

Load factor of hydropower plants and its importance in planning and design

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Abstract

In planning and design of new hydropower plants, the expected annual energy production is an important economical factor. The annual energy production of hydropower plants varies with availability of water and fluctuations in demand for electric power. Demand fluctuates between day and night, summer and winter and weekdays and holidays, sometimes in a deterministic manner, but there is considerable stochastic fluctuation too. Nevertheless, the statistical distribution of the instantaneous power load on the plant may be known, this is the power load duration curve. The water demand of the hydropower station does not have the same duration curve as the power demand due to the nonlinear effect of the energy losses in the water conduits. The duration curve for the water flow is derived from the duration curve of the demand. The load factor for the flow is calculated for the case when the duration curve of the market is according to Junges duration curve. Rules given on the estimation of available spinning reserve and the corrections that have to be made of the load factor if some peak power is produced by a thermal station the size of the hydropower station diminished accordingly. The effect of the difference of duration curves for power and water demand is demonstrated in a case study.

1. Introduction

One of the problems in hydropower is how great an installed capacity we need in a powerhouse to produce the energy we have calculated from information on waterflow and developed head. Because of the fluctuating nature of the energy demand, or the energy market, we need an installed capacity that is fairly larger than the average load on the power station. This is also true when we have ample possibilities of flow regulation in dams and storage reservoirs.

In the so called hydropower model of the National Energy Authority of Iceland (Loftur Þorsteinsson 1985) this problem is attacked by assuming that the specific properties of the energy market can be described by a load duration curve, and

installed capacity is calculated from this load duration curve. In this paper this method is revisited and we bypass the approximation formerly made that load factor of the energy load and flow factor of the water flow (later called the load factor of the water flow to stress the relationship between these two important factors) can be considered equal – which they are obviously not. The result is a relation between the load factors, average head loss and average energy loss.

The losses of hydraulic head make the duration curve of the flow through the powerstation different from the load duration curve as soon as we start regulating the energy production and thereby the flow through the turbines. In this paper a method to calculate the difference is devised, and it is shown how effects of spinning reserves and excess capacity installation can be counted for. By this, we don't need an exact estimation of the power production to be able to determine the installed capacity

The calculation of flow factor from the load factor is necessary in optimal design of power stations (Jónas Elíasson 1994). The load factor of the water flow through the power station is a important design parameter in the optimisation program HYDRA (Jónas Elíasson, Pall Jensson og Guðmundur Ludvigsson 1997) and the results presented here are used in HYDRA together with former design methods devised (Axel Hilmarrsson 1993) and (Jónas Elíasson *et al.* 1995).

2. Installed capacity and annual energy production

2.1. Definitions

Head water level is the highest possible water level at the station intake in full operation and with zero bypass flow.

Tail water level is the energy head of the water flowing out of the turbines.

Total (gross) head Vertical distance between head- and tail water.

Gross capacity Maximum capacity if all headlosses, hydraulic and otherwise, are considered zero. It is at any time:

$$F_{re}(t) = \gamma Q(t) H_{br} \quad (1)$$

F_{re}	Produced power
$Q(t)$	Flow through the station.
H_{br}	Gross head
γ	Unit weight of water
t	time

Produced energy is the actual energy production. Its maximum power is the installed capacity.

Design flow (Design discharge) is the maximum throughput through the turbines is the flow we design the turbines for, when this flow is applied the power station runs on installed capacity.

$$F_v = e_v \gamma Q_v H_{br} \quad (2)$$

F_v Installed capacity
 e_v Efficiency factor counting for losses
 Q_v Design flow

Effective head is losses subtracted from the gross head at installed capacity output. (2) shows the relation between gross head and installed capacity.

The efficiency factor can be dissolved in several factors, one for each type of head loss.

$$e_v = e_f e_{vh} e_{ra} \quad (3)$$

e_f Efficiency factor due to head losses in water conduits
 e_{vh} Efficiency factor due to energy losses in generator
 e_{ra} Efficiency factor due to other electrical losses

When an efficiency factor is mentioned, it usually refers to head losses at design flow Q_v . Head losses in water conduits are usually calculated by the know resistance formulas of hydraulics. This makes the head losses proportional to Q_v^2 (Nigam 1985). This also applies when the flow varies with time, then the head losses are proportional to $Q(t)^2$. This head loss refers to the efficiency factor e_f , which is calculated in the next chapter. However it must be noted that head loss in open channels is not proportional to $Q(t)^2$, the topographical gradient of the terrain mostly determines the difference in water level between the upper and the lower end of a water canal. Head losses in long canals should be subtracted in the beginning, they are mostly independent of the flow anyway, and gross head calculated from the water level at the intake into the penstock.

Effective head is now

$$\begin{aligned} H_v &= H_{br} - \Sigma \Delta H \\ &= H_{br} \left(1 - \frac{\Sigma \Delta H}{H_{br}} \right) = H_{br} \left(1 - C_{fa} Q_v^2 \right) = H_{br} e_f \end{aligned} \quad (4)$$

C_{fa} Resistance factor dependent on size and roughness of conduits
 ΔH Total head loss at flow Q_v

2.2. Energy production.

The rate of energy production changes with time. Even so, it is customary to assume a fixed annual production of a hydropower station in the design phase. The energy production may be calculated by (5). This equation also defines the parameters it introduces.

$$E = \int_0^T F(t) dt = F_v T_k = F_m T = \lambda_F F_v T \quad (5)$$

T_k	Load duration time
F_m	Average load on the station
λ_F	Load factor, a parameter characterising the <u>energy market</u>

T_k and F_m can be found by integrating the duration curve of the market the hydro project is supposed to serve. Fig. 1. shows examples of some basic duration curves. The result of the integral (5) is the same if the integration is over real time, or is taken over relative time (t/T), which is the horizontal axis in fig. 1. Fig. 1 is actually the probability function of the load, showing three basic load duration curves, Junges, constant maximum (60 % of the time then zero load), and constant average load when maximum load is applied for very short time only. Fig 1 will be discussed in greater detail later.

Now we insert (2), (3) and (4) into (5) and find:

$$E = \int_0^T e_{vh} e_{ra} \gamma Q(t) H_{br} (1 - C_{fa} Q(t)^2) dt = e_{vh} e_{ra} \gamma Q_{vm} H_{br} (1 - C_{fa} \frac{\beta}{\lambda_R} Q_v^2) \quad (6)$$

$$\beta \quad \text{Cubic average of the flow} = \frac{1}{T} \int_0^T \left(\frac{Q(t)}{Q_v} \right)^3 dt$$

$$\lambda_R \quad \text{Load factor of flow} = Q_{vm}/Q_v$$

$$Q_{vm} \quad \text{Average flow through the station, not equal to average river flow, } Q_m, \text{ due to by-pass during flood periods.}$$

Inserting (2) and (5) we get for the annual energy production and installed capacity:

$$E = \lambda_F F_v T = e_{vh} e_{ra} \gamma Q_{vm} (H_{br} - \Delta H \beta / \lambda_R) T \quad (7)$$

$$F_v = e_{vh} e_{ra} \gamma Q_v (H_{br} - \Delta H)$$

From this we deduct instantaneous load at any time t ,

$$\begin{aligned}
F(t) &= e_{vh}e_{ra}\gamma Q(t)H_{br}(1 - C_{afa}Q(t)^2) = \\
& e_{vh}e_{ra}\gamma Q(t)H_{br} \left(1 - C_{fa}Q_v^2 \left(\frac{Q(t)}{Q_v} \right)^2 \right) \\
&= \\
&= e_{vh}e_{ra}\gamma Q(t)H_{br} \left(1 - \frac{\Delta H}{H_{br}} \left(\frac{Q(t)}{Q_v} \right)^2 \right) \quad (8)
\end{aligned}$$

Flow duration

When the load has a certain duration curve we regulate the production of energy by regulating the flow through the station turbines in phase with the load fluctuations. Now we define the duration curve mathematically in the following manner:

$$\begin{aligned}
F(t) &\leq F_v J(x); \quad t = T - xT \text{ hours per year} \\
F(t) &\geq F_v J(x); \quad t = xT \text{ hours per year} \\
x &\quad \text{parameter } 0 \leq x \leq 1
\end{aligned}$$

Now inspection of (7) and (8) shows that following must hold:

$$J(x) = B(x) \left(\frac{1 - B(x)^2 \frac{\Delta H}{H_{br}}}{1 - \frac{\Delta H}{H_{br}}} \right) \quad (9)$$

$$B(x) = Q(x)/Q_v \quad (10)$$

The load duration curve $J(x)$ is known, it is deduced from study of the load fluctuations of the energy market the power station is built to serve. Eq. (9) defines $B(x)$ when $J(x)$ is known, we have to solve a cubic equation and the solution is:

$$(11) \quad B(x) = \frac{-2}{\sqrt{3\Delta H / H_{br}}} \cos \left(\frac{\text{Arc cos} \left(-J(x) 1,5 (1 - \Delta H / H_{br}) \sqrt{3\Delta H / H_{br}} \right)}{3} + \frac{\pi}{3} \right)$$

As the reader can see we have to know $\Delta H/H_{br}$ in order to find $B(x)$, which is related to the cubic average of flow and the load factor of the flow in the following manner:

$$\beta = \frac{1}{T} \int_0^T \left(\frac{Q(t)}{Q_v} \right)^3 dt = \int_0^1 B(x)^3 dx \quad (12)$$

$$\lambda_R = \frac{1}{T} \int_0^T \left(\frac{Q(t)}{Q_v} \right) dt = \int_0^1 B(x) dx$$

But integration of (9) gives us:

$$\lambda_R = \lambda_F \left(\frac{H_{br} - \Delta H}{H_{br} - \frac{\beta}{\lambda_R} \Delta H} \right) = \lambda_F \left(\frac{1 - \Delta H / H_{br}}{1 - \frac{\beta}{\lambda_R} \Delta H / H_{br}} \right)$$

or

$$\lambda_R = \lambda_F - (\lambda_F - \beta) \Delta H / H_{br} \quad (13)$$

So in order to find the relative head loss $\Delta H/H_{br}$ we have to solve (13) and in order to do that we have to know $B(x)$ that only comes from (9) where we need $\Delta H/H_{br}$.

This imposes a problem of course. Fortunately we do build our hydro stations with low relative head loss and then we can use approximate formulas. One of them is:

$$\beta = \lambda_R^3 \left(1 + 3,7 \frac{(1 - \lambda_R)^2}{1 + 2\lambda_R} \right) \quad (14)$$

Which may be used to find $\Delta H/H_{br}$ using an iteration process. Most experienced designers, will however choose to start with an educated guess of the relative headloss $\Delta H/H_{br}$, calculate $B(x)$ from (9), then use (12) and (13) to find λ_F must, correct the initial value and calculate once again. This is easy enough to do for any load duration curve $J(x)$, e.g. in a spreadsheet program.

Average utilized head is not the same as the time average of the station head. This is due to nonlinearities in the head losses. The average utilized head is (see (7)):

$$H_{nm} = H_{br} \left(1 - \frac{\beta}{\lambda_R} \Delta H / H_{br} \right) \quad (15)$$

▲ H_{nm} Average utilized head.

Average utilized head can be used directly to calculate the energy production using (7). Time average of the head is a little different and cannot be used for the same purpose. When $B(x)$ is found we can immediately find the time average of the head by averaging (4):

$$H_m = H_{br} (1 - \beta' \Delta H / H_{br}) \quad (16)$$

Here we have used

$$\beta' \cong \frac{\beta}{\lambda_R} - \frac{2}{3} \left(\frac{\beta}{\lambda_R} - \lambda_R^2 \right)$$

This is an approximate equation, similar to (14). Exact calculation of H_m is not a problem, it goes along the same lines as before and is left to the reader.

We now have a working procedure to design our hydro power station. We start by estimating an average design flow $Q_{vm} < Q_m$, then find the necessary dams and regulation works to maintain this flow and finally find the load factor of the flow as described and design the station. This gives us one station alternative and the associated Q_{vm} . This does not have to be the optimal one, but the globally optimal Q_{vm} can be calculated using the normal procedures of optimizing headlosses against construction costs. This is a very complicated process (Jónas Elíasson 1994), but it can be done automatically using the described process, and this is done by the software HYDRA (Jónas Elíasson, Páll Jensson og Guðmundur Ludvigsson 1997). This software uses the procedure described here.

2.3. Three types of load duration.

In discussions of hydropower station design the so-called Junges duration curve is often used as an example.

$$J(x) = 1 - (1 - \lambda_F^2) x^{\lambda_F} \quad (16)$$

This duration curve owes the popularity to the simple fact that it is a one-parameter distribution in the range $0 \leq x \leq 1$ and has the average λ_F as a duration curve for a market with the load factor λ_F must have. The maximum is 1 in $x = 0$ and the minimum is λ_F^2 in $x = 1$. As the duration curve has only one parameter we cannot change the minimum without changing the average. Table 1 gives λ_R and β coefficients for various values of λ_F and relative head loss $\Delta H/H_{br}$. Table 1 should give the reader an idea of how these coefficients change with load factor and relative head loss.

One can see that for low relative head loss ($\Delta H/H_{br} = 0,05$ or 5 %) the λ_R and λ_F are very close. But above $\Delta H/H_{br} = 10$ % the difference becomes significant. At first for the low load factors, but for higher relative head loss there is a significant difference in the load factors for power and water flow for the higher power load factors too. This shows us that the assumption $\lambda_F = \lambda_R$ is not at all justified.

The beta coefficient depends highly on the load duration. High load duration means small fluctuations in load (instantaneous power production). Now consider the following three load duration examples pictured in fig. 1.

First there is Junges load duration curve. β and λ_R calculated for moderate relative head loss. Second is when we produce the total annual energy demand at full capacity and keep the station idle for the rest of the year. This is the largest range of

fluctuations we can get, either full load or nothing. Thirdly, in contrast to this we can produce the annual energy output by keeping the load constant at average load for the whole year, but only at one instant we go up to full capacity for such a short time that the energy produced during that time is negligible. This is the smallest range of load fluctuations we can get.

Fig. 1 gives us an idea of how the beta coefficient that controls the energy losses depends on the load duration. The longer we run the station on full capacity, the higher is beta and with it the energy losses grow larger. The highest beta value is for the second example in fig. 1, all energy production on full capacity giving $\beta = \lambda_R$. This is maximum energy loss. In example three, all energy production is on average power output and that gives $\beta = \lambda_R^3$. And we can get the relationship between λ_R and the relative head losses by inserting this in (13).

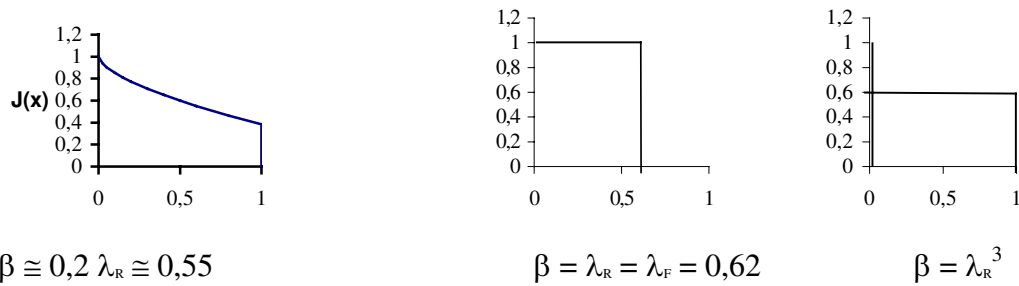


Fig. 1 Energy loss coefficients and load factors for water flow for $\lambda_F = 0,62$

Table 1
Load factor for station water flow for Junges load duration

$\Delta H/H$	λ_F	0,30	0,35	0,40	0,45	0,50	0,55	0,60	0,65	0,70	0,75	0,80	0,85	0,90	0,95
0,05	λ_R	0,29	0,34	0,39	0,43	0,48	0,53	0,58	0,63	0,68	0,73	0,79	0,84	0,89	0,94
	β	,052	,073	,098	,127	,162	,201	,247	,299	,359	,429	,511	,605	,716	,846
0,1	λ_R	0,28	0,32	0,37	0,42	0,46	0,51	0,56	0,61	0,66	0,72	0,77	0,82	0,88	0,94
	β	,047	,066	,088	,115	,147	,184	,226	,276	,334	,402	,481	,576	,690	,828
0,15	λ_R	0,26	0,31	0,35	0,40	0,44	0,49	0,54	0,59	0,64	0,69	0,75	0,80	0,86	0,93
	β	,041	,058	,078	,103	,132	,165	,205	,251	,306	,370	,447	,541	,656	,803
0,2	λ_R	0,25	0,29	0,33	0,38	0,42	0,47	0,52	0,56	0,61	0,67	0,72	0,78	0,84	0,91
	β	,036	,051	,068	,090	,116	,146	,182	,224	,274	,334	,407	,498	,613	,768
0,25	λ_R	0,23	0,27	0,31	0,36	0,40	0,44	0,49	0,54	0,59	0,64	0,69	0,75	0,81	0,89
	β	,031	,043	,059	,077	,100	,126	,158	,195	,240	,294	,361	,446	,557	,716

Table 1 is compiled as an example but not as a tool in practical design. Junges load duration curve is not a universal fact. But it is a usable “mean” in unclear situations and preliminary calculations. Fig. 1 also gives an idea of the uncertainty involved. It is compiled for $\lambda_f = 0,62$ or load duration time $T_k = 5432$ hours. Compilations for different load factors do not impose a problem.

2.4. Calculation examples.

Consider $Q_{vm} = 90$ m³/s, gross head 105 meters. $T_k = 6500$ hours. Relative head loss is assumed 20 %, efficiency of mechanical and electrical equipment is 0,92. Now we have

$$\lambda_f = 6500/8760 = 0,742; \quad \lambda_r = 0,66 \quad \beta = 0,322 \quad (\text{From table 1})$$

$$Q_v = 90/0,66 = 136,4 \text{ m}^3/\text{s} \quad (\text{Eq. (6)})$$

$$F_v = 0,92 \cdot 0,8 \cdot 9,81 \cdot 105 \cdot 136,4/1000 = 103,4 \text{ MW} \quad (\text{Eq. (7)})$$

$$E = 103,4 \cdot 6500/1000 = 672 \text{ GWh/ári} \quad (\text{Eq. (5)})$$

$$E = 0,92 (1 - 0,2 \cdot 0,322/0,66) \cdot 9,81 \cdot 90 \cdot 105 \cdot 8,76 = 674 \quad (\text{Eq. (7)})$$

Next we design water conduits for $Q_v = 136,4$ m³/s. Now we may find that the relative head loss is 15 % instead of the originally assumed 20 %. Now we can repeat the calculations.

$$\lambda_f = 6500/8760 = 0,742; \quad \lambda_r = 0,68 \quad \beta = 0,358$$

$$Q_v = 90/0,68 = 132,4 \text{ m}^3/\text{s}$$

$$F_v = 0,92 \cdot 0,85 \cdot 9,81 \cdot 105 \cdot 132,4/1000 = 106,6 \text{ MW}$$

$$E = 106,6 \cdot 6500/1000 = 693 \text{ GWh/ári}$$

$$E = 0,92 (1 - 0,15 \cdot 0,359/0,68) \cdot 9,81 \cdot 90 \cdot 105 \cdot 8,76 = 688$$

We see that the new Q_v differs from the first one by 3 % only.

We might want to install larger capacity in the station than we actually need to produce the energy. This extra capacity can be used e.g. for spinning reserve. Using (17) and (18) (see next chapter) we can find the largest possible capacity increase we can install without increasing the water conduits.

$$\alpha_{RA} = \frac{2}{3(1-0,15)\sqrt{3 \cdot 0,15}} = 1,17 \quad \alpha_{RR} = \frac{1}{\sqrt{3\Delta H/H}} = 1,49 \quad \text{max}$$

This means we can increase the capacity by 17% by installing larger electrical and mechanical equipment. However, in order to use it we have to increase the water flow by 49 %. This large increase in water demand in order to obtain a small increase in power output can be justified if the extra power is put in use for very short time only which is the case for spinning reserve.

3. Spinning reserve

All power systems need spinning reserves, that is idle excess capacity that can be put in use immediately in order to make up for large load fluctuations or sudden mechanical failures. Inadequate spinning reserves have caused enormous blackouts in large power systems.

Theories on how great the spinning reserves have to be is not the issue in this paper, but it may be pointed out that hydro power stations can more often than not provide economical spinning reserves by making the mechanical and electrical equipment larger than needed to produce the energy available. But if we make the dams and waterways larger also, but not the mechanical and electrical equipment (turbines and generators) only, this spinning reserve can be too expensive.

When the spinning reserve is put in use, this usually happens automatically, the governor of the system simply opens up for the water and lets into the turbines a water flow $Q > Q_v$. This means head losses above the design value of the relative head loss. The question is now, how large can this spinning reserve be.

Let us assume a spinning reserve $(\alpha_{RA} - 1) F_v$. Total capacity is now $\alpha_{RA} F$. We now need extra water flow, and we can find how much it is by inserting $\alpha_{RA} J(x)$ into (9) in stead of $J(x)$ and calculate exactly as before. We find increased λ_r , lets say it α_{RR} greater than the old value. The entire process is really unchanged, we only have new final values for capacity and maximum flow:

$$\begin{aligned} F_{RV} &= \alpha_{RA} F_v, \\ Q_{RV} &= \alpha_{RR} Q_v \end{aligned} \quad (17)$$

Mechanical and electrical equipment has to be designed for this new value. But there is little more to it, α_{RA} cannot take any value, the largest possible values we can use are:

$$\alpha_{RA} \leq \frac{2}{3(1 - \Delta H / H) \sqrt{3 \Delta H / H}} \quad \alpha_{RR} \leq \frac{1}{\sqrt{3 \Delta H / H}} \quad (18)$$

This may sound astonishing, but it is a hydraulic fact, that if we let all the water through the waterways that flow resistance allows, there would be no power left to drive the turbines. The gross head is totally used up by the waterways. In between our design flow and this theoretical “zero power” flow there is a discharge that gives us maximum power. That is the largest power output the station can provide with the waterways we have designed from the needs of economical power production.

4. Case study

Fljotsdalur Hydroelectric Project in east of Iceland is taken as an example to show that the lambda for flow is actually not equal to lambda for power and this assumption can lead to inaccurate results. This project consist in short of a large reservoir with a 31 km long headrace tunnel leading to a 450 m long vertical pressure shaft, powerhouse and 1200 m long tailrace tunnel. In this example different headrace tunnel diameters were assumed. For each diameter the system was firstly calculated with the assumption that lambda flow is equal to lambda power. Secondly the actual exact lambda for flow was found and the system then recalculated. In all cases a Junge load curve with a load duration time (see (5) and chapter 2.3) of 6500 hours was assumed. This is a common load duration time for average public utilities.

Table 2
Effect of the assumption

Headrace diameter	4,5		4,8		5,0	
Lambda for power λ_F	0,742	0,742	0,742	0,742	0,742	0,742
Lambda for flow λ_R	0,742	0,717	0,742	0,729	0,742	0,734
Cubic av. of flow β	0,4414	0,405	0,4414	0,4221	0,4414	0,4295
Design flow Q_v m ³ /s	36,44	37,72	36,44	37,09	36,44	36,84
Inst. cap. F_v MW	179,34	184,04	184,44	187,16	187,59	189,06
Energy E GWh/a	1.197,51	1.196,24	1.217,02	1.216,55	1.229,08	1.228,86

We see that the result of the assumption $\lambda_R = \lambda_F$ leads to a smaller turbine than we actually need. In this case it does not make o much the difference, so the example demonstrates clearly why most designers have not been concerned about the inaccuracy of this simplification. An experienced designer using this simplification in his handmade design would probably recommend somewhat bigger turbines to rule out the inaccuracy. But when we use software like HYDRA and loose the tracking we have in doing our calculations by hand the assumption $\lambda_R = \lambda_F$ is unacceptable.

We also see that the error increases by increasing head losses. The small tunnel gives a difference of almost 5 MW. A serious designer should not accept such a difference.

In case of shorter load duration time (smaller λ_F), as is the case for e.g. peak power stations the error becomes larger and more significant. In countries with combined systems, fossil/hydro or nuclear/fossil/hydro, the role of the hydropower is often to provide the peak power and some times pumping stations are built for providing peak power. In such cases it is essential not to use $\lambda_R = \lambda_F$.

One result of the case study is quite remarkable and deserves closer attention. It is that the optimized energy production is mostly unaffected by the simplification. The explanation is, that the time public utilities, like the one we are looking at, run their machinery at installed capacity power output is very short. The plant is unable to

deliver the required megawatts, but it does not suffer a significant reduction in sales, if the customers accept the capacity deficit, which is hardly the case.

5. Conclusion

In designing power stations with optimized waterways and capacity, it is necessary to use separate load factors for power and flow. When we have a load factor

$$\lambda_F = \frac{\text{Average load}}{\text{Maximum load}}$$

Then there is a corresponding flow factor

$$\lambda_R = \frac{\text{Average flow}}{\text{Maximum flow}}$$

And $\lambda_R > \lambda_F$, the difference depends *only* upon the relative head loss of the system and the approximation $\lambda_R = \lambda_F$ is neither necessary or justified. The difference can be up to 30 % in case of the one-parameter Junges load duration curve.

In the three different examples it is shown that the energy loss factor is:

$$\lambda_R \geq \beta \geq \lambda_R^3$$

This energy loss factor is important in finding the energy losses in optimisation.

When it is desirable and economical, extra power can be installed as spinning reserve with water conduits (waterways) unchanged. Then we have the possibility of a larger power output than the design capacity and can use more water than the design flow.

By increasing the power capacity by a factor α_{RA} we need to increase the water flow by a corresponding factor α_{RR} . Then the final power capacity and design flow can be

$$\begin{aligned} F_{RV} &= \alpha_{RA} F_v \\ Q_{RV} &= \alpha_{RR} Q_v \end{aligned}$$

But α_{RA} and α_{RR} cannot exceed the following values:

$$\alpha_{RA} \leq \frac{2}{3(1 - \Delta H / H) \sqrt{3 \Delta H / H}} \quad \alpha_{RR} \leq \frac{1}{\sqrt{3 \Delta H / H}} \quad (18)$$

A case study shows us that high loads factors as normally experienced by medium and large utilities produce errors in design that do not have to be serious. In designing peak power stations the effect will always be serious.

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