LOAD FLOW CONVERGENCE IN THE VICINITY OF A VOLTAGE STABILITY LIMIT

S. Abe N. Hamada A. Isono K. Okuda

Hitachi Research Lab., Hitachi, Ltd
Hitachi, Ibaraki, Japan

ABSTRACT

Because load flow problems are expressed as sets of nonlinear simultaneous equations, they have no unique solutions. In this paper a region where a set of initial values converges to a stable load flow solution under specified conditions, is investigated theoretically when the Newton-Raphson method is applied to a set of nodal power equations expressed in either polar or rectangular coordinates. The results are tested in load flow calculations on a 28-node power system and the convergence characteristics for the two types of coordinates are compared.

INTRODUCTION

Because of the increasing difficulty of obtaining power plant sites in the vicinity of power consumers, electrical power is now often transported through large capacity lines over long distances from power plants to consumers. Under these circumstances, in order to insure the stable operation of power systems, it is necessary to study the voltage stability of power systems, as well as the steady state and transient stabilities. Several papers on voltage stability[1-5] have been presented recently.

In load flow calculations, loads are usually represented by constant real and reactive powers, which have voltage characteristics severer than actual loads have from a standpoint of voltage stability. Therefore the voltage magnitudes at the voltage stability limit are higher than those under actual operating conditions, and the values obtained for a stable solution approach those obtained for unstable one. Therefore, a careless selection of initial values may cause load flow calculations to diverge or even to converge to an unstable solution.

Several attempts[5-7] have been made to clarify load flow convergence. In [5], a modified Newton-Raphson algorithm, which suppresses excessive voltage corrections by using the Hessian matrix, is proposed to improve load flow convergence. In [7], a region where a set of initial values converges to a stable solution under specified conditions, when the Newton-Raphson method is applied to a set of nodal power equations expressed in polar coordinates, is investigated theoretically.

F 78 007-7. A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE PES Winter Meeting, New York, NY, January 29 - February 3, 1978. Manuscript submitted July 22, 1977; made available for printing November 1, 1977.

In this paper the results obtained in [7] are first summarized. Then a convergence region for rectangular coordinates is discussed and compared with one for polar coordinates. The results are further exemplified and compared in load flow calculations on a 28-node power network by varying the set of initial values.

CONVERGENCE REGION OF LOAD FLOW CALCULATIONS EXPRESSED IN POLAR COORDINATES

For an (N+M)-node power system, let nodes 1 to N be nodes where real and reactive powers are specified, nodes N+1 to N+M-1 be nodes where real powers and voltage magnitudes are specified, and node N+M be the slack node. Then a set of equations for a load flow problem is given by (I3), (I4) and (I7), in Appendix I, where $k_i = 0$ and $c_i = 0$ in (I7). Under light load conditions, there is a solution whose reduced Jacobian matrix $F_1 - A_1A_2^{-1}F_2$ is an M-matrix as can be seen from Appendix I. By loading gradually, a solution whose Jacobian vanishes may be obtained, and a load condition heavier than that under which the Jacobian vanishes may give no load flow solution. But so long as solutions exist, there is a unique solution whose reduced Jacobian matrix is nearly equal to an M-matrix, because among the conditions which specify an M-matrix, the one that states that the reduced Jacobian be positive is the severest possible condition, and because the sign of the reduced Jacobian coincides with that of the Jacobian when a phase difference across any line is less than $\pi/2$. (See Appendices I and II.)

From a voltage stability standpoint, the load flow solution that is wanted is the solution whose reduced Jacobian matrix is nearly equal to an M-matrix. A solution which satisfies this restriction under specified conditions is called a stable solution, and solutions which do not satisfy it are called unstable solutions hereafter.

Applying the Newton-Raphson method to the set of equations (I3), (I4) and (I7), where $k_i = 0$ and $c_i = 0$ gives

$$V^{(n+1)} = V^{(n)} + \triangle V^{(n)} \qquad (1)$$

$$\theta^{(n+1)} = \theta^{(n)} + \triangle\theta^{(n)} \qquad (2)$$

$$F^{(n)}\begin{bmatrix} \triangle V \\ \triangle\theta \end{bmatrix}^{(n)} = \begin{bmatrix} F_1 & A_1 \\ F_2 & A_2 \end{bmatrix}^{(n)}\begin{bmatrix} \triangle V \\ \triangle\theta \end{bmatrix}^{(n)} = -\begin{bmatrix} f_Q \\ f_P \end{bmatrix}^{(n)} \qquad (3)$$

where n denotes the n-th iteration of the calculations.

In [7], a region where a set of voltage magnitudes converges to a stable solution is theoretically studied when initial phase angles are set to zero and the Newton-Raphson method is applied to the following equations, instead of (I3), (I4) and (I7).

$$g_Q = 0 \qquad (4)$$

$$g_P = 0 \qquad (5)$$

where $g_Q = (g_{Q,1}, \cdots, g_{Q,N})^T$, $g_P = (g_{P,1} \cdots, g_{P,N+M-1})^T$
and $g_{Q,i} = f_{Q,i}/V_i$, $g_{P,i} = f_{P,i}/V_i$

The results show that a stable solution is obtained when a set of initial voltage magnitudes is selected so that the reduced Jacobian matrix is nearly equal to an M-matrix for $\theta^{(0)} = 0$. (See Appendix I.) The process of converging to a stable solution is as follows: At the first iteration all voltage magnitudes are corrected higher than those characteristic of a stable solution. They then decrease monotonously and converge to those characteristic of a stable solution. The monotonous convergence of phase angles cannot always be guaranteed.

These results were strictly proved for the case given in (4), under the condition $P_i = 0 (i=1,\ldots,N+M-1)$. The analysis was then extended to the general case given in (4) and (5), under the assumption that the effect of small changes in g_P on ΔV is very small compared with that of small changes in g_Q. Load flow calculations carried out with (I3), (I4) and (I7), on a 28-node power network showed that the selection of low voltages made load flow calculations diverge even when the reduced Jacobian matrix was set nearly equal to an M-matrix. An examination of the calculation process revealed that this divergence was caused by large phase angle corrections at the first iteration. When the phase angles were not corrected at the first iteration, namely when $\Delta\theta^{(0)} = 0$ (hence $\theta^{(1)} = \theta^{(0)} = 0$), a stable solution was obtained, so long as the reduced Jacobian matrix was set nearly equal to an M-matrix. As the selection of high voltages causes the reduced Jacobian matrix to become an M-matrix, this selection is sufficient to provide for the stable convergence of a load flow problem.

CONVERGENCE REGION OF LOAD FLOW CALCULATIONS EXPRESSED IN RECTANGULAR COORDINATES

For an (N+M)-node power system, let nodes 1 to N be P-Q specified nodes, and nodes N+1 to N+M-1 be P-V specified nodes and node N+M be the slack node, as in the previous chapter. A set of equations for a load flow problem is

$$f_Q = 0 \qquad (6)$$

$$f_V = 0 \qquad (7)$$

$$f_P = 0 \qquad (8)$$

where $f_V = (f_{V,1}, \cdots, f_{V,M-1})^T$
and $f_{V,i} = \frac{1}{2}(E_{i+N}'^2 + E_{i+N}''^2) - L_{i+N}$

$E'_{i+N} + jE''_{i+N}$: complex voltage at node i+N

$\sqrt{2L_{i+N}}$: specified voltage magnitude at node i+N

Applying the Newton-Raphson method to (6), (7) and (8), where E_i' and E_i'' are independent variables, yields

$$E'^{(n+1)} = E'^{(n)} + \triangle E'^{(n)} \qquad (9)$$

$$E''^{(n+1)} = E''^{(n)} + \triangle E''^{(n)} \qquad (10)$$

$$\begin{bmatrix} \partial f_Q/\partial E' & \partial f_Q/\partial E'' \\ \partial f_V/\partial E' & \partial f_V/\partial E'' \\ \partial f_P/\partial E' & \partial f_P/\partial E'' \end{bmatrix}^{(n)} \begin{bmatrix} \triangle E' \\ \triangle E'' \end{bmatrix}^{(n)} = -\begin{bmatrix} f_Q \\ f_V \\ f_P \end{bmatrix}^{(n)} \qquad (11)$$

where $E' = (E_1', \cdots, E_{N+M-1}')^T$

$E'' = (E_1'', \cdots, E_{N+M-1}'')^T$

In order to clarify the correspondence between (3) and (11), (11) is expressed in polar coordinates.

Let
$$\left.\begin{aligned} E_i' &= V_i \cos\theta_i \\ E_i'' &= V_i \sin\theta_i \end{aligned}\right\} \quad i = 1, \cdots, N+M-1 \qquad (12)$$

or
$$\left.\begin{aligned} V_i &= \sqrt{E_i'^2 + E_i''^2} \\ \tan\theta_i &= E_i''/E_i' \end{aligned}\right\} \quad i = 1, \cdots, N+M-1 \qquad (13)$$

then
$$\begin{aligned} \frac{\partial V_i}{\partial E_i'} &= \frac{E_i'}{V_i} \\ \frac{\partial \theta_i}{\partial E_i'} &= -\frac{E_i''}{E_i'^2}\cos^2\theta_i \\ \frac{\partial V_i}{\partial E_i''} &= \frac{E_i''}{V_i} \\ \frac{\partial \theta_i}{\partial E_i''} &= \frac{1}{E_i'}\cos^2\theta_i \end{aligned} \qquad (14)$$

In view of (14), (11) becomes

$$\begin{bmatrix} \dfrac{\partial f_Q}{\partial V} & \dfrac{\partial f_Q}{\partial V'} & \dfrac{\partial f_Q}{\partial \theta} \\ 0 & \dfrac{\partial f_V}{\partial V'} & 0 \\ \dfrac{\partial f_P}{\partial V} & \dfrac{\partial f_P}{\partial V'} & \dfrac{\partial f_P}{\partial \theta} \end{bmatrix}^{(n)} \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \end{bmatrix}^{(n)} \begin{bmatrix} \triangle E' \\ \triangle E'' \end{bmatrix}^{(n)} = -\begin{bmatrix} f_Q \\ f_V \\ f_P \end{bmatrix}^{(n)} \qquad (15)$$

where $V = (V_1, \cdots, V_N)^T$

$$V' = (V_{N+1}, \cdots, V_{N+M-1})^T$$

and $H_1, \ldots, H_4$ are the diagonal matrices and,

$$H_1 = \mathrm{diag}(E_1'/V_1, \cdots, E'_{N+M-1}/V_{N+M-1})$$

$$H_2 = \mathrm{diag}(E_1''/V_1, \cdots, E''_{N+M-1}/V_{N+M-1})$$

$$H_3 = \mathrm{diag}(-E_1''\cos^2\theta_1/E_1'^2, \cdots, -E''_{N+M-1}\cos^2\theta_{N+M-1}/E'^2_{N+M-1})$$

$$H_4 = \mathrm{diag}(\cos^2\theta_1/E_1', \cdots, \cos^2\theta_{N+M-1}/E'_{N+M-1})$$

Defining

$$\begin{bmatrix} \triangle V \\ \triangle V' \\ \triangle\theta \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \end{bmatrix} \begin{bmatrix} \triangle E' \\ \triangle E'' \end{bmatrix} \quad \text{------------} (16)$$

Then, (15) becomes

$$\begin{bmatrix} \dfrac{\partial f_Q}{\partial V} & \dfrac{\partial f_Q}{\partial V'} & \dfrac{\partial f_Q}{\partial \theta} \\ 0 & \dfrac{\partial f_V}{\partial V'} & 0 \\ \dfrac{\partial f_P}{\partial V} & \dfrac{\partial f_P}{\partial V'} & \dfrac{\partial f_P}{\partial \theta} \end{bmatrix}^{(n)} \begin{bmatrix} \triangle V \\ \triangle V' \\ \triangle\theta \end{bmatrix}^{(n)} = - \begin{bmatrix} f_Q \\ f_V \\ f_P \end{bmatrix}^{(n)} \quad \text{------------} (17)$$

which is equivalent to

$$-\frac{\partial f_V}{\partial V'}^{(n)} \Delta V'^{(n)} = f_V^{(n)} \quad \text{------} (18)$$

$$\begin{bmatrix} F_1 & A_1 \\ F_2 & A_2 \end{bmatrix}^{(n)} \begin{bmatrix} \triangle V \\ \triangle\theta \end{bmatrix}^{(n)} = - \begin{bmatrix} f_Q \\ f_P \end{bmatrix}^{(n)} - \begin{bmatrix} \dfrac{\partial f_Q}{\partial V'} \triangle V' \\ \dfrac{\partial f_P}{\partial V'} \triangle V' \end{bmatrix}^{(n)} \quad \text{------------} (19)$$

When there are no P-V specified nodes, (19) coincides with (3). But this does not mean that the convergence processes for rectangular and polar coordinates are the same. Because the initial voltage magnitudes selected for P-V specified nodes are the same as the specified voltage magnitudes, $f_V^{(0)} = 0$, and therefore $\Delta V'^{(0)} = 0$. Therefore for the first convergence iteration (n = 0), (19) coincides with (3).

The set of equations (9), (10) and (11) can be solved for according to the procedure shown in Fig. 1, which is explained as follows:

1) Transform $E'^{(n)}$ and $E''^{(n)}$ into $V^{(n)}$, $V'^{(n)}$ and $\theta^{(n)}$ by (13).
2) Solve (17) for $\Delta V^{(n)}$, $\Delta V'^{(n)}$ and $\Delta\theta^{(n)}$.
3) Solve (16) for $E'^{(n)}$ and $E''^{(n)}$.
4) Obtain $E'^{(n+1)}$, and $E''^{(n+1)}$ from (9) and (10).

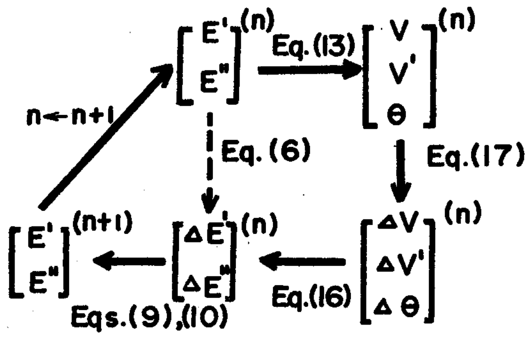


Fig. 1 Load Flow Calculations in Rectangular Coordinates

If the changes in voltage magnitudes and phase angles that occur at each iteration were small, a convergence process that uses rectangular coordinates would be almost the same as one that uses polar coordinates, but because the changes that occur during the first few iterations are usually large, a convergence process that uses rectangular coordinates is not the same as one that uses polar coordinates. And (1) and (2) do not generally hold for this procedure, but the following equations do hold for voltage magnitudes. (See Appendix III.)

$$V^{(n+1)} \geq V^{(n)} + \triangle V^{(n)}$$
$$V'^{(n+1)} \geq V'^{(n)} + \triangle V'^{(n)} \quad \text{------------} (20)$$

The following equations hold for the first iteration for the phase angle at node i. (See Appendix III.)

$$\theta_i^{(1)} = \frac{1}{\alpha_i} \frac{\triangle\theta_i^{(0)}}{1 + \dfrac{\triangle E_i'^{(0)}}{E_i'^{(0)}}} \quad \text{------------} (21)$$

As $\alpha_i > 1$, for $\Delta E_i^{(0)} > 0$

$$|\theta_i^{(1)}| \leq |\triangle\theta_i^{(0)}| \quad \text{------------} (22)$$

As (19) coincides with (3) for the first iteration, so the voltage magnitudes given by (9), (10) and (11) are higher than those given by (1), (2) and (3) for the first iteration under the conditions:

$$E''^{(0)} = \theta^{(0)} = 0$$

and

$$E'^{(0)} = \begin{bmatrix} V^{(0)} \\ E \end{bmatrix}$$

And if $\Delta E_i'^{(0)} > 0$, the corresponding absolute values for phase angles given by (9), (10) and (11) are smaller than those given by (1), (2) and (3). This generally enables the load flow calculations expressed in rectangular coordinates to converge, so long as the reduced Jacobian matrix $(F_1 - A_1 A_2^{-1} F_2)^{(0)}$ evaluated by $V^{(0)}$, $V_i'^{(0)}$ and $\theta^{(0)}$, which are transformed from $E'^{(0)}$ and $E''^{(0)}$, is set equal to an M-matrix. In the case where the reduced Jacobian matrix is set equal to an M-matrix, which causes the corrected voltage magnitudes to be higher than the magnitudes in a stable solution, and where a low voltage is selected, $\Delta E'^{(0)} > 0$ and, therefore, large phase angle corrections are suppressed. As the voltage magnitudes obtained with rectangular coordinates are always higher than those obtained with polar coordinates, from (20), the reduced Jacobian matrix will not deviate from an M-matrix after the second iteration. So it is concluded that if $E''^{(0)}$ is set equal to zero and $E'^{(0)}$ is set so that the corresponding reduced Jacobian matrix evaluated by $V^{(0)}$ $(= E'^{(0)})$ and $\theta^{(0)}$ $(= 0)$ is nearly equal to an M-matrix, the load flow calculations in rectangular coordinates will converge to a stable solution under the specified conditions. But a monotonous voltage decrease after the second iteration is not always guaranteed, as is the case with polar coordinates, although the corresponding changes in voltage magnitudes may be negative after the second iteration.[7]

EXAMPLE

Figure 2 is the 28-node sample system used in [7]. Load flow calculations were carried out for the same set of initial values with both polar and rectangular coordinates in order to clarify the differences in the convergence processes characteristic of the two types. In Fig. 2 node 28 was selected as the slack node and the voltage magnitudes at nodes 23 to 28 were set at 1.05 p.u.. Table I shows line impedences and Table II shows specified generation, loads and shunt capacitance. Table III shows two solutions obtained under the conditions given by Tables I and II. The reduced Jacobian matrix evaluated by SOL 1 was an M-matrix, but the one evaluated by SOL 2 was not. So SOL 1 and SOL 2 correspond to stable and unstable solutions, respectively.

A convergence test was made with

$$|\Delta E_i'^{(n)}| \leq 1.0 \times 10^{-5}$$

$$|\Delta E_i''^{(n)}| \leq 1.0 \times 10^{-5}$$

for rectangular coordinates, and with

$$|\Delta V_i^{(n)}| \leq 1.0 \times 10^{-5}$$

$$|\Delta \theta_i^{(n)}| \leq 1.0 \times 10^{-5}$$

for polar coordinates. A tolerence of 1.0×10^{-5} was selected for a monotonous voltage convergence test.

Table IV shows the number of convergence iterations for 5 cases where $\theta^{(0)}$ $(E''^{(0)}) = 0$ and $V_i^{(0)}$ $(E'^{(0)}) = 1.0$ to 0.58 p.u.. As can be seen from the table, when $\Delta\theta^{(0)}$ $(\Delta E''^{(0)}) \neq 0$, the phase angles (the imaginary parts of voltages) were corrected from the first iteration, and when $\Delta\theta^{(0)}$ $(\Delta E''^{(0)}) = 0$, they were fixed to zero for the first iteration. The reduced Jacobian matrices for the first four cases were M-matrices, but the matrix for the last was not. The theoretical investigation in the previous chapter indicates that whether the load flow calculations are expressed in polar or in rectangular coordinates, they will converge for cases 1 to 4, but not for case 5. However in actual calculations, while this was true for rectangular coordinates, it was not true for polar coordinates in cases 3 and 4 when $\Delta\theta^{(0)} \neq 0$. Although the corrected voltage magnitudes were higher than the magnitudes in a stable solution, large phase angle corrections occured, and therefore, the calculations diverged. This means that if the phase angles are not corrected at the first iteration(namely $\Delta\theta^{(0)} = 0$), the calculations will converge. Table IV shows this did indeed happen. The calculations expressed in rectangular coordinates for cases 3 and 4 converged even when

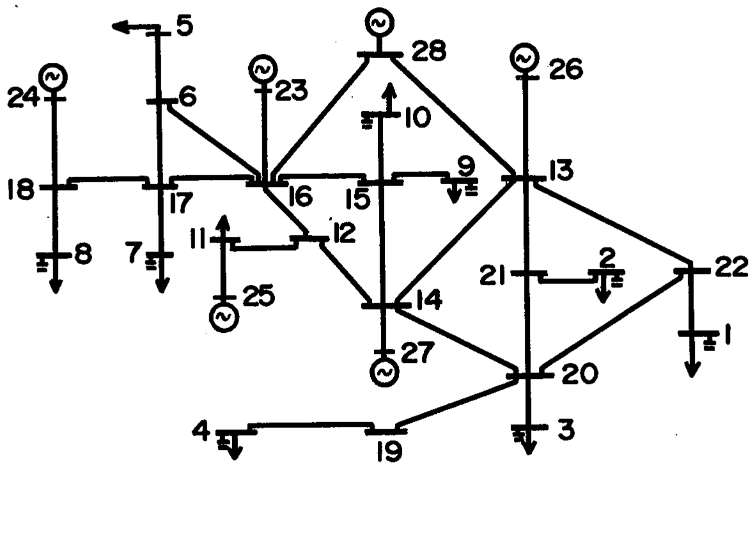


Fig. 2 Model Power System

Table I Line Impedences

Node i-j	Impedence	Node i-j	Impedence
1-22	0.0140 + j0.07	13-21	0.0296 + j0.148
2-21	0.0078 + j0.039	13-22	0.0242 + j0.121
3-20	0.0058 + j0.029	13-26	0.0070 + j0.035
4-19	0.0054 + j0.027	13-28	0.02 + j0.1
5- 6	0.0070 + j0.035	14-15	0.0068 + j0.034
6-16	0.0062 + j0.031	14-20	0.0038 + j0.019
6-17	0.0010 + j0.005	14-27	0.0112 + j0.056
7-17	0.0196 + j0.098	15-16	0.022 + j0.11
8-18	0.028 + j0.14	16-17	0.0054 + j0.027
9-15	0.0118 + j0.059	16-23	0.0070 + j0.035
10-15	0.0294 + j0.147	16-28	0.0054 + j0.027
11-12	0.028 + j0.14	17-18	0.0072 + j0.036
11-25	0.0258 + j0.129	18-24	0.0104 + j0.052
12-14	0.022 + j0.11	19-20	0.0038 + j0.019
12-16	0.0062 + j0.031	20-21	0.0076 + j0.038
13-14	0.006 + j0.03	20-22	0.0128 + j0.064

Table II Specified Generation, Loads and Shunt Capacitance (in p.u.)

Node	Susceptance of Shunt Capacitance	Generation P	Load P	Load Q
1	1.2069		1.45	0.5
2	1.0252		2.65	-0.22
3	2.0804		3.81	0.26
4	1.6665		3.84	-0.02
5	1.5422		3.74	0.34
6			1.29	0.03
7	1.0042		1.35	0.58
8	0.2080		0.608	0.07
9	0.4476		1.18	-0.18
10	0.7768		0.75	0.44
11	0.5868		1.39	0.52
23		1.91		
24		4.40		
25		0.49		
26		3.90		
27		1.52		

Table III Load Flow Solutions

Node	Voltage Magnitudes (in p.u.) SOL 1	SOL 2	Phase Angles (in degrees) SOL 1	SOL 2	Node	Voltage Magnitudes (in p.u.) SOL 1	SOL 2	Phase Angles (in degrees) SOL 1	SOL 2
1	1.0000	0.7409	-47.99	-62.55	15	0.9799	0.8499	-31.53	-37.42
2	1.0000	0.7626	-51.77	-68.50	16	1.0038	0.9730	-11.52	-12.64
3	1.0000	0.7578	-50.45	-66.15	17	0.9949	0.9654	-13.57	-14.85
4	1.0000	0.7311	-54.68	-73.86	18	1.0017	0.9838	-5.99	-7.05
5	1.0000	0.9655	-22.68	-24.52	19	0.9817	0.7426	-48.08	-62.36
6	0.9940	0.9637	-14.60	-15.94	20	0.9768	0.7666	-43.33	-54.64
7	1.0000	0.9628	-21.70	-23.47	21	0.9787	0.7617	-45.14	-57.61
8	1.0000	0.9809	-11.08	-12.32	22	0.9772	0.7659	-41.45	-51.71
9	1.0000	0.8644	-36.04	-43.34	23	(1.05)	(1.05)	-8.28	-9.66
10	1.0000	0.8290	-38.58	-46.65	24	(1.05)	(1.05)	6.25	5.21
11	1.0000	0.9697	-25.56	-28.71	25	(1.05)	(1.05)	-22.56	-25.97
12	0.9864	0.9312	-17.58	-19.62	26	(1.05)	(1.05)	-19.23	-24.80
13	0.9850	0.8864	-26.27	-31.37	27	(1.05)	(1.05)	-29.36	-37.52
14	0.9792	0.8424	-33.44	-40.42	28	(1.05)	(1.05)	(0.0)	(0.0)

( ): Specified Values.

Table IV Number of Iterations with Polar and Rectangular Coordinates

Case Number	$V_i^{(0)}, \theta_i^{(0)}$ $(E'^{(0)}_i, E''^{(0)}_i)$	det $F_1^{(0)}$	det $F^{(0)}$	Is G M-matrix ?	$\Delta\theta^{(0)} \neq 0$ Rectangular Coor.	$\Delta\theta^{(0)} \neq 0$ Polar Coor.	$\Delta\theta^{(0)} = 0$ Rectangular Coor.	$\Delta\theta^{(0)} = 0$ Polar Coor.
1	(1.00,0.0)	6.78×10^{31}	1.29×10^{70}	YES	12	7	10	8
2	(0.80,0.0)	1.70×10^{29}	7.98×10^{62}	YES	10	9	8	9
3	(0.70,0.0)	2.93×10^{27}	2.55×10^{58}	YES	15	Diverged	10	9
4	(0.60,0.0)	-7.20×10^{23}	1.07×10^{52}	YES	16	Diverged	15	14
5	(0.58,0.0)	-3.74×10^{24}	-3.06×10^{51}	NO	Diverged	Diverged	Diverged	Diverged

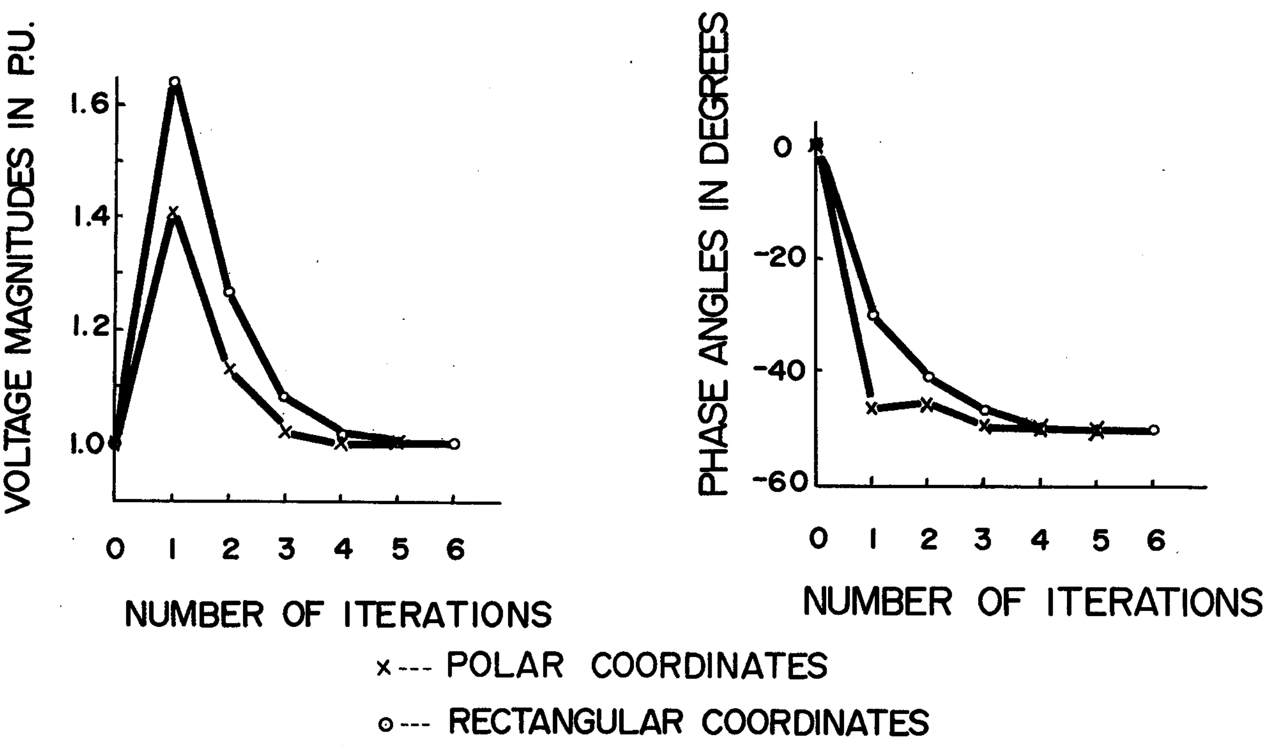


Fig. 3 Voltage Magnitudes and Phase Angles at Node 3 From the Newton-Raphson Solutions for Case 1 ($\Delta\theta^{(0)} \neq 0$)

$\Delta\theta^{(0)} \neq 0$. This was due to the suppression of large phase angle corrections described above. When polar coordinates were used, the number of iterations increased monotonously in inverse proportion to the magnitudes of the initial voltages that were selected. However, when rectangular coordinates were used, the increase was monotonous and in inverse proportion to the magnitudes of the initial voltages only for cases 2,3 and 4. This suggests that although a convergence region obtained with rectangular coordinates may be wider than one obtained with polar coordinates, the convergence with rectangular coordinates is less stable than that with polar coordinates. This is closely related to the fact that voltage magnitudes decrease monotonously with polar coordinates but not with rectangular coordinates. In the case of polar coordinates, all voltage magnitudes converged monotonously from the second iteration, for case 1 when $\Delta\theta^{(0)} \neq 0$, and for cases 1 to 4 when $\Delta\theta^{(0)} = 0$. But in the case of the rectangular coordinates, some voltage magnitudes of some iterations did not converge monotonously.

Figure 3 shows the convergence processes at node 3 for case 1. Voltage magnitudes with both types of coordinates converged monotonously from the second iteration. For the first iteration the voltage magnitude for rectangular coordinates was higher than that for polar coordinates, and the absolute value for the phase angle was smaller, which exemplified the first investigation.

DISCUSSION OF RESULTS

Theoretical investigations and load flow calculations show that a region where a set of initial values converges to a stable solution is given by $\theta^{(0)}(E''^{(0)}) = 0$ and $V^{(0)}(E'^{(0)})$, which is chosen so that the reduced Jacobian matrix is nearly equal to an M-matrix. Because setting the reduced Jacobian matrix nearly equal to an M-matrix requires the selection of high voltages, it is enough to select high initial voltage magnitudes, without carrying out matrix calculations.

The above method guarantees a stable solution under specified conditions and corresponds to an actual stable operating point when the P-V specified nodes correspond to generator nodes and the P-Q specified nodes to load nodes, since there will be no loads that have severer voltage characteristics than constant power characteristics. If there are some P-V specified nodes which correspond to load nodes, a stable solution will not necessarily coincide with an actual stable operating point.[7]

CONCLUSIONS

The Newton-Raphson method was applied to a set of nodal power equations and the region where a set of initial values converges to a stable load flow solution under specified conditions, was studied and clarified. The results indicate that the selection of high voltages, under the condition that the phase angles (or the imaginary parts of voltages) are set to zero, guarantees convergence to a stable solution, if one exists.

Theoretical investigations and load flow calculations showed that the stability of the convergence processes obtained with polar coordiantes is superior to that obtained with rectangular coordinates, although the size of the convergence region may be smaller.

ACKNOWLEDGEMENT

The authors wish to thank Mr. H. Kitanosono of Omika Works for his helpful discussions, and Dr. E. Kobayashi, Dr. Y. Kawamoto, Dr. K. Hirasawa and Mr. M. Goto of Hitachi Research Laboratory for their valuable suggestions and encouragement.

APPENDIX I

Power System Voltage Stability

For an (N+M)-node power system with M power sources and N nonlinear loads, voltage stability criteria have been given by the following:[1,3,4]

$$\frac{dV_j}{dE_i} > 0 \quad \text{for almost all the combinations of } (i,j), \text{ where } i = 1,\ldots,M,\ j = 1,\ldots,N \qquad (I1)$$

$$\frac{dV_j}{db_i} > 0 \quad \text{for almost all the combinations of } (i,j), \text{ where } i,j = 1,\ldots,N \qquad (I2)$$

where node numbers for loads are assigned from 1, to N, and those for sources from N+1 to N+M,

and V_j : voltage magnitude at node j

$E_i = V_{N+i}$: Source voltage magnitude at node N+i and is assumed to be constant

b_i : susceptance of shunt capacitance at node i.

Stability criteria (I1) and (I2) have been further sophisticated by using M-matrix properties.[3,4] For nodes i = 1,...,N, the following 2N nodal power equations hold:

$$f_{Q,i} = Q_i + Q_{bi} + \sum_{\ell \in S_i} Q_{i\ell} = 0 \qquad (I3)$$

$$f_{P,i} = P_i + \sum_{\ell \in S_i} P_{i\ell} = 0 \qquad (I4)$$

where S_i : set of node numbers with lines connected to node i

P_i : real power consumed at node i

Q_i : reactive power consumed at node i

Q_{bi} : reactive power fed from susceptance b_i at node i

P_{ik} : real power flow from node i to node k at node i

Q_{ik} : reactive power flow from node i to node k at node i

and they are given, respectively, by

$$P_i = P_{oi} V_i^{mi}$$

$$Q_i = Q_{oi} V_i^{ni}$$

$$Q_{bi} = -b_i V_i^2$$

$$Q_{ik} = -B_{ik} V_i^2 + B_{ik} V_i V_k \cos(\theta_i - \theta_k) - G_{ik} V_i V_k \sin(\theta_i - \theta_k)$$

$$P_{ik} = -B_{ik} V_i V_k \sin(\theta_i - \theta_k) + G_{ik}\{V_i^2 - V_i V_k \cos(\theta_i - \theta_k)\}$$

$$i,k = 1,\cdots,N+M \qquad (I5)$$

where θ_i ; voltage phase angle at node i in reference to node N+M

$G_{ik} + jB_{ik}$; complex admittance of line from node i to node k

Let the incremental generated outputs for the increase of the total transmission loss and of the total power consumed by loads be given by,

$$df_{P,N+i} = d(P_{N+i} + \sum_{\ell \in S_{N+i}} P_{N+i,\ell}) - k_i\, d(P_{LOSS} + \sum_{j=1}^{N+M} P_j) = 0$$

$$i = 1, \cdots, M-1 \qquad (I6)$$

where k_i are incremental factors of the generated outputs at node N+i and satisfy

$$\sum_{i=1}^{M} k_i = 1 \quad \text{and} \quad k_i \geq 0$$

and P_{LOSS} is the total transmission loss and is given by

$$P_{LOSS} = \sum_{\substack{i,j=1 \\ i \neq j}}^{N+M} P_{ij} = \sum_{(i,j)} (P_{ij} + P_{ji})$$

$$= \sum_{(i,j)} G_{ij}\{V_i^2 + V_j^2 - 2V_iV_j\cos(\theta_i - \theta_j)\}$$

where (i,j) are all combinations of node numbers corresponding to both ends of lines.

Let k_i be constant and integrate (I6)

$$f_{P,N+i} = P_{N+i} + \sum_{\ell \in S_{N+i}} P_{N+i,\ell} - k_i(P_{LOSS} + \sum_{j=1}^{N+M} P_j) + c_i = 0$$

$$i = 1, \cdots, M-1 \qquad (I7)$$

where c_i are integral constants and

$$\sum_{i=1}^{M} c_i = 0$$

Let,

$$f_Q = (f_{Q,1}, \cdots, f_{Q,N})^T$$

$$f_P = (f_{P,1}, \cdots, f_{P,J})^T$$

$$V = (V_1, \cdots, V_N)^T$$

$$\theta = (\theta_1, \cdots, \theta_J)^T$$

$$E = (E_1, \cdots, E_{M-1})^T$$

$$b = (b_1, \cdots, b_N)^T$$

$$J = N+M-1$$

Then taking small increments in (I3) and (I4), and combining them with (I6) yield

$$\begin{bmatrix} \partial f_Q/\partial V & \partial f_Q/\partial \theta \\ \partial f_P/\partial V & \partial f_P/\partial \theta \end{bmatrix} \begin{bmatrix} dV/dE \\ d\theta/dE \end{bmatrix} = -\begin{bmatrix} \partial f_Q/\partial E \\ \partial f_P/\partial E \end{bmatrix} \qquad (I8)$$

$$\begin{bmatrix} \partial f_Q/\partial V & \partial f_Q/\partial \theta \\ \partial f_P/\partial V & \partial f_P/\partial \theta \end{bmatrix} \begin{bmatrix} dV/db \\ d\theta/db \end{bmatrix} = -\begin{bmatrix} \partial f_Q/\partial b \\ \partial f_P/\partial b \end{bmatrix}$$

$$= \begin{bmatrix} V_1^2 & & 0 \\ & \ddots & \\ 0 & & V_N^2 \\ \hline & 0 & \end{bmatrix} \quad \text{------ (I9)}$$

Letting

$$F = \begin{bmatrix} F_1 & A_1 \\ F_2 & A_2 \end{bmatrix} = \begin{bmatrix} \partial f_Q/\partial V & \partial f_Q/\partial \theta \\ \partial f_P/\partial V & \partial f_P/\partial \theta \end{bmatrix}$$

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = -\begin{bmatrix} \partial f_Q/\partial E \\ \partial f_P/\partial E \end{bmatrix}, \quad \begin{bmatrix} D_1 \\ 0 \end{bmatrix} = -\begin{bmatrix} \partial f_Q/\partial b \\ \partial f_P/\partial b \end{bmatrix}$$

and eliminating $d\theta/dE$ and $d\theta/db$ in (I8) and (I9) give, respectively, to

$$G \cdot dV/dE = C_1 - A_1 A_2^{-1} C_2 \quad \text{------ (I10)}$$

$$G \cdot dV/db = D_1 \quad \text{------ (I11)}$$

where $G = F_1 - A_1 A_2^{-1} F_2$ and G is called the reduced Jacobian matrix.

From (I10) and (I11), it is clear that the stability criteria (I1) and (I2) are equivalent to the conditions that almost all the elements of G^{-1} and $C_1 - A_1 A_2^{-1} C_2$ are positive. Under light load conditions, $G \simeq F_1$ and $C_1 - A_1 A_2^{-1} C_2 \simeq C_1$ hold. Because the elements of C_1 are non-negative, the load voltage is stable, if the elements of G^{-1} are all non-negative. This is equivalent to the condition that G is an M-matrix, because the off-diagonal elements of G are all non-negative. (See Appendix II.)

Under heavy load conditions, there can be some positive off-diagonal elements in G due to the term $A_1 A_2^{-1} F_2$. Therfore, G cannot be an M-matrix in a strict sense. But so long as the absolute values of positive off-diagonal elements are small compared with those of negative off-diagonal elements, elements of G^{-1} can be non-negative from the continuity of matrix inversion. Hence the stability analysis is made by the following two steps:

(i) Off-diagonal elements of G corresponding to the non-zero, off-diagonal elements of F_1 are all non-positive, and elements of $C_1 - A_1 A_2^{-1} C_2$ corresponding to non-zero elements of C_1 are all non-negative.

(ii) All principal minors of G are positive.

The matrix G which satisfies the above conditions is called to be "nearly equal to an M-matrix".

Now assume that A_1 is an irreducible matrix and so is G. This is an appropriate assumption because if A_1 is reducible, the power system studied can be divided into two or more subsystems and voltage stability can be analyzed independently in each subsystem where corresponding matrix A_1 is irreducible.

Among the conditions that the principal minors of G are positive, the one that the determinant of G is positive is the severest, from Theorem 3 in Appendix II. Therefore, the following stability criterion can be used under the normal operating conditions:

$$\det G > 0 \quad \text{------ (I12)}$$

When a phase difference across any line is less than $\pi/2$, which holds for the normal operating conditions, matrix A_2 is an M-matrix from Theorem 4, therefore, $\det A_2 > 0$. And as

$$\det F = \det G \cdot \det A$$

stability criterion (I12) is equivalent to

$$\det F > 0 \quad \text{------ (I13)}$$

which coincides with one proposed by Venikov in [5], when $k_i = 0$, $i = 1, \ldots, M-1$ and $k_M = 1$ are assumed in (I6).

APPENDIX II

M-matrix[8,9]

Theorem 1 For a square matrix $A = (a_{ij})$ with non-positive off-diagonal elements, the following two conditions are equivalent:

1) All principal minors of A are positive.
2) Matrix A is non-singular and elements of A^{-1} are all non-negative.

Definition 1 A matrix which satisfies either of the above conditions is said to be an M-matrix.

Definition 2 An $n \times n$ matrix is said to be reducible if there exists an $n \times n$ permutation matrix P such that

$$PAP^T = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \quad \text{------ (II1)}$$

where A_{11}: $r \times r$ matrix and $1 \le r \le n$

A_{22}: $(n-r) \times (n-r)$ matrix

A matrix A is irreducible if no such permutation matrix exists.

Theorem 2 Let A be an M-matrix. Then elements of A^{-1} are all positive if and only if A is irreducible.

Theorem 3 Let A be an irreducible matrix. If for $\varepsilon > 0$, $A + \varepsilon E$ is an M-matrix where E is a unit matrix, then proper principal minors are all positive.

Definition 3 An $n \times n$ matrix $A = (a_{ij})$ is said to be diagonally dominant if

$$|a_{ii}| \ge \sum_{\substack{i=1 \\ j \ne i}}^{n} |a_{ij}| \quad \text{------ (II2)}$$

holds for all $1 \le i \le n$. An irreducible matrix A is said to be irreducibly diagonally dominant if for at least one i, the strict inequality holds.

Theorem 4 Let an irreducibly diagonally dominant matrix A have non-positive off-diagonal elements and positive diagonal elements, then A is an M-matrix and $A^{-1} > 0$.

APPENDIX III

Proof of (20)

From (16),

$$\triangle V_i^{(n)} = \frac{1}{V_i^{(n)}} \left(E_i'^{(n)} \triangle E_i'^{(n)} + E_i''^{(n)} \triangle E_i''^{(n)} \right) \quad \text{(III1)}$$

Also from Fig.1 and (III1),

$$(V_i^{(n+1)})^2 = (E_i'^{(n)} + \triangle E_i'^{(n)})^2 + (E_i''^{(n)} + \triangle E_i''^{(n)})^2$$
$$= (V_i^{(n)})^2 + 2V_i^{(n)} \triangle V_i^{(n)} + (\triangle E_i'^{(n)})^2 + (\triangle E_i''^{(n)})^2$$

Hence

$$(V_i^{(n+1)})^2 - (V_i^{(n)} + \triangle V_i^{(n)})^2 = (\triangle E_i'^{(n)})^2 + (\triangle E_i''^{(n)})^2 - (\triangle V_i^{(n)})^2$$
$$= \frac{1}{(V_i^{(n)})^2} \{ [(E_i'^{(n)})^2 + (E_i''^{(n)})^2][(\triangle E_i'^{(n)})^2 + (\triangle E_i''^{(n)})^2] - (E_i'^{(n)} \triangle E_i'^{(n)} + E_i''^{(n)} \triangle E_i''^{(n)})^2 \}$$
$$= \frac{1}{(V_i^{(n)})^2} (E_i''^{(n)} \triangle E_i'^{(n)} - E_i'^{(n)} \triangle E_i''^{(n)})^2 \geq 0$$

Thus (20) holds.

Proof of (21)

As $\theta_i^{(0)} = 0$, and $E_i''^{(0)} = 0$,

$$\tan\theta_i^{(1)} - \tan\theta_i^{(0)} = \tan\theta_i^{(1)} = \frac{\triangle E_i''^{(0)}}{E_i'^{(0)} + \triangle E_i'^{(0)}}$$
$$= \frac{\triangle E_i''^{(0)}}{E_i'^{(0)}} \frac{1}{1 + \frac{\triangle E_i'^{(0)}}{E_i'^{(0)}}} \quad \text{(III2)}$$

By Taylor's formula,

$$\tan\theta_i^{(1)} = \theta_i^{(1)} + \left(\tfrac{1}{3} + \tan^2\theta_i'^{(1)}\right)(\theta_i^{(1)})^3$$
$$= [1 + (\tfrac{1}{3} + \tan^2\theta_i'^{(1)})(\theta_i^{(1)})^2]\theta_i^{(1)}$$
$$\triangleq \alpha_i \theta_i^{(1)} \quad \text{(III3)}$$

where $0 \leq |\theta_i'^{(1)}| \leq |\theta_i^{(1)}|$

From(13),

$$\triangle\theta_i^{(0)} = \frac{\triangle E_i''^{(0)}}{E_i'^{(0)}} \quad \text{(III4)}$$

Hence from (III2), (III3) and (III4),

$$\theta_i^{(1)} = \frac{1}{\alpha_i} \frac{\triangle\theta_i^{(0)}}{1 + \frac{\triangle E_i'^{(0)}}{E_i'^{(0)}}}$$

As $\alpha_i > 1$,

$$|\theta_i^{(1)}| \leq |\triangle\theta_i^{(0)}|$$

holds for $\Delta E_i'^{(0)} > 0$.

REFERENCES

[1] S. Abe, J. Makino and A. Isono et al., "Criteria for Power System Voltage Stability by Steady-State Analysis," Paper A75 435-8, presented at IEEE Summer Meeting, San Francisco, July, 1975.

[2] O. Nagasaki, U. Moroe et al., "Power System Voltage Stability," Paper Nos. ET-75-5 - ET-75-16, presented at the Technical Meeting on Power Engineering , IEE of Japan, may 1975.

[3] S. Abe and A. Isono, "Determination of Power System Voltage Stability, Part I: Theory," IEE of Japan, Vol. 96-B, No.4, 1976.

[4] S. Abe and A. Isono, "Determination of Power System Voltage Stability, Part II: Digital Simulation," ibid.

[5] V. A. Venikov, V. A. Stroev et al.,"Estimation of Electrical Power System Steady-State Stability in Load Flow Calculations," IEEE Transaction on Power Apparatus and Systems, Vol. PAS-94, pp 1034-1041, May/June 1975.

[6] B. K. Johnson, "Extraneous and False Load Flow Solutions," IEEE Transaction on Power Apparatus and Systems, Vol. PAS-96, pp 524-534, March/April 1977

[7] S. Abe and A. Isono, "Initial Value Selection of Load Flow Calculations in the Vicinity of a Voltage Stability Limit," IEE of Japan, Vol. 97-B, No. 1, 1977

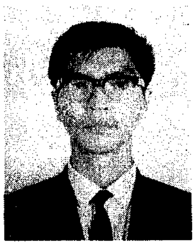
[8] R. S. Varga, Matrix Iterative Analysis. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962

[9] M. Fiedler and V. Pták, "On Matrices with Non-Positive Off-Diagonal Elements and Positive Principal Minors," Czech. Math. J., Vol. 12 No. 3 pp 382-400, 1962

S. Abe was born in Matsuyama, Japan, on July 19, 1947. He received the B.S. degree in electronics engineering and M.S. degree in electrical engineering both from Kyoto University, Kyoto, Japan, in 1970 and 1972, respectively.

Since 1972, he has been with Hitachi, Ltd., Tokyo, Japan, and has been working mainly in the field of power system analysis in Hitachi Research Laboratory.

Mr. Abe is a member of the Institute of Electrical Engineers of Japan.



N. Hamada (M'75) was born in Osaka, Japan, on February 3, 1944. He received the B.S., M.S. and Ph.D. degrees in electrical engineering from Osaka University in 1966, 1968, and 1977, respectively.

Since 1968, he has been with Hitachi, Ltd., Tokyo, Japan, where he has been working in the field of computer control systems. He is currently studying software engineering at Carnegie-Mellon University, Pittsburgh as a visitor. Dr. Hamada is a member of the Institute of Electrical Engineers of Japan and ACM.

A. Isono, photograph and biography not available at the time of publication.

K. Okuda, photograph and biography not available at the time of publication.

Discussion

G. K. Rao (Institute of Technology, Banaras Hindu University, Varanasi, India): The authors are to be complimented for yet another valuable addition to the literature on analysis of load flow convergence. They derived necessary conditions underwhich the Newton-Raphson (N–R) load flow solution converges to a stable solution space under certain operating conditions. Also investigated is the effect of the coordinate system (polar or rectangular) on convergence.

Venikov et al [5] examined a similar problem through a modified N-R method where, as the authors rightly pointed out, the over corrections to the state vector are avoided by multiplying the state vector corrections by a parameter λ. This parameter is computed through the Hessian matrix at the current iteration of the load flow equations. The Hessian matrix makes the procedure tedious although it ensures convergence where the usual N-R method does not converge because of the possibility of the Jacobian becoming singular during solution.

A similar parameter—corrected N-R method was suggested elsewhere [10] in Russian literature which to the discusser's knowledge did not appear in English.

The linearised recurrence equations of the N–R solution are written as

$$[x]^{(i+1)} = [x]^{(i)} - \lambda[J]^{-1}[\Delta f]. \qquad (D1)$$

For $\lambda = 1$, (D1) reduces to the usual N-R procedure. The parameter λ is computed as follows:

Let $x^{(0)}$ be the initial approximate solution

1. Compute $F^{(0)} = \Sigma_i \Delta f_i^2 (x^{(0)})$
2. With $\lambda = 1$, Compute $\Delta x^{(0)}$ and update x
   $x^{(1)} = x^{(0)} + \Delta x^{(0)}$
3. Compute $F^{(1)} = \Sigma_i \Delta f_i^2 (x^{(1)})$
4. If $F^{(1)} < F^{(0)}$, then $x^{(1)}$ is in solution and go to (2). If $F^{(1)} \geqslant F^{(0)}$ make $\lambda = ½$ and update $x^{(0)}$ by
   $x^{(1)} = x^{(0)} + ½ \Delta x^{(0)}$

Compute $F^{(1)}$ at this point and verify if $F^{(1)} < F^{(0)}$. If yes, go to (2) else reduce λ further by half until $F^{(1)} < F^{(0)}$ by

$$x^{(i)} = x^{(i-1)} + (½^n)\Delta x^{(i-1)}$$ before going to (2).

As the authors of [10] showed, the method always works provided the Jacobian does not change sign during the solution. The procedure converged for systems for which the usual N-R method did not. For systems where the Jacobian changes sign the procedure suggested in [5] or the favorable starting solutions as suggested in this paper could be effectively used.

Would the authors please comment if they tested their method in polar coordinates with all nodes (except slack) as P–Q nodes. The discusser thanks the authors for making available a copy of their paper.

REFERENCE

10. V. I. Idelchik, V. I. Tarasov, "Experimental Investigations on the convergence of the Load Flow Solution" (in Russian) in the research reports of the Irkutsk Polytechnic, Issue-72, Irkutsk, USSR 1971.

Manuscript received February 15, 1978.

Kavuru A. Ramarao (Cleveland Electric Illuminating Co., Cleveland, OH): We congratulate the authors for their interesting theoretical analysis on the load flow convergence.

It is interesting to note that the authors have success with load flow cases which have large phase angle corrections but not corrected in the first iteration.

The load flow problems, though expressed as a set of non-linear equations, do in general converge to a unique solution with a flat voltage start 1 + j0. The solutions could be non-unique in cases where the generator voltages are regulated by the control of MVARS within limits.

The results shown in Table IV indicate the cases of degradation of load flow convergence with voltage start from 1 + j0 down to .58 + j0. It is not clear as to why anybody would choose a low voltage start like 0.58 + j0 to perform load flow analysis.

It would be more interesting if the authors show an example of load flow which does not converge with the usual flat voltage start and a higher voltage start causes convergence to a stable solution.

In the "Discussion of Results", the authors indicate that it is enough to select high initial voltage magnitudes without carrying out matrix calculations. How do we choose this start for a system?

There is a minor correction in Appendix I on page 8 in the last line of the paragraph following eg (I11) to read as "Non-positive (see Appendix II)" instead of "Non-negative (see Appendix II)".

We appreciate the authors' comments.

Manuscript received February 27, 1978.

S. Abe, N. Hamada, A, Isono, and **K. Okuda:** The authors thank the discussors for their valuable contributions. The points raised by Mr. Rao will be discussed first. If the method proposed in [10] always works provided the Jacobian does not change sign during the solution, it would be enough to apply this method only to the first convergence iteration, because the load flow calculations will converge to a stable solution when a set of initial values is chosen so that the corresponding reduced Jacobian matrix is set equal to an M-matrix (In most cases it will correspond to the Jacobian being positive under the conditions that the directions of power flows are chosen as in I3 and I4.), and when the corrections for the first convergence iteration are suppressed.

In order to improve load flow convergence, it is useful to change specified conditions for nodes as follows:

(1) To specify real and reactive powers, by adding reactive power fed from a shunt capacitor at the node, to the specified value.

(2) to specify real power and a voltage magnitude.

The effect caused by item (2) is large, whereas that by item (1) is small. Therefore, if all generator nodes whose voltage magnitudes are usually regulated to be constant, are chosen to be P-Q specified nodes, convergence characteristics are affected significantly, and in extreme cases load flow calculations may give no solution, although there exists a stable solution in the case where generator nodes are P–V specified nodes.

As Mr. Ramarao pointed out, there would be no convergence problems under a present network configuration with present load conditions. However, for power system planning, load flow analysis is widely used and under future network conditions there will be no reason to believe that a stable solution is obtained by a flat voltage start. Therefore, the authors believe that it is useful to have clarified load flow convergence characteristics.

Although the authors were fully aware that it would have been more striking to show such an example as Mr. Ramarao indicated, what they wanted to show in Table IV was the validity of the theoretical convergence region. A low voltage start like 0.58 + j0.0 is, of course, meaningless in practical load flow analysis. What should be noted is that any voltage selection higher than 0.6 + j0.0 causes convergence to a stable solution. This fact indicates the method of an initial value selection. If the power system studied is under heavy load conditions, has long distance transmission lines, or has P–Q specified, generator nodes, a voltage selection higher than 1.0 + j0.0, e.g. 2.0 + j0.0 is more likely to cause load flow calculations to converge to a stable solution. Or if the calculations fail to converge with a flat voltage start, the best way to judge whether there exists a stable solution under specified conditions is to retry the calculations with the initial voltages much larger than 1.0 + j0.0.

The authors wish to thank Mr. Ramarao for pointing out a correction. There is also a correction as shown below:

Fig. 1 on page 3 Eq. (6) → Eq. (11)

Manuscript received April 27, 1978.