Littoral Tracking using Particle Filter[1]

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Abstract - *Littoral tracking refers to the tracking of targets on land and in sea near the boundary of the two regions. A ground-moving target continues to move on land and can not enter the sea. Similarly, a sea-moving target moves in the sea and the land serves as an infeasible region. Enforcing infeasible regions or hard constraints in the framework of the Kalman filter or interacting multiple model (IMM) estimator is not natural. However, these hard constraints can be easily enforced using the particle filter algorithm. We formulate the littoral tracking problem as a joint tracking and classification problem, where we assign a target class for each isolated land or water region. We use a reflecting boundary condition to enforce the region constraint. We demonstrate this concept for a single target using the airborne ground moving target indicator measurements. Numerical results show that the proposed algorithm produces robust classification probabilities using kinematic measurements.*

Keywords: Littoral tracking, Ground moving target indicator (GMTI) measurements, Joint tracking and classification (JTC), Particle Filter (PF), Constrained tracking.

1 Introduction

Tracking of targets on land and in sea near the boundary between the two regions is known as littoral tracking. In the ground-, air-, or sea- target tracking problem, the targets are assumed to belong in a given region only. Either no constraint is imposed or constraints specific to a given region are imposed on the target motion. For example in the ground-target tracking problem, targets are constrained to lie on the surface of the Earth. No specific constraints are usually imposed on the target motion in the air-target tracking problem. For the sea-target tracking problem explicit constraints can be imposed for surface ships and submarines. The littoral tracking problem is challenging due to two reasons. Firstly, given measurements in the first scan, we do not know the origin of the measurements (land- or sea-target) even in the absence of false alarms. Secondly, we need to address constrained tracking for the land- and sea-targets. Previous researchers have addressed the constrained tracking problem using the pseudo-measurements [5], [19] projection algorithm in the Kalman filter (KF) framework [4], and sampling with rejection using the particle filter (PF) [13]. Challa and Bergman [13] studied the constrained tracking problem of an aircraft where flight envelopes on speed and acceleration provide the necessary constraints. They used sampling importance resampling (SIR) with rejection (WR) in the PF. This approach is known as the SIR-WR algorithm. Their results show that the PF with SIR-WR gives improved results over a standard KF without constraints. A fairer approach would be to compare the KF with pseudo-measurements and PF with SIR-WR. However, imposing non-equality constraints in the KF is difficult, whereas equality constraints are easy to enforce.

In this paper, we address the single target tracking problem in the littoral region using kinematic measurements only (no feature measurements are used). The ground moving target indicator (GMTI) measurements [17]-[19] represent the kinematic measurements for our problem. For a land- or sea-target, the GMTI report location can be on land or sea due to measurement error. Therefore, given GMTI measurements alone, the target class (land- or sea-type) is unknown. This problem can be viewed as a joint tracking and classification (JTC) problem using kinematic measurements. Our objectives are to estimate the kinematic state and conditional target classification probabilities for the land- and sea-type targets using a Bayesian framework.

The PF [1], [2], [10]-[12], [14] has been shown to be a powerful algorithm for problems with nonlinear

[1] *Proc. Fifth International Conference on Information Fusion* (Fusion 2002), Annapolis, MD, U.S.A.

dynamics and measurement models and non-Gaussian distributions. In recent years, the PF has been successfully applied to the JTC problem [7], [15], [16]. It is now established that the PF algorithm is far superior compared with the grid-based algorithms that solve the Fokker-Plank equation [20]. Gordon, Maskell, and Kirubarajan have recently analyzed existing PF based JTC algorithms and proposed a new robust JTC algorithm [15]. We use this PF based JTC algorithm for the littoral tracking problem. According to this algorithm, a bank of PFs is used with one PF for each target class. We do not use the SIR-WR approach in the PF since this requires generation of a large number of particles due to rejection. For manageable computational complexity, we employ a reflecting boundary with SIR. We refer to this approach as the SIR-RB algorithm. Our numerical results show that this proves to be a reasonable method for the littoral tracking problem. The kinematic state for each target class is estimated using a PF where the state is constrained to the appropriate region. We compute the likelihood of each target class using all available measurements and then compute the target classification probabilities using the likelihoods from all PFs at a given observation time. The kinematic state of the target is determined by a linear combination of the individual PF estimated states.

In order to keep the numerical computation simple, we assume that the target moves in the XY plane with a nearly constant velocity model (NCVM) and the GMTI sensor moves with constant velocity at a fixed height above the XY plane. We use a land- and sea-region separated by a straight-line boundary in our numerical simulation.

The outline of the paper is as follows. Sections 2 and 3 present the kinematic model and GMTI measurement model, respectively. Section 4 describes the Bayesian JTC algorithm for the littoral tracking problem using kinematic measurements only. Section 5 presents the detailed steps of the PF based JTC algorithm applicable to littoral tracking. Finally, Sections 5 and 6 present numerical results and conclusions.

2 Kinematic Model

Let $x_k \in \Re^n$ denote the target state at time $k := t_k$. The state consists of two-dimensional position and velocity:

$$(2-1) \qquad x_k := \begin{bmatrix} p_{kx} & p_{ky} & v_{kx} & v_{ky} \end{bmatrix}',$$

where $p_{kx} := p_x(t_k)$, $p_{ky} := p_y(t_k)$, $v_{kx} := v_x(t_k)$, and $v_{ky} := v_y(t_k)$. Here, (p_{kx}, p_{ky}) and (v_{kx}, v_{ky}) represent the position and velocity at time k, respectively. The discrete-time kinematic model derived from the continuous-time dynamics of the state for the NCVM is described by [3], [5], [13]

$$(2-2) \qquad x_j = \Phi(j, j-1)x_{j-1} + w(j, j-1),$$

where $\Phi(j, j-1) := \Phi(t_j, t_{j-1})$ is the state transition matrix and $w(j, k-1) := w(t_j, t_{j-1})$ is the integrated process noise [3], [13]. The state transition matrix and integrated process noise for the NCVM are given by [3], [5]

$$(2-3) \quad \Phi(j, j-1) := \begin{bmatrix} 1 & 0 & \Delta_j & 0 \\ 0 & 1 & 0 & \Delta_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Delta_j = (t_j - t_{j-1}),$$

$$(2-4) \qquad E\{w(j, j-1)\} = 0,$$

$$(2-5) \qquad E\{w(j, j-1)w'(l, l-1)\} = \delta_{jl} Q(\Delta_j).$$

$Q(\Delta_j)$ is the covariance of the integrated process noise with the following form [3], [5]

$$(2-6) \qquad Q(\Delta_j) = \begin{bmatrix} \frac{1}{3}q_1(\Delta_j)^3 & 0 & \frac{1}{2}q_1(\Delta_j)^2 & 0 \\ 0 & \frac{1}{3}q_2(\Delta_j)^3 & 0 & \frac{1}{2}q_2(\Delta_j)^2 \\ \frac{1}{2}q_1(\Delta_j)^2 & 0 & q_1\Delta_j & 0 \\ 0 & \frac{1}{2}q_2(\Delta_j)^2 & 0 & q_2\Delta_j \end{bmatrix},$$

where q_1 and q_2 are the power spectral densities of the continuous-valued process noise along the X and Y directions, respectively [13].

3 GMTI Measurement Model

A GMTI radar sensor measures the range (r), azimuth (α), and range-rate $(\dot{r})$ of a target [17]-[19]. The GMTI measurement model at time *j* is described by [18], [19]

$$(3-1) \qquad z_j = h(x_j, s_j) + v_j,$$

where $s_j \in \Re^3$ and $v_j \in \Re^3$ are sensor position and measurement noise at time *j*, respectively:

$$(3-2) \qquad s_j := \begin{bmatrix} s_{jx} & s_{jy} & s_{jz} \end{bmatrix}',$$

$$(3-3) \qquad v_j := \begin{bmatrix} v_{jr} & v_{j\alpha} & v_{j\dot{r}} \end{bmatrix}'.$$

We assume that the sensor position s_j is error-free. In addition, we assume that v_j is a zero-mean independent Gaussian noise with diagonal covariance R_j:

$$(3-4) \qquad v_j \sim N(0, R_j),$$

$$(3-5) \qquad R_j = \begin{bmatrix} \sigma_{jr}^2 & 0 & 0 \\ 0 & \sigma_{j\alpha}^2 & 0 \\ 0 & 0 & \sigma_{j\dot{r}}^2 \end{bmatrix}.$$

Dropping the subscript *j*, the GMTI measurement model for range, azimuth, and range-rate is described by [18]

$$(3-6) \qquad h_r(x,s) = [(x-s_x)^2 + (y-s_y)^2 + s_z^2]^{1/2},$$

$$(3-7) \quad h_\alpha(x,s) = \begin{cases} \tan^{-1}(x-s_x, y-s_y), & \text{if } \tan^{-1}(x-s_x, y-s_y) > 0, \\ \tan^{-1}(x-s_x, y-s_y) + 2\pi, & \text{if } \tan^{-1}(x-s_x, y-s_y) < 0, \end{cases}$$

$$(3-8) \quad h_{\dot{r}}(x,s) = \frac{(x-s_x)\dot{x} + (y-s_y)\dot{y}}{[(x-s_x)^2 + (y-s_y)^2 + s_z^2]^{1/2}}.$$

4 Bayesian JTC Algorithm using Kinematic Measurements

A number of JTC algorithms are presented in [7], [9], [15], [16]. We summarize the Bayesian JTC algorithm using only the kinematic measurements.

We assume that *M* possible target classes exist, where $C = \{c_1, \; c_2, \; \ldots \; c_M\}$ represents the set of target classes. Let the discrete random variable $c \in C$ denote the class of a target. Let $Z^k := \{z_1, z_2, ..., z_k\}$ be the cumulative set of kinematic measurements at time *k*. Our objective is to estimate the continuous state x_k and the posterior classification probabilities $P(c \mid Z^k), \; c \in C$ using Z^k. Formally, we need the posterior joint state-class probability density

$$(4-1) \qquad p(x_k, c \mid Z^k), \; c \in C,$$

where

$$(4-2) \qquad P(c \mid Z^k) = \int_{x_k} p(x_k, c \mid Z^k) dx_k$$

represents the probability for target class *c*. Our goal is to estimate x_k and $P(c \mid Z^k), \; c \in C$. We assume that the prior density for each class

$$(4-3) \qquad p_0(x_0, c \mid Z^k), \; c \in C,$$

is known before any measurement is available. Suppose we know $p(x_{k-1}, c \mid Z^{k-1}), \; c \in C$. Then $p(x_k, c \mid Z^k)$, $c \in C$ can be computed from $p(x_{k-1}, c \mid Z^{k-1}), \; c \in C$ using the prediction and the measurement update steps. Since x_k does not depend explicitly on Z^{k-1}, using the prediction step, the prior density at time *k* is given by

$$(4-4) \quad p(x_k, c \mid Z^{k-1}) = \int_{x_{k-1}} p(x_k \mid x_{k-1}, c) p(x_{k-1}, c \mid Z^{k-1}) dx_{k-1},$$

where $p(x_k \mid x_{k-1}, c)$ is the state transition for the class *c*. The prior density of the state-class $p(x_k, c \mid Z^{k-1})$ can be updated at time *k* by Bayes' rule using the measurement z_k:

$$(4-5) \quad p(x_k, c \mid Z^k) = \frac{p(z_k \mid x_k, c) p(x_k, c \mid Z^{k-1})}{p(z_k \mid Z^{k-1})}, \; c \in C,$$

$$(4-6) \quad p(z_k \mid Z^{k-1}) = \sum_{c \in C} \int_{x_k} p(z_k \mid x_k, c) p(x_k, c \mid Z^{k-1}) dx_k,$$

where $p(z_k \mid x_k, c)$ is the likelihood for the class *c*. We observe from (4-4) and (4-5) that the prediction and measurement update steps of the state-class density for each class can be determined independent of the other classes. This implies that we need a filter (e.g. EKF or PF) for each target class. In the PF approach we need a bank of *C* particle filters.

Using Bayes' theorem, the general recursive equation for target classification probability $P(c \mid Z^k)$ is

$$(4-7) \quad P(c \mid Z^k) = \frac{p(z_k \mid c, Z^{k-1}) P(c \mid Z^{k-1})}{p(z_k \mid \mathbf{Z}^{k-1})}, \quad c \in C,$$

$$= \frac{p(z_k \mid c, Z^{k-1}) P(c \mid Z^{k-1})}{\sum_{c' \in C} p(z_k \mid c', Z^{k-1}) P(c' \mid Z^{k-1})}, \quad c \in C.$$

We have the prior target classification probabilities

$$(4-8) \quad P_0(c), \quad c \in C, \text{ with } \sum_{c \in C} P_0(c) = 1.$$

Each filter first computes

$$(4-9) \quad L(c \mid Z^k) = p(z_k \mid c, Z^{k-1}) P(c \mid Z^{k-1}), \; c \in C,$$

independently and then from M filters, we obtain the classification probabilities

$$(4-10) \quad P(c \mid Z^k) = L(c \mid Z^k) / \sum_{c' \in C} L(c' \mid Z^k), \quad c \in C.$$

5 Littoral Tracking using PF

In this section, we present an algorithm for the littoral tracking of a single moving target using the PF. In general, the area interest (AOI) in the littoral region consists of multiple land and water regions as shown in Figure 1. We restrict our analysis to a single target. Future work will address the littoral tracking of multiple targets. The total number of target classes M is equal to the total number of land and water regions in the AOI.

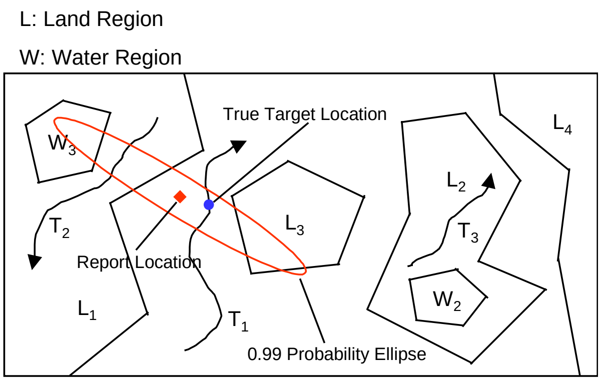

Figure 1. Multi-target littoral tracking scenario with multiple land and water regions.

For simplicity, we assume that the probability of detection is unity and false alarms are absent. GMTI measurements are available from one or more sensors. Given that the target is a land-target, the GMTI report location can lie in the land or water region due to large cross-range error. Therefore, it is not possible to determine the region in which the target moves from the location of GMTI reports. We assume that prior probability of the target type (land- or sea-type) is proportional to the area of the region. Let A_i, $i = 1,2,...,M$ be area of the i^{th} region . Then the prior probability of target class is

$$(5-1) \quad P_0(c_i) = A_i / \sum_{l=1}^{M} A_l, \; i = 1,2,...,M.$$

We use a bank of M particle filters, one corresponding to each region. The operation of each filter in the bank is similar to a conventional unconstrained PF except two differences:

- Each individual PF imposes the region constraint such that the position of each particle lies inside the region.
- Each individual PF computes $L(c \mid Z^k)$.

Let $w_k^{i,j}$ and $\overline{w}_k^{i,j}$ be the unnormalized and normalized weights for the j^{th} particle in the i^{th} PF at time k, respectively. Using the optimal proposal distribution $q(x_k^{i,j} \mid x_{k-1}^{i,j}, z_k)$ [2], [10]-[12]

$$(5-2) \qquad w_k^{i,j} = \overline{w}_{k-1}^{i,j} \frac{p(z_k \mid x_k^{i,j}) p(x_k^{i,j} \mid x_{k-1}^{i,j}, c_i)}{q(x_k^{i,j} z_k \mid x_{k-1}^{i,j}, c_i)}.$$

Using the simple proposal distribution $q(x_k^{i,j} \mid x_{k-1}^{i,j}, z_k)$ $\approx p(x_k^{i,j} \mid x_{k-1}^{i,j})$, which is valid for small process noise,

$$(5-3) \qquad w_k^{i,j} = \overline{w}_{k-1}^{i,j} p(z_k \mid x_k^{i,j}).$$

Using the PF approach [15]

$$(5-4) \qquad p(z_k \mid c_i, Z^{k-1}) = \sum_{j=1}^{N} w_k^{i,j}.$$

Two possible methods can be used to impose the region constraints in a PF. The simplest approach is the *rejection sampling*. While generating the particles, if a particle falls outside the desired region, then the particle is rejected. This process is repeated until all particles fall inside the region. This approach can be computationally intensive. The second approach is the use of a *reflecting boundary* such that the particle position lies inside the region. The direction of the component of velocity perpendicular to the boundary is also reversed. We present the detailed steps of the PF based littoral tracking algorithm below.

- Select the number of particles (N) to be generated and the threshold for resampling (N_{thres}) [1], [2], [11]

- **Initialization**

For class $i = 1,2,\ldots, M$

Set measurement scan index $k = 0$.

Generate N samples $\{x_0^{i,j}\}_{j=1}^N$ from the prior density $p_0(x_0^i)$ for class i.

Set initial weights: $\{\overline{w}_0^{i,j} = 1/N\}_{j=1}^N$.

Set prior class probabilities $\{P_0(i)\}_{i=1}^M$ using (5-1).

End

- **Process Measurements**

For class $i = 1,2,\ldots, M$

Set measurement index $k = 0$

I. Increment k: $k = k + 1$.

For $j = 1,2,\ldots, N$

If $k > 1$

Prediction

Generate $w^{i,j}(k,k-1) \sim N(0, Q(k,k-1))$.

Generate $x_k^{i,j} = \Phi(k,k-1)x_{k-1}^{i,j} + w^{i,j}(k,k-1)$.

If the position of $x_k^{i,j}$ lies outside the region

Apply the reflecting boundary condition to $x_k^{i,j}$.

EndiIf

EndIf

Update state with measurement z_k

Compute the likelihood $p(z_k \mid x_k^{i,j})$.

Calculate $w_k^{i,j} = \overline{w}_{k-1}^{i,j} p(z_k \mid x_k^{i,j})$.

EndFor

Calculate $p(z_k \mid c_i, Z^{k-1}) = \sum_{j=1}^{N} w_k^{i,j}$.

Normalize the weights:

$$\left\{\overline{w}_k^{i,j} = w_k^{i,j} / p(z_k \mid c_i, Z^{k-1})\right\}_{j=1}^N.$$

Compute updated state estimate for class i

$$\hat{x}_{k|k}^i = \sum_{j=1}^{N} \overline{w}_k^{i,j} x_k^{i,j}.$$

Compute effective sample size (N_{eff}^i):

$$N_{\text{eff}}^i = 1 / \sum_{j=1}^{N} (w_k^{i,j})^2.$$

If $N_{\text{eff}}^i < N_{\text{thres}}$

- Resample a new set $\{x_k^{i,j^*}\}_{j=1}^N$ by sampling with replacement N times from the discrete set $\{x_k^{i,j}\}_{j=1}^N$ where $\Pr(x_k^{i,j^*} = x_k^{i,l}) = w_k^{i,l}$.
- Set $\{w_k^{i,j} = 1/N\}_{j=1}^N$.

EndIf

Calculate $L(c_i \mid Z^k) = p(z_k \mid c_i, Z^{k-1}) P(c_i \mid Z^{k-1})$.

EndFor

Calculate class probabilities:

$$\left\{P(c_i \mid Z^k) = L(c_i \mid Z^k) / \sum_{l=1}^{M} L(c_l \mid Z^k)\right\}_{i=1}^M.$$

Calculate combined state estimate:

$$\hat{x}_{k|k} = \sum_{j=1}^{N} P(c_i \mid Z^k) \hat{x}_{k|k}^i.$$

6 Simulation and Numerical Results

Figure 2 shows the littoral tracking AOI with one land and one sea region and the trajectory of the GMTI sensor used in our simulation. We generated the truth target trajectory in two dimensions using the NCVM. We consider three different trajectories of the target. Table 1 presents parameters for the truth target trajectories. The target trajectories in Cases 1 and 3 are well inside the land and sea regions, respectively. The trajectory in Case 2 is in the land region near the boundary. Sensor trajectory parameters and GMTI measurement error standard deviations are presented in Table 2. We assume that the sensor trajectory is error-free. We generate 50 GMTI measurements for the target and use 5,000 particles for each PF. Target truth trajectory and GMTI report locations are shown in Figure 3 from a single Monte Carlo run for Case 1. We observe in Figure 2 that the report locations lie both on land and sea due to large cross-range error, although the true target trajectory lies on land. This property of the data makes it difficult to determine the target type (land- or sea-type) from GMTI report locations.

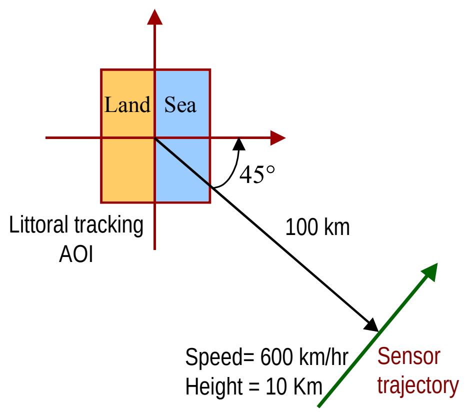

Figure 2. Littoral tracking area of interest (AOI) and GMTI sensor trajectory.

Table 1. Target truth trajectory parameters.

Parameter	Value
Initial position (x, y) for Case 1	(-40, -800) (m)
Initial position (x, y) for Case 2	(-20, -800) (m)
Initial position (x, y) for Case 3	(50, -800) (m)
Initial speed	40 (km/hr)
Azimuth of initial velocity	90 (deg)
Power spectral density of acceleration-x process noise, q_1	0.002 (m^2/s^3)
Power spectral density of acceleration-y process noise, q_2	0.002 (m^2/s^3)

Table 2. Sensor trajectory parameters and measurement error standard deviations.

Parameter	Value
Sensor ground range	100 (km)
Sensor height	10 (km)
Sensor speed	600 (km/hr)
Sensor revisit time	2 (s)
Range standard deviation	20 (m)
Azimuth standard deviation	1 (milli-radian)
Range-rate standard deviation	1 (m/s)

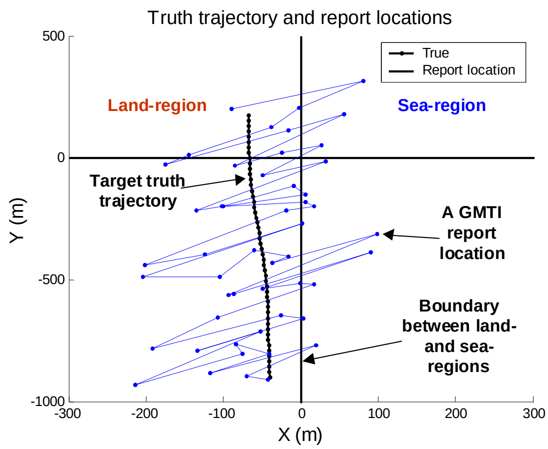

Figure 3. Target truth trajectory and GMTI report locations for Case 1.

Figures 4, 7, and 10, present the target classification probabilities for Cases 1, 2, and 3 respectively. Results in these figures indicate that after processing five to eight measurements, the classification algorithm achieves a probability of unity for the correct target type. Figures 5, 8, and 11, present the true trajectory and estimated trajectories for land- and sea-type targets for Cases 1, 2, and 3, respectively. The true trajectory and combined PF trajectories for the three cases are presented in Figures 6, 9, and 12.

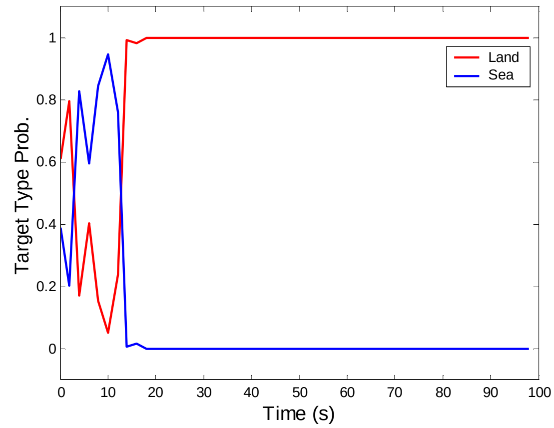

Figure 4. Probability of land- and sea-type target for Case 1.

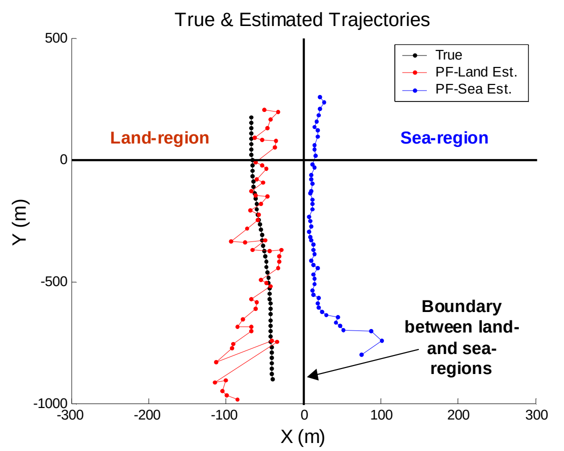

Figure 5. True, land- and sea-type target trajectories for Case 1.

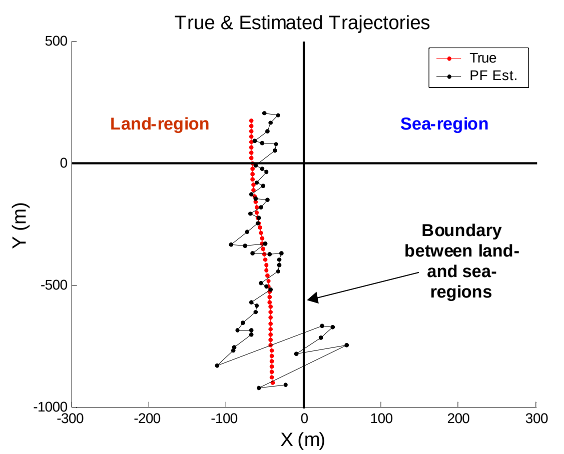

Figure 6. True and combined PF trajectories for Case 1.

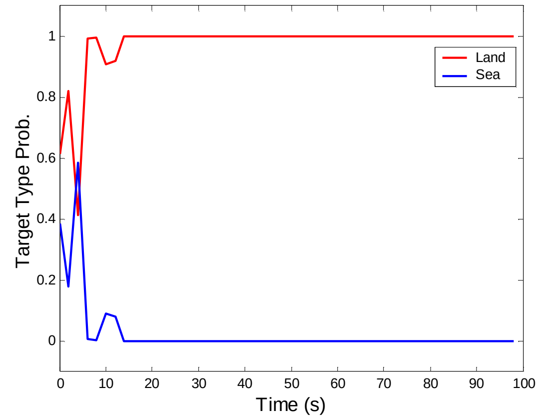

Figure 7. Probability of land- and sea-type target for Case 2.

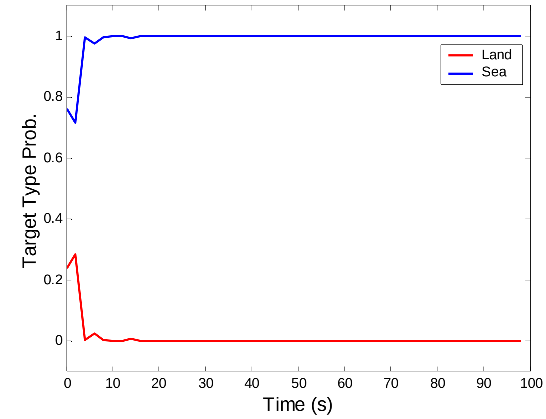

Figure 10. Probability of land- and sea-type target for Case 3.

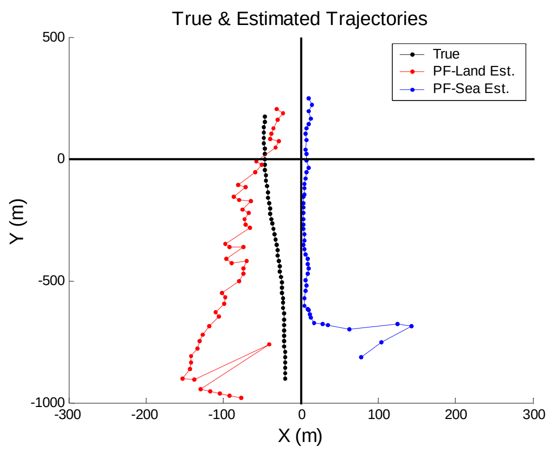

Figure 8. True, land- and sea-type target trajectories for Case 2.

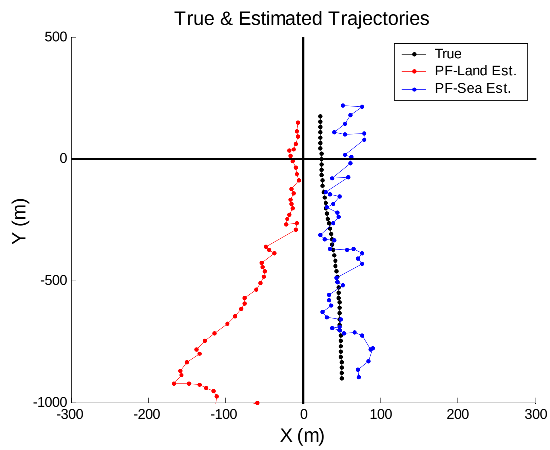

Figure 11. True, land- and sea-type target trajectories for Case 3.

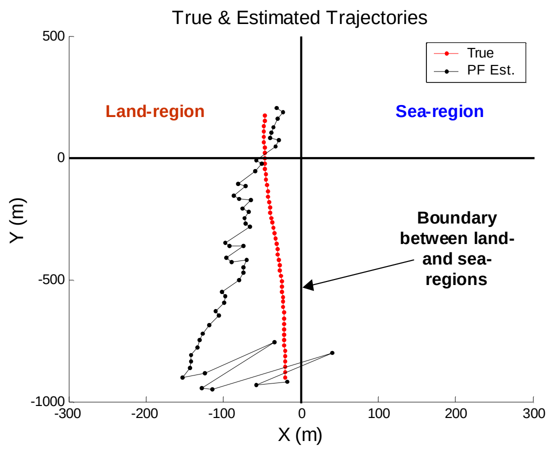

Figure 9. True and combined PF trajectories for Case 2.

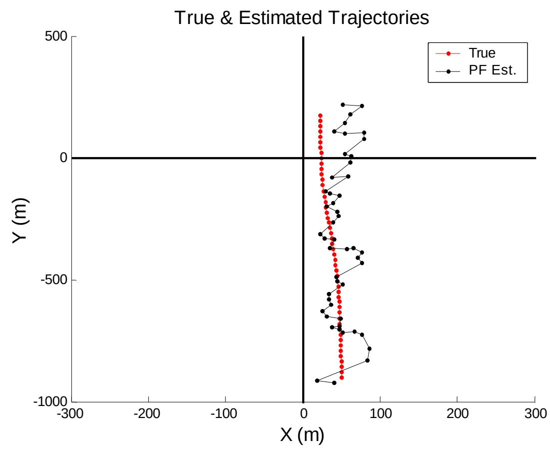

Figure 12. True and combined PF trajectories for Case 3.

7 Conclusions

In this paper we have developed a littoral target tracking and classification algorithm using the particle filter (PF). This algorithm is based on the previous work presented in [15] for the joint tracking and classification problem using the PF. We have used a reflecting boundary condition to impose the region constraint for a land or sea type target. The numerical results show that the algorithm determines the target class in a robust manner after processing five to eight GMTI measurements. We plan to extend the current approach to the multi-target littoral tracking problem with realistic data.

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