement, etc., for aircraft structures and components in order to fulfil the requirements of safety, durability, damage tolerance, and reliability is of major importance [72].

The outputs of probabilistic analysis can provide information regarding the expected costs of completing maintenance scenarios. Such outputs are the probability of failure, the expected percentage of cracks detected and repaired, and the cost of NDI technique employed and estimates of downtime, which in turn depends on the extent of the repair. Providing the cost for each of these items is known, the cost for a particular maintenance procedure can be estimated. Completing this information for several maintenance strategies or scenarios will allow comparison of the estimate cost for each scenario or technique, assisting managers to determine the most economical and most effective maintenance decisions, regarding inspection timing and inspection methods whilst maintaining high level of safety. This comparison can be made for various types of NDI techniques, or inspection intervals or repair/replacement techniques. Figure 4 is taken from reference [5], demonstrating a typical example for cost analysis by varying inspection intervals over a period of operation (4000 hours). It clearly showed that having too short or too long maintenance interval is not cost effective, and an optimal maintenance interval, most cost effective, can be find using cost analysis.

The mathematical procedures to perform a cost analysis are relatively simple, and it can be found in detail in reference [5,73,74]. However, the cost of repair of fastener holes varies depending on the crack size. This is because the most common repair technique involves reaming fastener holes at increments of hole sizes until the crack is completely removed or no longer detected by NDI technique. Thus the cost of repair increases depending on the location, crack size and the number of hole size increments required before the embedded crack can be complete removed. This must be taken into account appropriately.

PRISM [75] is a commercially available probabilistic tool for aircraft fleet management, which addresses cost directly within its analysis procedures. It is capable of addressing cost associated with new and existing maintenance intervals, cost effectiveness and feasibility of replacing a component or by inspection and repair.

Although PROF [5,6] does not address cost directly however, its output provides all the necessary information for determining the cost of a particular maintenance scenario.

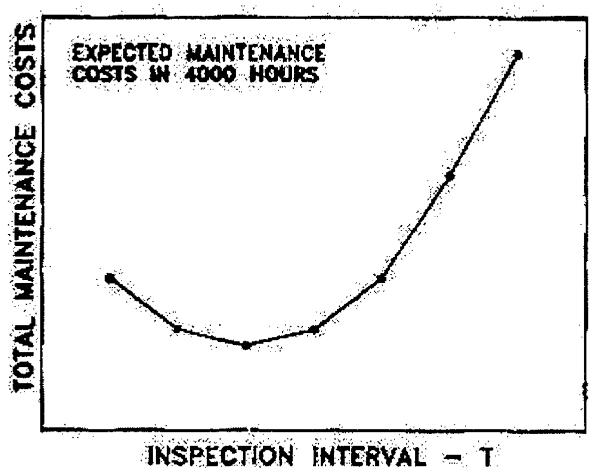

Figure 4: A hypothetical example showing the total maintenance costs over 4000 hours versus inspection interval for determining the most cost effective inspection interval.

7.3 Economic Life Determination

The economic life is the time when the increase in the number of crack damages exceeds the economic repair crack size and/or when the increase of maintenance cost, including the costs of inspection and repair, is so rapid that it is no longer economical to maintain the aircraft [72]. The determination of the economic life from maintenance costs is an important factor for aircraft managers for their decisions in determining whether the aircraft is still competitive or whether other alternatives or replacement of the aircraft should be considered.

The probabilistic approach is capable of quantitatively predicting this economic life for aircraft structures as it was demonstrated by Rudd and Gray [19], Yang [72] and addressed by Lincoln *et. al.* [86]. Rudd and Gray [19] utilised the Equivalent Initial Flaw (Quality) Method for determining the economic lives for an airframe structure. The probabilistic approach is capable of providing valuable information for evaluating maintenance decisions, where inspection and repairs are performed at scheduled intervals in effect to extend the economic life of the aircraft. However, the effect of scheduled maintenance plans on economic life of a component is shown to be limited [72].

8. Limitations/Discussions

The risk analysis methodologies considered in this review have been primarily concerned with the fatigue of aircraft structures. It has been shown that the procedures can be readily applied to structures in general subjected to some form of time dependent wear out or cumulative damage processes. The term "wear out process" refers to any form of degradation process in material properties over a period of time '*t*'. Corrosion and fretting were good examples of such processes. The probabilistic analysis methods are quite generic in most cases. The fundamentals of the reliability analyses are the realistic modelling of the problem in hand and the representative input data of the problem.

The 'reliability' of input data is one important aspect of structural risk and reliability analysis. It has been emphasised throughout this literature review that the collection of representative data and accumulation of practical data is a valuable asset for risk and reliability assessment. The fact that reliability assessment deals with relatively rare events and working with such small probabilities, a large number of data or the accurate prediction of the population is highly necessary for accurate output. From the discussions above, the exact reliability would require the true population to be known, which it could require some sampling of a million specimens or by pooling tens of thousands equivalent groups of data. Of course, this is impossible in term of practicality, cost and availability of data that are equivalent for pooling. However, one must keep in mind that statistical and probabilistic theories are implemented and employed for approximating the true population in the best suitable manner using limited amount of data and small sampling, rather than for determining the exact distribution of the population. This indicates that uncertainty exists even in reliability analysis such as promoted here, and it is the analysts' duty to approximate the population in the best possible manner.

The tail sensitivity problem is a well-known problem in reliability analysis since the very beginning. As indicated by Matthew and Neal [18] the estimates of the extreme tail quartiles and their corresponding reliabilities can be unstable unless large data sets are used. This is the justification for accurately fitting the tail of the data sample. In addition, a computation of high reliability may have little or no association with actual component reliability since no data set is capable of validating the statistical approximation assumed. This is particularly important for dynamic components such as those for helicopter rotor parts, where a cumulative reliability of 9_6 is required. The inadequacy of this aspect in structural risk and reliability analysis has been shown through the recent literature, where investigations have been mainly based upon data collection techniques that could provide more representative risk predictions.

In structural risk and reliability analysis, it is a common mistake to make unreliable assumptions regarding the probability distributions of the variables when data is unavailable or scarce. Only in the form of sensitivity analysis, when the interest is to investigate the effect various distribution functions of a random variable have on the failure probability, may the use of non-representative distribution functions be acceptable. We must avoid the use of non-representative distribution functions when not enough evidence or data are available for supporting the assumption, for non-representative input data indicates unreliable risk assessment.

The technique of Equivalent Initial Flaw Size Distribution (EIFS) is particularly useful when considering the fatigue failure of airframe structures. This is because airframe structures do not have extremely pristine surface finish as compared to those of helicopter and engine components, and it is designed to sustain significant 'macro' crack growth so structural components can be inspected and maintained to avoid failure. The EIFS technique assumes some initial flaw exists in all structures, thus ignoring the crack initiation period, or the period of micro crack propagation. Engine and helicopter components have a very small critical crack size in comparison to airframe structures and the period of micro crack propagation makes up a large proportion of the components' design life. The use of the power law, as mentioned in section 4.2.3, for representing the crack growth at the small crack regime is not an acceptable assumption for this case. For this reason, the EIFS technique often seems inadequate in comparison to the TTIC technique in engine and helicopter components unless fracture mechanics allows accurate prediction of crack growth in the small crack regime. Crack growth prediction in the small fatigue crack regime is extremely important not only for life prediction and risk and reliability of rotor discs, engine blades and dynamic component, but also for airframe structures. This is because data from fractographic analyses and the derivation of EIFS distribution from TTCI distribution and inspection data often show fatigue cracks initiate from equivalent initial flaw sizes in the small crack and mechanically short crack regime (10μm-100μm). Thus, it is important that the complete crack growth function beginning from microstructurally short cracks (≈10μm) up to the critical crack size a_c is known to allow reliable prediction for future fleet management and strategies.

The probabilistic techniques, as discussed in section 5, employed for risk assessment also play an important part on the outcome of the analysis. The various assumptions and approximation techniques utilised in each method contributes to the failure probabilities. This can appear to be rather unsatisfactory, for it implies that different analysts can obtain different estimates of the probability of structural failure, depending on the models they care to use [13]. Thus, it is important to focus on the reliability of the procedures of the methods in estimating the probability of failure, to ensure that the selected technique can provide reliable, conservative and satisfactory solutions for the problems by realistically representing the problem in hand. Validation of the failure probabilities of aircraft structures and components can be difficult for the failure statistics of the same structure or components under the similar conditions are scarce and in many cases, impossible to obtain. This is due to the fact that we do not allow structural components to simply wear out and fail. They are usually inspected, maintained and repaired when cracks are found. For this reason, the probability of failure should only be seen as a nominal or estimated measure of risk, rather than an absolute measure of risk. One way of validating the risk of failure can be by using inspection and maintenance data, for it constitutes the most important feed back for reliability analyses. The information and data obtained from these inspections and maintenance procedure can also be used for updating the input variables and for updating the assessment, and hence providing more representative continual and future predictions.

9. Concluding Remarks

Structural reliability analysis is an engineering method for predicting the probability of failure of a population of structural details under equivalent usage. This analysis technique is capable of predicting fatigue lives for a population of structural details by taking into account the stochastic variables without the use of any implicit and over-conservative factors. From the standpoint of practicality, the probabilistic analysis methodologies should be as simple and as cost effective as possible while maintaining reasonable accuracy for predicting the failure probability of fatigue critical components [49].

It has been shown that the output is particularly useful for fleet management. The probability of failure per hour/flight provides operators with information regarding the expected chance of failure of their fleet within the next hours of operations or flights, and hence allows better management of the use of the aircraft. In the cases when inspection techniques are employed in structural reliability analysis, inspection intervals can be optimised based on the type of NDI inspection-repair technique or replacement procedure.

It must be kept in mind that statistical and probabilistic theories are not implemented for determining the exact variation of the true population but it is to allow reliable approximation of the population in a practical and prevailing fashion when data are scarce. The ability of the analyst to ensure the population and important variables are statistically and realistically estimated by utilising the available approximation techniques effectively to avoid over-conservative assumptions is an important element in reliability analysis. These are some of the important factors that increase the readiness and feasibility of risk and reliability analysis for airframe structures, component and fleet management.

Validation of the results of a risk assessment can be most difficult since we cannot allow components to fail in service simply to justify the risk prediction. Thus, this makes the inspection data the most important feedback for validating the results and for making improvement to the future prediction and for improvement of the analysis technique.

Risk and Reliability analysis has shown a large application to airframe structures and aircraft components management for inspection and replacement planning. Its application potential had been shown to extend beyond structural fatigue cracking. It has the ability to model the uncertainties in the mechanics of other phenomena such as corrosion/corrosion fatigue, fretting and for economic life determination. This clearly showed that risk and reliability analysis is capable of addressing the key areas that will allow for more efficient and better utilisation of the airframe structures and component, and allow risk analysis and ultimately risk management to be preformed on these expensive equipment.

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Appendix A: Methods for Determining the Probability of Stress Exceedance Function

The probability of stress exceedance function describes the probability that a given stress is exceeded over the time '*t*', usually given in terms of per flight or per flight hour. Table A1 summarises the several techniques that are been utilised in the literature and their difference are displayed.

Table A1: Determining the Probability of Stress exceedance per flight using various techniques

A	B	C	D[1]	E[2]	F[3]
Peak Stress KSI	Exceed. Per 1180 hours	Exceed. Per Flight hour	Nerf: Exceed. per Peak L.R.=52.32	PROF Prob. Exceed. per flight	Truncation (Exceed>1)=1
3	61738	52.32034	1	1	1
4	59513	50.43475	0.96396	1	1
6	57566	48.78475	0.93242	1	1
8	56538	47.91356	0.91577	1	1
10	43174	36.58814	0.69931	1	1
12	28547	24.19237	0.46239	1	1
14	18832	15.95932	0.30503	1.00000	1
16	8474	7.18136	0.13726	0.99956	1
18	4621	3.91610	0.07485	0.98293	1
20	1182	1.00169	0.01915	0.63629	1
22	701	0.59407	0.01135	0.44980	0.59407
24	352	0.29831	0.00570	0.25856	0.29831
26	21	0.01780	0.00034	0.01764	0.01780
28	9	0.00763	0.00015	0.00760	0.00763

Column B From Spectrum
Column C = Column B / 1180 Hours
Column D = Column C / 52.32 (52.32 = Load rate per hour or Average peaks per hour)
Column E = 1-(1-Colume D) ^ 52.32
Column F: if Column C > 1, Column F = 1, else Column F = Column C
L.R. = Load Rate

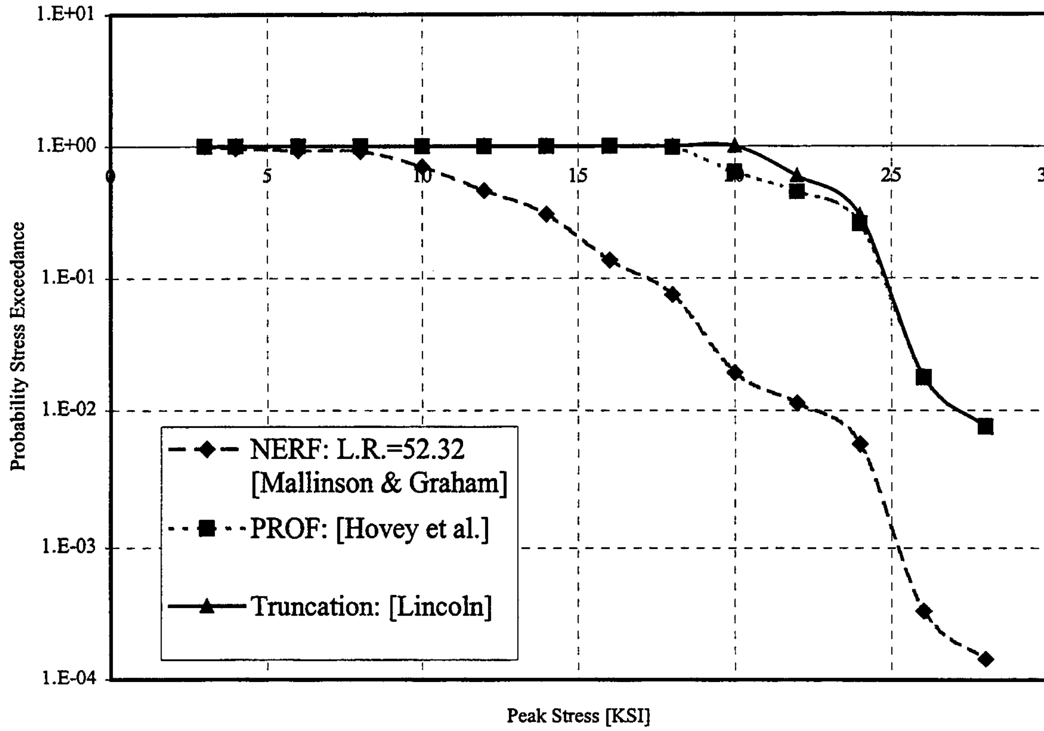

Figure A1: Comparison of the Probability of Stress Exceedance Function for various techniques.

1: A.D. Graham, G.D. Mallinson and Y.C. Tong [7]

2: P.W. Hovey, A.P. Berens and D.A. Skinns [5]

3: J.W. Lincoln [10]

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19. ABSTRACT

Aircraft structural risk and reliability analysis has gained considerable interest in the area of aircraft structural integrity and fleet management in recent times. In this report, a literature review of the current approaches and methodologies that have been utilised in the area of structural risk and reliability analysis for aircraft structures and components is conducted.

The structural risk and reliability approach discussed here deals mainly with the probability of failure due to aircraft structural fatigue. It reviews the engineering techniques and probabilistic methodologies that have been reported in the literature. It also discuss the advantages, disadvantage and the limitations of this approach and the reason for its need in aircraft structural and component integrity. The application of structural risk and reliability analysis is not limited to structural fatigue problem, its implication in areas other than airframe and component fatigue are also presented. This review shows that risk and reliability analysis can be a very useful tool for fleet management and it has potential implications for structural design.

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